TYBA COACHING CENTRE, AJMER

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CLASS 9 MATHEMATICS

THE ULTIMATE COMPLETE MASTER TEXTBOOK

Dual-Board Edition: Rationalised NCERT / RBSE & CBSE Foundation (2026–27)

12 Complete Chapters • 12-Section Mastery Architecture • 100% English Edition

Concept Derivations • 12 Custom Vector Diagrams • Formula Banks • Smart Shortcuts • 192 Solved Examples • 144 Competency MCQs • 3-Tier Chapter Tests • Full 80-Mark Sample Paper

1. Edition Scope, Copyright & How to Use This Textbook

This master textbook is engineered for Class 9 students preparing for **RBSE Class 9 Annual Examinations (2026–27)**, **CBSE Class 9 NCERT Curriculum**, and foundational **NTSE / Olympiad / Class 10 Board** mathematics. Every chapter follows a uniform **12-Section Mastery Architecture (Sections A–L)** so students build deep conceptual clarity alongside rapid exam-hall execution.

- **Sections A–C (Concept & Formula Core):** Exam relevance, intuitive derivations, custom vector figures, and complete formula banks with boundary conditions.
- **Sections D–F (Problem Classification & Worked Mastery):** High-frequency problem patterns, mathematically validated shortcuts, and 16 graded solved examples per chapter (Beginner to Challenge).
- **Sections G–J (MCQ Mastery, Strategy & Quick Revision):** 12 competency MCQs with distractor analysis, exam-hall tactics, common error fixes, and a dedicated 1-page revision sheet.
- **Sections K–L (3-Tier Assessment & Faculty Guide):** Basic, Exam Standard, and Challenge tests with separate answer keys plus TYBA faculty classroom notes.

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2. Official Class 9 Syllabus & Unit Weightage (2026–27)

RBSE Class 9 Mathematics (100 Marks, 3 Hours 15 Minutes) and **Rationalised NCERT Class 9** organize the 12 core chapters across 6 units (CBSE Class 9 annual exams carry 80 Marks Theory + 20 Marks Internal Assessment).

Unit	Unit Name & Included Chapters	RBSE Marks (100M)
Unit I	**Number Systems** (Ch. 1 Number Systems)	**08**
Unit II	**Algebra** (Ch. 2 Polynomials, Ch. 4 Linear Equations in Two Variables)	**28**
Unit III	**Coordinate Geometry** (Ch. 3 Coordinate Geometry)	**06**
Unit IV	**Geometry** (Ch. 5 Euclid's Geometry, Ch. 6 Lines & Angles, Ch. 7 Triangles, Ch. 8 Quadrilaterals, Ch. 9 Circles)	**28**
Unit V	**Mensuration** (Ch. 10 Heron's Formula, Ch. 11 Surface Areas & Volumes)	**20**
Unit VI	**Statistics** (Ch. 12 Statistics)	**10**
Total	**All 6 Units / 12 Rationalised NCERT Chapters**	**100**

80-Mark Standard Blueprint (CBSE / School Annual Pattern)

- **Section A:** 18 Multiple Choice Questions (1M each) + 2 Assertion-Reason Questions (1M each) = **20 Marks**.
- **Section B:** 5 Very Short Answer (VSA) Questions (2M each) = **10 Marks**.
- **Section C:** 6 Short Answer (SA) Questions (3M each) = **18 Marks**.
- **Section D:** 4 Long Answer (LA) Questions (5M each) = **20 Marks**.
- **Section E:** 3 Source/Case-Based Questions (4M each) = **12 Marks**.

3. Complete 12-Chapter Table of Contents & Rationalised Scope

Ch.	Chapter Title	Core Focus & Rationalised Exclusions (2026–27)
01	Number Systems	Rational/irrational numbers, decimal expansions, rationalization, laws of exponents. *(Excl: Successive magnification)*
02	Polynomials	Degree, zeroes, Factor Theorem, factorisation, algebraic identities. *(Excl: Remainder Theorem proof)*
03	Coordinate Geometry	Cartesian system, quadrants, plotting points. *(Full chapter retained)*
04	Linear Equations in Two Variables	Standard form $ax + by + c = 0$, infinitely many solutions, graphs. *(Excl: Equations of lines parallel to axes)*
05	Intro to Euclid's Geometry	Euclid's definitions, 7 axioms, 5 postulates. *(Excl: Equivalent versions of 5th postulate)*
06	Lines and Angles	Linear pair, vertically opposite, parallel lines & transversal. *(Excl: Angle sum property theorem proof)*
07	Triangles	SAS, ASA, AAS, SSS, RHS congruence, isosceles properties. *(Excl: Triangle inequalities)*
08	Quadrilaterals	Parallelogram properties, Mid-point Theorem & converse. *(Excl: Angle sum proof)*
09	Circles	Equal chords, perpendicular from centre, angle subtended by arc, cyclic quads. *(Excl: 3 non-collinear points circle)*
10	Heron's Formula	Area of triangle using $\sqrt{s(s-a)(s-b)(s-c)}$. *(Excl: Area of quadrilateral applications)*
11	Surface Areas and Volumes	Right circular cone, sphere, hemisphere CSA, TSA, Volume. *(Excl: Cuboid, cube, cylinder)*
12	Statistics	Bar graphs, histograms (equal/varying widths), frequency polygons. *(Excl: Mean/median/mode, collection)*

UNIT I: NUMBER SYSTEMS TYBA COACHING CENTRE

Chapter 1: Number Systems

Rational & Irrational Numbers, Decimal Expansions, Rationalization & Exponents (2026–27)

A. Chapter Introduction & Examination Relevance

Every measurement in science and everyday life lives on the Real Number Line. Chapter 1 extends integers and fractions to irrational numbers like $\sqrt{2}$ and π, unifying them into the Real Number System $\mathbb{R}$.

- **What You Will Learn:** Natural, Whole, Integer, Rational, and Irrational numbers; finding rationals between two numbers; terminating vs non-terminating recurring decimals (p/q conversion); locating $\sqrt{n}$ on the number line; operations on real numbers; rationalizing denominators; and laws of exponents for real numbers.
- **Why This Chapter Matters:** Surds, rationalization, and exponent laws are used directly in Class 10 (Real Numbers, Quadratic Equations, Trigonometry) and Class 11 (Complex Numbers, Binomial Theorem).
- **Real-Life Applications:** Engineering tolerances, computer floating-point numbers, musical pitch ratios, and compound-interest fractional exponents.
- **Cross-Chapter Connections:** Supplies exponent laws and algebraic radicals to Chapter 2 (Polynomials) and Chapter 10 (Heron's Formula).
- **Exam & Foundation Relevance:** Unit I carries 8/100 marks in RBSE (7/80 in CBSE). Expect 1M MCQs on decimal types/exponents and 2M/3M questions on p/q conversion of $0.\overline{47}$ and rationalizing $\frac{1}{\sqrt{7}-\sqrt{6}}$.

B. Complete Concept Explanation & Derivations

1. Rational vs Irrational Numbers & Decimal Expansions

A **rational number** can be written as $\frac{p}{q}$ ($p, q \in \mathbb{Z}, q \neq 0$). Its decimal expansion is either **terminating** (when $q = 2^m 5^n$ in lowest form) or **non-terminating repeating (recurring)**. An **irrational number** cannot be written as $\frac{p}{q}$; its decimal expansion is **non-terminating non-repeating** (e.g., $\sqrt{2}, \sqrt{3}, \pi, 0.1010010001\ldots$).

[CONCEPT MASTER – 1. Rational vs Irrational Numbers & Decimal Expansions] Between any two distinct real numbers, there are infinitely many rational AND infinitely many irrational numbers!

2. Converting Recurring Decimals to p/q Form

To convert $x = 0.\overline{ab}$ (k repeating digits), multiply by 10^k and subtract x so the infinite decimal tail cancels completely: $10^k x - x = ab \implies x = \frac{ab}{10^k - 1}$. If non-repeating digits precede the bar (like $0.4\overline{7}$), multiply by 10 first to align the bar immediately after the decimal point.

[CONCEPT MASTER – 2. Converting Recurring Decimals to p/q Form] Interesting Fact: $0.\overline{9} = \frac{9}{9} = 1$ exactly—there is no real number between $0.\overline{9}$ and 1!

3. Operations on Real Numbers & Rationalization

The sum or difference of a rational and an irrational number is ALWAYS irrational; the product or quotient of a non-zero rational and an irrational is ALWAYS irrational. To **rationalize** $\frac{1}{\sqrt{a}+\sqrt{b}}$, multiply numerator and denominator by its conjugate $\sqrt{a} - \sqrt{b}$, using $(x+y)(x-y) = x^2 - y^2$.

[CONCEPT MASTER – 3. Operations on Real Numbers & Rationalization] Remember: the sum or product of TWO irrationals can be rational! E.g., $(2+\sqrt{3}) + (2-\sqrt{3}) = 4$ and $\sqrt{2} \times \sqrt{8} = 4$.

4. Laws of Exponents for Real Numbers

For $a > 0$ and rational exponents p, q: $a^p \cdot a^q = a^{p+q}$, $(a^p)^q = a^{pq}$, $\frac{a^p}{a^q} = a^{p-q}$, $(ab)^p = a^p b^p$, $a^{-p} = \frac{1}{a^p}$, $a^0 = 1$, and $a^{m/n} = (\sqrt[n]{a})^m$.

[CONCEPT MASTER – 4. Laws of Exponents for Real Numbers] Rationalised 2026–27 Scope: Visualizing real numbers on the number line via successive magnification is excluded from the 2026–27 syllabus.

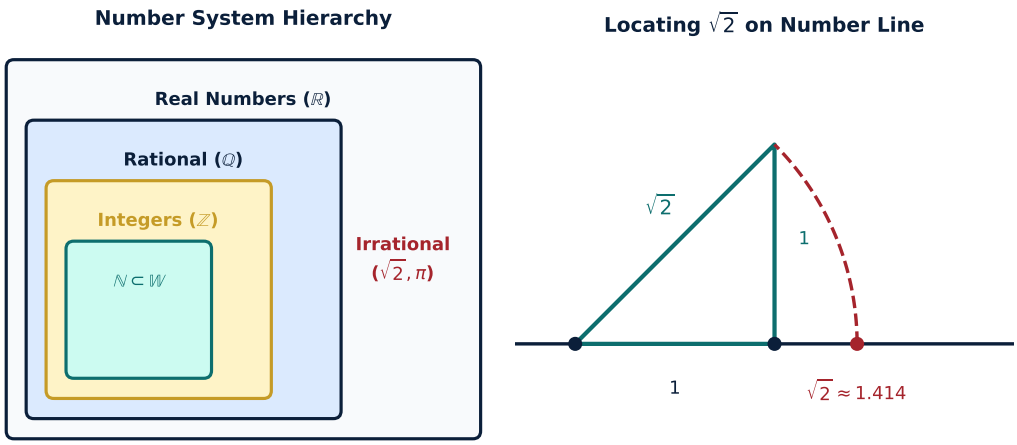


Figure 1: **Visual Model:** Real number hierarchy ($\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$) and geometric construction of $\sqrt{2}$ on the number line.

C. Complete Formula Bank

[FORMULA BANK – 1. Midpoint / Equivalent Rational] **Expression:** $r_{\text{mid}} = \frac{a+b}{2}$; n rationals: $\frac{p(n+1)}{q(n+1)}$
Symbols: $a, b \in \mathbb{Q}$.
Conditions & When to Use: Denominator $q \neq 0$.
Solved Mini-Example: Between 1 and 2: $3/2$.
Common Mistake: Dividing by n instead of $n+1$ when scaling denominators.

[FORMULA BANK – 2. Terminating Decimal Test] **Expression:** $q = 2^m \cdot 5^n$ (in lowest form $\gcd(p, q) = 1$)
Symbols: $m, n \in \mathbb{W}$.
Conditions & When to Use: Reduce $\frac{p}{q}$ to lowest terms FIRST.
Solved Mini-Example: $\frac{6}{15} = \frac{2}{5}$ terminates (even though 15 has 3).
Common Mistake: Checking q before canceling common factors.

[FORMULA BANK – 3. Conjugate Rationalization] **Expression:** $\frac{1}{\sqrt{a}\pm\sqrt{b}} = \frac{\sqrt{a}\mp\sqrt{b}}{a-b}$
Symbols: $a, b > 0, a \neq b$.
Conditions & When to Use: Flip ONLY the sign between the two terms.
Solved Mini-Example: $\frac{1}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{5}-\sqrt{3}}{2}$.
Common Mistake: Forgetting to multiply the numerator by the conjugate too.

[FORMULA BANK – 4. Radical Algebra Identities] **Expression:** $\sqrt{ab} = \sqrt{a}\sqrt{b}$; $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a-b$
Symbols: $a, b \geq 0$.
Conditions & When to Use: Non-negative radicands.
Solved Mini-Example: $(3+\sqrt{3})(3-\sqrt{3}) = 9-3 = 6$.
Common Mistake: Writing $\sqrt{a+b} = \sqrt{a}+\sqrt{b}$.

[FORMULA BANK – 5. Product & Quotient Exponents] **Expression:** $a^m \cdot a^n = a^{m+n}$; $\frac{a^m}{a^n} = a^{m-n}$
Symbols: $a > 0$, $m, n \in \mathbb{Q}$.
Conditions & When to Use: Same base a.
Solved Mini-Example: $2^{2/3} \cdot 2^{1/3} = 2^1 = 2$.
Common Mistake: Multiplying exponents instead of adding in $a^m \cdot a^n$.

[FORMULA BANK – 6. Power & Fractional Exponent] **Expression:** $(a^m)^n = a^{mn}$; $a^{m/n} = (\sqrt[n]{a})^m$
Symbols: $a > 0, n \in \mathbb{N}$.
Conditions & When to Use: Take the n-th root FIRST to keep numbers small.
Solved Mini-Example: $32^{2/5} = (32^{1/5})^2 = 2^2 = 4$.
Common Mistake: Computing $32^2 = 1024$ first.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Olympiad/NTSE Foundation problem structures.

Pattern 1: Inserting n Rationals Between Two Fractions

- **Identification Cue:** Find 5 rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$. | **Concept & Formula:** Scale numerator and denominator by $(n+1)$.
- **Standard Step-by-Step:** Make denominators equal, then multiply top and bottom of both by $(n+1)$.
- **Fastest Valid Method:** Multiply by 10 when $n \leq 9$ for effortless mental fractions. | **Alternative:** Repeated midpoint $\frac{a+b}{2}$.
- **Common Trap:** Scaling before equalizing denominators.
- **Practice Drill:** Ex. 1 & Ex. 5.

Pattern 2: Converting $p.\overline{qr}$ to p/q Form

- **Identification Cue:** Express $0.\overline{6}$ or $0.4\overline{7}$ or $1.2\overline{7}$ in p/q form. | **Concept & Formula:** Multiply by 10^k to cancel repeating tail.
- **Standard Step-by-Step:** Let $x =$ decimal, shift non-repeating digits first if any, then shift one full period and subtract.
- **Fastest Valid Method:** MCQ check: $\frac{\text{All digits} - \text{Non-repeating digits}}{9\ldots90\ldots0}$. | **Alternative:** Infinite G.P. (in higher classes).
- **Common Trap:** Subtracting x when the bar doesn't start right after the decimal point.
- **Practice Drill:** Ex. 6 & Ex. 9.

Pattern 3: Rationalizing & Finding $a + b\sqrt{c}$

- **Identification Cue:** If $\frac{\sqrt{3}-1}{\sqrt{3}+1} = a + b\sqrt{3}$, find a, b. | **Concept & Formula:** Multiply by conjugate, then compare rational and irrational parts.
- **Standard Step-by-Step:** Rationalize LHS into $A + B\sqrt{c}$ and equate $a = A, b = B$.
- **Fastest Valid Method:** Remember $(\sqrt{a} \pm \sqrt{b})^2 = a + b \pm 2\sqrt{ab}$. | **Alternative:** Combine fractions via cross-multiplication.
- **Common Trap:** Sign error when subtracting $(a + b\sqrt{c})$.
- **Practice Drill:** Ex. 10 & Ex. 11.

Pattern 4: Simplifying Fractional Exponents

- **Identification Cue:** Evaluate $(125)^{-1/3}$ or $\frac{64^{1/2} \cdot 27^{2/3}}{8^{1/3}}$. | **Concept & Formula:** Prime factorize bases first.
- **Standard Step-by-Step:** Write bases as prime powers ($125 = 5^3, 64 = 2^6, 27 = 3^3$), cancel denominators in exponents.
- **Fastest Valid Method:** Take n-th root immediately: $125^{1/3} = 5$. | **Alternative:** Combine same-base exponents.
- **Common Trap:** Writing $a^{-n} = -a^n$ instead of $\frac{1}{a^n}$.
- **Practice Drill:** Ex. 4 & Ex. 12.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Instant Recurring Decimal p/q Formula] **Core Idea & Why It Works:** $0.\overline{AB} = \frac{AB}{99}$ (one 9 per bar digit); $0.A\overline{B} = \frac{AB-A}{90}$ (one 9 per bar digit, one 0 per non-bar digit).
Valid Domain: All recurring decimals. | **Where It Fails:** Non-repeating irrationals.
Easy Example: $0.\overline{3} = 3/9 = 1/3$; $0.4\overline{7} = \frac{47-4}{90} = \frac{43}{90}$.
Exam-Level Example: Instant verification for 3-mark NCERT p/q questions.
Advanced Example: $0.2\overline{35} = \frac{235-2}{990} = \frac{233}{990}$.

[SMART SHORTCUT – 2. Reciprocal of $\sqrt{n+1}+\sqrt{n}$] **Core Idea & Why It Works:** Whenever $a-b=1$, $\frac{1}{\sqrt{a}+\sqrt{b}} = \sqrt{a}-\sqrt{b}$ directly!
Valid Domain: Consecutive integers or squares differing by 1 (like $2+\sqrt{3}$, since $2^2-3=1$). | **Where It Fails:** When $a-b \neq 1$.
Easy Example: $\frac{1}{\sqrt{7}-\sqrt{6}} = \sqrt{7}+\sqrt{6}$.
Exam-Level Example: If $x = 3+2\sqrt{2}$ $(9-8=1)$, then $\frac{1}{x} = 3-2\sqrt{2}$, so $x+\frac{1}{x} = 6$!
Advanced Example: Telescoping sum $\sum_{k=1}^{99} \frac{1}{\sqrt{k+1}+\sqrt{k}} = \sqrt{100}-1 = 9$.

[SMART SHORTCUT – 3. Zero-Mistake Scaling for n Rationals] **Core Idea & Why It Works:** To insert $n \leq 9$ rationals between $\frac{a}{b}$ and $\frac{c}{b}$, just multiply numerator and denominator by 10!
Valid Domain: Any two distinct fractions. | **Where It Fails:** If gap is tiny (< 0.01), use 100.
Easy Example: Between $3/5$ and $4/5$: $30/50$ and $40/50$ give $31/50, 32/50, \ldots$
Exam-Level Example: Clean, fast, and zero division arithmetic.
Advanced Example: Works for negative rationals too.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Find five rational numbers between 1 and 2.
Given & Required: $1 = \frac{6}{6}, 2 = \frac{12}{6}$. | **Concept & Formula:** Scale by $n+1=6$.
Detailed Solution: $\frac{7}{6}, \frac{8}{6}, \frac{9}{6}, \frac{10}{6}, \frac{11}{6}$. **Final Answer:** $\frac{7}{6}, \frac{4}{3}, \frac{3}{2}, \frac{5}{3}, \frac{11}{6}$
Fastest Alternative / Shortcut: $1.1, 1.2, 1.3, 1.4, 1.5$ also works!

SOLVED EXAMPLE 2 [Beginner]
Question: Classify as rational or irrational: (i) $2-\sqrt{5}$, (ii) $(3+\sqrt{23})-\sqrt{23}$, (iii) $\frac{2\sqrt{7}}{7\sqrt{7}}$.
Given & Required: Three expressions. | **Concept & Formula:** Simplify first!
Detailed Solution: (i) Irrational (rational − irrational); (ii) 3 (Rational); (iii) $2/7$ (Rational). **Final Answer: (i) Irrational, (ii) Rational, (iii) Rational**
Fastest Alternative / Shortcut: Always simplify before classifying.

SOLVED EXAMPLE 3 [Beginner]
Question: Rationalize the denominator of $\frac{1}{\sqrt{7}-\sqrt{6}}$.
Given & Required: $\sqrt{7}-\sqrt{6}$. | **Concept & Formula:** Multiply by $\sqrt{7}+\sqrt{6}$.
Detailed Solution: $\frac{\sqrt{7}+\sqrt{6}}{7-6} = \sqrt{7}+\sqrt{6}$. **Final Answer:** $\sqrt{7}+\sqrt{6}$
Fastest Alternative / Shortcut: $7-6=1$.

SOLVED EXAMPLE 4 [Beginner]
Question: Evaluate $16^{3/4}$ and $125^{-1/3}$.
Given & Required: Fractional exponents. | **Concept & Formula:** $a^{m/n} = (a^{1/n})^m$.
Detailed Solution: $16^{3/4} = (2^4)^{3/4} = 2^3 = 8$. $125^{-1/3} = (5^3)^{-1/3} = 5^{-1} = \frac{1}{5}$. **Final Answer:** 8 **and** $\frac{1}{5}$
Fastest Alternative / Shortcut: Prime bases 2 and 5.

SOLVED EXAMPLE 5 [Intermediate]

Question: Find three different irrational numbers between $\frac{5}{7}$ and $\frac{9}{11}$.

Given & Required: $5/7 = 0.\overline{714285}$, $9/11 = 0.\overline{81}$. | **Concept & Formula:** Non-terminating non-repeating decimals.

Detailed Solution: Any non-repeating decimals between $0.714\ldots$ and $0.818\ldots$: $0.730730073000\ldots$, $0.750750075\ldots$, $0.801800180001\ldots$ **Final Answer:** $0.7307300\ldots$, $0.7507500\ldots$, $0.8018001\ldots$

Fastest Alternative / Shortcut: Use increasing zeros $0, 00, 000$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Express $0.\overline{6}$ in the form $\frac{p}{q}$.

Given & Required: $x = 0.666\ldots$ | **Concept & Formula:** Multiply by 10.

Detailed Solution: $10x = 6.666\cdots \implies 10x - x = 6 \implies 9x = 6 \implies x = \frac{2}{3}$. **Final Answer:** $\frac{2}{3}$

Fastest Alternative / Shortcut: $6/9 = 2/3$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Simplify $(5+\sqrt{7})(2+\sqrt{5})$ and $(\sqrt{5}+\sqrt{2})^2$.

Given & Required: Binomial surds. | **Concept & Formula:** Distributive & $(a+b)^2$.

Detailed Solution: $(5+\sqrt{7})(2+\sqrt{5}) = 10+5\sqrt{5}+2\sqrt{7}+\sqrt{35}$. $(\sqrt{5}+\sqrt{2})^2 = 5+2+2\sqrt{10} = 7+2\sqrt{10}$. **Final Answer:** $10+5\sqrt{5}+2\sqrt{7}+\sqrt{35}$; $7+2\sqrt{10}$

Fastest Alternative / Shortcut: Do not forget $2ab$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Rationalize $\frac{5}{\sqrt{3}-\sqrt{5}}$.

Given & Required: Denominator $\sqrt{3}-\sqrt{5}$. | **Concept & Formula:** Conjugate $\sqrt{3}+\sqrt{5}$.

Detailed Solution: $\frac{5(\sqrt{3}+\sqrt{5})}{3-5} = -\frac{5}{2}(\sqrt{3}+\sqrt{5})$. **Final Answer:** $-\frac{5}{2}(\sqrt{3}+\sqrt{5})$

Fastest Alternative / Shortcut: Watch the negative denominator $3-5=-2$.

SOLVED EXAMPLE 9 [Board Standard]

Question: Express $0.4\overline{7}$ in the form $\frac{p}{q}$.

Given & Required: $x = 0.4777\ldots$ | **Concept & Formula:** Multiply by 10 and 100.

Detailed Solution: $10x = 4.777\ldots$ and $100x = 47.777\ldots$. Subtract: $90x = 43 \implies x = \frac{43}{90}$. **Final Answer:** $\frac{43}{90}$

Fastest Alternative / Shortcut: $(47-4)/90 = 43/90$.

SOLVED EXAMPLE 10 [Board Standard]

Question: Find a and b if $\frac{3+\sqrt{7}}{3-\sqrt{7}} = a+b\sqrt{7}$.

Given & Required: LHS fraction. | **Concept & Formula:** Rationalize LHS.

Detailed Solution: $\frac{(3+\sqrt{7})^2}{9-7} = \frac{9+7+6\sqrt{7}}{2} = 8+3\sqrt{7}$. Thus $a=8, b=3$. **Final Answer:** $a=8, b=3$

Fastest Alternative / Shortcut: $(16+6\sqrt{7})/2 = 8+3\sqrt{7}$.

SOLVED EXAMPLE 11 [Board Standard]

Question: If $x = 3+2\sqrt{2}$, find the value of $x^2+\frac{1}{x^2}$.

Given & Required: $x = 3+2\sqrt{2}$. | **Concept & Formula:** $x+\frac{1}{x}$ first.

Detailed Solution: $\frac{1}{x} = \frac{3-2\sqrt{2}}{9-8} = 3-2\sqrt{2}$. So $x+\frac{1}{x} = 6$. Squaring: $x^2+\frac{1}{x^2}+2 = 36 \implies x^2+\frac{1}{x^2} = 34$. **Final Answer:** 34

Fastest Alternative / Shortcut: $6^2-2 = 34$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Simplify $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$.
Given & Required: Three surd fractions. | **Concept & Formula:** Rationalize each term.
Detailed Solution: Term 1: $\frac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{7} = \sqrt{30}-3$. Term 2: $\frac{2\sqrt{5}(\sqrt{6}-\sqrt{5})}{1} = 2\sqrt{30}-10$. Term 3: $\frac{3\sqrt{2}(\sqrt{15}-3\sqrt{2})}{15-18} = -(\sqrt{30}-6) = 6-\sqrt{30}$. Sum $= (\sqrt{30}-3)-(2\sqrt{30}-10)-(6-\sqrt{30}) = 1$. **Final Answer:** 1
Fastest Alternative / Shortcut: Watch minus signs carefully!

SOLVED EXAMPLE 13 [Board Standard]

Question: Prove that $\frac{1}{1+x^{b-a}+x^{c-a}} + \frac{1}{1+x^{a-b}+x^{c-b}} + \frac{1}{1+x^{a-c}+x^{b-c}} = 1$.
Given & Required: Symmetric exponents. | **Concept & Formula:** Multiply top/bottom by x^a, x^b, x^c.
Detailed Solution: First term $= \frac{x^a}{x^a+x^b+x^c}$, second $= \frac{x^b}{x^a+x^b+x^c}$, third $= \frac{x^c}{x^a+x^b+x^c}$. Sum $= \frac{x^a+x^b+x^c}{x^a+x^b+x^c} = 1$. **Final Answer:** 1
Fastest Alternative / Shortcut: Common denominator $x^a+x^b+x^c$.

SOLVED EXAMPLE 14 [Challenge]

Question: Evaluate $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{8}+\sqrt{9}}$.
Given & Required: 8 telescoping terms. | **Concept & Formula:** Rationalize each.
Detailed Solution: $(\sqrt{2}-1)+(\sqrt{3}-\sqrt{2})+\cdots+(\sqrt{9}-\sqrt{8}) = \sqrt{9}-1 = 3-1 = 2$. **Final Answer:** 2
Fastest Alternative / Shortcut: Last root minus first root: $\sqrt{9}-1=2$.

SOLVED EXAMPLE 15 [Challenge]

Question: If $2^x = 3^y = 6^{-z}$, prove that $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0$.
Given & Required: $2^x = 3^y = 6^{-z} = k$. | **Concept & Formula:** Express $2, 3, 6$ in terms of k.
Detailed Solution: $2 = k^{1/x}$, $3 = k^{1/y}$, $6 = k^{-1/z}$. Since $2\times 3 = 6$, $k^{1/x+1/y} = k^{-1/z} \implies \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 0$. **Final Answer:** **Proved**
Fastest Alternative / Shortcut: Use $2\times 3 = 6$.

SOLVED EXAMPLE 16 [Challenge]

Question: Find the value of x if $5^{x-3}\cdot 3^{2x-8} = 225$.
Given & Required: $225 = 5^2\cdot 3^2$. | **Concept & Formula:** Equate prime exponents.
Detailed Solution: $x-3 = 2 \implies x = 5$, and check $2(5)-8 = 2$. Verified! **Final Answer:** $x = 5$
Fastest Alternative / Shortcut: Prime factorize $225 = 25\times 9$.

G. Competency MCQ Mastery Section

Q1. Every rational number is:

(A) A natural number (B) An integer (C) A real number (D) A whole number

Answer: (C) $\mathbb{Q}\subset\mathbb{R}$. *Fast Method:* Hierarchy $\mathbb{N}\subset\mathbb{W}\subset\mathbb{Z}\subset\mathbb{Q}\subset\mathbb{R}$. *Distractor Note:* Reversing subset direction.

Q2. Which of the following is an irrational number?

(A) $\sqrt{4/9}$ (B) $\sqrt{12}/\sqrt{3}$ (C) $\sqrt{7}$ (D) $\sqrt{81}$

Answer: (C) $\sqrt{12}/\sqrt{3} = \sqrt{4} = 2$, while $\sqrt{7}$ is non-square. *Fast Method:* Simplify $\sqrt{12}/\sqrt{3}$ first. *Distractor Note:* Picking (B) without dividing.

Q3. Decimal representation of a rational number CANNOT be:

(A) Terminating (B) Non-terminating (C) Non-terminating repeating (D) Non-terminating non-repeating

Answer: (D) Non-terminating non-repeating decimals are irrational. *Fast Method:* Definition of irrational. *Distractor Note:* Picking (B) or (C).

Q4. The p/q form of $0.\overline{9}$ is:

(A) $9/10$ (B) 1 (C) $9/100$ (D) $1/9$

Answer: (B) $10x - x = 9 \implies x = 1$. *Fast Method:* $9/9 = 1$. *Distractor Note:* Picking $9/10$.

Q5. Value of $(256)^{0.16} \times (256)^{0.09}$ is:

(A) 4 (B) 16 (C) 64 (D) 256.25

Answer: (A) $256^{0.25} = 256^{1/4} = 4$. *Fast Method:* Add exponents: $0.16 + 0.09 = 0.25$. *Distractor Note:* Multiplying exponents.

Q6. Rationalizing factor of $\sqrt{5} - 2$ is:

(A) $\sqrt{5} - 2$ (B) $\sqrt{5} + 2$ (C) $2 - \sqrt{5}$ (D) $\sqrt{5}$

Answer: (B) $(\sqrt{5} - 2)(\sqrt{5} + 2) = 5 - 4 = 1$. *Fast Method:* Conjugate. *Distractor Note:* Picking $\sqrt{5}$.

Q7. Product of any two irrational numbers is:

(A) Always irrational (B) Always rational (C) Always an integer (D) Sometimes rational, sometimes irrational

Answer: (D) $\sqrt{2} \cdot \sqrt{2} = 2$ (rational), $\sqrt{2} \cdot \sqrt{3} = \sqrt{6}$ (irrational). *Fast Method:* Test counterexamples. *Distractor Note:* Picking 'Always irrational'.

Q8. Value of $2^0 + 7^0$ divided by 5^0 is:

(A) 2 (B) 1 (C) $9/5$ (D) 0

Answer: (A) $(1 + 1)/1 = 2$. *Fast Method:* $a^0 = 1$. *Distractor Note:* Taking $a^0 = 0$.

Q9. If $x = 2 + \sqrt{3}$, then $x + \frac{1}{x}$ equals:

(A) $2\sqrt{3}$ (B) 4 (C) 2 (D) $4 + 2\sqrt{3}$

Answer: (B) $\frac{1}{x} = 2 - \sqrt{3}$, sum $= 4$. *Fast Method:* $2^2 - 3 = 1$, double rational part. *Distractor Note:* Doubling $\sqrt{3}$.

Q10. $\sqrt[4]{\sqrt[3]{2^2}}$ equals:

(A) $2^{-1/6}$ (B) 2^{-6} (C) $2^{1/6}$ (D) 2^6

Answer: (C) $(2^{2/3})^{1/4} = 2^{2/12} = 2^{1/6}$. *Fast Method:* Multiply exponents $2 \times \frac{1}{3} \times \frac{1}{4}$. *Distractor Note:* Adding $4 + 3 = 7$.

Q11. Which number has a terminating decimal expansion?

(A) $7/15$ (B) $11/24$ (C) $13/80$ (D) $5/12$

Answer: (C) $80 = 2^4 \times 5^1$ (only $2, 5$). *Fast Method:* Check $q = 2^m 5^n$. *Distractor Note:* Ignoring factor 3 in $15, 24, 12$.

Q12. Value of $(\sqrt{11} - \sqrt{7})(\sqrt{11} + \sqrt{7})$ is:

(A) 4 (B) 18 (C) $\sqrt{4}$ (D) 77

Answer: (A) $11 - 7 = 4$. *Fast Method:* $a - b$. *Distractor Note:* Adding $11 + 7 = 18$.

H. Special Exam & MCQ Strategies

- Always simplify radicals (e.g., $\sqrt{12}/\sqrt{3} = 2$) before deciding whether a number is rational or irrational.
- For fractional exponents like $64^{2/3}$, ALWAYS apply the denominator root first ($64^{1/3} = 4$), then the numerator power ($4^2 = 16$).
- In rationalization sums with 3 terms, rationalize each term cleanly on a separate line to prevent sign mistakes.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Writing $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ (e.g., $\sqrt{9+16} = 3 + 4 = 7$). $\longrightarrow$ **Correction:** $\sqrt{9+16} = \sqrt{25} = 5$; square roots NEVER distribute over addition!

[COMMON MISTAKE] **Error:** Writing $5^{-2} = -25$. $\longrightarrow$ **Correction:** Negative exponents mean reciprocal, NOT negative value: $5^{-2} = \frac{1}{25}$.

[COMMON MISTAKE] **Error:** Converting $0.4\overline{7}$ as $\frac{47}{99}$. $\longrightarrow$ **Correction:** Only 7 repeats, so $0.4\overline{7} = \frac{47-4}{90} = \frac{43}{90}$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $\frac{1}{\sqrt{a}\pm\sqrt{b}} = \frac{\sqrt{a}\mp\sqrt{b}}{a-b}$; $a^m a^n = a^{m+n}$; $(a^m)^n = a^{mn}$; $a^{m/n} = (\sqrt[n]{a})^m$.

[QUICK REVISION] Key Theorems & Results: Terminating iff $q = 2^m 5^n$; $0.\overline{ab} = \frac{ab}{99}$; $0.a\overline{b} = \frac{ab-a}{90}$.

[QUICK REVISION] Shortcut Summary: If $a - b = 1$, $\frac{1}{\sqrt{a}+\sqrt{b}} = \sqrt{a} - \sqrt{b}$; Multiply by 10 to insert rationals.

[QUICK REVISION] High-Frequency Exam Patterns: Inserting rationals/irrationals, p/q conversion, $a + b\sqrt{c}$ rationalization, Exponent simplification.

Last-Minute Revision Checklist:

- ☐ Simplify radicals before classifying
- ☐ Root first in $a^{m/n}$
- ☐ Conjugate sign flip
- ☐ Bar alignment in p/q

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Which is rational? (A) $\sqrt{2}$ (B) π (C) $\sqrt{25}$ (D) $\sqrt{3}$
2. Value of $27^{2/3}$ is: (A) 3 (B) 9 (C) 18 (D) 81
3. Rationalizing factor of $\sqrt{3}+1$ is: (A) $\sqrt{3}+1$ (B) $\sqrt{3}-1$ (C) $\sqrt{3}$ (D) 2
4. $10^0 + 20^0$ equals: (A) 0 (B) 1 (C) 2 (D) 30

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. p/q form of $0.\overline{27}$ is: (A) $27/100$ (B) $3/11$ (C) $27/90$ (D) $9/33$
2. If $\frac{2+\sqrt{3}}{2-\sqrt{3}} = a + b\sqrt{3}$, then $a + b$ is: (A) 7 (B) 11 (C) 4 (D) 1
3. Value of $(64)^{-2/3}$ is: (A) 16 (B) $1/16$ (C) -16 (D) $1/8$
4. Between $1/3$ and $1/2$, which rational lies? (A) $1/4$ (B) $5/12$ (C) $2/3$ (D) $1/6$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If $x = 5 + 2\sqrt{6}$, then $\sqrt{x} + \frac{1}{\sqrt{x}}$ is: (A) $2\sqrt{3}$ (B) $2\sqrt{2}$ (C) 10 (D) $2\sqrt{6}$
2. Sum $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \cdots + \frac{1}{\sqrt{15}+\sqrt{16}}$: (A) 3 (B) 4 (C) $\sqrt{15} - 1$ (D) 5
3. If $3^x = 5^y = 15^z$, then $\frac{1}{x} + \frac{1}{y}$ equals: (A) z (B) $1/z$ (C) $2/z$ (D) $-1/z$
4. Last digit of 7^{2026} is: (A) 1 (B) 3 (C) 7 (D) 9

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (C):** $\sqrt{25} = 5$. **Q2 (B):** $(27^{1/3})^2 = 3^2 = 9$. **Q3 (B):** Conjugate $\sqrt{3} - 1$. **Q4 (C):** $1 + 1 = 2$.
Exam Std	**Q1 (B):** $27/99 = 3/11$. **Q2 (B):** $(2+\sqrt{3})^2 = 7 + 4\sqrt{3} \implies a = 7, b = 4, a + b = 11$. **Q3 (B):** $(4)^{-2} = 1/16$. **Q4 (B):** $4/12 < 5/12 < 6/12$.
Challenge	**Q1 (A):** $(\sqrt{x} + \frac{1}{\sqrt{x}})^2 = x + \frac{1}{x} + 2 = 10 + 2 = 12 \implies 2\sqrt{3}$. **Q2 (A):** $\sqrt{16} - 1 = 4 - 1 = 3$. **Q3 (B):** $3 \times 5 = 15 \implies \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$. **Q4 (D):** $2026 \equiv 2 \pmod 4 \implies 7^2 = 49 \to 9$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Real number hierarchy & $\sqrt{n}$ construction. Day 2: Decimal expansions & p/q. Day 3: Rationalization. Day 4: Laws of exponents.
- **How to Introduce the Concept:** Ask: 'Can you write $\sqrt{2}$ as an exact fraction?' Introduce Pythagoras' discovery of irrationals.
- **Difficult Areas & Remediation:** $0.4\overline{7}$ alignment and negative/fractional exponents.
- **Blackboard Teaching Examples:** Show $\sqrt{2}$ right-triangle arc on the number line step by step.
- **Classroom Questions to Ask Students:** Why is $\pi \approx 22/7$ rational while π itself is irrational?
- **Common Student Doubts & Answers:** Students ask why $0.\overline{9} = 1$; prove it cleanly via $10x - x = 9$.
- **Homework & Revision Strategy:** Assign 4 p/q conversions, 5 rationalization problems, and 5 exponent simplifications.

[TEACHER'S TIP] Strong rationalization skills here make Class 10 Trigonometry and Class 11 Limits effortless.

UNIT II: ALGEBRA TYBA COACHING CENTRE

Chapter 2: Polynomials

Degree, Zeroes, Factor Theorem, Middle-Term Splitting & Algebraic Identities (2026–27)

A. Chapter Introduction & Examination Relevance

Polynomials are the building blocks of algebra. Chapter 2 classifies polynomials by degree and terms, finds their zeroes, establishes the Factor Theorem, and equips you with 8 master algebraic identities.

- **What You Will Learn:** Definition & degree of polynomials, linear/quadratic/cubic types, zeroes of a polynomial, Factor Theorem, factorisation of quadratics (middle-term splitting) and cubics, and all 8 standard algebraic identities.
- **Why This Chapter Matters:** Polynomials carry massive weightage in Class 9 (14/100 in RBSE) and directly continue in Class 10 Chapter 2 (Polynomials) and Chapter 4 (Quadratic Equations).
- **Real-Life Applications:** Area/volume modeling, projectile trajectories, engineering curve fitting, and computer error-correcting codes.
- **Cross-Chapter Connections:** Uses exponents from Chapter 1 and feeds into Chapter 4 (Linear Equations) and Class 10 Quadratic Equations.
- **Exam & Foundation Relevance:** Unit II carries 28/100 marks in RBSE (20/80 in CBSE). Expect 1M MCQs on degree/zeroes, 2M/3M Factor Theorem k-finding, and 3M/5M cubic factorisation or $x^3+y^3+z^3-3xyz$ identities. (Remainder Theorem proof is excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Polynomials in One Variable, Degree & Zeroes

An algebraic expression $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$ is a **polynomial** iff every exponent of x is a **whole number** $(0, 1, 2, \ldots)$. Expressions with x^{-1}, $\sqrt{x}$, $\frac{1}{x+1}$ are NOT polynomials. The highest power with non-zero coefficient is the **degree** (degree of non-zero constant is 0; degree of zero polynomial 0 is **not defined**). A number c is a **zero** of $p(x)$ iff $p(c) = 0$.

[CONCEPT MASTER – 1. Polynomials in One Variable, Degree & Zeroes] A linear polynomial $ax + b$ $(a \neq 0)$ has exactly 1 zero $(x = -b/a)$. A polynomial of degree n has at most n real zeroes.

2. Factor Theorem & Factorisation of Quadratics/Cubics

Factor Theorem: For polynomial $p(x)$ of degree ≥ 1 and $a \in \mathbb{R}$: (1) $(x - a)$ is a factor of $p(x)$ iff $p(a) = 0$. To factorise $ax^2 + bx + c$, split $b = p + q$ such that $pq = ac$. To factorise a cubic $p(x)$, find one root r by testing factors of the constant term, divide by $(x - r)$, and factorise the remaining quadratic.

[CONCEPT MASTER – 2. Factor Theorem & Factorisation of Quadratics/Cubics] Quick Root Check for Cubics: Always test $x = 1$ (sum of all coefficients $= 0$) and $x = -1$ (sum of even-power coeffs $=$ sum of odd-power coeffs) first!

3. Master Algebraic Identities (I–VIII)

Beyond $(x \pm y)^2$ and $x^2 - y^2$: $(x + a)(x + b) = x^2 + (a + b)x + ab$; $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$; $(x \pm y)^3 = x^3 \pm y^3 \pm 3xy(x \pm y)$; $x^3 \pm y^3 = (x \pm y)(x^2 \mp xy + y^2)$; and the 3-variable cubic identity $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$.

[CONCEPT MASTER – 3. Master Algebraic Identities (I–VIII)] Special Conditional Identity: If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$!

4. Geometric & Numerical Applications

Identities evaluate numerical cubes/products rapidly (e.g., 103×107, 99^3, $(-12)^3 + 7^3 + 5^3$) and factorise area/volume expressions of rectangles and cuboids into length, breadth, and height.

[CONCEPT MASTER – 4. Geometric & Numerical Applications] Rationalised 2026–27 Scope: Formal statement/proof of Remainder Theorem is excluded, while Factor Theorem and all 8 identities remain central.

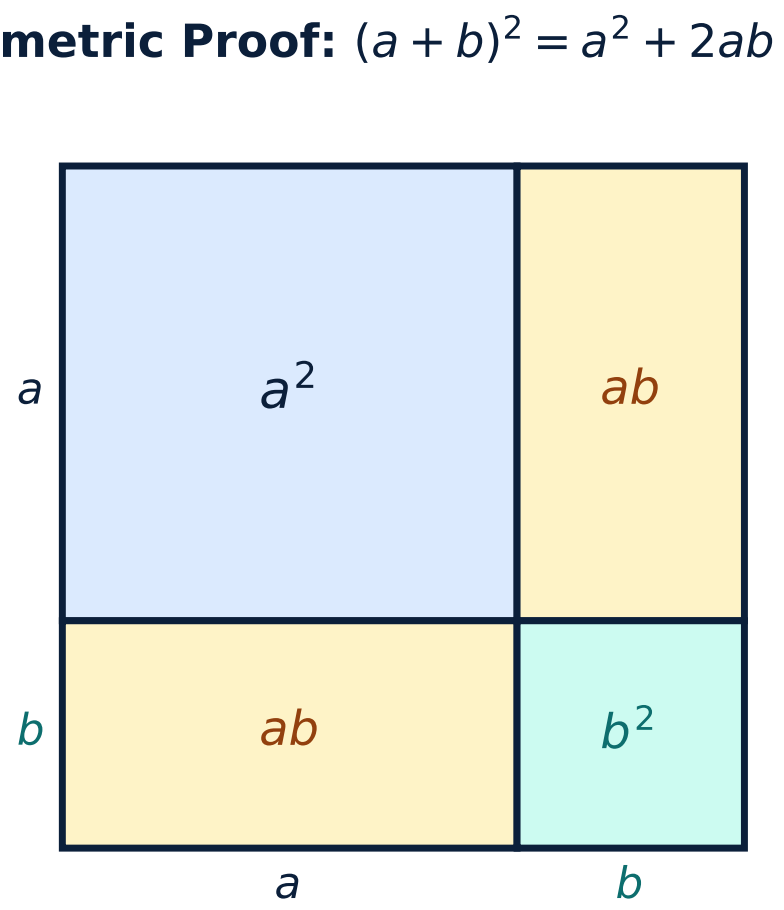


Figure 1: **Visual Model:** Geometric area proof of $(a+b)^2 = a^2 + 2ab + b^2$ forming the foundation of algebraic factorisation.

C. Complete Formula Bank

[FORMULA BANK – 1. Zero of Linear Polynomial] **Expression:** $p(x) = ax + b = 0 \implies x = -\frac{b}{a}$ $(a \neq 0)$
Symbols: $a, b \in \mathbb{R}$.
Conditions & When to Use: $a \neq 0$.
Solved Mini-Example: Zero of $3x - 6$ is $x = 2$.
Common Mistake: Writing $x = b/a$ without the minus sign.

[FORMULA BANK – 2. Factor Theorem] **Expression:** $(x - a)$ is a factor of $p(x) \iff p(a) = 0$
Symbols: $a \in \mathbb{R}$, $\deg(p) \geq 1$.
Conditions & When to Use: For $(ax + b)$, check $p(-b/a) = 0$.
Solved Mini-Example: $(x - 1)$ is a factor of $x^3 - 1$ since $1^3 - 1 = 0$.
Common Mistake: Testing $p(-a)$ instead of $p(a)$ for $(x - a)$.

[FORMULA BANK – 3. Trinomial Square Identity] **Expression:** $(x+y+z)^2 = x^2+y^2+z^2+2xy+2yz+2zx$
Symbols: x, y, z: any terms.
Conditions & When to Use: Keep signs inside x, y, z when expanding.
Solved Mini-Example: $(a-2b+c)^2 = a^2+4b^2+c^2-4ab-4bc+2ac$.
Common Mistake: Putting minus signs on x^2, y^2, z^2.

[FORMULA BANK – 4. Cube of Binomial] **Expression:** $(x \pm y)^3 = x^3 \pm y^3 \pm 3xy(x \pm y)$
Symbols: x, y: any terms.
Conditions & When to Use: Watch signs in $(x-y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$.
Solved Mini-Example: $(2x-1)^3 = 8x^3 - 12x^2 + 6x - 1$.
Common Mistake: Writing $(x-y)^3 = x^3 - y^3$.

[FORMULA BANK – 5. Sum & Difference of Cubes] **Expression:** $x^3+y^3 = (x+y)(x^2-xy+y^2)$; $x^3-y^3 = (x-y)(x^2+xy+y^2)$
Symbols: x, y: any terms.
Conditions & When to Use: Middle term is $\mp xy$ (NO factor of 2!).
Solved Mini-Example: $27x^3+8 = (3x+2)(9x^2-6x+4)$.
Common Mistake: Writing $2xy$ inside the quadratic factor.

[FORMULA BANK – 6. Three-Cube Identity] **Expression:** $x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-zx)$
Symbols: x, y, z.
Conditions & When to Use: If $x+y+z=0$, then $x^3+y^3+z^3 = 3xyz$.
Solved Mini-Example: $(-12)^3+7^3+5^3 = 3(-12)(7)(5) = -1260$.
Common Mistake: Forgetting $3xyz$ when $x+y+z=0$.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Olympiad/NTSE Foundation problem structures.

Pattern 1: Polynomial Check, Degree & Zeroes

- **Identification Cue:** Is $y+\frac{2}{y}$ a polynomial? Find degree of 5 or 0, or find $p(2)$. | **Concept & Formula:** Exponents $\in \mathbb{W}$ and substitution.
- **Standard Step-by-Step:** Rewrite fractions like $2/y = 2y^{-1}$ to check exponents; substitute $x=c$ to evaluate $p(c)$.
- **Fastest Valid Method:** Any x in denominator or under $\sqrt{}$ means NOT a polynomial. | **Alternative:** Expand brackets first.
- **Common Trap:** Saying degree of 0 is 0 (it is NOT defined!).
- **Practice Drill:** Ex. 1 & Ex. 2.

Pattern 2: Finding Unknown k via Factor Theorem

- **Identification Cue:** Find k if $(x-1)$ is a factor of $4x^3+3x^2-4x+k$. | **Concept & Formula:** $p(a)=0$.
- **Standard Step-by-Step:** Set $x-a=0 \implies x=a$, plug into $p(a)=0$, solve for k.
- **Fastest Valid Method:** For $(x-1)$, simply set sum of coefficients $=0$! | **Alternative:** Long division.
- **Common Trap:** Plugging $x=-1$ for $(x-1)$.
- **Practice Drill:** Ex. 5 & Ex. 6.

Pattern 3: Factorising Quadratics & Cubics

- **Identification Cue:** Factorise $6x^2+17x+5$ or x^3-2x^2-x+2. | **Concept & Formula:** Middle-term split ($pq=ac$) and trial root + division.
- **Standard Step-by-Step:** For cubics, find root r, factor out $(x-r)$ by grouping or division, then factor quadratic.

- **Fastest Valid Method:** Check $x = 1$ and $x = -1$ first in cubics. | **Alternative:** Grouping $2 + 2$ terms when ratios match (like $1 : -2$ and $-1 : 2$).
- **Common Trap:** Sign error when splitting negative middle term.
- **Practice Drill:** Ex. 7 & Ex. 10.

Pattern 4: Expanding & Factorising via Identities V–VIII

- **Identification Cue:** Factorise $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$ or prove $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[\dots]$. | **Concept & Formula:** Reverse-engineering squares/cubes.
- **Standard Step-by-Step:** Identify the negative term(s) to assign the minus sign to the shared variable, then apply identity.
- **Fastest Valid Method:** In $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$, z appears in BOTH negative terms ($-24yz, -16xz$), so z is negative: $(2x + 3y - 4z)^2$! | **Alternative:** Test a numerical point $(1, 1, 1)$.
- **Common Trap:** Assigning minus sign to the wrong variable.
- **Practice Drill:** Ex. 11 & Ex. 13.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. $(x - 1)$ and $(x + 1)$ Factor Tests in 2 Seconds] **Core Idea & Why It Works:** $(x - 1)$ is a factor of $p(x) \iff$ Sum of all coefficients $= 0$. $(x + 1)$ is a factor $\iff$ Sum of even-power coeffs = Sum of odd-power coeffs.
Valid Domain: All polynomials. | **Where It Fails:** Nothing.
Easy Example: In $x^3 - 2x^2 - x + 2$: $1 - 2 - 1 + 2 = 0 \implies (x - 1)$ is a factor!
Exam-Level Example: Finds the first factor of almost every Class 9 cubic in 2 seconds.
Advanced Example: Also finds k instantly when $(x \pm 1)$ is a factor.

[SMART SHORTCUT – 2. Shared-Variable Sign Rule for $(a + b + c)^2$] **Core Idea & Why It Works:** When factorising $a^2 + b^2 + c^2 \pm 2ab \pm 2bc \pm 2ca$, look at the TWO negative cross-terms; the variable common to BOTH negative terms gets the minus sign!
Valid Domain: All 3-variable perfect squares. | **Where It Fails:** If all cross-terms are positive, all signs are $+$.
Easy Example: $-24yz$ and $-16xz$ share $z \implies$ term is $-4z$.
Exam-Level Example: Zero trial-and-error in NCERT Ex. 2.4 Q5.
Advanced Example: If ab is positive, a and b have the same sign.

[SMART SHORTCUT – 3. Zero-Sum Cube Shortcut ($a + b + c = 0$)] **Core Idea & Why It Works:** Whenever $a + b + c = 0$, $a^3 + b^3 + c^3 = 3abc$ immediately!
Valid Domain: Numbers, algebraic expressions like $(x - y)^3 + (y - z)^3 + (z - x)^3$. | **Where It Fails:** If $a + b + c \neq 0$.
Easy Example: $(-12)^3 + 7^3 + 5^3 = 3(-12)(7)(5) = -1260$.
Exam-Level Example: $(a - b)^3 + (b - c)^3 + (c - a)^3 = 3(a - b)(b - c)(c - a)$ in 1 line!
Advanced Example: Check $28^3 - 15^3 - 13^3 = 3(28)(-15)(-13) = 16380$.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Find the degree of: (i) $5x^3 + 4x^2 + 7x$, (ii) 3, (iii) $5t - \sqrt{7}$.
Given & Required: Three polynomials. | **Concept & Formula:** Highest power of variable.
Detailed Solution: (i) 3; (ii) $3 = 3x^0$, so degree 0; (iii) 1. **Final Answer:** $3, 0, 1$
Fastest Alternative / Shortcut: Non-zero constant has degree 0.

SOLVED EXAMPLE 2 [Beginner]

Question: Find $p(0), p(1), p(-2)$ for $p(x) = x^2 - 3x + 2$.
Given & Required: $p(x) = x^2 - 3x + 2$. | **Concept & Formula:** Polynomial evaluation.
Detailed Solution: $p(0) = 2$; $p(1) = 1 - 3 + 2 = 0$; $p(-2) = 4 + 6 + 2 = 12$. **Final Answer:** $2, 0, 12$
Fastest Alternative / Shortcut: 1 is a zero of $p(x)$.

SOLVED EXAMPLE 3 [Beginner]

Question: Find the zero of $p(x) = 3x - 2$ and $q(x) = cx + d$ $(c \neq 0)$.
Given & Required: Linear polynomials. | **Concept & Formula:** Set $p(x) = 0$.
Detailed Solution: $3x - 2 = 0 \implies x = \frac{2}{3}$; $cx + d = 0 \implies x = -\frac{d}{c}$. **Final Answer:** $\frac{2}{3}$ **and** $-\frac{d}{c}$
Fastest Alternative / Shortcut: $-$const/coeff.

SOLVED EXAMPLE 4 [Beginner]

Question: Evaluate 103×107 without direct multiplication.
Given & Required: $(100 + 3)(100 + 7)$. | **Concept & Formula:** $(x + a)(x + b) = x^2 + (a + b)x + ab$.
Detailed Solution: $100^2 + (3 + 7)(100) + 21 = 10000 + 1000 + 21 = 11021$. **Final Answer:** $11,021$
Fastest Alternative / Shortcut: $x = 100$.

SOLVED EXAMPLE 5 [Intermediate]

Question: Check whether $x + 2$ is a factor of $x^3 + 3x^2 + 5x + 6$.
Given & Required: $p(-2)$ check. | **Concept & Formula:** Factor Theorem.
Detailed Solution: $p(-2) = (-2)^3 + 3(-2)^2 + 5(-2) + 6 = -8 + 12 - 10 + 6 = 0$. Yes, it is a factor. **Final Answer: Yes** **($p(-2) = 0$)**
Fastest Alternative / Shortcut: Plug $x = -2$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Find k if $x - 1$ is a factor of $p(x) = 2x^2 + kx + \sqrt{2}$.
Given & Required: $p(1) = 0$. | **Concept & Formula:** Factor Theorem.
Detailed Solution: $2(1)^2 + k(1) + \sqrt{2} = 0 \implies k = -(2 + \sqrt{2})$. **Final Answer:** $-(2 + \sqrt{2})$
Fastest Alternative / Shortcut: Sum of coefficients $= 0$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Factorise $6x^2 + 17x + 5$ by splitting the middle term.
Given & Required: $ac = 30, b = 17$. | **Concept & Formula:** $15 + 2 = 17, 15 \times 2 = 30$.
Detailed Solution: $6x^2 + 15x + 2x + 5 = 3x(2x + 5) + 1(2x + 5) = (3x + 1)(2x + 5)$. **Final Answer:** $(3x + 1)(2x + 5)$
Fastest Alternative / Shortcut: Pair 15 and 2.

SOLVED EXAMPLE 8 [Intermediate]

Question: Expand $(3a - 7b - c)^2$ and $(2x + 1)^3$.
Given & Required: Identities V and VI. | **Concept & Formula:** Trinomial square & cube.
Detailed Solution: $(3a - 7b - c)^2 = 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca$. $(2x + 1)^3 = 8x^3 + 12x^2 + 6x + 1$. **Final Answer:** $9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca$; $8x^3 + 12x^2 + 6x + 1$
Fastest Alternative / Shortcut: $(-7b)(-c) = +14bc$.

SOLVED EXAMPLE 9 [Board Standard]

Question: Factorise $x^3 - 3x^2 - 9x - 5$.

Given & Required: Cubic polynomial. | **Concept & Formula:** Trial root + Factor Theorem.

Detailed Solution: Check $x = -1$: $-1-3+9-5 = 0 \implies (x+1)$ is a factor. Divide: $x^3-3x^2-9x-5 = (x+1)(x^2-4x-5) = (x+1)(x-5)(x+1) = (x+1)^2(x-5)$. **Final Answer:** $(x+1)^2(x-5)$

Fastest Alternative / Shortcut: Even-coeff sum $-3-5 = -8$, odd-coeff $1-9 = -8 \implies x = -1$.

SOLVED EXAMPLE 10 [Board Standard]

Question: Factorise $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$.

Given & Required: 3-variable square. | **Concept & Formula:** Shared-variable sign rule.

Detailed Solution: Negative cross-terms $-2\sqrt{2}xy$ and $-8xz$ share x, so x-term is negative: $(-\sqrt{2}x + y + 2\sqrt{2}z)^2$. **Final Answer:** $(-\sqrt{2}x + y + 2\sqrt{2}z)^2$

Fastest Alternative / Shortcut: Shared variable x gets minus sign!

SOLVED EXAMPLE 11 [Board Standard]

Question: Factorise $27x^3 + y^3 + z^3 - 9xyz$.

Given & Required: $(3x)^3 + y^3 + z^3 - 3(3x)yz$. | **Concept & Formula:** Identity VIII.

Detailed Solution: $(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3zx)$. **Final Answer:** $(3x+y+z)(9x^2+y^2+z^2-3xy-yz-3zx)$

Fastest Alternative / Shortcut: Match $a = 3x, b = y, c = z$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Without actually calculating the cubes, find the value of $(-12)^3 + (7)^3 + (5)^3$.

Given & Required: $a = -12, b = 7, c = 5$. | **Concept & Formula:** $a + b + c = 0 \implies a^3 + b^3 + c^3 = 3abc$.

Detailed Solution: $-12 + 7 + 5 = 0 \implies 3(-12)(7)(5) = -1260$. **Final Answer:** -1260

Fastest Alternative / Shortcut: Zero-sum cube shortcut.

SOLVED EXAMPLE 13 [Board Standard]

Question: Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (z - x)^2]$.

Given & Required: NCERT Identity proof. | **Concept & Formula:** Expand RHS squares.

Detailed Solution: $(x-y)^2 + (y-z)^2 + (z-x)^2 = 2(x^2+y^2+z^2-xy-yz-zx)$. Dividing by 2 gives $(x+y+z)(x^2+y^2+z^2-xy-yz-zx) = x^3+y^3+z^3-3xyz$. **Final Answer: Verified**

Fastest Alternative / Shortcut: Start from RHS.

SOLVED EXAMPLE 14 [Challenge]

Question: If $x + y = 12$ and $xy = 27$, find $x^3 + y^3$.

Given & Required: $x + y = 12, xy = 27$. | **Concept & Formula:** $x^3 + y^3 = (x + y)^3 - 3xy(x + y)$.

Detailed Solution: $12^3 - 3(27)(12) = 1728 - 972 = 756$. **Final Answer:** 756

Fastest Alternative / Shortcut: Also $x, y = 9, 3 \implies 729 + 27 = 756$!

SOLVED EXAMPLE 15 [Challenge]

Question: Find possible dimensions for a cuboid whose volume is $12ky^2 + 8ky - 20k$.

Given & Required: $V = 12ky^2 + 8ky - 20k$. | **Concept & Formula:** Factorise into 3 factors.

Detailed Solution: $4k(3y^2+2y-5) = 4k(3y+5)(y-1)$. Dimensions: $4k$, $(3y+5)$, $(y-1)$. **Final Answer:** $4k,\ (3y+5),\ (y-1)$

Fastest Alternative / Shortcut: Pull out GCF $4k$ first.

SOLVED EXAMPLE 16 [Challenge]

Question: If $ax^3 + bx^2 + x - 6$ has $(x+2)$ as a factor and leaves remainder 4 when divided by $(x-2)$, find a and b.

Given & Required: $p(-2) = 0, p(2) = 4$. | **Concept & Formula:** Factor/evaluation system.

Detailed Solution: $p(-2) : -8a + 4b - 8 = 0 \implies -2a + b = 2$. $p(2) : 8a + 4b - 4 = 4 \implies 2a + b = 2$. Adding: $2b = 4 \implies b = 2, a = 0$. **Final Answer:** $a = 0, b = 2$

Fastest Alternative / Shortcut: Add equations to eliminate a.

G. Competency MCQ Mastery Section

Q1. Which of the following is a polynomial?

(A) $x + \frac{1}{x}$ (B) $\sqrt{x} + 3$ (C) $x^2 + \sqrt{2}$ (D) $x^{-2} + 1$

Answer: (C) Exponents of x in (C) are $2, 0 \in \mathbb{W}$. *Fast Method:* Radical on constant $\sqrt{2}$ is allowed! *Distractor Note:* Thinking $\sqrt{2}$ disqualifies (C).

Q2. Degree of the zero polynomial 0 is:

(A) 0 (B) 1 (C) Any natural number (D) Not defined

Answer: (D) $0 = 0 \cdot x^n$ for any n, so degree is undefined. *Fast Method:* Non-zero constant $= 0$; zero $=$ undefined. *Distractor Note:* Picking 0.

Q3. Zero of the polynomial $p(x) = 2x + 5$ is:

(A) $-2/5$ (B) $-5/2$ (C) $2/5$ (D) $5/2$

Answer: (B) $2x + 5 = 0 \implies x = -5/2$. *Fast Method:* $-b/a$. *Distractor Note:* Missing minus sign.

Q4. If $x^{51} + 51$ is divided by $x + 1$, the remainder is:

(A) 0 (B) 1 (C) 49 (D) 50

Answer: (D) $p(-1) = (-1)^{51} + 51 = -1 + 51 = 50$. *Fast Method:* Plug $x = -1$. *Distractor Note:* Writing $(-1)^{51} = +1$.

Q5. If $x + y + z = 0$, then $x^3 + y^3 + z^3$ equals:

(A) 0 (B) xyz (C) $3xyz$ (D) $-3xyz$

Answer: (C) Identity VIII when $x + y + z = 0$. *Fast Method:* $3xyz$. *Distractor Note:* Picking 0.

Q6. Factorisation of $4x^2 - 9y^2$ is:

(A) $(2x - 3y)^2$ (B) $(2x + 3y)(2x - 3y)$ (C) $(4x + 9y)(4x - 9y)$ (D) $(2x + 3y)^2$

Answer: (B) $a^2 - b^2 = (a + b)(a - b)$. *Fast Method:* Difference of squares. *Distractor Note:* Picking (A).

Q7. If $(x - 2)$ is a factor of $x^2 - kx + 6$, then k is:

(A) 5 (B) -5 (C) 1 (D) 3

Answer: (A) $4 - 2k + 6 = 0 \implies k = 5$. *Fast Method:* Plug $x = 2$. *Distractor Note:* Sign slip.

Q8. Value of $249^2 - 248^2$ is:

(A) 1 (B) 477 (C) 487 (D) 497

Answer: (D) $(249 + 248)(249 - 248) = 497$. *Fast Method:* $a + b$ when $a - b = 1$. *Distractor Note:* Arithmetic slip.

Q9. Coefficient of x^2 in $(2 + x^2 + x)$ is:

(A) 2 (B) 1 (C) -1 (D) 0

Answer: (B) Coefficient of x^2 is 1. *Fast Method:* Read coeff of x^2. *Distractor Note:* Reading constant 2.

Q10. If $x + \frac{1}{x} = 3$, then $x^2 + \frac{1}{x^2}$ is:

(A) 9 (B) 7 (C) 11 (D) 6

Answer: (B) $3^2 - 2 = 7$. *Fast Method:* $k^2 - 2$. *Distractor Note:* Answering 9.

Q11. Value of $\frac{x}{y} + \frac{y}{x} = -1$ $(x, y \neq 0)$ implies $x^3 - y^3$ is:

(A) 1 (B) -1 (C) 0 (D) $1/2$

Answer: (C) $x^2 + xy + y^2 = 0 \implies x^3 - y^3 = (x - y)(0) = 0$. *Fast Method:* Cross-multiply to $x^2 + xy + y^2 = 0$. *Distractor Note:* Guessing 1 or -1.

Q12. Number of zeroes of a cubic polynomial is at most:

(A) 1 (B) 2 (C) 3 (D) 4

Answer: (C) Degree $3 \implies$ at most 3 zeroes. *Fast Method:* Degree = max zeroes. *Distractor Note:* Saying 2.

H. Special Exam & MCQ Strategies

- For any cubic factorisation, ALWAYS test $x = 1$ and $x = -1$ first before trying $\pm 2, \pm 3, \ldots$
- In numerical evaluations like 99^3 or 103×97, pick the round base (100) so mental powers stay clean.
- Double-check every polynomial factorisation by substituting $x = 1$ into both the original polynomial and your factored form.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Saying $\sqrt{2}x + 5$ is not a polynomial because of $\sqrt{2}$. $\longrightarrow$ **Correction:** Coefficients can be irrational; only EXPONENTS of x must be whole numbers.

[COMMON MISTAKE] **Error:** Writing $(x + y)^3 = x^3 + y^3$. $\longrightarrow$ **Correction:** $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$.

[COMMON MISTAKE] **Error:** Writing degree of 0 as 0. $\longrightarrow$ **Correction:** Degree of a non-zero constant is 0; degree of the zero polynomial is NOT defined.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $(x \pm y)^2$; $x^2 - y^2$; $(x + y + z)^2$; $(x \pm y)^3$; $x^3 \pm y^3$; $x^3 + y^3 + z^3 - 3xyz$.

[QUICK REVISION] Key Theorems & Results: Factor Theorem: $(x - a)$ factor $\iff p(a) = 0$; If $x + y + z = 0 \implies x^3 + y^3 + z^3 = 3xyz$.

[QUICK REVISION] Shortcut Summary: $x = 1$ (sum of coeffs $= 0$); Shared-variable sign rule for $(a + b + c)^2$; $x + 1/x = k \implies x^2 + 1/x^2 = k^2 - 2$.

[QUICK REVISION] High-Frequency Exam Patterns: Degree/polynomial test, Factor Theorem k, Cubic factorisation, 8 Identities.

Last-Minute Revision Checklist:

- ☐ Whole-number exponents only
- ☐ Degree of 0 undefined
- ☐ Factor $2xy$ in $(x \pm y)^2$, NO 2 in $x^3 \pm y^3$
- ☐ Verify at $x = 1$

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Degree of $4y^3 - 2y + 7$ is: (A) 1 (B) 2 (C) 3 (D) 4
2. Zero of $p(x) = x - 7$ is: (A) -7 (B) 7 (C) 0 (D) $1/7$
3. Value of $p(1)$ for $p(x) = 5x^2 - 3x + 7$: (A) 9 (B) 15 (C) 5 (D) 7
4. Expand $(x - 3)(x + 3)$: (A) $x^2 - 6$ (B) $x^2 - 9$ (C) $x^2 + 9$ (D) $x^2 - 6x + 9$

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Factorise $x^2 - 5x + 6$: (A) $(x - 2)(x - 3)$ (B) $(x + 2)(x + 3)$ (C) $(x - 1)(x - 6)$ (D) $(x + 1)(x - 6)$
2. If $x + 1$ is a factor of $x^3 + x^2 + x + k$, then k is: (A) 1 (B) -1 (C) 0 (D) 3
3. Value of $10^3 + (-6)^3 + (-4)^3$ is: (A) 720 (B) -720 (C) 0 (D) 240
4. Factorise $8x^3 + 27y^3$: (A) $(2x + 3y)(4x^2 - 6xy + 9y^2)$ (B) $(2x + 3y)^3$ (C) $(2x - 3y)(4x^2 + 6xy + 9y^2)$ (D) $(2x + 3y)(4x^2 + 6xy + 9y^2)$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If $a + b + c = 9$ and $ab + bc + ca = 26$, find $a^2 + b^2 + c^2$: (A) 29 (B) 55 (C) 81 (D) 19
2. If $x - \frac{1}{x} = 4$, then $x^3 - \frac{1}{x^3}$ is: (A) 64 (B) 76 (C) 52 (D) 68
3. Factorise $x^3 - 6x^2 + 11x - 6$: (A) $(x - 1)(x - 2)(x - 3)$ (B) $(x + 1)(x + 2)(x + 3)$ (C) $(x - 1)(x - 2)(x + 3)$ (D) $(x - 1)^2(x - 6)$
4. Value of $\frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}$ $(a \neq b \neq c)$ is: (A) 0 (B) 1 (C) 3 (D) -3

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (C):** Highest power is 3. **Q2 (B):** $x-7=0 \implies x=7$. **Q3 (A):** $5-3+7=9$. **Q4 (B):** $x^2-3^2=x^2-9$.
Exam Std	**Q1 (A):** $-2-3=-5, (-2)(-3)=6$. **Q2 (A):** $-1+1-1+k=0 \implies k=1$. **Q3 (A):** $10-6-4=0 \implies 3(10)(-6)(-4)=720$. **Q4 (A):** $a=2x, b=3y$ in a^3+b^3.
Challenge	**Q1 (A):** $81-2(26)=29$. **Q2 (B):** $4^3+3(4)=76$. **Q3 (A):** Roots $1,2,3$. **Q4 (C):** Sum $=0 \implies$ Num $=3(a-b)(b-c)(c-a)$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Degree, zeroes. Day 2: Factor Theorem & quadratics. Day 3: Cubic factorisation. Day 4: Identities V–VIII.
- **How to Introduce the Concept:** Show the $a^2+2ab+b^2$ square-tile area model so students SEE why the $2ab$ term exists!
- **Difficult Areas & Remediation:** Middle-term signs in quadratics and missing xy factor in $x^3 \pm y^3$.
- **Blackboard Teaching Examples:** Write all 8 identities in a numbered grid on the board during the whole chapter.
- **Classroom Questions to Ask Students:** Why can't $x+\frac{1}{x}$ be called a polynomial?
- **Common Student Doubts & Answers:** Students ask about Remainder Theorem; explain Factor Theorem is the $R=0$ special case and evaluation $p(a)$ gives the remainder.
- **Homework & Revision Strategy:** Assign 5 quadratic factorisations, 3 cubics, and 5 identity expansions/factorisations.

[TEACHER'S TIP] Mastery of Chapter 2 Identities makes Class 10 Algebra 3x faster.

UNIT III: COORDINATE GEOMETRY TYBA COACHING CENTRE

Chapter 3: Coordinate Geometry

Cartesian System, Abscissa, Ordinate, Quadrants & Plotting Points (2026–27)

A. Chapter Introduction & Examination Relevance

René Descartes unified algebra and geometry by assigning every point on a flat plane a unique ordered pair (x, y). Chapter 3 introduces the Cartesian coordinate system, quadrants, and plotting points.

- **What You Will Learn:** Horizontal x-axis, vertical y-axis, Origin $(0, 0)$, Abscissa (x-coordinate), Ordinate (y-coordinate), sign conventions in Quadrants I–IV, points on axes, and plotting/reading coordinates.
- **Why This Chapter Matters:** Full chapter is retained in 2026–27! It is the easiest 100% scoring unit in Class 9 and is the direct base for Chapter 4 (Linear Graphs), Class 10 Distance/Section Formulas, and Class 11 Conics.
- **Real-Life Applications:** GPS map coordinates (latitude/longitude), computer screen pixel grids (x, y), chess boards, and data scatter plots.
- **Cross-Chapter Connections:** Immediately powers Chapter 4 (graphing $ax + by + c = 0$) and Chapter 12 (graphical statistics).
- **Exam & Foundation Relevance:** Unit III carries 6/100 marks in RBSE (4/80 in CBSE). Expect 1M MCQs on quadrants/abscissa/ordinate and 2M/3M plotting & figure-area questions.

B. Complete Concept Explanation & Derivations

1. The Cartesian Plane, Origin, Abscissa & Ordinate

Two perpendicular number lines—the horizontal x**-axis** ($X'OX$) and vertical y**-axis** ($Y'OY$)—intersect at the **Origin** $O(0, 0)$. For any point $P(x, y)$: x is the **abscissa** (perpendicular distance from y-axis with sign) and y is the **ordinate** (perpendicular distance from x-axis with sign).

[CONCEPT MASTER – 1. The Cartesian Plane, Origin, Abscissa & Ordinate] Watch the Distance Twist: Distance of $P(x, y)$ from the y-axis is $|x|$, and distance from the x-axis is $|y|$!

2. The Four Quadrants & Sign Conventions

The axes divide the plane into 4 **Quadrants** (counter-clockwise): **Q-I** $(+, +)$, **Q-II** $(-, +)$, **Q-III** $(-, -)$, and **Q-IV** $(+, -)$.

[CONCEPT MASTER – 2. The Four Quadrants & Sign Conventions] Points with $x = 0$ or $y = 0$ lie ON an axis and do NOT belong to any quadrant!

3. Coordinates of Points on the Axes

Every point on the x-axis has $y = 0$, so its form is $(x, 0)$. Every point on the y-axis has $x = 0$, so its form is $(0, y)$. The Origin $(0, 0)$ lies on both axes.

[CONCEPT MASTER – 3. Coordinates of Points on the Axes] Ordered Pair Rule: $(a, b) \neq (b, a)$ unless $a = b$. Thus $(3, 5)$ and $(5, 3)$ are two different points.

4. Plotting Points & Geometric Figures on the Grid

To plot (x, y), move x units along the x-axis from O, then y units parallel to the y-axis. Joining plotted vertices lets us identify rectangles, squares, triangles, and find their perimeter or area directly from grid units.

[CONCEPT MASTER – 4. Plotting Points & Geometric Figures on the Grid] Full Chapter Retained (2026–27): Every topic in Chapter 3 is examinable.

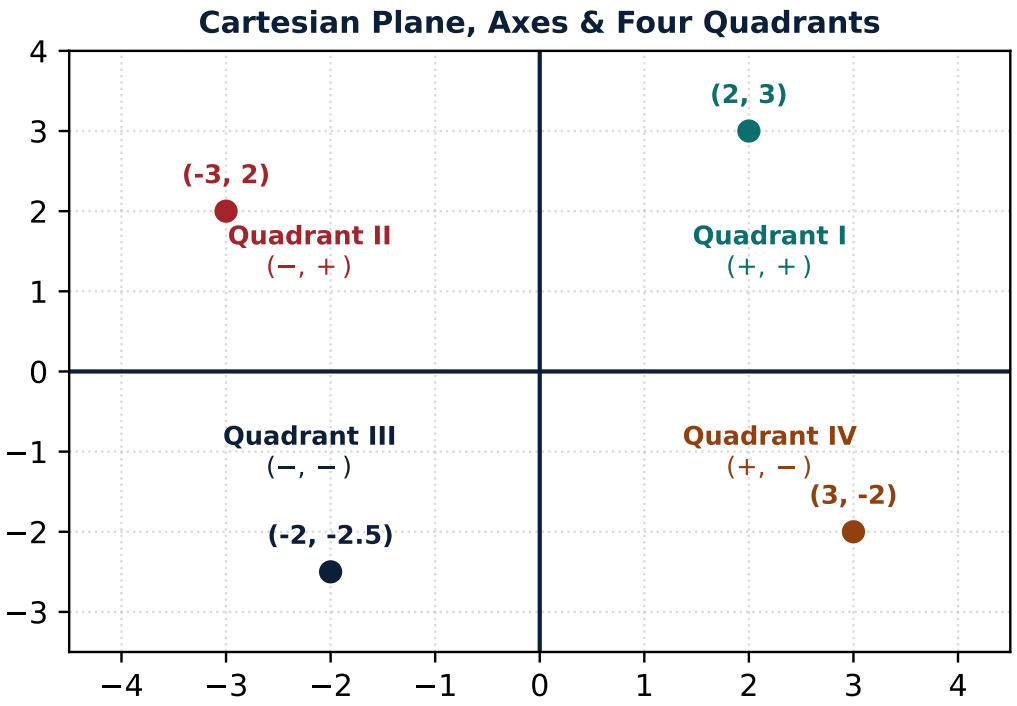


Figure 1: **Visual Model:** Cartesian coordinate plane showing the Origin $(0,0)$, X- and Y-axes, and sign rules across all four Quadrants.

C. Complete Formula Bank

[FORMULA BANK – 1. Ordered Pair Representation] **Expression:** $P(x,y)$: x = abscissa, y = ordinate
Symbols: $x, y \in \mathbb{R}$.
Conditions & When to Use: Always write x first, y second.
Solved Mini-Example: In $(-4,7)$, abscissa $= -4$, ordinate $= 7$.
Common Mistake: Swapping abscissa and ordinate.

[FORMULA BANK – 2. Perpendicular Distances from Axes] **Expression:** From x-axis $= |y|$; From y-axis $= |x|$
Symbols: $|y| \geq 0, |x| \geq 0$.
Conditions & When to Use: Distance is always non-negative.
Solved Mini-Example: Distance of $(-5,-3)$ from x-axis is $|-3| = 3$.
Common Mistake: Writing -3 as a distance or picking $|x| = 5$.

[FORMULA BANK – 3. Quadrant Sign Table] **Expression:** Q-I: $(+,+)$, Q-II: $(-,+)$, Q-III: $(-,-)$, Q-IV: $(+,-)$
Symbols: $x \neq 0, y \neq 0$.
Conditions & When to Use: Counter-clockwise from top-right.
Solved Mini-Example: $(3,-4)$ is in Q-IV.
Common Mistake: Numbering quadrants clockwise.

[FORMULA BANK – 4. Points on Coordinate Axes] **Expression:** x-axis: $(x,0)$; y-axis: $(0,y)$; Origin: $(0,0)$
Symbols: $x, y \in \mathbb{R}$.
Conditions & When to Use: Axis points are in NO quadrant.
Solved Mini-Example: $(0,-6)$ lies on the negative y-axis.
Common Mistake: Saying $(0,-6)$ lies in Q-III or Q-IV.

[FORMULA BANK – 5. Mirror Images in Axes] **Expression:** In x-axis: $(x, -y)$; In y-axis: $(-x, y)$
Symbols: $P(x, y)$.
Conditions & When to Use: Perpendicular reflection.
Solved Mini-Example: Mirror of $(2, 5)$ in x-axis is $(2, -5)$.
Common Mistake: Flipping x for x-axis reflection.

[FORMULA BANK – 6. Distance from Origin] **Expression:** $OP = \sqrt{x^2 + y^2}$
Symbols: $O(0, 0), P(x, y)$.
Conditions & When to Use: Pythagoras theorem.
Solved Mini-Example: Distance of $(3, 4)$ from origin is $\sqrt{9 + 16} = 5$.
Common Mistake: Adding $|x| + |y| = 7$ instead of $\sqrt{x^2 + y^2}$.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Olympiad/NTSE Foundation problem structures.

Pattern 1: Quadrant & Axis Identification

- **Identification Cue:** In which quadrant or on which axis do $(-2, 4), (3, -1), (-1, 0), (0, -5)$ lie? | **Concept & Formula:** Sign rules + zero-coordinate check.
- **Standard Step-by-Step:** Check if either coordinate is 0 (axis); otherwise match (x, y) signs to Q-I..IV.
- **Fastest Valid Method:** Zero in $y \implies x$-axis; zero in $x \implies y$-axis. | **Alternative:** Sketch quick cross axes.
- **Common Trap:** Putting $(0, -5)$ into Q-IV instead of y-axis.
- **Practice Drill:** Ex. 1 & Ex. 3.

Pattern 2: Abscissa, Ordinate & Axis Distance

- **Identification Cue:** Find (abscissa of P) $-$ (ordinate of Q), or distance of $(-3, 7)$ from x-axis. | **Concept & Formula:** $P(x, y) \implies$ abscissa x, ordinate y.
- **Standard Step-by-Step:** Extract x and y with signs; take $|\cdot|$ for distance from opposite axis.
- **Fastest Valid Method:** Distance from X-axis uses Y-coordinate! | **Alternative:** Plot point on grid.
- **Common Trap:** Giving negative distance or using x for x-axis distance.
- **Practice Drill:** Ex. 2 & Ex. 5.

Pattern 3: Plotting Vertices & Finding Area/4th Vertex

- **Identification Cue:** Plot $A(1, 1), B(4, 1), C(4, 5)$, find D to complete rectangle and its area. | **Concept & Formula:** Opposite sides parallel to axes.
- **Standard Step-by-Step:** Match x- and y-coordinates of parallel sides; Area = width $\times$ height.
- **Fastest Valid Method:** Rectangle width $= |x_2 - x_1|$, height $= |y_2 - y_1|$. | **Alternative:** Count grid squares.
- **Common Trap:** Subtracting coordinates in wrong order.
- **Practice Drill:** Ex. 7 & Ex. 9.

Pattern 4: Mirror Image & Symmetry

- **Identification Cue:** Find mirror image of $(-3, 4)$ in x-axis and y-axis. | **Concept & Formula:** Keep axis coordinate, flip the other.
- **Standard Step-by-Step:** In x-axis keep x, flip y; in y-axis keep y, flip x.
- **Fastest Valid Method:** Name of axis tells which coordinate stays SAME! | **Alternative:** Plot and fold along axis.
- **Common Trap:** Flipping both coordinates.
- **Practice Drill:** Ex. 8.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Opposite-Letter Distance Rule] **Core Idea & Why It Works:** Distance from X-axis $= |Y|$; Distance from Y-axis $= |X|$.
Valid Domain: All points (x, y). | **Where It Fails:** Nothing.
Easy Example: Distance of $(-6, 8)$ from x-axis is 8; from y-axis is 6.
Exam-Level Example: Prevents the #1 trick MCQ in Class 9.
Advanced Example: Extends to 3D in Class 11!

[SMART SHORTCUT – 2. Axis-Name-Stays-Same Reflection Rule] **Core Idea & Why It Works:** Reflecting across the X-axis keeps X the same ($y \to -y$); reflecting across the Y-axis keeps Y the same ($x \to -x$).
Valid Domain: All points. | **Where It Fails:** Nothing.
Easy Example: Reflect $(3, -5)$ in y-axis: keep -5, flip $3 \implies (-3, -5)$.
Exam-Level Example: Instant 1-mark MCQ solver.
Advanced Example: Reflecting in origin flips both.

[SMART SHORTCUT – 3. Quadrant Count Counter-Clockwise] **Core Idea & Why It Works:** Start top-right (I: $+, +$) and move counter-clockwise: II $(-, +)$, III $(-, -)$, IV $(+, -)$.
Valid Domain: All 4 quadrants. | **Where It Fails:** Axis points.
Easy Example: $(-2, -9)$ is bottom-left = Q-III.
Exam-Level Example: Matches Trigonometry ASTC quadrant numbering in Class 10–11.
Advanced Example: Zero confusion.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Write the abscissa and ordinate of $P(-5, 8)$ and $Q(3, -2)$.
Given & Required: Two points. | **Concept & Formula:** $(x, y) = (\text{abscissa}, \text{ordinate})$.
Detailed Solution: For P: abscissa $= -5$, ordinate $= 8$. For Q: abscissa $= 3$, ordinate $= -2$. **Final Answer:** $P(-5, 8)$ **and** $Q(3, -2)$
Fastest Alternative / Shortcut: First is abscissa, second is ordinate.

SOLVED EXAMPLE 2 [Beginner]
Question: Name the quadrant or axis for: $(2, 5), (-3, 4), (-1, -6), (4, -2), (0, 5), (-7, 0)$.
Given & Required: 6 points. | **Concept & Formula:** Signs & zeros.
Detailed Solution: $(2, 5)$ Q-I; $(-3, 4)$ Q-II; $(-1, -6)$ Q-III; $(4, -2)$ Q-IV; $(0, 5)$ y-axis; $(-7, 0)$ x-axis. **Final Answer: I, II, III, IV, y-axis, x-axis**
Fastest Alternative / Shortcut: Check 0's first.

SOLVED EXAMPLE 3 [Beginner]
Question: Find the distance of $(4, -7)$ from the x-axis and y-axis.
Given & Required: $(4, -7)$. | **Concept & Formula:** $|y|$ and $|x|$.
Detailed Solution: From x-axis $= |-7| = 7$ units; from y-axis $= |4| = 4$ units. **Final Answer: 7 and 4 units**
Fastest Alternative / Shortcut: Opposite-letter rule.

SOLVED EXAMPLE 4 [Beginner]
Question: What are the coordinates of the Origin?
Given & Required: Intersection of axes. | **Concept & Formula:** Definition.
Detailed Solution: $(0, 0)$. **Final Answer:** $(0, 0)$
Fastest Alternative / Shortcut: Both $x = 0, y = 0$.

SOLVED EXAMPLE 5 [Intermediate]

Question: Find (abscissa of A) $-$ (ordinate of B) if $A(-2,5)$ and $B(4,-3)$.

Given & Required: $A(-2,5), B(4,-3)$. | **Concept & Formula:** Extract x_A and y_B.

Detailed Solution: Abscissa of $A = -2$; Ordinate of $B = -3$. Difference $= -2-(-3) = 1$. **Final Answer:** 1

Fastest Alternative / Shortcut: Watch double negative!

SOLVED EXAMPLE 6 [Intermediate]

Question: A point lies on the x-axis at a distance of 7 units to the left of the y-axis. Write its coordinates.

Given & Required: On x-axis, 7 units left. | **Concept & Formula:** Left of y-axis $\implies x < 0$.

Detailed Solution: $(-7,0)$. **Final Answer:** $(-7,0)$

Fastest Alternative / Shortcut: Left = negative x.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find the mirror image of $(-4,6)$ in (i) the x-axis, (ii) the y-axis.

Given & Required: $(-4,6)$. | **Concept & Formula:** Axis-stays-same rule.

Detailed Solution: (i) $(-4,-6)$; (ii) $(4,6)$. **Final Answer:** $(-4,-6)$ **and** $(4,6)$

Fastest Alternative / Shortcut: Flip the other coordinate.

SOLVED EXAMPLE 8 [Intermediate]

Question: Find the distance of $P(6,-8)$ from the origin.

Given & Required: $P(6,-8)$. | **Concept & Formula:** $\sqrt{x^2+y^2}$.

Detailed Solution: $\sqrt{6^2+(-8)^2} = \sqrt{36+64} = 10$ units. **Final Answer:** 10 **units**

Fastest Alternative / Shortcut: 6-8-10 Pythagorean triple!

SOLVED EXAMPLE 9 [Board Standard]

Question: Three vertices of a rectangle $ABCD$ are $A(2,1), B(6,1), C(6,4)$. Find the coordinates of D and the area of the rectangle.

Given & Required: A, B, C. | **Concept & Formula:** Horizontal & vertical sides.

Detailed Solution: D shares $x = 2$ with A and $y = 4$ with C, so $D(2,4)$. Width $= 6-2 = 4$, height $= 4-1 = 3$. Area $= 4 \times 3 = 12$ sq. units. **Final Answer:** $D(2,4)$**, Area** $= 12$ **sq. units**

Fastest Alternative / Shortcut: Match $x_D = x_A, y_D = y_C$.

SOLVED EXAMPLE 10 [Board Standard]

Question: Plot $P(0,0), Q(3,0), R(0,4)$ and find the area and perimeter of $\triangle PQR$.

Given & Required: Right triangle on axes. | **Concept & Formula:** $\frac{1}{2} \times$ base $\times$ height.

Detailed Solution: Base $PQ = 3$, Height $PR = 4$, Hypotenuse $QR = \sqrt{3^2+4^2} = 5$. Area $= \frac{1}{2}(3)(4) = 6$ sq. units; Perimeter $= 3+4+5 = 12$ units. **Final Answer: Area** $= 6$**, Perimeter** $= 12$

Fastest Alternative / Shortcut: 3-4-5 right triangle.

SOLVED EXAMPLE 11 [Board Standard]

Question: If $(x+2,4) = (5,y-2)$, find x and y and state the quadrant of (x,y).

Given & Required: Equal ordered pairs. | **Concept & Formula:** Component equality.

Detailed Solution: $x+2 = 5 \implies x = 3$; $y-2 = 4 \implies y = 6$. Point $(3,6)$ lies in Quadrant I. **Final Answer:** $x = 3, y = 6$**; Quadrant I**

Fastest Alternative / Shortcut: Equate x's and y's.

SOLVED EXAMPLE 12 [Board Standard]

Question: Points $A(5,3), B(-2,3), D(5,-4)$ are three vertices of a square $ABCD$. Find the coordinates of C and the perimeter.

Given & Required: Side length $= 5-(-2) = 7$. | **Concept & Formula:** Square symmetry.

Detailed Solution: C has $x=-2$ and $y=-4 \implies C(-2,-4)$. Perimeter $= 4 \times 7 = 28$ units. **Final Answer:** $C(-2,-4)$, **Perimeter** $= 28$

Fastest Alternative / Shortcut: Side $= 7$.

SOLVED EXAMPLE 13 [Board Standard]

Question: Write the coordinates of a point whose ordinate is -5 and which lies on the y-axis.

Given & Required: On y-axis ($x=0$), $y=-5$. | **Concept & Formula:** Axis point.

Detailed Solution: $(0,-5)$. **Final Answer:** $(0,-5)$

Fastest Alternative / Shortcut: y-axis means $x=0$.

SOLVED EXAMPLE 14 [Challenge]

Question: Find the area of the triangle with vertices $A(1,2), B(5,2), C(3,7)$.

Given & Required: Horizontal base AB. | **Concept & Formula:** Base along $y=2$.

Detailed Solution: Base $AB = 5-1 = 4$. Height from $C(3,7)$ to $y=2$ is $7-2=5$. Area $= \frac{1}{2}(4)(5) = 10$ sq. units. **Final Answer:** 10 **sq. units**

Fastest Alternative / Shortcut: Since $y_A = y_B = 2$, base is horizontal!

SOLVED EXAMPLE 15 [Challenge]

Question: If $a<0$ and $b>0$, in which quadrant does $(-b,a)$ lie?

Given & Required: $a<0, b>0$. | **Concept & Formula:** Sign analysis.

Detailed Solution: $-b<0$ and $a<0$, so $(-b,a)$ is $(-,-) \implies$ Quadrant III. **Final Answer: Quadrant III**

Fastest Alternative / Shortcut: Test $a=-1, b=1 \implies (-1,-1)$.

SOLVED EXAMPLE 16 [Challenge]

Question: A point is at distance 3 units from x-axis and 4 units from y-axis in Quadrant II. Find its reflection across the x-axis.

Given & Required: Q-II: $(-4,3)$. | **Concept & Formula:** Opposite-letter rule + reflection.

Detailed Solution: In Q-II, $x=-4, y=3 \implies (-4,3)$. Reflection in x-axis $= (-4,-3)$. **Final Answer:** $(-4,-3)$

Fastest Alternative / Shortcut: $-x$ is 4, $+y$ is 3.

G. Competency MCQ Mastery Section

Q1. Point $(-3,-5)$ lies in Quadrant:

(A) I (B) II (C) III (D) IV

Answer: (C) Both coordinates negative. *Fast Method:* $(-,-) \to$ Q-III. *Distractor Note:* Confusing II and III.

Q2. Abscissa of all points on the y-axis is:

(A) 0 (B) 1 (C) -1 (D) Any number

Answer: (A) $x=0$ on the y-axis. *Fast Method:* y-axis $\implies x=0$. *Distractor Note:* Saying 'Any number'.

Q3. Distance of $(3,-4)$ from the x-axis is:

(A) 3 (B) -4 (C) 4 (D) 5

Answer: (C) $|y| = |-4| = 4$. *Fast Method:* Opposite-letter rule. *Distractor Note:* Picking -4 or 3.

Q4. Point $(0,-7)$ lies on:

(A) Quadrant III (B) Quadrant IV (C) x-axis (D) y-axis

Answer: (D) $x=0$ means on the y-axis. *Fast Method:* $x=0 \to y$-axis. *Distractor Note:* Picking Q-III or Q-IV.

Q5. If $P(-1,1), Q(3,-4), R(1,-1), S(-2,-3), T(-4,4)$, which lie in Q-IV?

(A) P, T (B) Q, R (C) S only (D) Q only

Answer: (B) Q-IV is $(+,-)$: $Q(3,-4)$ and $R(1,-1)$. *Fast Method:* Check $+x, -y$. *Distractor Note:* Missing R.

Q6. Mirror image of $(5,-2)$ in the y-axis is:

(A) $(-5,-2)$ (B) $(5,2)$ (C) $(-5,2)$ (D) $(-2,5)$

Answer: (A) Keep $y=-2$, flip $x \to -5$. *Fast Method:* Flip x. *Distractor Note:* Flipping y.

Q7. Ordinate of $(7,-3)$ is:

(A) 7 (B) -3 (C) 3 (D) 4

Answer: (B) y-coordinate with sign is -3. *Fast Method:* 2nd coordinate. *Distractor Note:* Dropping minus sign.

Q8. Points $(2,3)$ and $(3,2)$ represent:

(A) Same point (B) Different points (C) Origin (D) Points on x-axis

Answer: (B) Ordered pairs: $2 \neq 3$. *Fast Method:* $(a,b) \neq (b,a)$. *Distractor Note:* Thinking order doesn't matter.

Q9. If abscissa and ordinate are equal and negative, the point lies in:

(A) Q-I (B) Q-II (C) Q-III (D) Q-IV

Answer: (C) $(-a,-a)$ is in Q-III. *Fast Method:* Both negative. *Distractor Note:* Picking Q-I.

Q10. Distance of $(5,12)$ from the origin is:

(A) 17 (B) 7 (C) 13 (D) 169

Answer: (C) $\sqrt{25+144}=13$. *Fast Method:* 5-12-13 triple. *Distractor Note:* Adding $5+12$.

Q11. Perpendicular distance of $(-2,6)$ from the y-axis is:

(A) -2 (B) 2 (C) 6 (D) 4

Answer: (B) $|-2|=2$. *Fast Method:* $|x|$. *Distractor Note:* Giving -2.

Q12. If y-coordinate of a point is 0, it always lies on:

(A) Q-I (B) Q-IV (C) x-axis (D) y-axis

Answer: (C) $y=0$ defines the x-axis. *Fast Method:* $y=0 \to x$-axis. *Distractor Note:* Picking y-axis.

H. Special Exam & MCQ Strategies

- Always check for 0 in either coordinate before naming a quadrant—axis points belong to NO quadrant.
- Remember 'Opposite Letter' for axis distances: distance from X-axis is $|y|$, distance from Y-axis is $|x|$.
- When finding areas of plotted rectangles or triangles, align bases along grid lines and count units carefully.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Saying $(0,4)$ lies in Quadrant I. $\longrightarrow$ **Correction:** Any point with $x=0$ or $y=0$ lies ON an axis (y-axis here), not in any quadrant.

[COMMON MISTAKE] **Error:** Writing distance of $(-3,5)$ from y-axis as -3. $\longrightarrow$ **Correction:** Distances are always non-negative: $|-3|=3$ units.

[COMMON MISTAKE] **Error:** Confusing abscissa (x) and ordinate (y). $\longrightarrow$ **Correction:** Alphabetical order: Abscissa comes before Ordinate, just like x comes before y!

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $P(x,y)$: x = abscissa, y = ordinate; Dist from x-axis $= |y|$; from y-axis $= |x|$; $OP = \sqrt{x^2+y^2}$.

[QUICK REVISION] Key Theorems & Results: Q-I $(+,+)$, Q-II $(-,+)$, Q-III $(-,-)$, Q-IV $(+,-)$; x-axis $(x,0)$, y-axis $(0,y)$.

[QUICK REVISION] Shortcut Summary: Opposite-letter distance; Axis-stays-same mirror image; Alphabetical Abscissa=x, Ordinate=y.

[QUICK REVISION] High-Frequency Exam Patterns: Quadrant/axis naming, Abscissa-ordinate arithmetic, Mirror images, Rectangle/triangle area on grid.

Last-Minute Revision Checklist:

- ☐ 0 means axis, not quadrant
- ☐ Distances are positive
- ☐ x before y
- ☐ Counter-clockwise I-II-III-IV

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Quadrant of $(-5,2)$ is: (A) I (B) II (C) III (D) IV
2. Abscissa of $(4,-9)$ is: (A) -9 (B) 4 (C) 9 (D) -4
3. Point $(8,0)$ lies on: (A) Q-I (B) Q-IV (C) x-axis (D) y-axis
4. Origin coordinates are: (A) $(1,1)$ (B) $(0,1)$ (C) $(1,0)$ (D) $(0,0)$

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Distance of $(-7,3)$ from x-axis is: (A) 7 (B) 3 (C) -7 (D) $\sqrt{58}$
2. Mirror image of $(4,-6)$ in x-axis is: (A) $(-4,-6)$ (B) $(4,6)$ (C) $(-4,6)$ (D) $(6,-4)$
3. Area of rectangle with vertices $(0,0),(5,0),(5,3),(0,3)$: (A) 8 (B) 15 (C) 16 (D) 30
4. (Ordinate of $(2,9)$) + (Abscissa of $(-3,4)$) is: (A) 6 (B) 13 (C) -1 (D) 1

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If $P(a,b)$ lies in Q-IV, then $Q(-a,-b)$ lies in: (A) Q-I (B) Q-II (C) Q-III (D) Q-IV
2. Perimeter of triangle with vertices $(0,0),(6,0),(0,8)$: (A) 14 (B) 24 (C) 48 (D) 10
3. Point on y-axis equidistant from $(0,2)$ and $(0,-6)$: (A) $(0,-2)$ (B) $(0,2)$ (C) $(-2,0)$ (D) $(0,-4)$
4. If $(x-1,y+3)=(4,1)$, then (x,y) lies in: (A) Q-I (B) Q-II (C) Q-III (D) Q-IV

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $(-,+)$ is Q-II. **Q2 (B):** x-coordinate is 4. **Q3 (C):** $y=0 \implies x$-axis. **Q4 (D):** $(0,0)$.
Exam Std	**Q1 (B):** $\|y\|=3$. **Q2 (B):** Keep x, flip y. **Q3 (B):** $5\times 3=15$. **Q4 (A):** $9+(-3)=6$.
Challenge	**Q1 (B):** $a>0,b<0 \implies -a<0,-b>0$ (Q-II). **Q2 (B):** $6+8+10=24$. **Q3 (A):** Midpoint $(0,\frac{2-6}{2})=(0,-2)$. **Q4 (D):** $x=5,y=-2 \implies (+,-)$ Q-IV.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Axes, origin, abscissa/ordinate, quadrants. Day 2: Plotting points & geometric figures.
- **How to Introduce the Concept:** Use classroom seating rows and columns ('Row 3, Column 2') to introduce ordered pairs (x, y)!
- **Difficult Areas & Remediation:** Axis points vs quadrants and distance from x-axis ($|y|$) vs y-axis ($|x|$).
- **Blackboard Teaching Examples:** Draw a large 4-quadrant grid and play a quick 2-minute coordinate-guessing game.
- **Classroom Questions to Ask Students:** Why does $(0, 5)$ not belong to Quadrant I or Quadrant II?
- **Common Student Doubts & Answers:** Students swap (x, y) order; reinforce 'walk across the hall (x) before taking the elevator (y)'.
- **Homework & Revision Strategy:** Assign plotting a mystery polygon on graph paper and finding its area.

[TEACHER'S TIP] This short chapter is a guaranteed full-marks scorer in Class 9.

UNIT II: ALGEBRA TYBA COACHING CENTRE

Chapter 4: Linear Equations in Two Variables

Standard Form $ax + by + c = 0$, Infinitely Many Solutions & Linear Graphs (2026–27)

A. Chapter Introduction & Examination Relevance

While a linear equation in one variable has a unique solution, an equation like $x + y = 4$ has infinitely many pairs (x, y) that satisfy it. Chapter 4 connects algebraic equations in x and y to straight lines on the Cartesian plane.

- **What You Will Learn:** Standard form $ax + by + c = 0$ (a, b not both zero), writing one-variable equations in two-variable form, finding ordered-pair solutions, testing whether a point lies on a line, and word problems.
- **Why This Chapter Matters:** Chapter 4 is the direct prerequisite for Class 10 Pair of Linear Equations in Two Variables and Class 11 Straight Lines.
- **Real-Life Applications:** Cost-and-quantity modeling, taxi-fare formulas ($y = mx + c$), currency conversion lines, and physics motion graphs.
- **Cross-Chapter Connections:** Builds directly on Chapter 2 (linear polynomials) and Chapter 3 (Cartesian plane), leading into Class 10 Chapter 3.
- **Exam & Foundation Relevance:** Unit II carries 28/100 marks in RBSE (14M for Ch. 4). Expect 1M MCQs on a, b, c and infinitely many solutions, plus 2M/3M k-finding and word-problem equation framing. (Equations of lines parallel to x- and y-axes are excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Standard Form $ax + by + c = 0$

Any equation that can be put in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{R}$ and a, b are **not both zero** ($a^2 + b^2 \neq 0$), is a linear equation in two variables x and y. Even a one-variable equation like $2x = 5$ can be written in two variables as $2x + 0 \cdot y - 5 = 0$.

[CONCEPT MASTER – 1. Standard Form $ax + by + c = 0$] Always move ALL terms to the LHS before reading a, b, c! In $y = 3x - 2$, rewrite as $3x - y - 2 = 0$, giving $a = 3, b = -1, c = -2$.

2. Infinitely Many Solutions

A solution of $ax + by + c = 0$ is an ordered pair (x, y) that makes LHS = RHS. Because we can pick ANY real value for x and solve for y, a linear equation in two variables has **infinitely many solutions**.

[CONCEPT MASTER – 2. Infinitely Many Solutions] The easiest two solutions to compute are ALWAYS the axis intercepts: set $x = 0$ to find $(0, -c/b)$ and set $y = 0$ to find $(-c/a, 0)$!

3. Geometric Meaning & Line Through Origin

Every solution (x, y) plots as a point on a straight line, and every point on that line is a solution of $ax + by + c = 0$. If $c = 0$ (i.e., $ax + by = 0$ or $y = mx$), the line **passes through the Origin** $(0, 0)$.

[CONCEPT MASTER – 3. Geometric Meaning & Line Through Origin] Quick Test: A line $ax + by + c = 0$ passes through $(0, 0)$ if and only if the constant term $c = 0$.

4. Word Problems & Parameter k

Given that a specific point (x_0, y_0) is a solution of $ax + by + c = 0$, substitute $x = x_0, y = y_0$ directly to find an unknown parameter k. In word problems (notebook vs pen cost, auto-rickshaw fare, Yamini-Fatima contribution), define two clear variables x and y and translate the relation into $ax + by + c = 0$.

[CONCEPT MASTER – 4. Word Problems & Parameter k] Rationalised 2026–27 Scope: Equations of lines parallel to the x-axis and y-axis ($x = a, y = b$) are excluded from the 2026–27 NCERT syllabus.

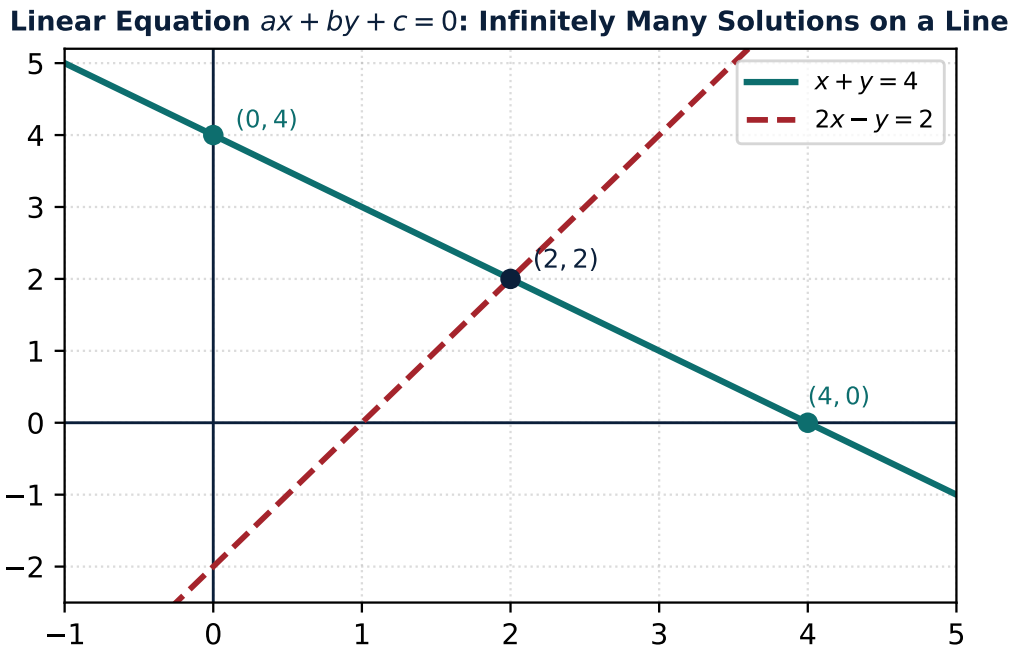


Figure 1: **Visual Model:** Straight-line graphs of $x + y = 4$ and $2x - y = 2$ demonstrating that every point on the line is a solution of the equation.

C. Complete Formula Bank

[FORMULA BANK – 1. Standard Form] **Expression:** $ax + by + c = 0$ $(a^2 + b^2 \neq 0)$
Symbols: $a, b, c \in \mathbb{R}$; x, y: variables.
Conditions & When to Use: a and b cannot both be 0.
Solved Mini-Example: $x = -5 \implies 1 \cdot x + 0 \cdot y + 5 = 0$ $(a = 1, b = 0, c = 5)$.
Common Mistake: Reading c from the RHS without changing its sign.

[FORMULA BANK – 2. x- and y-Intercept Solutions] **Expression:** At $x = 0$: $(0, -c/b)$; At $y = 0$: $(-c/a, 0)$
Symbols: $a \neq 0, b \neq 0$.
Conditions & When to Use: Fastest two points to plot a line.
Solved Mini-Example: For $2x + 3y = 6$: $(0, 2)$ and $(3, 0)$.
Common Mistake: Mixing up $(x, 0)$ and $(0, y)$.

[FORMULA BANK – 3. Line Through Origin] **Expression:** $ax + by = 0$ (i.e., $c = 0$)
Symbols: a, b not both zero.
Conditions & When to Use: Satisfied by $(0, 0)$.
Solved Mini-Example: $y = 3x$ passes through $(0, 0), (1, 3), (-1, -3)$.
Common Mistake: Thinking $y = 3x + 1$ passes through $(0, 0)$.

[FORMULA BANK – 4. Solution Verification] **Expression:** (x_0, y_0) is a solution $\iff ax_0 + by_0 + c = 0$
Symbols: $x_0, y_0 \in \mathbb{R}$.
Conditions & When to Use: Check exact equality.
Solved Mini-Example: $(2, 0)$ satisfies $x - 2y = 2$ since $2 - 0 = 2$.
Common Mistake: Plugging x_0 into y and y_0 into x.

[FORMULA BANK – 5. Fare / Linear Cost Model] **Expression:** $y = F + r(x-1)$ or $y = mx + c$
Symbols: F: fixed first-km charge, r: rate per extra km.
Conditions & When to Use: $x \geq 1$.
Solved Mini-Example: Rs. 8 for 1st km + Rs. 5/km after: $y = 8 + 5(x-1) = 5x + 3$.
Common Mistake: Writing $y = 8 + 5x$ (double-charging the 1st km!).

[FORMULA BANK – 6. One-Variable to Two-Variable Form] **Expression:** $px = q \implies px + 0 \cdot y - q = 0$; $py = q \implies 0 \cdot x + py - q = 0$
Symbols: $p \neq 0$.
Conditions & When to Use: Missing variable has coefficient 0.
Solved Mini-Example: $y = 2 \implies 0x + 1y - 2 = 0$.
Common Mistake: Writing coefficient 1 instead of 0 for the missing variable.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Olympiad/NTSE Foundation problem structures.

Pattern 1: Converting to $ax + by + c = 0$ & Finding a, b, c

- **Identification Cue:** Express $y - 2 = 0$ or $2x = -5y$ in $ax + by + c = 0$ and state a, b, c. | **Concept & Formula:** Standard form alignment.
- **Standard Step-by-Step:** Move all terms to LHS, include $0 \cdot x$ or $0 \cdot y$ if a variable is missing, then read a, b, c.
- **Fastest Valid Method:** Check signs carefully when transposing across $=$. | **Alternative:** Keep RHS 0.
- **Common Trap:** Omitting the 0 coefficient for a missing variable.
- **Practice Drill:** Ex. 1 & Ex. 2.

Pattern 2: Finding 4 Solutions & Testing Candidate Pairs

- **Identification Cue:** Find 4 solutions of $\pi x + y = 9$, or check if $(4, 0)$ is a solution of $x - 2y = 4$. | **Concept & Formula:** Plug simple x values $(0, 1, -1, 2)$.
- **Standard Step-by-Step:** Isolate y (or the variable with coeff 1), choose convenient values for the other variable.
- **Fastest Valid Method:** If coeff of x is 3 and y is 1, pick $x = 0, 1, 2, -1$; for $\pi x + y = 9$, pick $x = 0, 1, 2, -1$ and leave π exact! | **Alternative:** Use $(0, -c/b)$ and $(-c/a, 0)$ first.
- **Common Trap:** Replacing π with 3.14 unnecessarily.
- **Practice Drill:** Ex. 3 & Ex. 5.

Pattern 3: Finding Unknown k Given a Solution

- **Identification Cue:** Find k if $x = 2, y = 1$ is a solution of $2x + 3y = k$, or $(2k - 1, k)$ lies on $10x - 9y = 12$. | **Concept & Formula:** Substitute (x_0, y_0) into equation.
- **Standard Step-by-Step:** Replace x and y with the given values/expressions and solve the resulting linear equation in k.
- **Fastest Valid Method:** Direct substitution. | **Alternative:** None needed.
- **Common Trap:** Swapping x and y values.
- **Practice Drill:** Ex. 6 & Ex. 10.

Pattern 4: Framing Linear Equations from Word Problems

- **Identification Cue:** Cost of a notebook is twice a pen; or taxi fare Rs. 8 for 1st km and Rs. 5 per subsequent km. | **Concept & Formula:** Assign x, y clearly and translate.
- **Standard Step-by-Step:** State 'Let $x = \ldots$ and $y = \ldots$', write equation, convert to $ax + by + c = 0$.
- **Fastest Valid Method:** Check with a concrete number (e.g., $x = 2$ km $\implies y = 8 + 5 = 13$). | **Alternative:** Table of values.
- **Common Trap:** Writing $y = 8 + 5x$ instead of $8 + 5(x - 1)$.
- **Practice Drill:** Ex. 4 & Ex. 9.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Zero-Test for Origin & Intercepts] **Core Idea & Why It Works:** Line $ax+by+c=0$ passes through $(0,0)$ iff $c=0$; cuts x-axis at $(-c/a,0)$ and y-axis at $(0,-c/b)$.
Valid Domain: All linear equations. | **Where It Fails:** If $a=0$ or $b=0$ (no corresponding intercept).
Easy Example: $3x+4y=12$ cuts axes at $(4,0)$ and $(0,3)$.
Exam-Level Example: Finds triangle area with axes $=\frac{1}{2}|\frac{c}{a}\cdot\frac{c}{b}|$ in 5 seconds.
Advanced Example: Check MCQ options in 2 seconds.

[SMART SHORTCUT – 2. Multiple-of-Coefficient Picking] **Core Idea & Why It Works:** To get integer solutions for $ax+by=c$, isolate the smaller-coefficient variable and choose multiples of its denominator.
Valid Domain: All integer-coefficient linear equations. | **Where It Fails:** Irrational coefficients like π.
Easy Example: $2x+3y=12 \implies y=\frac{12-2x}{3}$; pick $x=0,3,6,-3$ to get integers $y=4,2,0,6$!
Exam-Level Example: Eliminates messy fractions in tables.
Advanced Example: Clean integer points make graphing exact.

[SMART SHORTCUT – 3. Taxi-Fare $(x-1)$ Template] **Core Idea & Why It Works:** If first unit costs F and each additional unit costs r, total cost for $x\geq 1$ units is $y=F+r(x-1)=rx+(F-r)$.
Valid Domain: All tiered-rate word problems. | **Where It Fails:** If $x=0$.
Easy Example: 1st km Rs. 10, subsequent Rs. 6/km: $y=6x+4$.
Exam-Level Example: Instant check: at $x=1$, $y=6+4=10$!
Advanced Example: Works for parking, electricity, and call-rate problems.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Write $2x+3y=9.3\overline{5}$ in the form $ax+by+c=0$ and indicate a,b,c.
Given & Required: $2x+3y=9.3\overline{5}$. | **Concept & Formula:** Standard form.
Detailed Solution: $2x+3y-9.3\overline{5}=0$, so $a=2,b=3,c=-9.3\overline{5}$. **Final Answer:** $a=2,b=3,c=-9.3\overline{5}$
Fastest Alternative / Shortcut: Transpose RHS to LHS.

SOLVED EXAMPLE 2 [Beginner]
Question: Write $y-2=0$ in the form $ax+by+c=0$ and state a,b,c.
Given & Required: $y-2=0$. | **Concept & Formula:** Missing x has coeff 0.
Detailed Solution: $0\cdot x+1\cdot y-2=0 \implies a=0,b=1,c=-2$. **Final Answer:** $a=0,b=1,c=-2$
Fastest Alternative / Shortcut: Insert $0\cdot x$.

SOLVED EXAMPLE 3 [Beginner]
Question: Check whether $(4,0)$ and $(\sqrt{2},4\sqrt{2})$ are solutions of $x-2y=4$.
Given & Required: $x-2y=4$. | **Concept & Formula:** Direct substitution.
Detailed Solution: For $(4,0)$: $4-2(0)=4$ (Yes). For $(\sqrt{2},4\sqrt{2})$: $\sqrt{2}-8\sqrt{2}=-7\sqrt{2}\neq 4$ (No). **Final Answer:** $(4,0)$ **Yes;** $(\sqrt{2},4\sqrt{2})$ **No**
Fastest Alternative / Shortcut: Plug x,y into LHS.

SOLVED EXAMPLE 4 [Beginner]

Question: The cost of a notebook is twice the cost of a pen. Write a linear equation in two variables to represent this.

Given & Required: Notebook $= x$, Pen $= y$. | **Concept & Formula:** Algebraic translation.

Detailed Solution: $x = 2y \implies x - 2y = 0$. **Final Answer:** $x - 2y = 0$

Fastest Alternative / Shortcut: Notebook is $2\times$ pen.

SOLVED EXAMPLE 5 [Intermediate]

Question: Find four different solutions of the equation $x + 2y = 6$.

Given & Required: $x = 6 - 2y$. | **Concept & Formula:** Choose 4 values of y.

Detailed Solution: $y = 0 \implies (6, 0)$; $y = 1 \implies (4, 1)$; $y = 2 \implies (2, 2)$; $y = 3 \implies (0, 3)$. **Final Answer:** $(6, 0), (4, 1), (2, 2), (0, 3)$

Fastest Alternative / Shortcut: Isolate x since its coeff is 1.

SOLVED EXAMPLE 6 [Intermediate]

Question: Find the value of k, if $x = 2, y = 1$ is a solution of the equation $2x + 3y = k$.

Given & Required: $x = 2, y = 1$. | **Concept & Formula:** Substitution.

Detailed Solution: $2(2) + 3(1) = k \implies k = 7$. **Final Answer:** $k = 7$

Fastest Alternative / Shortcut: $4 + 3 = 7$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find four solutions of $\pi x + y = 9$.

Given & Required: $y = 9 - \pi x$. | **Concept & Formula:** Leave π exact.

Detailed Solution: $x = 0 \implies (0, 9)$; $x = 1 \implies (1, 9 - \pi)$; $x = 2 \implies (2, 9 - 2\pi)$; $x = -1 \implies (-1, 9 + \pi)$. **Final Answer:** $(0, 9), (1, 9 - \pi), (2, 9 - 2\pi), (-1, 9 + \pi)$

Fastest Alternative / Shortcut: Isolate y.

SOLVED EXAMPLE 8 [Intermediate]

Question: Find the coordinates where $3x + 4y = 12$ meets the x-axis and y-axis.

Given & Required: $3x + 4y = 12$. | **Concept & Formula:** Axis intercepts.

Detailed Solution: On x-axis ($y = 0$): $3x = 12 \implies (4, 0)$. On y-axis ($x = 0$): $4y = 12 \implies (0, 3)$. **Final Answer:** $(4, 0)$ **and** $(0, 3)$

Fastest Alternative / Shortcut: Set $y = 0$, then $x = 0$.

SOLVED EXAMPLE 9 [Board Standard]

Question: The taxi fare in a city is Rs. 8 for the first kilometre and Rs. 5 per km for subsequent distance. Taking distance as x km and total fare as Rs. y, write a linear equation and find the fare for 10 km.

Given & Required: $F = 8, r = 5$. | **Concept & Formula:** Taxi-fare model.

Detailed Solution: $y = 8 + 5(x - 1) = 5x + 3 \implies 5x - y + 3 = 0$. At $x = 10$: $y = 5(10) + 3 =$ Rs. 53. **Final Answer:** $5x - y + 3 = 0$**; Rs.** 53

Fastest Alternative / Shortcut: Check $x = 1$: $5(1) + 3 = 8$.

SOLVED EXAMPLE 10 [Board Standard]

Question: If the point $(3, 4)$ lies on the graph of the equation $3y = ax + 7$, find the value of a.

Given & Required: $x = 3, y = 4$. | **Concept & Formula:** Point on graph = solution.

Detailed Solution: $3(4) = a(3) + 7 \implies 12 = 3a + 7 \implies 3a = 5 \implies a = \frac{5}{3}$. **Final Answer:** $a = \frac{5}{3}$

Fastest Alternative / Shortcut: Plug $(3, 4)$ directly.

SOLVED EXAMPLE 11 [Board Standard]

Question: If $(2k-1, k)$ is a solution of $10x - 9y = 12$, find k.

Given & Required: $x = 2k-1, y = k$. | **Concept & Formula:** Linear equation in k.

Detailed Solution: $10(2k-1) - 9k = 12 \implies 20k - 10 - 9k = 12 \implies 11k = 22 \implies k = 2$. **Final Answer:** $k = 2$

Fastest Alternative / Shortcut: Check $(3, 2)$: $30 - 18 = 12$.

SOLVED EXAMPLE 12 [Board Standard]

Question: The force applied on a body is directly proportional to acceleration produced ($F = ma$). Taking mass $m = 5$ kg, write the equation and find force when acceleration is 6 m/s^2.

Given & Required: $y = 5x$. | **Concept & Formula:** Proportionality line through origin.

Detailed Solution: $y = 5x$ (or $5x - y = 0$). At $x = 6$, $y = 30$ N. **Final Answer:** $y = 5x$**; 30 N**

Fastest Alternative / Shortcut: $c = 0$ passes through $(0, 0)$.

SOLVED EXAMPLE 13 [Board Standard]

Question: Yamini and Fatima together contributed Rs. 100 towards the PM Relief Fund. Write a linear equation and find how much Fatima contributed if Yamini gave Rs. 45.

Given & Required: $x + y = 100$. | **Concept & Formula:** Sum model.

Detailed Solution: $x + y = 100$. If $x = 45$, $y = 100 - 45 =$ Rs. 55. **Final Answer:** $x + y = 100$**; Rs.** 55

Fastest Alternative / Shortcut: $100 - 45 = 55$.

SOLVED EXAMPLE 14 [Challenge]

Question: In countries like USA and Canada, temperature is measured in Fahrenheit (F), whereas in India it is in Celsius (C). Using $F = \frac{9}{5}C + 32$, find the temperature that is numerically the same in both Fahrenheit and Celsius.

Given & Required: $F = C$. | **Concept & Formula:** Set $F = C = x$.

Detailed Solution: $x = \frac{9}{5}x + 32 \implies -\frac{4}{5}x = 32 \implies x = -40$. **Final Answer:** $-40°$

Fastest Alternative / Shortcut: $-40°$C $= -40°$F.

SOLVED EXAMPLE 15 [Challenge]

Question: Find the area of the triangle formed by the line $2x + 3y = 12$ with the coordinate axes.

Given & Required: Intercepts $(6, 0)$ and $(0, 4)$. | **Concept & Formula:** $\frac{1}{2} \times$ base $\times$ height.

Detailed Solution: Area $= \frac{1}{2} \times 6 \times 4 = 12$ sq. units. **Final Answer:** 12 **sq. units**

Fastest Alternative / Shortcut: $\frac{c^2}{2|ab|} = \frac{144}{12} = 12$.

SOLVED EXAMPLE 16 [Challenge]

Question: For what value of c does the linear equation $2x + cy = 8$ have equal values of x and y for its solution $(2, 2)$?

Given & Required: $x = y = 2$. | **Concept & Formula:** Substitution.

Detailed Solution: $2(2) + c(2) = 8 \implies 4 + 2c = 8 \implies c = 2$. **Final Answer:** $c = 2$

Fastest Alternative / Shortcut: $2c = 4 \implies c = 2$.

G. Competency MCQ Mastery Section

Q1. The linear equation $2x - 5y = 7$ has:

(A) A unique solution (B) Two solutions (C) Infinitely many solutions (D) No solution

Answer: (C) Every linear equation in two variables has infinitely many solutions. *Fast Method:* Line has ∞ points. *Distractor Note:* Confusing with 1-variable equation.

Q2. Equation $x = 7$ in two variables can be written as:

(A) $1 \cdot x + 1 \cdot y = 7$ (B) $1 \cdot x + 0 \cdot y = 7$ (C) $0 \cdot x + 1 \cdot y = 7$ (D) $0 \cdot x + 0 \cdot y = 7$

Answer: (B) y-coefficient must be 0. *Fast Method:* Missing $y \to 0 \cdot y$. *Distractor Note:* Writing $1 \cdot y$.

Q3. Which point lies on the line $x + y = 5$?

(A) $(2, 4)$ (B) $(3, 2)$ (C) $(1, 3)$ (D) $(4, 0)$

Answer: (B) $3 + 2 = 5$. *Fast Method:* Check sum $= 5$. *Distractor Note:* Arithmetic slip.

Q4. If $(2, 0)$ is a solution of $2x + 3y = k$, then k is:

(A) 4 (B) 6 (C) 5 (D) 2

Answer: (A) $2(2) + 3(0) = 4$. *Fast Method:* Plug $(2, 0)$. *Distractor Note:* Adding $2 + 3 = 5$.

Q5. Graph of $ax + by + c = 0$ passes through the origin if:

(A) $a = 0$ (B) $b = 0$ (C) $c = 0$ (D) $a = b$

Answer: (C) Plugging $(0, 0)$ gives $c = 0$. *Fast Method:* $c = 0$. *Distractor Note:* Setting $a = 0$.

Q6. How many lines pass through $(2, 14)$?

(A) 1 (B) 2 (C) 7 (D) Infinitely many

Answer: (D) Infinitely many lines pass through one point. *Fast Method:* E.g., $x + y = 16, 7x - y = 0, \ldots$ *Distractor Note:* Picking 1 (2 points determine 1 line).

Q7. Any solution of $ax + by + c = 0$ ($a \neq 0, b \neq 0$) on the x-axis has the form:

(A) $(0, y)$ (B) $(x, 0)$ (C) (a, b) (D) $(0, 0)$

Answer: (B) Every x-axis point has $y = 0$. *Fast Method:* $y = 0$. *Distractor Note:* Picking $(0, y)$.

Q8. If $y = 3x + 5$, then:

(A) Unique solution (B) Only two solutions (C) Infinitely many solutions (D) No solution

Answer: (C) Linear equation in 2 variables. *Fast Method:* Standard property. *Distractor Note:* Thinking $y = mx + c$ is different.

Q9. Point where $2x + 5y = 10$ meets the y-axis is:

(A) $(5, 0)$ (B) $(0, 2)$ (C) $(2, 0)$ (D) $(0, 5)$

Answer: (B) Set $x = 0 \implies 5y = 10 \implies y = 2$. *Fast Method:* $(0, 10/5)$. *Distractor Note:* Swapping x and y intercepts.

Q10. In $x - \frac{y}{5} - 10 = 0$, the values of a, b, c are:

(A) $1, -5, -10$ (B) $1, -1/5, -10$ (C) $1, 1/5, 10$ (D) $5, -1, -10$

Answer: (B) Coeff of y is $-1/5$. *Fast Method:* Read $-\frac{1}{5}y$. *Distractor Note:* Writing $b = -5$.

Q11. If $(1, -2)$ is a solution of $3x - ky = 9$, then k is:

(A) 3 (B) -3 (C) 6 (D) $3/2$

Answer: (A) $3(1) - k(-2) = 9 \implies 2k = 6 \implies k = 3$. *Fast Method:* Watch $-k(-2) = +2k$. *Distractor Note:* Sign error $-2k = 6$.

Q12. If $F = \frac{9}{5}C + 32$ and $C = 30°$, then F is:

(A) $86°$ (B) $62°$ (C) $54°$ (D) $90°$

Answer: (A) $\frac{9}{5}(30) + 32 = 54 + 32 = 86°$. *Fast Method:* $54 + 32 = 86$. *Distractor Note:* Forgetting $+32$.

H. Special Exam & MCQ Strategies

- Always rewrite any equation in $ax + by + c = 0$ form on the LHS BEFORE reading a, b, c.
- When finding 4 solutions, isolate the variable whose coefficient is 1 or -1 (or smallest) to avoid fractions.
- Remember: 1 equation in 2 variables has infinitely many solutions; you need 2 equations (Class 10) for a unique intersection.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Writing $a = 2, b = 3, c = 5$ for $2x + 3y = 5$. $\longrightarrow$ **Correction:** Move 5 to LHS: $2x + 3y - 5 = 0 \implies c = -5$.

[COMMON MISTAKE] **Error:** Translating taxi fare (Rs. 8 first km, Rs. 5 per extra km) as $y = 8 + 5x$. $\longrightarrow$ **Correction:** First km is already paid in Rs. 8, so $y = 8 + 5(x - 1) = 5x + 3$.

[COMMON MISTAKE] **Error:** Writing $2x = 5$ in two variables as $2x + y - 5 = 0$. $\longrightarrow$ **Correction:** Missing variable y has coefficient 0: $2x + 0 \cdot y - 5 = 0$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $ax + by + c = 0$ ($a^2 + b^2 \neq 0$); x-int $(-c/a, 0)$; y-int $(0, -c/b)$; Origin iff $c = 0$.

[QUICK REVISION] Key Theorems & Results: Linear eq in 2 variables has ∞ solutions; Fare: $y = F + r(x - 1)$.

[QUICK REVISION] Shortcut Summary: Set $x = 0$ and $y = 0$ for quickest 2 points; Pick multiples of denominator for integer points.

[QUICK REVISION] High-Frequency Exam Patterns: Reading a, b, c, Finding 4 solutions, Finding k from (x_0, y_0), Fare/temperature word problems.

Last-Minute Revision Checklist:

- ☐ Move c to LHS for sign
- ☐ 0 coeff for missing variable
- ☐ $(x - 1)$ in first-km fare
- ☐ Check x, y order in (x_0, y_0)

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Number of solutions of $x - 3y = 5$ is: (A) 1 (B) 2 (C) 0 (D) Infinitely many
2. For $3x + 2y = 6$, (a, b, c) in $ax + by + c = 0$ is: (A) $(3, 2, 6)$ (B) $(3, 2, -6)$ (C) $(2, 3, -6)$ (D) $(3, -2, 6)$
3. Which is a solution of $x + y = 7$? (A) $(3, 4)$ (B) $(2, 6)$ (C) $(1, 5)$ (D) $(4, 4)$
4. Does $y = 4x$ pass through $(0, 0)$? (A) Yes (B) No (C) Only if $x = 1$ (D) Cannot say

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. If $(3, 2)$ lies on $2x + ky = 10$, then k is: (A) 1 (B) 2 (C) 3 (D) 4
2. y-intercept of $4x - 3y + 12 = 0$ is: (A) $(0, 4)$ (B) $(0, -4)$ (C) $(-3, 0)$ (D) $(3, 0)$
3. Equation $3y = 7$ in two variables is: (A) $3x + 0y - 7 = 0$ (B) $0x + 3y - 7 = 0$ (C) $x + 3y - 7 = 0$ (D) $0x + 3y + 7 = 0$
4. Auto fare: Rs. 15 for 1st km, Rs. 8/km after. Fare for 5 km: (A) Rs. 47 (B) Rs. 55 (C) Rs. 40 (D) Rs. 50

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Area of triangle formed by $x + y = 6$ with coordinate axes: (A) 12 (B) 18 (C) 36 (D) 9
2. If $(k + 1, 2k)$ satisfies $3x - 2y = 5$, then k is: (A) -2 (B) 2 (C) 1 (D) -1
3. Point on $x + 2y = 9$ whose ordinate is twice its abscissa: (A) $(1.8, 3.6)$ (B) $(3, 3)$ (C) $(2, 4)$ (D) $(1, 4)$
4. At what temperature is Fahrenheit double of Celsius ($F = 2C$)? (A) $160°$C (B) $100°$C (C) $80°$C (D) $-40°$C

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (D):** Linear eq in 2 variables. **Q2 (B):** $3x + 2y - 6 = 0$. **Q3 (A):** $3 + 4 = 7$. **Q4 (A):** $c = 0 \implies (0, 0)$ satisfies it.
Exam Std	**Q1 (B):** $6 + 2k = 10 \implies k = 2$. **Q2 (A):** $x = 0 \implies -3y + 12 = 0 \implies y = 4$. **Q3 (B):** $0 \cdot x + 3y - 7 = 0$. **Q4 (A):** $15 + 8(4) = 47$.
Challenge	**Q1 (B):** $\frac{1}{2}(6)(6) = 18$. **Q2 (A):** $3k + 3 - 4k = 5 \implies -k = 2 \implies k = -2$. **Q3 (A):** $y = 2x \implies x + 4x = 9 \implies x = 1.8, y = 3.6$. **Q4 (A):** $2C = \frac{9}{5}C + 32 \implies \frac{1}{5}C = 32 \implies C = 160°$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Standard form $ax + by + c = 0$ & ∞ solutions. Day 2: Graphing & word problems.
- **How to Introduce the Concept:** Ask: 'If two pens cost Rs. 10 total ($x + y = 10$), how many price combinations are possible?'
- **Difficult Areas & Remediation:** Reading c with the right sign and $(x - 1)$ in first-km fare problems.
- **Blackboard Teaching Examples:** Show how $2x + 3y = 6$ gives a table of points that all snap onto one straight line.
- **Classroom Questions to Ask Students:** Why does $ax + by + c = 0$ say 'a and b are not both zero' instead of '$a \neq 0, b \neq 0$'?
- **Common Student Doubts & Answers:** Students ask about $x = a$ and $y = b$ graphs; mention they are excluded from 2026–27 exams.
- **Homework & Revision Strategy:** Assign 5 standard-form conversions, 3 k-value problems, and 3 word problems.

[TEACHER'S TIP] Emphasize that every point on a line is a solution—this idea returns in Class 10, 11, and 12!

UNIT IV: GEOMETRY TYBA COACHING CENTRE

Chapter 5: Introduction to Euclid's Geometry

Euclid's Definitions, 7 Common Axioms & 5 Fundamental Postulates (2026–27)

A. Chapter Introduction & Examination Relevance

Why can we trust a geometric theorem? Around 300 BCE, the Greek mathematician Euclid organized all of geometry in his famous book *Elements* from a handful of self-evident truths: Axioms and Postulates.

- **What You Will Learn:** Euclid's 7 basic definitions (point, line, surface, plane), 7 general Axioms (common notions), 5 geometric Postulates, and deductive proofs using them.
- **Why This Chapter Matters:** Chapter 5 teaches how mathematical proof works—separating assumed axioms from proven theorems—setting the logical foundation for Chapters 6, 7, 8, and 9.
- **Real-Life Applications:** Deductive logic, computer-aided geometric theorem provers, architectural drafting, and formal mathematical reasoning.
- **Cross-Chapter Connections:** Directly leads into Chapter 6 (Lines & Angles), Chapter 7 (Triangles), and Chapter 9 (Circles).
- **Exam & Foundation Relevance:** Unit IV carries 28/100 marks in RBSE (25/80 in CBSE). Chapter 5 contributes 2–4 marks via 1M MCQs distinguishing Axioms vs Postulates and 2M/3M deductive proofs (e.g., $AC = BD \implies AB = CD$). (Equivalent versions of the 5th postulate are excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Euclid's Definitions & Undefined Terms

Euclid gave 23 definitions in *Elements*, including: (1) A **point** is that which has no part; (2) A **line** is breadthless length; (3) The ends of a line are points; (4) A **surface** has length and breadth only; (5) The edges of a surface are lines. Modern geometry treats *point, line, plane* as **undefined terms** because defining every word leads to circular definitions.

[CONCEPT MASTER – 1. Euclid's Definitions & Undefined Terms] Dimensions Ladder: Solid (3D) $\rightarrow$ Surface (2D) $\rightarrow$ Line (1D) $\rightarrow$ Point (0D).

2. Euclid's Seven Axioms (Common Notions)

Axioms apply throughout ALL of mathematics (not just geometry): (1) Things equal to the same thing are equal to one another ($a = c, b = c \implies a = b$). (2) Equals added to equals give equals. (3) Equals subtracted from equals give equals. (4) Things which coincide with one another are equal. (5) **The whole is greater than the part.** (6) Things which are double of the same thing are equal. (7) Things which are halves of the same thing are equal.

[CONCEPT MASTER – 2. Euclid's Seven Axioms (Common Notions)] Axiom vs Postulate: **Axioms** are universal truths across all branches of math; **Postulates** are assumptions specific to *Geometry*.

3. Euclid's Five Postulates (Specific to Geometry)

(1) A straight line may be drawn from any one point to any other point (uniquely!). (2) A terminated line can be produced indefinitely. (3) A circle can be drawn with any centre and any radius. (4) All right angles are equal to one another ($90°$). (5) If a straight line falling on two straight lines makes interior angles on the same side $< 180°$, the two lines, if produced indefinitely, meet on that side.

[CONCEPT MASTER – 3. Euclid's Five Postulates (Specific to Geometry)] Postulate 5 is the famous **Parallel Criterion**: if $\angle 1 + \angle 2 = 180°$, the lines NEVER meet (they are parallel)!

4. Deductive Theorems & Midpoint Uniqueness

Using Postulate 1, two distinct lines cannot have more than one point in common. Using Axiom 3 (subtracting equals from equals), if $AC = BD$ on a line A–B–C–D, subtracting BC from both sides proves $AB = CD$. Using Axiom 5, every line segment has one and only one midpoint.

[CONCEPT MASTER – 4. Deductive Theorems & Midpoint Uniqueness] Rationalised 2026–27 Scope: Equivalent versions of Euclid's 5th Postulate (Playfair's Axiom) are excluded from the 2026–27 NCERT syllabus.

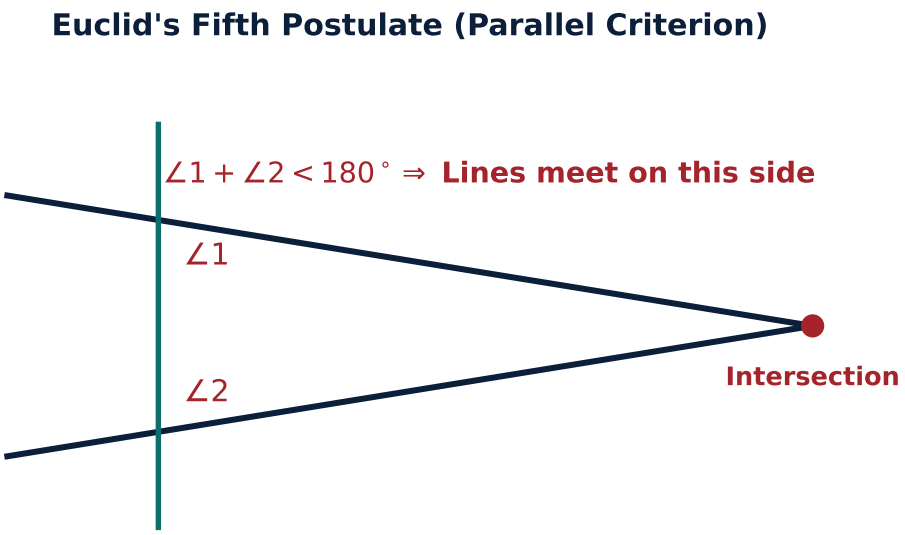


Figure 1: **Visual Model:** Euclid's Fifth Postulate: if interior angles $\angle 1 + \angle 2 < 180°$, the two lines intersect on that side.

C. Complete Formula Bank

[FORMULA BANK – 1. Axiom 1 (Transitivity)] **Expression:** $a = c$ and $b = c \implies a = b$
Symbols: a, b, c: magnitudes.
Conditions & When to Use: Universal across algebra & geometry.
Solved Mini-Example: If $AB = PQ$ and $XY = PQ$, then $AB = XY$.
Common Mistake: Calling this a postulate instead of an axiom.

[FORMULA BANK – 2. Axiom 2 & 3 (Add/Subtract Equals)] **Expression:** $a = b \implies a + c = b + c$ and $a - c = b - c$
Symbols: a, b, c: magnitudes.
Conditions & When to Use: Valid for segments, angles, areas.
Solved Mini-Example: If $AC = BD$, then $AC - BC = BD - BC \implies AB = CD$.
Common Mistake: Confusing Axiom 2 (addition) with Axiom 3 (subtraction).

[FORMULA BANK – 3. Axiom 4 (Superposition / Coincidence)] **Expression:** Things which coincide with one another are equal
Symbols: Geometric figures.
Conditions & When to Use: Basis of triangle congruence.
Solved Mini-Example: If C lies between A and B, $AC + CB$ coincides with AB, so $AC + CB = AB$.
Common Mistake: Thinking coincidence only checks area.

[FORMULA BANK – 4. Axiom 5 (Whole > Part)] **Expression:** $a = b + c\ (c > 0) \implies a > b$
Symbols: a: whole, b: part.
Conditions & When to Use: Positive magnitudes.
Solved Mini-Example: Proves uniqueness of midpoint of a segment.
Common Mistake: Forgetting $c > 0$ requirement.

[FORMULA BANK – 5. Postulate 1 & 2 (Lines)] **Expression:** 2 distinct points $\implies$ 1 unique line; segment extends indefinitely
Symbols: Points P, Q.
Conditions & When to Use: Euclidean plane.
Solved Mini-Example: Two distinct lines intersect in at most 1 point.
Common Mistake: Saying infinite lines pass through 2 distinct points.

[FORMULA BANK – 6. Postulate 5 (Intersection Criterion)] **Expression:** $\angle 1 + \angle 2 < 180^\circ \implies$ lines meet on that side
Symbols: $\angle 1, \angle 2$: same-side interior angles.
Conditions & When to Use: If sum $= 180^\circ$, lines are parallel.
Solved Mini-Example: If $\angle 1 = 80^\circ, \angle 2 = 95^\circ$, sum $= 175^\circ < 180^\circ \implies$ lines intersect.
Common Mistake: Thinking lines meet on the $> 180^\circ$ side.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Olympiad/NTSE Foundation problem structures.

Pattern 1: True/False Conceptual Statements

- **Identification Cue:** Only one line can pass through a single point; terminated line can be produced indefinitely. | **Concept & Formula:** Postulates 1–5 & Axioms 1–7.
- **Standard Step-by-Step:** Check each statement against Euclid's exact axioms/postulates and give a 1-line reason.
- **Fastest Valid Method:** 1 point $\to \infty$ lines; 2 distinct points $\to$ 1 unique line. | **Alternative:** Counterexample sketch.
- **Common Trap:** Confusing 'one point' with 'two distinct points'.
- **Practice Drill:** Ex. 1 & Ex. 2.

Pattern 2: Segment Proofs ($AC = BD \implies AB = CD$)

- **Identification Cue:** If $AC = BD$, prove $AB = CD$; or if C is midpoint of AB, prove $AC = \frac{1}{2}AB$. | **Concept & Formula:** Axioms 2, 3, 4, 7.
- **Standard Step-by-Step:** State the geometric relation ($AC = AB + BC$), apply the named Euclid Axiom explicitly.
- **Fastest Valid Method:** Always cite the exact Axiom number or statement in brackets! | **Alternative:** Algebraic substitution.
- **Common Trap:** Writing steps without citing Euclid's Axiom (loses 1 mark in school exams).
- **Practice Drill:** Ex. 5 & Ex. 6.

Pattern 3: Uniqueness of Midpoint Proof

- **Identification Cue:** Prove that every line segment has one and only one midpoint. | **Concept & Formula:** Proof by contradiction + Axiom 5.
- **Standard Step-by-Step:** Assume two midpoints C and D, get $AC = \frac{1}{2}AB = AD$, contradicting Axiom 5 unless $C = D$.
- **Fastest Valid Method:** Contradiction via Axiom 5. | **Alternative:** Direct coordinate check.
- **Common Trap:** Forgetting to state contradiction with Axiom 5.
- **Practice Drill:** Ex. 8.

Pattern 4: Axiom vs Postulate Classification

- **Identification Cue:** Identify which is an axiom vs postulate, or verify Postulate 5. | **Concept & Formula:** Universal math vs geometry-specific.

- **Standard Step-by-Step:** Check if statement involves circles/lines/right angles (Postulate) or general equality/parts (Axiom).
- **Fastest Valid Method:** Geometry terms (line, circle, 90°) $\rightarrow$ Postulate; general quantities $\rightarrow$ Axiom. | **Alternative:** Recall the 5 Postulates list.
- **Common Trap:** Calling 'All right angles are equal' an axiom (it is Postulate 4!).
- **Practice Drill:** Ex. 3 & Ex. 4.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Axiom vs Postulate 2-Second Filter] **Core Idea & Why It Works:** If the statement mentions specific geometric objects (points, lines, circles, right angles), it is a **Postulate**; if it talks about general 'things, equals, wholes, parts', it is an **Axiom**.

Valid Domain: All 7 Axioms and 5 Postulates. | **Where It Fails:** Nothing.

Easy Example: 'All right angles are equal' mentions right angles $\implies$ Postulate (No. 4).

Exam-Level Example: Instant 1-mark MCQ accuracy.

Advanced Example: Zero confusion.

[SMART SHORTCUT – 2. Dimension Countdown ($3 \rightarrow 2 \rightarrow 1 \rightarrow 0$)] **Core Idea & Why It Works:** Solid (3D) $\xrightarrow{\text{boundary}}$ Surface (2D) $\xrightarrow{\text{boundary}}$ Line (1D) $\xrightarrow{\text{ends}}$ Point (0D).

Valid Domain: All NCERT Ex. 5.1 conceptual MCQs. | **Where It Fails:** Nothing.

Easy Example: Boundaries of a surface are curves/lines (1D).

Exam-Level Example: Answers 3 textbook questions at once.

Advanced Example: Point has 0 dimensions.

[SMART SHORTCUT – 3. Postulate 5 Sum-Test] **Core Idea & Why It Works:** Add the two same-side interior angles: $< 180^\circ$ means lines MEET on that side; $= 180^\circ$ means PARALLEL.

Valid Domain: Any two coplanar lines cut by a transversal. | **Where It Fails:** 3D skew lines.

Easy Example: $70^\circ + 100^\circ = 170^\circ < 180^\circ \implies$ intersect on that side.

Exam-Level Example: Direct link to Chapter 6 co-interior angles!

Advanced Example: On the other side, sum is $360^\circ - 170^\circ = 190^\circ > 180^\circ$.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: State whether True or False: (i) Only one line can pass through a single point. (ii) Two distinct lines cannot have more than one point in common.

Given & Required: Two statements. | **Concept & Formula:** Postulate 1.

Detailed Solution: (i) False—infinitely many lines pass through one point. (ii) True—if two lines shared 2 points, they would be the same line by Postulate 1. **Final Answer: (i) False, (ii) True**

Fastest Alternative / Shortcut: 1 point $\rightarrow \infty$ lines; 2 points $\rightarrow$ 1 line.

SOLVED EXAMPLE 2 [Beginner]

Question: How many dimensions do a solid, a surface, a line, and a point have?

Given & Required: Geometric entities. | **Concept & Formula:** Dimension countdown.

Detailed Solution: Solid: 3, Surface: 2, Line: 1, Point: 0. **Final Answer:** $3, 2, 1, 0$

Fastest Alternative / Shortcut: $3 \rightarrow 2 \rightarrow 1 \rightarrow 0$.

SOLVED EXAMPLE 3 [Beginner]

Question: Which Euclid Axiom justifies: 'If $x = y$, then $x + 5 = y + 5$'?

Given & Required: Adding 5 to both sides. | **Concept & Formula:** Axiom 2.

Detailed Solution: Euclid's Axiom 2: If equals are added to equals, the wholes are equal. **Final Answer: Axiom 2**

Fastest Alternative / Shortcut: Equals added to equals.

SOLVED EXAMPLE 4 [Beginner]

Question: State Euclid's Fourth Postulate.

Given & Required: Right angles. | **Concept & Formula:** Postulate 4.

Detailed Solution: All right angles are equal to one another. **Final Answer: All right angles are equal**

Fastest Alternative / Shortcut: Postulate 4.

SOLVED EXAMPLE 5 [Intermediate]

Question: If a point C lies between two points A and B such that $AC = BC$, prove that $AC = \frac{1}{2}AB$.

Given & Required: $AC = BC$, C between A, B. | **Concept & Formula:** Axiom 2 & Axiom 7.

Detailed Solution: Add AC to both sides (Axiom 2): $AC + AC = BC + AC$. Since $BC + AC$ coincides with AB (Axiom 4), $2AC = AB \implies AC = \frac{1}{2}AB$ (Axiom 7). **Final Answer: Proved**

Fastest Alternative / Shortcut: Cite Axioms 2, 4, 7.

SOLVED EXAMPLE 6 [Intermediate]

Question: In a line segment A–B–C–D, if $AC = BD$, prove that $AB = CD$.

Given & Required: $AC = BD$. | **Concept & Formula:** Axiom 3.

Detailed Solution: $AC = AB + BC$ and $BD = BC + CD$. Thus $AB + BC = BC + CD$. Subtracting BC from both sides (Axiom 3: equals subtracted from equals leave equals) gives $AB = CD$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Subtract BC via Axiom 3.

SOLVED EXAMPLE 7 [Intermediate]

Question: If $\angle 1 = \angle 3$ and $\angle 2 = \angle 3$, what relation holds between $\angle 1$ and $\angle 2$? State the axiom.

Given & Required: $\angle 1 = \angle 3, \angle 2 = \angle 3$. | **Concept & Formula:** Axiom 1.

Detailed Solution: $\angle 1 = \angle 2$ by Euclid's Axiom 1 (things equal to the same thing are equal to one another). **Final Answer:** $\angle 1 = \angle 2$ **(Axiom 1)**

Fastest Alternative / Shortcut: Transitivity = Axiom 1.

SOLVED EXAMPLE 8 [Intermediate]

Question: Why is Axiom 5 ('The whole is greater than the part') considered a universal truth?

Given & Required: Axiom 5. | **Concept & Formula:** Universal scope.

Detailed Solution: It holds not only in geometry (segments, angles, areas) but in arithmetic, sets, and everyday quantities ($a + b > a$ for $b > 0$). **Final Answer: Holds in all branches of mathematics**

Fastest Alternative / Shortcut: Not restricted to geometry.

SOLVED EXAMPLE 9 [Board Standard]

Question: Prove that every line segment has one and only one midpoint.

Given & Required: Segment AB. | **Concept & Formula:** Contradiction + Axiom 5.

Detailed Solution: Suppose C and D are two distinct midpoints of AB with C between A and D. Then $AC = \frac{1}{2}AB$ and $AD = \frac{1}{2}AB$, so $AC = AD$ (Axiom 1). But AC is a part of AD, so $AD > AC$ (Axiom 5), a contradiction. Thus C and D coincide. **Final Answer: Proved**

Fastest Alternative / Shortcut: Axiom 1 vs Axiom 5 contradiction.

SOLVED EXAMPLE 10 [Board Standard]

Question: If C is the midpoint of AB and D is the midpoint of AC, prove that $AD = \frac{1}{4}AB$.

Given & Required: $AC = \frac{1}{2}AB$, $AD = \frac{1}{2}AC$. | **Concept & Formula:** Axiom 7 (halves of equals).

Detailed Solution: Since $AC = \frac{1}{2}AB$ and $AD = \frac{1}{2}AC$, substituting gives $AD = \frac{1}{2}(\frac{1}{2}AB) = \frac{1}{4}AB$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Half of a half is a quarter.

SOLVED EXAMPLE 11 [Board Standard]

Question: Prove that an equilateral triangle can be constructed on any given line segment AB.

Given & Required: Segment AB. | **Concept & Formula:** Postulate 3 & Axiom 1.

Detailed Solution: By Postulate 3, draw a circle with centre A and radius AB, and another with centre B and radius BA. Let them meet at C. Then $AC = AB$ and $BC = AB$ (radii). By Axiom 1, $AB = BC = AC$, so $\triangle ABC$ is equilateral. **Final Answer: Proved**

Fastest Alternative / Shortcut: Two circles of radius AB.

SOLVED EXAMPLE 12 [Board Standard]

Question: Two lines are cut by a transversal making same-side interior angles 78° and 97°. Will the lines intersect? On which side?

Given & Required: $78^\circ, 97^\circ$. | **Concept & Formula:** Postulate 5.

Detailed Solution: $78^\circ + 97^\circ = 175^\circ < 180^\circ$. By Euclid's 5th Postulate, the lines will intersect on the side of these two angles. **Final Answer: Yes, on the 78°–97° side**

Fastest Alternative / Shortcut: Sum $< 180^\circ$.

SOLVED EXAMPLE 13 [Board Standard]

Question: In $\triangle ABC$, $AB = AC$ and X, Y are points on AB, AC such that $AX = AY$. Prove $XB = YC$.

Given & Required: $AB = AC, AX = AY$. | **Concept & Formula:** Axiom 3.

Detailed Solution: $AB - AX = AC - AY$ (Axiom 3) $\implies XB = YC$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Subtract equals from equals.

SOLVED EXAMPLE 14 [Challenge]

Question: Ram and Shyam have the same weight. If they each gain weight by 2 kg, how do their new weights compare? State the axiom used.

Given & Required: $R = S$. | **Concept & Formula:** Axiom 2.

Detailed Solution: $R + 2 = S + 2$; their new weights remain equal by Euclid's Axiom 2. **Final Answer: Equal (Axiom 2)**

Fastest Alternative / Shortcut: Add equals to equals.

SOLVED EXAMPLE 15 [Challenge]

Question: Prove that two distinct lines cannot have more than one point in common.

Given & Required: Two distinct lines l, m. | **Concept & Formula:** Postulate 1.

Detailed Solution: Suppose l and m share two distinct points P and Q. By Postulate 1, there is a UNIQUE line passing through P and Q, so l and m must be the same line—a contradiction. **Final Answer: Proved**

Fastest Alternative / Shortcut: Uniqueness of line through 2 points.

SOLVED EXAMPLE 16 [Challenge]

Question: How many lines can pass through (i) 1 point, (ii) 2 distinct points, (iii) 3 non-collinear points (taken in pairs)?

Given & Required: Point counts. | **Concept & Formula:** Postulate 1 + ${}^3C_2$.

Detailed Solution: (i) Infinitely many; (ii) 1; (iii) 3 lines (AB, BC, CA). **Final Answer: (i) ∞, (ii) 1, (iii) 3**

Fastest Alternative / Shortcut: 3 non-collinear points form a triangle.

G. Competency MCQ Mastery Section

Q1. Euclid belonged to which country?

(A) Babylonia (B) Egypt (C) Greece (D) India

Answer: (C) Euclid was a Greek mathematician (Alexandria). *Fast Method:* Greek. *Distractor Note:* Picking Egypt.

Q2. Number of dimensions a solid has is:

(A) 1 (B) 2 (C) 3 (D) 0

Answer: (C) Length, breadth, height. *Fast Method:* 3D. *Distractor Note:* Confusing with surface (2D).

Q3. Boundaries of surfaces are:

(A) Surfaces (B) Curves/Lines (C) Points (D) Solids

Answer: (B) Edges of a surface are lines/curves. *Fast Method:* 2D $\rightarrow$ 1D. *Distractor Note:* Picking points.

Q4. Things which are double of the same thing are:

(A) Equal to one another (B) Halves (C) Unequal (D) Parallel

Answer: (A) Euclid's Axiom 6. *Fast Method:* Axiom 6. *Distractor Note:* Overthinking.

Q5. 'A terminated line can be produced indefinitely' is Euclid's:

(A) Axiom 2 (B) Postulate 1 (C) Postulate 2 (D) Postulate 5

Answer: (C) Postulate 2. *Fast Method:* Mentions line $\rightarrow$ Postulate. *Distractor Note:* Calling it Axiom 2.

Q6. Two distinct lines in a plane can intersect at most at:

(A) 0 points (B) 1 point (C) 2 points (D) Infinite points

Answer: (B) At most 1 common point. *Fast Method:* Postulate 1. *Distractor Note:* Picking 2.

Q7. In $l \parallel m$, if same-side interior angles are $\angle 1$ and $\angle 2$, then $\angle 1 + \angle 2$ equals:

(A) 90° (B) 180° (C) $< 180^\circ$ (D) 360°

Answer: (B) Parallel lines have co-interior sum $= 180^\circ$. *Fast Method:* Postulate 5 boundary. *Distractor Note:* Picking $< 180^\circ$.

Q8. Number of postulates in Euclid's *Elements* is:

(A) 3 (B) 5 (C) 7 (D) 23

Answer: (B) 5 Postulates (and 7 Axioms, 23 Definitions). *Fast Method:* 5 Postulates. *Distractor Note:* Confusing 5 with 7.

Q9. 'The whole is greater than the part' is:

(A) Postulate 5 (B) Axiom 5 (C) Definition (D) Theorem

Answer: (B) Euclid's Axiom 5. *Fast Method:* Universal truth $\rightarrow$ Axiom. *Distractor Note:* Calling it Postulate 5.

Q10. A pyramid is a solid figure whose base is:

(A) Only a triangle (B) Only a square (C) Only a rectangle (D) Any polygon

Answer: (D) Base can be any polygon; lateral faces are triangles. *Fast Method:* Any polygon. *Distractor Note:* Restricting to square/triangle.

Q11. Side faces of a pyramid are:

(A) Triangles (B) Squares (C) Polygons (D) Trapeziums

Answer: (A) All side faces meet at an apex and are triangles. *Fast Method:* Triangles. *Distractor Note:* Confusing with prism (rectangles).

Q12. Maximum number of lines through 1 given point is:

(A) 1 (B) 2 (C) 4 (D) Infinitely many

Answer: (D) Infinite lines pass through a single point. *Fast Method:* ∞. *Distractor Note:* Confusing with 2 points.

H. Special Exam & MCQ Strategies

- Always cite the exact Axiom or Postulate statement in brackets when writing a 2-mark or 3-mark proof.
- Keep the distinction clear: 7 Axioms = universal math; 5 Postulates = geometry-specific.
- Remember the dimension ladder: Solid (3) $\rightarrow$ Surface (2) $\rightarrow$ Line (1) $\rightarrow$ Point (0).

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Calling 'All right angles are equal' an Axiom. $\longrightarrow$ **Correction:** It is Euclid's 4th Postulate because it refers to geometric right angles.

[COMMON MISTAKE] **Error:** Saying only one line passes through one point. $\longrightarrow$ **Correction:** Infinitely many lines pass through 1 point; a UNIQUE line passes through 2 distinct points.

[COMMON MISTAKE] **Error:** Writing proofs like $AC = BD \implies AB = CD$ without mentioning Axiom 3. $\longrightarrow$ **Correction:** Explicitly state 'Subtracting BC from both sides (Euclid's Axiom 3)'.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: 7 Axioms (transitivity, $+$, $-$, coincidence, whole>part, doubles, halves); 5 Postulates.

[QUICK REVISION] Key Theorems & Results: Solid (3D) $\rightarrow$ Surface (2D) $\rightarrow$ Line (1D) $\rightarrow$ Point (0D).

[QUICK REVISION] Shortcut Summary: Geometry words $\rightarrow$ Postulate; General quantities $\rightarrow$ Axiom; $\angle 1 + \angle 2 < 180^\circ \implies$ intersect.

[QUICK REVISION] High-Frequency Exam Patterns: True/False conceptual checks, $AC = BD$ segment proofs, Midpoint uniqueness, Postulate 5.

Last-Minute Revision Checklist:

- ☐ Axiom vs Postulate difference
- ☐ Dimension counts 3-2-1-0
- ☐ Cite Axiom in proofs
- ☐ Postulate 5 $< 180^\circ$

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. A point has how many dimensions? (A) 0 (B) 1 (C) 2 (D) 3
2. Number of Euclid's Postulates is: (A) 3 (B) 5 (C) 7 (D) 10
3. How many lines pass through 2 distinct points? (A) 0 (B) 1 (C) 2 (D) Infinite
4. Euclid's Axiom 1 states things equal to the same thing are: (A) Unequal (B) Equal to one another (C) Parallel (D) Perpendicular

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Which postulate says all right angles are equal? (A) Postulate 1 (B) Postulate 2 (C) Postulate 4 (D) Postulate 5
2. If $AB = PQ$ and $PQ = XY$, then $AB = XY$ uses: (A) Axiom 1 (B) Axiom 2 (C) Axiom 3 (D) Axiom 5
3. Interior angles on same side are 85° and 90°. Lines will: (A) Never meet (B) Meet on that side (C) Be perpendicular (D) Coincide
4. If $AC = BD$ on line $ABCD$, proving $AB = CD$ uses: (A) Axiom 2 (B) Axiom 3 (C) Axiom 4 (D) Axiom 5

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. How many planes can pass through 3 non-collinear points? (A) 1 (B) 2 (C) 3 (D) Infinite
2. If $x + y = 10$ and $x = z$, then $z + y = 10$ uses: (A) Axiom 1 (B) Axiom 2 (C) Axiom 3 (D) Postulate 1
3. Total lines determined by 4 points, no 3 collinear: (A) 4 (B) 6 (C) 8 (D) 12
4. Number of midpoints a line segment can have: (A) 0 (B) 1 (C) 2 (D) Infinite

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (A):** 0 dimensions. **Q2 (B):** 5 postulates. **Q3 (B):** Postulate 1: unique line. **Q4 (B):** Transitivity.
Exam Std	**Q1 (C):** Postulate 4. **Q2 (A):** Axiom 1. **Q3 (B):** $85 + 90 = 175 < 180$ (Postulate 5). **Q4 (B):** Subtracting BC (Axiom 3).
Challenge	**Q1 (A):** 3 non-collinear points determine 1 unique plane. **Q2 (B):** Adding y to equals $x = z$ (Axiom 2). **Q3 (B):** ${}^4C_2 = 6$ lines. **Q4 (B):** Unique midpoint (Axiom 5 proof).

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Definitions, undefined terms, 7 Axioms. Day 2: 5 Postulates & deductive proofs.
- **How to Introduce the Concept:** Ask students to define 'point' without using 'dot/mark', and 'dot' without 'point'—showing why undefined terms are needed!
- **Difficult Areas & Remediation:** Distinguishing Axioms from Postulates and writing formal contradiction for unique midpoint.
- **Blackboard Teaching Examples:** Draw Postulate 5 with $\angle 1 + \angle 2 < 180^\circ$ vs $= 180^\circ$.
- **Classroom Questions to Ask Students:** Why couldn't Euclid prove his 5th Postulate from the first 4?
- **Common Student Doubts & Answers:** Students ask about Playfair's Axiom; note it is excluded in 2026–27.
- **Homework & Revision Strategy:** Assign 4 True/False justifications and 3 segment proofs citing exact Axioms.

[TEACHER'S TIP] Citing the exact Axiom/Postulate number makes answers stand out to examiners.

UNIT IV: GEOMETRY TYBA COACHING CENTRE

Chapter 6: Lines and Angles

Linear Pair, Vertically Opposite Angles, Parallel Lines & Transversal Theorems (2026–27)

A. Chapter Introduction & Examination Relevance

Chapter 6 turns Euclid's foundations into practical angle-chasing tools. Mastering Linear Pairs, Vertically Opposite Angles, and Parallel-Line Transversals unlocks almost every geometry proof in Classes 9 and 10.

- **What You Will Learn:** Types of angles (acute, obtuse, reflex, complementary, supplementary), Linear Pair Axiom, Vertically Opposite Angles Theorem, and Parallel Lines with Transversal (corresponding, alternate interior, co-interior angles).
- **Why This Chapter Matters:** Every theorem in Triangles (Ch. 7), Quadrilaterals (Ch. 8), Circles (Ch. 9), and Class 10 Similar Triangles relies on Chapter 6 angle-chasing.
- **Real-Life Applications:** Bridge truss geometry, periscope/mirror reflection angles, carpentry miter cuts, and solar panel tilt angles.
- **Cross-Chapter Connections:** Directly extends Chapter 5 (Postulate 5) and feeds into Chapters 7, 8, and 9.
- **Exam & Foundation Relevance:** Unit IV carries 28/100 marks in RBSE (25/80 in CBSE). Expect 1M complementary/supplementary MCQs and 3M/5M parallel-line angle-chasing proofs. (Angle sum property theorem proof is excluded in 2026–27, though the 180° triangle fact is used in calculations.)

B. Complete Concept Explanation & Derivations

1. Basic Angle Pairs & Linear Pair Axiom

Two angles are **complementary** if their sum is 90°, and **supplementary** if their sum is 180°. **Linear Pair Axiom:** If a ray stands on a line, the sum of the two adjacent angles so formed is 180° (and conversely, if adjacent angles sum to 180°, their non-common arms form a straight line).

[CONCEPT MASTER – 1. Basic Angle Pairs & Linear Pair Axiom] Reflex Angle Alert: If $\angle A = 40^\circ$, its reflex angle is $360^\circ - 40^\circ = 320^\circ$.

2. Vertically Opposite Angles Theorem

When two lines intersect at a point, the **vertically opposite angles are equal** (proved in 2 lines using the Linear Pair Axiom: $\angle 1 + \angle 2 = 180^\circ$ and $\angle 2 + \angle 3 = 180^\circ \implies \angle 1 = \angle 3$).

[CONCEPT MASTER – 2. Vertically Opposite Angles Theorem] The bisectors of a linear pair are ALWAYS perpendicular (90°) to each other!

3. Parallel Lines & Transversal Angle Rules

If a transversal intersects two **parallel lines** ($l \parallel m$): (1) **Corresponding angles** are equal (F-shape); (2) **Alternate interior angles** are equal (Z-shape); (3) **Co-interior (consecutive interior) angles** are **supplementary** (sum $= 180^\circ$, C-shape). The converses are also true and used to prove lines parallel.

[CONCEPT MASTER – 3. Parallel Lines & Transversal Angle Rules] Lines parallel to the same line are parallel to each other ($l \parallel n$ and $m \parallel n \implies l \parallel m$).

4. Construction Line Trick & Angle Chasing

When two parallel lines have a 'bent' V-shape or M-shape vertex between them (e.g., $\angle PQR$ between $PQ \parallel ST$), draw a construction line through the vertex parallel to both lines! This splits the bend into two alternate-interior or co-interior pairs instantly.

[CONCEPT MASTER – 4. Construction Line Trick & Angle Chasing] Rationalised 2026–27 Scope: The formal theorem proof of Angle Sum Property of a Triangle is excluded from the 2026–27 NCERT syllabus.

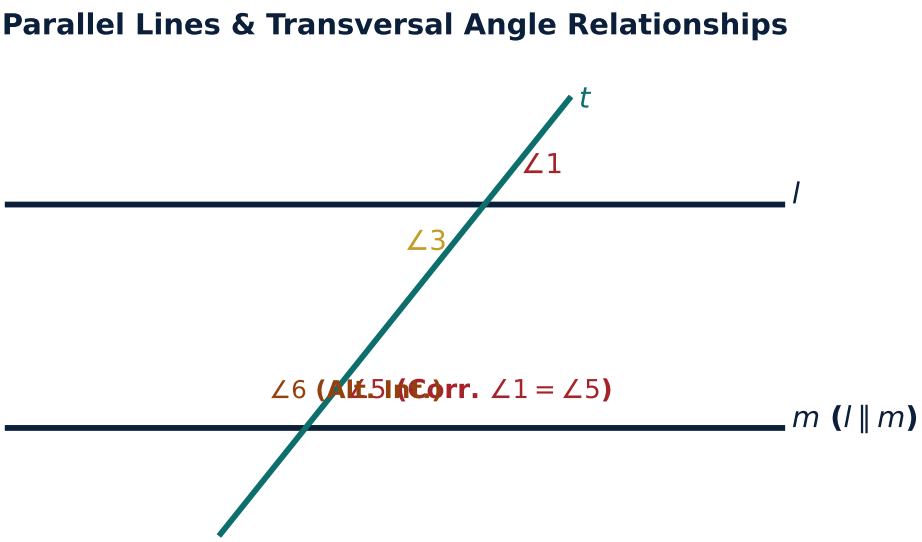


Figure 1: **Visual Model:** Parallel lines $l \parallel m$ cut by transversal t, showing equal corresponding angles ($\angle 1 = \angle 5$) and alternate interior angles ($\angle 3 = \angle 6$).

C. Complete Formula Bank

[FORMULA BANK – 1. Complementary & Supplementary] **Expression:** Comp: $x + y = 90°$; Supp: $x + y = 180°$
Symbols: $x, y > 0$.
Conditions & When to Use: Angle equal to its complement is $45°$; to its supplement is $90°$.
Solved Mini-Example: Supplement of $65°$ is $115°$.
Common Mistake: Confusing $90°$ and $180°$.

[FORMULA BANK – 2. Linear Pair & Straight Line] **Expression:** $\angle AOC + \angle BOC = 180°$
Symbols: Ray OC on line AB.
Conditions & When to Use: Adjacent angles on a straight line.
Solved Mini-Example: If ratio is $5:7$, angles are $75°$ and $105°$.
Common Mistake: Assuming any two supplementary angles form a linear pair (they must be adjacent!).

[FORMULA BANK – 3. Vertically Opposite Angles] **Expression:** $\angle AOC = \angle BOD$; $\angle AOD = \angle BOC$
Symbols: Intersecting lines AB, CD.
Conditions & When to Use: Non-adjacent opposite pairs.
Solved Mini-Example: If one angle at intersection is $50°$, the other three are $130°, 50°, 130°$.
Common Mistake: Equating adjacent angles instead of opposite ones.

[FORMULA BANK – 4. Parallel-Transversal Equalities] **Expression:** Corresponding: $\angle 1 = \angle 5$; Alt. Interior: $\angle 3 = \angle 6$
Symbols: $l \parallel m$.
Conditions & When to Use: Only when lines are parallel.
Solved Mini-Example: Z-shape gives $\angle 3 = \angle 6$.
Common Mistake: Calling co-interior angles equal (they sum to $180°$!).

[FORMULA BANK – 5. Co-Interior Angles] **Expression:** $\angle 3 + \angle 5 = 180^\circ$
Symbols: Interior angles on same side of transversal.
Conditions & When to Use: $l \parallel m$.
Solved Mini-Example: If $\angle 3 = 110^\circ$, $\angle 5 = 70^\circ$.
Common Mistake: Setting $\angle 3 = \angle 5$.

[FORMULA BANK – 6. Reflex Angle] **Expression:** Reflex$(\theta) = 360^\circ - \theta$
Symbols: $0^\circ < \theta < 180^\circ$.
Conditions & When to Use: Between 180° and 360°.
Solved Mini-Example: If $\angle QOP = 110^\circ$, reflex $\angle QOP = 250^\circ$.
Common Mistake: Writing $180^\circ - \theta$ for reflex.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Olympiad/NTSE Foundation problem structures.

Pattern 1: Ratio & Linear-Pair / Vertically Opposite Problems

- **Identification Cue:** Two lines intersect; angles are in ratio $5:7$, find all 4 angles; or find reflex $\angle QOP$. | **Concept & Formula:** Linear Pair $= 180^\circ$ + VOA.
- **Standard Step-by-Step:** Set angles as $5x, 7x$, solve $5x + 7x = 180^\circ$, use VOA for opposite pair.
- **Fastest Valid Method:** Sum of ratio parts divides 180°: $180/12 = 15^\circ$. | **Alternative:** Use 360° around a point.
- **Common Trap:** Forgetting to compute $360^\circ - \theta$ when 'reflex' is asked.
- **Practice Drill:** Ex. 1 & Ex. 5.

Pattern 2: Ray Bisector Proofs ($\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$)

- **Identification Cue:** Prove $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$ or bisectors of linear pair are $\perp$. | **Concept & Formula:** Express angles as sums/differences of adjacent parts.
- **Standard Step-by-Step:** Write $\angle QOS = 90^\circ + \angle ROS$ and $\angle POS = 90^\circ - \angle ROS$, then subtract.
- **Fastest Valid Method:** Subtracting the two 90° equations cancels 90° immediately! | **Alternative:** Substitute $\angle POQ = 180^\circ$.
- **Common Trap:** Sign error when subtracting $(90^\circ - \angle ROS)$.
- **Practice Drill:** Ex. 9 & Ex. 10.

Pattern 3: Parallel Lines with Bent Vertex (Draw Parallel Line)

- **Identification Cue:** In $PQ \parallel ST$, $\angle PQR = 110^\circ, \angle RST = 130^\circ$, find $\angle QRS$. | **Concept & Formula:** Draw line through R parallel to PQ and ST.
- **Standard Step-by-Step:** Use co-interior $= 180^\circ$ on both sides to find adjacent angles at R, then subtract from 180°.
- **Fastest Valid Method:** For M-shape ($AB \parallel CD$ with bend E), middle angle $= \angle 1 + \angle 2$! | **Alternative:** Extend PQ to intersect SR.
- **Common Trap:** Assuming Q, R, S form a triangle with PQ and ST.
- **Practice Drill:** Ex. 11 & Ex. 12.

Pattern 4: Mirror Reflection & Parallelism Proof

- **Identification Cue:** Two parallel mirrors $PQ \parallel RS$, incident ray AB reflects along BC and CD; prove $AB \parallel CD$. | **Concept & Formula:** Angle of incidence = angle of reflection + alternate interior angles.
- **Standard Step-by-Step:** Draw normals $BM \perp PQ$ and $CN \perp RS$ ($BM \parallel CN$), equate alternate interior angles $\angle 2 = \angle 3$, double to get $\angle ABC = \angle BCD$.
- **Fastest Valid Method:** Normals to parallel lines are parallel! | **Alternative:** Use complementary base angles.
- **Common Trap:** Forgetting to justify WHY $BM \parallel CN$.
- **Practice Drill:** Ex. 13.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. F, Z, C Letter Pattern for Transversals] **Core Idea & Why It Works:** **F**-shape = Corresponding (Equal); **Z**-shape = Alternate Interior (Equal); **C**-shape = Co-interior (Sum = $180°$).

Valid Domain: All parallel-line diagrams. | **Where It Fails:** Non-parallel lines.

Easy Example: Spot a Z between $AB \parallel CD \implies$ angles are equal!

Exam-Level Example: Speeds up diagram reading in 2 seconds.

Advanced Example: Works even when rotated or flipped.

[SMART SHORTCUT – 2. M-Shape Bend Angle = Sum of Acute Arms] **Core Idea & Why It Works:** If $AB \parallel CD$ and a point E lies between them forming $\angle BAE = \alpha$ and $\angle DCE = \beta$ (both opening right), then $\angle AEC = \alpha + \beta$.

Valid Domain: All M-shape bends between parallel lines. | **Where It Fails:** If one angle opens the opposite way (then use co-interior $180° - \alpha$).

Easy Example: $\alpha = 35°, \beta = 45° \implies \angle AEC = 80°$.

Exam-Level Example: Instant MCQ check for NCERT Ex. 6.2!

Advanced Example: Draw parallel through E for full 3-mark proof.

[SMART SHORTCUT – 3. Linear-Pair Bisector = Always $90°$] **Core Idea & Why It Works:** If OE bisects $\angle AOC$ and OF bisects $\angle BOC$ on a straight line AOB, then $\angle EOF = 90°$ ALWAYS.

Valid Domain: Every linear pair. | **Where It Fails:** Non-supplementary adjacent angles.

Easy Example: $\frac{1}{2}(x) + \frac{1}{2}(180° - x) = 90°$.

Exam-Level Example: Instant 1-mark MCQ answer.

Advanced Example: Similarly, bisectors of co-interior angles meet at $90°$!

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 **[Beginner]**

Question: Find the complement of $37°$ and the supplement of $112°$.

Given & Required: $37°, 112°$. | **Concept & Formula:** $90° - \theta$ and $180° - \theta$.

Detailed Solution: Complement $= 90° - 37° = 53°$. Supplement $= 180° - 112° = 68°$. **Final Answer:** $53°$ **and** $68°$

Fastest Alternative / Shortcut: Subtract from $90°$ and $180°$.

SOLVED EXAMPLE 2 **[Beginner]**

Question: If an angle is $20°$ less than its complement, find the angle.

Given & Required: $x = (90° - x) - 20°$. | **Concept & Formula:** Complementary equation.

Detailed Solution: $2x = 70° \implies x = 35°$. **Final Answer:** $35°$

Fastest Alternative / Shortcut: $(90 - 20)/2 = 35°$.

SOLVED EXAMPLE 3 **[Beginner]**

Question: Lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70°$ and $\angle BOD = 40°$, find $\angle BOE$ and reflex $\angle COE$.

Given & Required: $\angle BOD = 40°$. | **Concept & Formula:** VOA + Linear Pair.

Detailed Solution: $\angle AOC = \angle BOD = 40°$ (VOA). Thus $\angle BOE = 70° - 40° = 30°$. On line AB: $\angle COE = 180° - 70° = 110°$, so reflex $\angle COE = 360° - 110° = 250°$. **Final Answer:** $30°$ **and** $250°$

Fastest Alternative / Shortcut: Watch 'reflex' $= 360° - 110°$.

SOLVED EXAMPLE 4 [Beginner]

Question: Two angles of a linear pair are in the ratio $4:5$. Find the angles.

Given & Required: $4x + 5x = 180°$. | **Concept & Formula:** Linear pair.

Detailed Solution: $9x = 180° \implies x = 20°$. Angles: $80°$ and $100°$. **Final Answer:** $80°$ **and** $100°$

Fastest Alternative / Shortcut: $180/9 = 20$.

SOLVED EXAMPLE 5 [Intermediate]

Question: Lines XY and MN intersect at O. If $\angle POY = 90°$ and $a : b = 2 : 3$ (where $a = \angle POM, b = \angle MOX$), find $c = \angle XON$.

Given & Required: $a + b = 90°, a : b = 2 : 3$. | **Concept & Formula:** Linear pair on XY and MN.

Detailed Solution: $5k = 90° \implies k = 18° \implies a = 36°, b = 54°$. On line MN: $b + c = 180° \implies c = 180° - 54° = 126°$. **Final Answer:** $c = 126°$

Fastest Alternative / Shortcut: $180° - 54° = 126°$.

SOLVED EXAMPLE 6 [Intermediate]

Question: If $AB \parallel CD$ and $CD \parallel EF$, and $y : z = 3 : 7$ (where x, y are co-interior and $x = z$ by alternate interior), find x.

Given & Required: $y + z = 180°$ since $x = z$. | **Concept & Formula:** Parallel transitivity.

Detailed Solution: $3k + 7k = 180° \implies k = 18° \implies z = 126° \implies x = 126°$. **Final Answer:** $126°$

Fastest Alternative / Shortcut: $7 \times 18 = 126°$.

SOLVED EXAMPLE 7 [Intermediate]

Question: If $AB \parallel CD$, $EF \perp CD$, and $\angle GED = 126°$, find $\angle AGE$, $\angle GEF$, and $\angle FGE$.

Given & Required: $\angle GED = 126°, \angle FED = 90°$. | **Concept & Formula:** Alternate interior + linear pair.

Detailed Solution: $\angle AGE = \angle GED = 126°$ (Alt. Int.). $\angle GEF = 126° - 90° = 36°$. $\angle FGE = 180° - 126° = 54°$. **Final Answer:** $126°, 36°, 54°$

Fastest Alternative / Shortcut: Check $36 + 54 + 90 = 180°$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Prove that vertically opposite angles are equal when two lines AB and CD intersect at O.

Given & Required: Intersecting lines. | **Concept & Formula:** Linear Pair Axiom.

Detailed Solution: $\angle AOC + \angle AOD = 180°$ and $\angle AOD + \angle BOD = 180° \implies \angle AOC = \angle BOD$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Equate both $180°$ sums.

SOLVED EXAMPLE 9 [Board Standard]

Question: In a figure, POQ is a line and ray $OR \perp PQ$. Ray OS lies between OP and OR. Prove that $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$.

Given & Required: $\angle POR = \angle QOR = 90°$. | **Concept & Formula:** Subtract two $90°$ expressions.

Detailed Solution: $\angle QOS = 90° + \angle ROS$ and $\angle POS = 90° - \angle ROS$. Subtracting: $\angle QOS - \angle POS = 2\angle ROS \implies \angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Classic NCERT 3-mark proof.

SOLVED EXAMPLE 10 [Board Standard]

Question: If $PQ \parallel ST$, $\angle PQR = 110°$, and $\angle RST = 130°$, find $\angle QRS$.

Given & Required: Bent vertex R below PQ, ST. | **Concept & Formula:** Draw $XY \parallel ST$ through R.

Detailed Solution: Co-interior with PQ: $180° - 110° = 70°$. Co-interior with ST: $180° - 130° = 50°$. On line XY: $\angle QRS = 180° - (70° + 50°) = 60°$. **Final Answer:** $60°$

Fastest Alternative / Shortcut: $110° + 130° - 180° = 60°$!

SOLVED EXAMPLE 11 [Board Standard]

Question: If a transversal intersects two lines such that the bisectors of a pair of corresponding angles are parallel, prove that the two lines are parallel.

Given & Required: Bisectors $BE \parallel CG$. | **Concept & Formula:** Double equal angles.

Detailed Solution: Since $BE \parallel CG$, corresponding angles $\angle ABE = \angle BCG$. Multiplying by 2 gives $2\angle ABE = 2\angle BCG \implies \angle ABQ = \angle BCS$, which are corresponding angles for PQ, RS. Hence $PQ \parallel RS$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Multiply corresponding angles by 2.

SOLVED EXAMPLE 12 [Board Standard]

Question: Prove that the bisectors of a linear pair are perpendicular to each other.

Given & Required: Linear pair $\angle AOC + \angle BOC = 180^\circ$. | **Concept & Formula:** Halve 180°.

Detailed Solution: $\angle EOF = \frac{1}{2}\angle AOC + \frac{1}{2}\angle BOC = \frac{1}{2}(180^\circ) = 90^\circ$. **Final Answer: Proved (90°)**

Fastest Alternative / Shortcut: Half of 180° is 90°.

SOLVED EXAMPLE 13 [Board Standard]

Question: Two parallel mirrors PQ and RS face each other. Incident ray AB strikes PQ at B, reflects along BC to RS at C, and reflects along CD. Prove $AB \parallel CD$.

Given & Required: $PQ \parallel RS$. | **Concept & Formula:** Normals + reflection law.

Detailed Solution: Draw normals $BM \perp PQ, CN \perp RS \implies BM \parallel CN \implies \angle 2 = \angle 3$ (alt. int.). By reflection, $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$, so $\angle 1 + \angle 2 = \angle 3 + \angle 4 \implies \angle ABC = \angle BCD$ (alt. int.), hence $AB \parallel CD$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Equal alternate interior angles.

SOLVED EXAMPLE 14 [Challenge]

Question: In $\triangle ABC$, BO and CO are bisectors of exterior angles at B and C. Express $\angle BOC$ in terms of $\angle A$.

Given & Required: Exterior bisectors meet at O. | **Concept & Formula:** Triangle sum 180°.

Detailed Solution: $\angle OBC = 90^\circ - \frac{B}{2}$, $\angle OCB = 90^\circ - \frac{C}{2}$. Thus $\angle BOC = 180^\circ - (180^\circ - \frac{B+C}{2}) = \frac{B+C}{2} = 90^\circ - \frac{\angle A}{2}$. **Final Answer:** $90^\circ - \frac{\angle A}{2}$

Fastest Alternative / Shortcut: Interior bisectors give $90^\circ + \frac{A}{2}$; exterior give $90^\circ - \frac{A}{2}$!

SOLVED EXAMPLE 15 [Challenge]

Question: If $AB \parallel CD$, $\angle APQ = 50^\circ$, and $\angle PRD = 127^\circ$, find $x = \angle PQR$ and $y = \angle QPR$.

Given & Required: Q, R on CD; P on AB. | **Concept & Formula:** Alternate interior + exterior angle.

Detailed Solution: $x = \angle APQ = 50^\circ$ (Alt. Int.). Exterior $\angle PRD = x + y \implies 127^\circ = 50^\circ + y \implies y = 77^\circ$. **Final Answer:** $x = 50^\circ, y = 77^\circ$

Fastest Alternative / Shortcut: $127 - 50 = 77^\circ$.

SOLVED EXAMPLE 16 [Challenge]

Question: Find x if $(2x + 10)^\circ$ and $(3x - 20)^\circ$ are co-interior angles between two parallel lines.

Given & Required: Sum $= 180^\circ$. | **Concept & Formula:** Co-interior supplementary.

Detailed Solution: $(2x + 10) + (3x - 20) = 180 \implies 5x - 10 = 180 \implies 5x = 190 \implies x = 38^\circ$. **Final Answer:** $x = 38^\circ$

Fastest Alternative / Shortcut: $5x = 190$.

G. Competency MCQ Mastery Section

Q1. An angle equal to its supplement is:

(A) 45° (B) 90° (C) 180° (D) 60°

Answer: (B) $x + x = 180^\circ \implies x = 90^\circ$. *Fast Method:* Half of 180°. *Distractor Note:* Picking 45° (complement).

Q2. If an angle is 4 times its complement, the angle is:

(A) 18° (B) 36° (C) 72° (D) 144°

Answer: (C) $4k + k = 90^\circ \implies k = 18^\circ \implies 72^\circ$. *Fast Method:* $4/5$ of 90°. *Distractor Note:* Picking 18° (the complement).

Q3. When two parallel lines are cut by a transversal, co-interior angles are:

(A) Equal (B) Complementary (C) Supplementary (D) Reflex

Answer: (C) Sum is 180°. *Fast Method:* C-shape $= 180^\circ$. *Distractor Note:* Picking 'Equal'.

Q4. Reflex angle of 75° is:

(A) 15° (B) 105° (C) 285° (D) 275°

Answer: (C) $360^\circ - 75^\circ = 285^\circ$. *Fast Method:* $360 - 75$. *Distractor Note:* Subtracting from 180°.

Q5. Angle between the bisectors of a linear pair is:

(A) 45° (B) 60° (C) 90° (D) 180°

Answer: (C) Half of $180^\circ = 90^\circ$. *Fast Method:* 90°. *Distractor Note:* Picking 180°.

Q6. If $(3x - 10)^\circ$ and $(2x + 30)^\circ$ are vertically opposite angles, then x is:

(A) 40 (B) 20 (C) 32 (D) 10

Answer: (A) $3x - 10 = 2x + 30 \implies x = 40$. *Fast Method:* Equate VOA. *Distractor Note:* Adding to 180°.

Q7. If $l \parallel m$ and corresponding angles are $(4x - 15)^\circ$ and $(3x + 5)^\circ$, then x is:

(A) 10 (B) 20 (C) 30 (D) 40

Answer: (B) $4x - 15 = 3x + 5 \implies x = 20$. *Fast Method:* Corresponding angles are equal. *Distractor Note:* Adding to 180°.

Q8. Sum of all angles around a point is:

(A) 90° (B) 180° (C) 270° (D) 360°

Answer: (D) Complete revolution $= 360^\circ$. *Fast Method:* 360°. *Distractor Note:* Picking 180°.

Q9. If two lines are perpendicular to the same line, they are:

(A) Perpendicular (B) Parallel (C) Intersecting (D) Coincident

Answer: (B) Corresponding angles are $90^\circ = 90^\circ \implies$ parallel. *Fast Method:* Parallel. *Distractor Note:* Picking perpendicular.

Q10. Angles of a triangle are in ratio $2 : 3 : 4$. The largest angle is:

(A) 40° (B) 60° (C) 80° (D) 100°

Answer: (C) $9k = 180^\circ \implies k = 20^\circ \implies 4k = 80^\circ$. *Fast Method:* $4 \times 20 = 80^\circ$. *Distractor Note:* Picking 40° (smallest).

Q11. If an exterior angle of a triangle is 115° and one interior opposite angle is 45°, the other is:

(A) 65° (B) 70° (C) 75° (D) 160°

Answer: (B) $115^\circ - 45^\circ = 70^\circ$. *Fast Method:* Ext. angle = sum of opp. interior. *Distractor Note:* Subtracting from 180°.

Q12. A ray stands on a line. If one angle is 108°, the other is:

(A) 72° (B) 82° (C) 18° (D) 252°

Answer: (A) $180^\circ - 108^\circ = 72^\circ$. *Fast Method:* Linear pair. *Distractor Note:* Arithmetic slip.

H. Special Exam & MCQ Strategies

- Whenever parallel lines have a bend or zig-zag between them, immediately draw a dashed parallel line through the bend vertex.
- Label every known angle directly on your diagram before writing a single line of proof.
- Remember F (Corresponding =), Z (Alternate Interior =), and C (Co-interior = 180°).

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Setting co-interior angles equal to each other. $\longrightarrow$ **Correction:** Co-interior angles (C-shape) are SUPPLEMENTARY ($\angle 1 + \angle 2 = 180^\circ$), not equal (unless 90°).

[COMMON MISTAKE] **Error:** Computing reflex angle as $180^\circ - \theta$. $\longrightarrow$ **Correction:** Reflex angle is $360^\circ - \theta$.

[COMMON MISTAKE] **Error:** Assuming two lines are parallel just because they look parallel in the sketch. $\longrightarrow$ **Correction:** Only use parallel-line properties when $l \parallel m$ is explicitly given or proved.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: Comp: $90°$; Supp: $180°$; Linear Pair $= 180°$; VOA $=$ equal; Reflex $= 360° - \theta$.

[QUICK REVISION] Key Theorems & Results: Parallel $l \parallel m$: Corr. $=$, Alt. Int. $=$, Co-int. $= 180°$; Linear-pair bisectors $\perp$ $(90°)$.

[QUICK REVISION] Shortcut Summary: F-Z-C patterns; M-shape bend $= \alpha + \beta$; Ext. bisectors meet at $90° - A/2$.

[QUICK REVISION] High-Frequency Exam Patterns: Linear pair & VOA ratios, $\angle ROS = \frac{1}{2}(\angle QOS - \angle POS)$, Parallel bend construction, Mirror proof.

Last-Minute Revision Checklist:

- ☐ Co-interior sums to $180°$
- ☐ Reflex $= 360° - \theta$
- ☐ Draw parallel through bend vertex
- ☐ Check given vs to-prove

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Supplement of $55°$ is: (A) $35°$ (B) $125°$ (C) $135°$ (D) $305°$
2. Angle equal to its complement is: (A) $30°$ (B) $45°$ (C) $60°$ (D) $90°$
3. If two lines intersect and one angle is $90°$, all 4 angles are: (A) $90°$ (B) $45°$ (C) $180°$ (D) Different
4. Alternate interior angles for $l \parallel m$ are: (A) Complementary (B) Supplementary (C) Equal (D) Reflex

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Angles on a straight line are $2x, 3x, 5x$. Value of x: (A) $18°$ (B) $36°$ (C) $20°$ (D) $15°$
2. If $l \parallel m$ and co-interior angles are in ratio $2:3$, larger angle is: (A) $72°$ (B) $108°$ (C) $90°$ (D) $120°$
3. Reflex of angle $105°$ is: (A) $75°$ (B) $255°$ (C) $265°$ (D) $15°$
4. If $AB \parallel CD$, $\angle A = 40°$ and $\angle C = 50°$ in an M-bend at E, $\angle AEC$ is: (A) $90°$ (B) $10°$ (C) $130°$ (D) $140°$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Angle which is $30°$ more than twice its supplement: (A) $130°$ (B) $50°$ (C) $110°$ (D) $120°$
2. In $\triangle ABC$, interior bisectors of B, C meet at O. If $\angle A = 70°$, $\angle BOC$ is: (A) $125°$ (B) $110°$ (C) $55°$ (D) $140°$
3. If $PQ \parallel RS$, $\angle PQR = 100°, \angle QRS = 40°$, then $\angle RST$ (opening right) is: (A) $120°$ (B) $140°$ (C) $100°$ (D) $80°$
4. Complement of $(90° - a)$ is: (A) a (B) $90° + a$ (C) $180° - a$ (D) $-a$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $180 - 55 = 125°$. **Q2 (B):** $90/2 = 45°$. **Q3 (A):** VOA + Linear pair give all $90°$. **Q4 (C):** Z-shape equality.
Exam Std	**Q1 (A):** $10x = 180 \implies x = 18°$. **Q2 (B):** $5k = 180 \implies k = 36 \implies 3k = 108°$. **Q3 (B):** $360 - 105 = 255°$. **Q4 (A):** $40 + 50 = 90°$.
Challenge	**Q1 (A):** $x = 2(180 - x) + 30 \implies 3x = 390 \implies x = 130°$. **Q2 (A):** $90° + 70°/2 = 125°$. **Q3 (A):** $80 + 40 = 120$? Wait: $100 + \angle RST - 180 = 40 \implies \angle RST = 120°$. **Q4 (A):** $90 - (90 - a) = a$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Angle pairs, Linear Pair, VOA. Day 2: Parallel lines, F-Z-C rules, bend constructions.
- **How to Introduce the Concept:** Hold two rulers as parallel lines and a third as a transversal—ask students to spot F, Z, and C!
- **Difficult Areas & Remediation:** Knowing when to draw a construction parallel line and distinguishing co-interior from alternate interior.
- **Blackboard Teaching Examples:** Use colored chalk for F, Z, and C shapes on the same transversal diagram.
- **Classroom Questions to Ask Students:** Why can't two lines be parallel if their co-interior angles are 80° and 95°?
- **Common Student Doubts & Answers:** Students ask about the triangle angle-sum theorem proof; clarify its formal proof is excluded in 2026–27 though $A + B + C = 180^\circ$ is used.
- **Homework & Revision Strategy:** Assign 5 angle-chasing numericals and 3 formal parallel-line proofs.

[TEACHER'S TIP] Drawing a clean, large diagram solves half of every geometry problem.

UNIT IV: GEOMETRY TYBA COACHING CENTRE

Chapter 7: Triangles

SAS, ASA, AAS, SSS, RHS Congruence Criteria, CPCTC & Isosceles Triangle Theorems (2026–27)

A. Chapter Introduction & Examination Relevance

Triangles are the most rigid and fundamental polygons in engineering and geometry. Chapter 7 establishes the exact criteria under which two triangles are identical in shape and size (congruent) and proves key properties of isosceles triangles.

- **What You Will Learn:** Concept of congruence ($\cong$), vertex correspondence, SAS, ASA, AAS, SSS, and RHS congruence rules, CPCTC (Corresponding Parts of Congruent Triangles), and isosceles triangle theorems (angles opposite equal sides are equal, and converse).
- **Why This Chapter Matters:** Triangle congruence is the #1 proof engine in Class 9 (Chapters 7, 8, and 9) and sets the stage for Class 10 Triangle Similarity.
- **Real-Life Applications:** Bridge and roof truss stability, surveying triangulation, foldable structures, and geometric CAD verification.
- **Cross-Chapter Connections:** Builds on Chapter 6 (VOA, alternate interior angles) and directly powers Chapter 8 (Quadrilaterals) and Chapter 9 (Circles).
- **Exam & Foundation Relevance:** Unit IV carries 28/100 marks in RBSE (25/80 in CBSE). Expect 1M congruence-rule MCQs and 3M/5M two-triangle congruence proofs. (Inequalities in a triangle are excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Congruence of Triangles & Vertex Correspondence

Two triangles are **congruent** ($\triangle ABC \cong \triangle PQR$) if one can be superposed exactly onto the other. Writing $\triangle ABC \cong \triangle PQR$ enforces strict **vertex order**: $A \leftrightarrow P$, $B \leftrightarrow Q$, $C \leftrightarrow R$. Once two triangles are proved congruent, all remaining corresponding sides and angles are equal by **CPCTC** (Corresponding Parts of Congruent Triangles).

[CONCEPT MASTER – 1. Congruence of Triangles & Vertex Correspondence] Warning: $\triangle ABC \cong \triangle PQR$ does NOT mean $\triangle ABC \cong \triangle QPR$! Always match vertices in the exact order of equal angles and sides.

2. The Five Congruence Criteria (SAS, ASA, AAS, SSS, RHS)

SAS: Two sides and the *included* angle. **ASA:** Two angles and the *included* side. **AAS:** Any two angles and a corresponding non-included side (follows from ASA since the 3rd angle is fixed). **SSS:** All three corresponding sides equal. **RHS:** In two right-angled triangles, the hypotenuse and one side are equal.

[CONCEPT MASTER – 2. The Five Congruence Criteria (SAS, ASA, AAS, SSS, RHS)] Why SSA / ASS and AAA are NOT valid: AAA guarantees same shape (similarity, Class 10) but NOT same size; SSA can produce two different triangles!

3. Isosceles Triangle Theorems

Theorem 7.2: Angles opposite to equal sides of an isosceles triangle are equal ($AB = AC \implies \angle B = \angle C$). **Theorem 7.3 (Converse):** Sides opposite to equal angles of a triangle are equal ($\angle B = \angle C \implies AB = AC$). Moreover, the altitude from the vertex angle of an isosceles triangle bisects both the base and the vertex angle.

[CONCEPT MASTER – 3. Isosceles Triangle Theorems] Every equilateral triangle has all three angles equal to 60°.

4. Multi-Step Congruence & Common-Side/Common-Angle Strategy

In complex figures (overlapping triangles or shared diagonals), look for: (1) a **common side** ($BD = BD$), (2) a **common angle** ($\angle A = \angle A$), (3) **vertically opposite angles**, or (4) **adding/subtracting a common angle** (e.g., $\angle BAD = \angle EAC \implies \angle BAC = \angle EAD$).

[CONCEPT MASTER – 4. Multi-Step Congruence & Common-Side/Common-Angle Strategy] Rationalised 2026–27 Scope: Inequalities in a triangle (Theorem 7.6, 7.7, 7.8) are excluded from the 2026–27 NCERT syllabus.

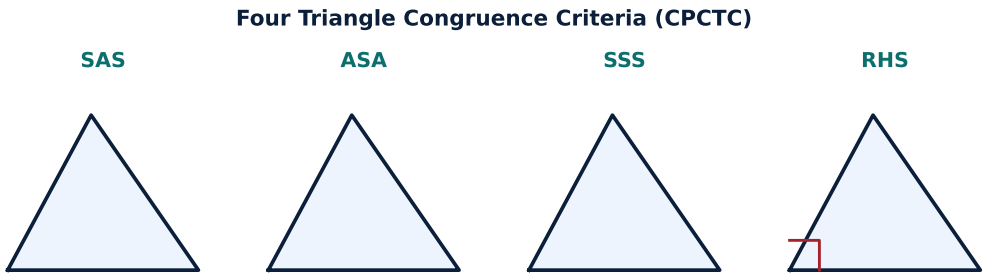


Figure 1: **Visual Model:** Four core triangle congruence criteria (SAS, ASA, SSS, RHS) guaranteeing exact superposition and CPCTC.

C. Complete Formula Bank

[FORMULA BANK – 1. SAS Congruence Axiom] **Expression:** $AB = PQ,\ \angle B = \angle Q,\ BC = QR \implies \triangle ABC \cong \triangle PQR$
Symbols: $\angle B$ is included between AB, BC.
Conditions & When to Use: Angle MUST lie between the two sides.
Solved Mini-Example: Two radii and included central angle prove chord congruence.
Common Mistake: Using SSA as SAS.

[FORMULA BANK – 2. ASA & AAS Congruence] **Expression:** $\angle B = \angle Q, BC = QR, \angle C = \angle R$ (ASA); $\angle A = \angle P, \angle B = \angle Q, BC = QR$ (AAS)
Symbols: Any 2 angles + 1 corresponding side.
Conditions & When to Use: Works for both included and non-included sides.
Solved Mini-Example: Used when VOA and alternate interior angles give two equal angles.
Common Mistake: Matching non-corresponding sides in AAS.

[FORMULA BANK – 3. SSS Congruence] **Expression:** $AB = PQ, BC = QR, CA = RP \implies \triangle ABC \cong \triangle PQR$
Symbols: All 3 pairs of sides equal.
Conditions & When to Use: No angles needed.
Solved Mini-Example: Diagonal of a kite/rhombus divides it into 2 SSS congruent triangles.
Common Mistake: Ignoring vertex order in the final statement.

[FORMULA BANK – 4. RHS Congruence] **Expression:** $\angle B = \angle Q = 90^\circ,\ AC = PR$ (Hyp), $AB = PQ \implies \triangle ABC \cong \triangle PQR$
Symbols: R: 90°, H: Hypotenuse, S: Side.
Conditions & When to Use: ONLY for right triangles with equal hypotenuses.
Solved Mini-Example: Altitude AD in isosceles $\triangle ABC$ gives $\triangle ABD \cong \triangle ACD$ by RHS.
Common Mistake: Using RHS when the two legs (not hypotenuse) are equal—that's SAS!

[FORMULA BANK – 5. Isosceles Triangle Property] **Expression:** $AB = AC \iff \angle B = \angle C$
Symbols: $\angle B, \angle C$: base angles.
Conditions & When to Use: Applies within the SAME triangle.
Solved Mini-Example: If $AB = AC$ and $\angle A = 50^\circ$, then $\angle B = \angle C = 65^\circ$.
Common Mistake: Equating $\angle A$ with $\angle B$ when $AB = AC$.

[FORMULA BANK – 6. Equilateral Triangle] **Expression:** $AB = BC = CA \iff \angle A = \angle B = \angle C = 60^\circ$
Symbols: All sides equal.
Conditions & When to Use: Height $= \frac{\sqrt{3}}{2}a$, Area $= \frac{\sqrt{3}}{4}a^2$.
Solved Mini-Example: Each angle is 60°.
Common Mistake: Forgetting all 3 angles are 60°.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Olympiad/NTSE Foundation problem structures.

Pattern 1: Identifying Valid Congruence Criteria & Vertex Order

- **Identification Cue:** In $\triangle ABC$ and $\triangle DEF$, $AB = FD, \angle A = \angle D$, which extra condition gives SAS? | **Concept & Formula:** Included angle between the two sides.
- **Standard Step-by-Step:** Draw both triangles side by side, mark equal parts, check whether the angle is strictly between the sides.
- **Fastest Valid Method:** For $\angle A$, the two sides meeting at A are AB and AC! | **Alternative:** Check vertex correspondence letters.
- **Common Trap:** Picking $BC = EF$ (gives SSA, invalid!).
- **Practice Drill:** Ex. 1 & Ex. 5.

Pattern 2: Single-Pair Congruence + CPCTC Proofs

- **Identification Cue:** Prove $\triangle ABD \cong \triangle BAC$ or AD bisects BC. | **Concept & Formula:** Mark Given + Common/VOA parts $\rightarrow$ apply SAS/ASA/RHS.
- **Standard Step-by-Step:** List 'In $\triangle_1$ and $\triangle_2$:', write 3 equalities with reasons, conclude congruence + CPCTC.
- **Fastest Valid Method:** Always mark ticks/arcs on the diagram first to spot the 3rd hidden part. | **Alternative:** Use isosceles base angles.
- **Common Trap:** Forgetting the 3rd hidden part (common side or VOA).
- **Practice Drill:** Ex. 6 & Ex. 7.

Pattern 3: Angle/Segment Addition-Subtraction Congruence

- **Identification Cue:** Given $AB = AD, AC = AE$, and $\angle BAD = \angle EAC$, prove $BC = DE$. | **Concept & Formula:** Add $\angle DAC$ to both sides.
- **Standard Step-by-Step:** Add common angle $\angle DAC$ (or segment) to obtain $\angle BAC = \angle DAE$, then prove $\triangle ABC \cong \triangle ADE$ by SAS.
- **Fastest Valid Method:** Spot the overlapping middle wedge $\angle DAC$. | **Alternative:** Label angles $1, 2, 3$.
- **Common Trap:** Applying SAS to $\triangle ABC$ using $\angle BAD$ directly.
- **Practice Drill:** Ex. 9.

Pattern 4: Two-Stage Congruence (Isosceles / Median Proofs)

- **Identification Cue:** Two sides AB, BC and median AM equal PQ, QR, PN; prove $\triangle ABC \cong \triangle PQR$. | **Concept & Formula:** Prove half-triangles congruent first by SSS, get $\angle B = \angle Q$ by CPCTC, then SAS for full triangles.
- **Standard Step-by-Step:** Stage 1: $\triangle ABM \cong \triangle PQN$ (SSS) $\implies \angle B = \angle Q$. Stage 2: $\triangle ABC \cong \triangle PQR$ (SAS).
- **Fastest Valid Method:** Break big triangle into smaller sub-triangles first. | **Alternative:** Extend median (for altitude/median properties).
- **Common Trap:** Trying to prove the full triangles congruent in 1 step.
- **Practice Drill:** Ex. 11 & Ex. 12.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Vertex-Letter Matching Check] **Core Idea & Why It Works:** In $\triangle ABC \cong \triangle PQR$, read corresponding pairs directly by position: $(1,2) \to AB = PQ$, $(2,3) \to BC = QR$, $(1,3) \to AC = PR$.
Valid Domain: All congruence statements. | **Where It Fails:** Nothing.
Easy Example: If $\triangle ABC \cong \triangle FDE$, then $AB = FD$ and $\angle B = \angle D$.
Exam-Level Example: Instant 1-mark MCQ solver.
Advanced Example: Prevents writing wrong CPCTC conclusions.

[SMART SHORTCUT – 2. Invalid Criteria Blacklist (AAA & SSA)] **Core Idea & Why It Works:** NEVER select **AAA** or **SSA** as a congruence rule—only SAS, ASA, AAS, SSS, and RHS are valid.
Valid Domain: All congruence MCQs. | **Where It Fails:** Right triangle SSA with hypotenuse IS RHS.
Easy Example: If two triangles have equal angles $50°, 60°, 70°$, they need NOT be congruent (AAA fails).
Exam-Level Example: Eliminates 2 wrong options immediately in MCQs.
Advanced Example: AAA gives Similarity in Class 10.

[SMART SHORTCUT – 3. Isosceles Base-Angle Formula] **Core Idea & Why It Works:** In isosceles $\triangle ABC$ with $AB = AC$, each base angle is $\frac{180° - \angle A}{2} = 90° - \frac{\angle A}{2}$.
Valid Domain: All isosceles triangles. | **Where It Fails:** If base angle is given instead of vertex angle (then vertex $= 180° - 2\theta$).
Easy Example: Vertex $\angle A = 40° \implies \angle B = \angle C = 70°$.
Exam-Level Example: Fast angle-chasing in Ch. 7, 8, and 9.
Advanced Example: If base angle is $45°$, triangle is right isosceles.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: In $\triangle ABC$, $AB = AC$ and $\angle B = 50°$. Find $\angle C$ and $\angle A$.
Given & Required: $AB = AC, \angle B = 50°$. | **Concept & Formula:** Isosceles triangle theorem.
Detailed Solution: $\angle C = \angle B = 50°$. $\angle A = 180° - (50° + 50°) = 80°$. **Final Answer:** $\angle C = 50°, \angle A = 80°$
Fastest Alternative / Shortcut: $180 - 100 = 80°$.

SOLVED EXAMPLE 2 [Beginner]
Question: In quadrilateral $ACBD$, $AC = AD$ and AB bisects $\angle A$. Show that $\triangle ABC \cong \triangle ABD$.
Given & Required: $AC = AD, \angle CAB = \angle DAB$. | **Concept & Formula:** SAS congruence.
Detailed Solution: In $\triangle ABC$ and $\triangle ABD$: $AC = AD$ (Given), $\angle CAB = \angle DAB$ (Given), $AB = AB$ (Common). By SAS, $\triangle ABC \cong \triangle ABD$. **Final Answer: Proved by SAS**
Fastest Alternative / Shortcut: Common side AB.

SOLVED EXAMPLE 3 [Beginner]
Question: If $\triangle ABC \cong \triangle PQR$, $AB = 5$ cm, $\angle B = 40°$, $\angle A = 80°$, find PQ and $\angle R$.
Given & Required: Congruent triangles. | **Concept & Formula:** CPCTC.
Detailed Solution: $PQ = AB = 5$ cm. $\angle C = 180° - (80° + 40°) = 60°$, so $\angle R = \angle C = 60°$. **Final Answer:** $PQ = 5$ **cm,** $\angle R = 60°$
Fastest Alternative / Shortcut: Match positions $1, 2, 3$.

SOLVED EXAMPLE 4 [Beginner]
Question: Find each angle of an equilateral triangle.
Given & Required: $AB = BC = CA$. | **Concept & Formula:** Isosceles theorem twice.
Detailed Solution: $\angle A = \angle B = \angle C = x \implies 3x = 180° \implies x = 60°$. **Final Answer:** $60°$ **each**
Fastest Alternative / Shortcut: $180/3 = 60°$.

SOLVED EXAMPLE 5 [Intermediate]

Question: AD and BC are equal perpendiculars to a line segment AB. Show that CD bisects AB.

Given & Required: $AD = BC$, $\angle A = \angle B = 90°$. | **Concept & Formula:** AAS congruence.

Detailed Solution: Let CD meet AB at O. In $\triangle OAD$ and $\triangle OBC$: $\angle AOD = \angle BOC$ (VOA), $\angle A = \angle B = 90°$, $AD = BC$. By AAS, $\triangle OAD \cong \triangle OBC \implies OA = OB$ (CPCTC). **Final Answer: Proved ($OA = OB$)**

Fastest Alternative / Shortcut: VOA + $90°$ + side = AAS.

SOLVED EXAMPLE 6 [Intermediate]

Question: Line l is the bisector of $\angle A$ and B is any point on l. BP and BQ are perpendiculars from B to the arms of $\angle A$. Show that $BP = BQ$.

Given & Required: $\angle PAB = \angle QAB$, $\angle P = \angle Q = 90°$. | **Concept & Formula:** AAS + CPCTC.

Detailed Solution: In $\triangle APB$ and $\triangle AQB$: $\angle P = \angle Q = 90°$, $\angle PAB = \angle QAB$, $AB = AB$ (Common). By AAS, $\triangle APB \cong \triangle AQB \implies BP = BQ$. **Final Answer: Proved ($BP = BQ$)**

Fastest Alternative / Shortcut: Every point on angle bisector is equidistant from arms!

SOLVED EXAMPLE 7 [Intermediate]

Question: In right triangle ABC right-angled at C, M is the midpoint of AB. C is joined to M and produced to D so $DM = CM$. Show $\triangle AMC \cong \triangle BMD$.

Given & Required: $AM = BM, CM = DM$. | **Concept & Formula:** SAS via VOA.

Detailed Solution: In $\triangle AMC$ and $\triangle BMD$: $AM = BM$, $\angle AMC = \angle BMD$ (VOA), $CM = DM$. By SAS, $\triangle AMC \cong \triangle BMD$. **Final Answer: Proved by SAS**

Fastest Alternative / Shortcut: Midpoint + VOA.

SOLVED EXAMPLE 8 [Intermediate]

Question: In $\triangle ABC$, AD is the perpendicular bisector of BC. Show that $\triangle ABC$ is isosceles ($AB = AC$).

Given & Required: $BD = CD, \angle ADB = \angle ADC = 90°$. | **Concept & Formula:** SAS + CPCTC.

Detailed Solution: In $\triangle ABD$ and $\triangle ACD$: $BD = CD$, $\angle ADB = \angle ADC = 90°$, $AD = AD$ (Common). By SAS, $\triangle ABD \cong \triangle ACD \implies AB = AC$. **Final Answer: Proved ($AB = AC$)**

Fastest Alternative / Shortcut: SAS (not RHS, since $AB = AC$ isn't given yet!).

SOLVED EXAMPLE 9 [Board Standard]

Question: In a figure, $AC = AE$, $AB = AD$, and $\angle BAD = \angle EAC$. Show that $BC = DE$.

Given & Required: $AC = AE, AB = AD, \angle BAD = \angle EAC$. | **Concept & Formula:** Add common $\angle DAC$.

Detailed Solution: Add $\angle DAC$ to both sides: $\angle BAD + \angle DAC = \angle EAC + \angle DAC \implies \angle BAC = \angle DAE$. In $\triangle ABC$ and $\triangle ADE$: $AB = AD$, $\angle BAC = \angle DAE$, $AC = AE$. By SAS, $\triangle ABC \cong \triangle ADE \implies BC = DE$. **Final Answer: Proved ($BC = DE$)**

Fastest Alternative / Shortcut: Add middle angle $\angle DAC$.

SOLVED EXAMPLE 10 [Board Standard]

Question: BE and CF are two equal altitudes of $\triangle ABC$. Using RHS congruence rule, prove that $\triangle ABC$ is isosceles.

Given & Required: $BE = CF$, $\angle BEC = \angle CFB = 90°$. | **Concept & Formula:** RHS + converse of isosceles.

Detailed Solution: In right $\triangle BEC$ and $\triangle CFB$: $\angle BEC = \angle CFB = 90°$, $BC = CB$ (Common Hypotenuse), $BE = CF$. By RHS, $\triangle BEC \cong \triangle CFB \implies \angle B = \angle C$ (CPCTC) $\implies AB = AC$. **Final Answer: Proved ($AB = AC$)**

Fastest Alternative / Shortcut: Common base BC is hypotenuse!

SOLVED EXAMPLE 11 [Board Standard]

Question: Two sides AB and BC and median AM of $\triangle ABC$ are respectively equal to sides PQ and QR and median PN of $\triangle PQR$. Show that $\triangle ABC \cong \triangle PQR$.

Given & Required: $AB = PQ, BC = QR, AM = PN$. | **Concept & Formula:** Two-stage SSS $\rightarrow$ SAS.

Detailed Solution: Since $BC = QR$, $BM = \frac{1}{2}BC = \frac{1}{2}QR = QN$. In $\triangle ABM$ and $\triangle PQN$: $AB = PQ, BM = QN, AM = PN \implies \triangle ABM \cong \triangle PQN$ (SSS) $\implies \angle B = \angle Q$ (CPCTC). Now in $\triangle ABC$ and $\triangle PQR$: $AB = PQ, \angle B = \angle Q, BC = QR \implies \triangle ABC \cong \triangle PQR$ (SAS). **Final Answer: Proved**

Fastest Alternative / Shortcut: Halve BC, QR first.

SOLVED EXAMPLE 12 [Board Standard]

Question: $\triangle ABC$ is an isosceles triangle with $AB = AC$. Side BA is produced to D such that $AD = AB$. Show that $\angle BCD = 90^\circ$.

Given & Required: $AB = AC = AD$. | **Concept & Formula:** Isosceles angles + triangle sum.

Detailed Solution: In $\triangle ABC$, $AB = AC \implies \angle ACB = \angle ABC$. In $\triangle ACD$, $AC = AD \implies \angle ACD = \angle ADC$. In $\triangle BCD$: $\angle B + (\angle ACB + \angle ACD) + \angle D = 180^\circ \implies 2(\angle ACB + \angle ACD) = 180^\circ \implies \angle BCD = 90^\circ$. **Final Answer: Proved (90°)**

Fastest Alternative / Shortcut: $2\angle BCD = 180^\circ$.

SOLVED EXAMPLE 13 [Board Standard]

Question: $\triangle ABC$ and $\triangle DBC$ are two isosceles triangles on the same base BC ($AB = AC, DB = DC$). Show that $\angle ABD = \angle ACD$.

Given & Required: $AB = AC, DB = DC$. | **Concept & Formula:** Join AD for SSS (or subtract base angles).

Detailed Solution: In $\triangle ABD$ and $\triangle ACD$: $AB = AC, BD = CD, AD = AD \implies \triangle ABD \cong \triangle ACD$ (SSS) $\implies \angle ABD = \angle ACD$ (CPCTC). **Final Answer: Proved**

Fastest Alternative / Shortcut: Join AD for 3-line SSS!

SOLVED EXAMPLE 14 [Challenge]

Question: In right $\triangle ABC$ ($\angle C = 90^\circ$), M is midpoint of AB and $CM = DM$ (Ex. 7). Prove $\angle DBC = 90^\circ$ and $CM = \frac{1}{2}AB$.

Given & Required: $\triangle AMC \cong \triangle BMD$. | **Concept & Formula:** Alternate interior + SAS.

Detailed Solution: $\angle ACM = \angle BDM$ (CPCTC) $\implies AC \parallel BD \implies \angle DBC + \angle ACB = 180^\circ \implies \angle DBC = 90^\circ$. Then $\triangle DBC \cong \triangle ACB$ (SAS) $\implies DC = AB \implies CM = \frac{1}{2}AB$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Median to hypotenuse $= \frac{1}{2}$ hypotenuse!

SOLVED EXAMPLE 15 [Challenge]

Question: If E and F are midpoints of equal sides AB and AC of $\triangle ABC$, show that $BF = CE$.

Given & Required: $AB = AC, AE = EB, AF = FC$. | **Concept & Formula:** SAS with common $\angle A$.

Detailed Solution: $AE = \frac{1}{2}AB = \frac{1}{2}AC = AF$. In $\triangle ABF$ and $\triangle ACE$: $AB = AC, \angle A = \angle A, AF = AE \implies \triangle ABF \cong \triangle ACE$ (SAS) $\implies BF = CE$. **Final Answer: Proved ($BF = CE$)**

Fastest Alternative / Shortcut: Shared $\angle A$.

SOLVED EXAMPLE 16 [Challenge]

Question: In $\triangle ABC$, $AB = AC$ and $\angle A = 100^\circ$. If BD bisects $\angle B$, find $\angle BDC$.

Given & Required: $AB = AC, \angle A = 100^\circ$. | **Concept & Formula:** Isosceles + triangle sum.

Detailed Solution: $\angle B = \angle C = 40^\circ$. Since BD bisects $\angle B$, $\angle DBC = 20^\circ$. In $\triangle BDC$: $\angle BDC = 180^\circ - (20^\circ + 40^\circ) = 120^\circ$.

Final Answer: 120°

Fastest Alternative / Shortcut: $180 - 60 = 120^\circ$.

G. Competency MCQ Mastery Section

Q1. Which of the following is NOT a congruence criterion?

(A) SAS (B) ASA (C) SSA (D) RHS

Answer: (C) SSA (or ASS) does not guarantee congruence. *Fast Method:* Blacklist: SSA, AAA. *Distractor Note:* Confusing SSA with SAS.

Q2. If $\triangle ABC \cong \triangle PQR$, then BC equals:

(A) PQ (B) QR (C) PR (D) AC

Answer: (B) Positions 2 and 3 give $BC = QR$. *Fast Method:* Letter position match. *Distractor Note:* Picking PQ.

Q3. In $\triangle ABC$, $BC = AB$ and $\angle B = 80^\circ$. Then $\angle A$ equals:

(A) 80° (B) 40° (C) 50° (D) 100°

Answer: (C) $\angle A = \angle C = \frac{180^\circ - 80^\circ}{2} = 50^\circ$. *Fast Method:* $(180 - 80)/2$. *Distractor Note:* Saying 80°.

Q4. Each angle of an equilateral triangle is:

(A) 45° (B) 60° (C) 90° (D) 120°

Answer: (B) $180^\circ/3 = 60^\circ$. *Fast Method:* 60°. *Distractor Note:* None.

Q5. In $\triangle PQR$, $\angle R = \angle P$, $QR = 4$ cm, $PR = 5$ cm. Length of PQ is:

(A) 4 cm (B) 5 cm (C) 2 cm (D) 2.5 cm

Answer: (A) Sides opposite $\angle R$ (PQ) and $\angle P$ (QR) are equal: 4 cm. *Fast Method:* $PQ = QR = 4$. *Distractor Note:* Picking $PR = 5$.

Q6. If $\triangle ABC \cong \triangle FDE$ with $AB = 5$ cm, $\angle B = 40^\circ, \angle A = 80^\circ$, then DF and $\angle E$ are:

(A) 5 cm, 60° (B) 5 cm, 80° (C) 5 cm, 40° (D) Cannot say

Answer: (A) $DF = AB = 5$, $\angle E = \angle C = 60^\circ$. *Fast Method:* Position match. *Distractor Note:* Picking 80°.

Q7. Two right triangles are congruent if hypotenuse and one side are equal by:

(A) SAS (B) SSS (C) RHS (D) ASA

Answer: (C) Definition of RHS. *Fast Method:* RHS. *Distractor Note:* Saying SAS.

Q8. If altitudes BE and CF of $\triangle ABC$ are equal, then $\triangle ABC$ is:

(A) Scalene (B) Isosceles (C) Right-angled (D) Obtuse

Answer: (B) $AB = AC$ by RHS on $\triangle BEC, \triangle CFB$. *Fast Method:* Isosceles. *Distractor Note:* Saying equilateral.

Q9. If $\triangle ABC \cong \triangle ACB$, then $\triangle ABC$ is isosceles with:

(A) $AB = BC$ (B) $AB = AC$ (C) $BC = AC$ (D) None

Answer: (B) Positions $(1, 2)$ and $(1, 3)$ give $AB = AC$. *Fast Method:* Swap $B \leftrightarrow C \implies AB = AC$. *Distractor Note:* Picking $AB = BC$.

Q10. In isosceles $\triangle ABC$ ($AB = AC$), if $\angle B = 65^\circ$, then $\angle A$ is:

(A) 65° (B) 50° (C) 130° (D) 40°

Answer: (B) $180^\circ - 2(65^\circ) = 50^\circ$. *Fast Method:* $180 - 130 = 50^\circ$. *Distractor Note:* Saying 65°.

Q11. Which rule CANNOT prove congruence of two triangles?

(A) AAS (B) SSS (C) AAA (D) ASA

Answer: (C) AAA only proves similarity, not congruence. *Fast Method:* AAA fails. *Distractor Note:* Thinking AAS fails.

Q12. In $\triangle ABC$, if $AB = AC$ and D is midpoint of BC, then $\angle ADB$ is:

(A) 45° (B) 60° (C) 90° (D) 120°

Answer: (C) $\triangle ABD \cong \triangle ACD$ (SSS) $\implies \angle ADB = \angle ADC = 90^\circ$. *Fast Method:* Median to base is $\perp$. *Distractor Note:* Picking 60°.

H. Special Exam & MCQ Strategies

- Before writing a congruence proof, mark equal sides with ticks and equal angles with arcs on the diagram.
- Always write the vertex correspondence in exact order (e.g., $\triangle ABD \cong \triangle ACD$), then read CPCTC pairs directly.
- Remember the 3 hidden equalities: Common side ($AD = AD$), Common angle ($\angle A = \angle A$), and Vertically Opposite Angles.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Writing 'By SSA congruence' or 'By AAA congruence'. $\longrightarrow$ **Correction:** SSA and AAA are INVALID! Use only SAS, ASA, AAS, SSS, or RHS.

[COMMON MISTAKE] **Error:** Using RHS when two legs of right triangles are equal (not the hypotenuse). $\longrightarrow$ **Correction:** Equal legs with 90° between them is SAS, not RHS.

[COMMON MISTAKE] **Error:** Mixing up vertex order in $\triangle ABC \cong \triangle PQR$. $\longrightarrow$ **Correction:** Verify that $A \leftrightarrow P, B \leftrightarrow Q, C \leftrightarrow R$ match the equal angles/sides.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: Valid rules: SAS, ASA, AAS, SSS, RHS; Invalid: SSA, AAA.

[QUICK REVISION] Key Theorems & Results: $AB = AC \iff \angle B = \angle C$; Equilateral $\implies 60^\circ$; CPCTC gives remaining equal parts.

[QUICK REVISION] Shortcut Summary: Position-match vertex letters; Base angle $= 90^\circ - A/2$; Angle bisector equidistant from arms.

[QUICK REVISION] High-Frequency Exam Patterns: Single-pair congruence + CPCTC, Common-angle addition, Two-stage median SSS→SAS, Equal altitudes RHS.

Last-Minute Revision Checklist:

- ☐ Never use SSA or AAA
- ☐ Exact vertex order
- ☐ Check common side/angle/VOA
- ☐ Cite CPCTC for final step

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Which is NOT a valid congruence rule? (A) SAS (B) AAA (C) SSS (D) AAS
2. In $\triangle ABC$, $AB = AC$ and $\angle A = 70^\circ$. Value of $\angle B$: (A) 70° (B) 55° (C) 110° (D) 35°
3. If $\triangle PQR \cong \triangle XYZ$, then $\angle Q$ equals: (A) $\angle X$ (B) $\angle Y$ (C) $\angle Z$ (D) $\angle P$
4. Number of equal sides in an equilateral triangle: (A) 1 (B) 2 (C) 3 (D) 0

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. In $\triangle ABC$ and $\triangle DEF$, $AB = DE, BC = EF$. For SAS, we need: (A) $\angle A = \angle D$ (B) $\angle B = \angle E$ (C) $\angle C = \angle F$ (D) $AC = DF$
2. In $\triangle ABC$, $AB = AC$ and $\angle B = 48^\circ$. Then $\angle A$ is: (A) 48° (B) 84° (C) 96° (D) 132°
3. If $\triangle ABC \cong \triangle PQR$ and $AB = 6, BC = 8, AC = 10$, perimeter of $\triangle PQR$ is: (A) 14 (B) 18 (C) 24 (D) 48
4. Altitude AD of isosceles $\triangle ABC$ ($AB = AC$) divides it into 2 congruent triangles by: (A) RHS (B) SSA (C) AAA (D) None

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. In $\triangle ABC$, $AB = AC$ and B is produced to D so $BD = BC$. If $\angle A = 40^\circ$, $\angle ACD$ is: (A) 105° (B) 70° (C) 35° (D) 110°
2. If $\triangle ABC \cong \triangle BCA$, then $\triangle ABC$ is: (A) Scalene (B) Isosceles only (C) Equilateral (D) Right-angled
3. In right $\triangle ABC$ ($\angle B = 90^\circ$), M is midpoint of AC and $AC = 10$ cm. Length of BM is: (A) 5 cm (B) 10 cm (C) 2.5 cm (D) 4 cm
4. Angle between two medians to equal sides of an equilateral triangle is: (A) 30° (B) 60° (C) 90° (D) 120°

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** AAA proves similarity, not congruence. **Q2 (B):** $(180-70)/2 = 55°$. **Q3 (B):** Middle letter $Q \leftrightarrow Y$. **Q4 (C):** All 3 sides equal.
Exam Std	**Q1 (B):** Included angle between AB, BC is $\angle B$. **Q2 (B):** $180-96=84°$. **Q3 (C):** $6+8+10=24$. **Q4 (A):** $90°$, Hypotenuse $AB=AC$, Common AD.
Challenge	**Q1 (A):** $\angle B = \angle ACB = 70°$; in $\triangle BCD$, $\angle D = \angle BCD = 35° \implies \angle ACD = 70+35 = 105°$. **Q2 (C):** $AB=BC$ and $BC=CA \implies AB=BC=CA$. **Q3 (A):** Median to hypotenuse $= \frac{1}{2}AC = 5$ cm. **Q4 (B):** Acute angle $= 60°$ (obtuse $120°$; standard acute $60°$).

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Congruence & SAS/ASA/AAS. Day 2: Isosceles theorems. Day 3: SSS/RHS & multi-stage proofs.
- **How to Introduce the Concept:** Hold two identical A4 sheets—when superposed, every corner and edge matches. That is congruence!
- **Difficult Areas & Remediation:** Why SSA fails and matching vertex order in $\triangle ABC \cong \triangle PQR$.
- **Blackboard Teaching Examples:** Draw two triangles with SSA (one acute, one obtuse) to show visually why SSA is invalid.
- **Classroom Questions to Ask Students:** Why does AAS work if ASA is already true?
- **Common Student Doubts & Answers:** Students ask about $AB > AC \implies \angle C > \angle B$; clarify Triangle Inequalities are excluded in 2026–27.
- **Homework & Revision Strategy:** Assign 5 congruence proofs (1 from each rule: SAS, ASA, AAS, SSS, RHS).

[TEACHER'S TIP] Always insist students write the reason in brackets after every single line of a geometry proof.

UNIT IV: GEOMETRY TYBA COACHING CENTRE

Chapter 8: Quadrilaterals

Properties of Parallelograms, Rectangles, Rhombuses, Squares & Mid-point Theorem (2026–27)

A. Chapter Introduction & Examination Relevance

Four-sided polygons—especially parallelograms, rectangles, rhombuses, and squares—are everywhere in architecture and design. Chapter 8 proves the exact properties of parallelograms using Chapter 7 congruence and establishes the famous Mid-point Theorem.

- **What You Will Learn:** Properties of a parallelogram (opposite sides equal, opposite angles equal, diagonals bisect each other), sufficient conditions for a parallelogram, special parallelograms (rectangle, rhombus, square), and the Mid-point Theorem + its converse.
- **Why This Chapter Matters:** The Mid-point Theorem is a favorite 5-mark proof in Class 9 and directly prepares students for the Basic Proportionality Theorem (Thales' Theorem) in Class 10.
- **Real-Life Applications:** Scissor-lift mechanisms (parallelogram linkages), tiling & tessellations, structural bracing, and surveying.
- **Cross-Chapter Connections:** Uses Chapter 6 (parallel lines) and Chapter 7 (triangle congruence) heavily; feeds into Class 10 Similar Triangles.
- **Exam & Foundation Relevance:** Unit IV carries 28/100 marks in RBSE (25/80 in CBSE). Expect 1M quadrilateral-property MCQs and 3M/5M Mid-point Theorem applications (e.g., Varignon's theorem: mid-points of any quadrilateral form a parallelogram). (Angle-sum proof of quadrilateral is excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Properties of a Parallelogram

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel. Core theorems: (1) A diagonal divides it into two congruent triangles; (2) Opposite sides are equal; (3) Opposite angles are equal; (4) Diagonals bisect each other; (5) Adjacent angles are supplementary ($180°$).

[CONCEPT MASTER – 1. Properties of a Parallelogram] Golden Criterion (Theorem 8.8): A quadrilateral is a parallelogram if ONE pair of opposite sides is BOTH **equal and parallel**!

2. Special Parallelograms: Rectangle, Rhombus & Square

Rectangle: Parallelogram with one $90°$ angle $\implies$ all angles $90°$ and **diagonals are equal**. **Rhombus:** Parallelogram with all sides equal $\implies$ **diagonals bisect each other at** $90°$. **Square:** Both a rectangle and a rhombus $\implies$ all sides equal, all angles $90°$, and diagonals are **equal AND perpendicular bisectors**.

[CONCEPT MASTER – 2. Special Parallelograms: Rectangle, Rhombus & Square] Diagonal Summary: Parallelogram (bisect), Rectangle (bisect + equal), Rhombus (bisect + $90°$), Square (bisect + equal + $90°$).

3. The Mid-point Theorem (Theorem 8.8 / 8.9)

The line segment joining the midpoints of any two sides of a triangle is **parallel to the third side** and **equal to half of it**: if E, F are midpoints of AB, AC in $\triangle ABC$, then $EF \parallel BC$ and $EF = \frac{1}{2}BC$.

[CONCEPT MASTER – 3. The Mid-point Theorem (Theorem 8.8 / 8.9)] Proof Idea: Extend EF to D with $FD = EF$, join CD, prove $\triangle AEF \cong \triangle CDF$ (SAS) to show $BCDE$ is a parallelogram!

4. Converse of Mid-point Theorem & Varignon's Application

Converse: A line drawn through the midpoint of one side of a triangle, parallel to another side, **bisects the third side**. Applying the Mid-point Theorem twice to the diagonals of ANY quadrilateral proves that joining the midpoints of its four sides always forms a **parallelogram**!

[CONCEPT MASTER – 4. Converse of Mid-point Theorem & Varignon's Application] Rationalised 2026–27 Scope: The formal theorem proof of Angle Sum Property of a Quadrilateral ($360°$) is excluded, though the $360°$ sum fact is still used.

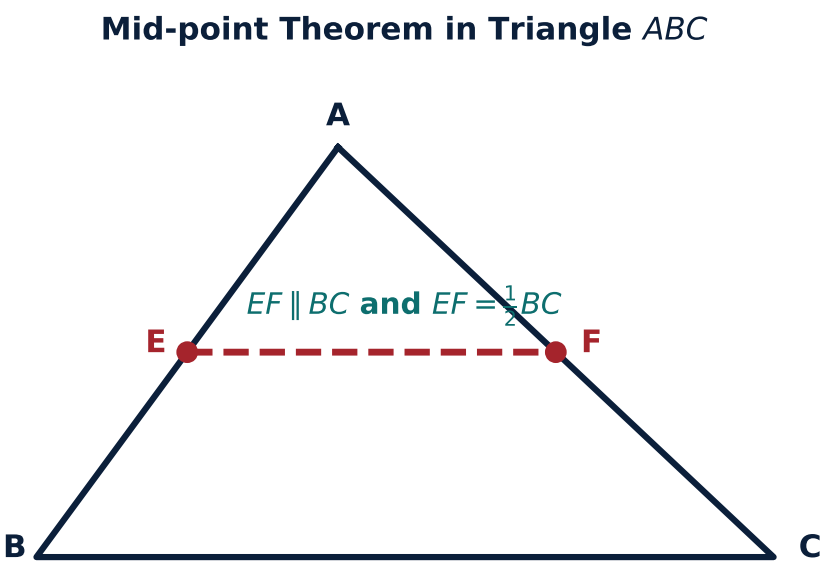


Figure 1: **Visual Model:** Mid-point Theorem in $\triangle ABC$: the segment joining midpoints E, F of two sides is parallel to BC and half its length.

C. Complete Formula Bank

[FORMULA BANK – 1. Quadrilateral Angle Sum] **Expression:** $\angle A + \angle B + \angle C + \angle D = 360°$
Symbols: Interior angles.
Conditions & When to Use: All simple quadrilaterals.
Solved Mini-Example: Ratio $3:5:9:13 \implies 30x = 360° \implies x = 12°$.
Common Mistake: Using $180°$ instead of $360°$.

[FORMULA BANK – 2. Parallelogram Adjacent Angles] **Expression:** $\angle A + \angle B = 180°$; $\angle A = \angle C,\ \angle B = \angle D$
Symbols: Consecutive & opposite angles.
Conditions & When to Use: Any parallelogram.
Solved Mini-Example: If one angle is $70°$, the four angles are $70°, 110°, 70°, 110°$.
Common Mistake: Equating adjacent angles.

[FORMULA BANK – 3. One-Pair Sufficient Test] **Expression:** $AB \parallel CD$ and $AB = CD \implies ABCD$ is a parallelogram
Symbols: Same pair of opposite sides.
Conditions & When to Use: Very useful in proofs.
Solved Mini-Example: Used in the Mid-point Theorem proof.
Common Mistake: Using $AB \parallel CD$ and $AD = BC$ (that could be an isosceles trapezium!).

[FORMULA BANK – 4. Mid-point Theorem] **Expression:** $AE = EB,\ AF = FC \implies EF \parallel BC$ and $EF = \frac{1}{2}BC$
Symbols: E, F: midpoints of AB, AC.
Conditions & When to Use: Any triangle ABC.
Solved Mini-Example: If $BC = 14$ cm, then $EF = 7$ cm.
Common Mistake: Writing $EF = 2BC$.

[FORMULA BANK – 5. Medial Triangle Perimeter] **Expression:** $\text{Perimeter}(\triangle DEF) = \frac{1}{2}\text{Perimeter}(\triangle ABC)$
Symbols: D, E, F: midpoints of BC, CA, AB.
Conditions & When to Use: Applies Mid-point Theorem to all 3 sides.
Solved Mini-Example: If $\triangle ABC$ has sides $6, 8, 10$, $\triangle DEF$ has perimeter $\frac{24}{2} = 12$.
Common Mistake: Forgetting Area is $\frac{1}{4}$, while Perimeter is $\frac{1}{2}$.

[FORMULA BANK – 6. Rhombus Area from Diagonals] **Expression:** $\text{Area} = \frac{1}{2}d_1 d_2$; Side $a = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$
Symbols: d_1, d_2: diagonals.
Conditions & When to Use: Diagonals intersect at $90°$.
Solved Mini-Example: Diagonals $6, 8 \implies$ Side $= \sqrt{3^2 + 4^2} = 5$, Area $= 24$.
Common Mistake: Using $\sqrt{d_1^2 + d_2^2}$ without halving.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Olympiad/NTSE Foundation problem structures.

Pattern 1: Angle Ratios & Bisector Angles in Parallelograms

- **Identification Cue:** Angles of a quadrilateral are in ratio $3 : 5 : 9 : 13$, or prove bisectors of parallelogram angles form a rectangle. | **Concept & Formula:** Sum $= 360°$; adjacent $= 180°$.
- **Standard Step-by-Step:** Halve adjacent angles $\frac{A}{2} + \frac{B}{2} = 90°$, so the interior angle of the bisector quad is $90°$.
- **Fastest Valid Method:** Bisectors of adjacent angles of a parallelogram ALWAYS meet at $90°$. | **Alternative:** Use co-interior angles.
- **Common Trap:** Assuming the bisectors form a square instead of a rectangle.
- **Practice Drill:** Ex. 1 & Ex. 6.

Pattern 2: Proving Rectangle / Rhombus / Square from Diagonals

- **Identification Cue:** Show that if diagonals of a quadrilateral bisect each other at right angles, it is a rhombus. | **Concept & Formula:** Two-triangle congruence (SAS) + parallelogram test.
- **Standard Step-by-Step:** Prove opposite triangles congruent (for parallelogram), then adjacent triangles congruent (for equal sides).
- **Fastest Valid Method:** Check both pairs: opposite for $\parallel$, adjacent for equal sides. | **Alternative:** Pythagoras on half-diagonals.
- **Common Trap:** Forgetting to prove it is a parallelogram first before calling it a rectangle.
- **Practice Drill:** Ex. 7 & Ex. 8.

Pattern 3: Mid-point Theorem Proofs (Varignon's Parallelogram)

- **Identification Cue:** Show that the line segments joining the midpoints of opposite sides of a quadrilateral bisect each other (or midpoints form a parallelogram/rhombus/rectangle). | **Concept & Formula:** Draw diagonal AC, apply Mid-point Theorem to $\triangle ABC$ and $\triangle ADC$.
- **Standard Step-by-Step:** Show $PQ \parallel AC, PQ = \frac{1}{2}AC$ and $SR \parallel AC, SR = \frac{1}{2}AC \implies PQ \parallel SR$ and $PQ = SR$.
- **Fastest Valid Method:** To show midpoints of a rectangle form a rhombus, just add $AC = BD \implies PQ = QR$! | **Alternative:** Use both diagonals AC, BD.
- **Common Trap:** Not drawing the diagonal first.
- **Practice Drill:** Ex. 10 & Ex. 11.

Pattern 4: Converse of Mid-point Theorem (Trapezium / Median Trisection)

- **Identification Cue:** In trapezium $ABCD$ ($AB \parallel DC$), E is midpoint of AD and $EF \parallel AB$; prove F is midpoint of BC. Or trisecting diagonal BD. | **Concept & Formula:** Draw diagonal BD to split trapezium into 2 triangles.
- **Standard Step-by-Step:** Apply Converse of Mid-point Theorem in $\triangle ABD$ (midpoint G on BD), then again in $\triangle BCD$ (midpoint F on BC).
- **Fastest Valid Method:** Always draw a diagonal to turn a trapezium into two triangles! | **Alternative:** Intercept theorem.
- **Common Trap:** Applying Mid-point Theorem directly to a 4-sided trapezium.
- **Practice Drill:** Ex. 12 & Ex. 13.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Quadrilateral Midpoint Transformation Table] **Core Idea & Why It Works:** Joining the midpoints of consecutive sides of: ANY Quad $\rightarrow$ Parallelogram; Rectangle $\rightarrow$ Rhombus; Rhombus $\rightarrow$ Rectangle; Square $\rightarrow$ Square!

Valid Domain: All 4 standard quadrilaterals. | **Where It Fails:** Nothing.

Easy Example: Rectangle has equal diagonals ($AC = BD$) $\implies$ midpoint quad has equal sides ($\frac{1}{2}AC = \frac{1}{2}BD$) $\implies$ Rhombus!

Exam-Level Example: Answers 4 classic MCQs in 1 second!

Advanced Example: Notice Rectangle $\leftrightarrow$ Rhombus swap!

[SMART SHORTCUT – 2. Diagonal Trisection via Midpoint Converse] **Core Idea & Why It Works:** In parallelogram $ABCD$ with midpoints E, F of opposite sides AB, CD, segments DE and BF trisect diagonal AC ($AP = PQ = QC$).

Valid Domain: All parallelograms. | **Where It Fails:** Non-parallelograms.

Easy Example: First prove $BFDE$ is a parallelogram, then apply Converse of Mid-point Theorem in $\triangle ABP$ and $\triangle DCQ$.

Exam-Level Example: Instant check for NCERT Ex. 8.2 Q5.

Advanced Example: Each piece is $\frac{1}{3}AC$.

[SMART SHORTCUT – 3. Rhombus Side from Diagonals] **Core Idea & Why It Works:** Since diagonals bisect at 90°, side $= \sqrt{(d_1/2)^2 + (d_2/2)^2}$.

Valid Domain: All rhombuses and squares. | **Where It Fails:** Rectangles (diagonals don't meet at 90°).

Easy Example: $d_1 = 12, d_2 = 16 \implies$ side $= \sqrt{6^2 + 8^2} = 10$.

Exam-Level Example: Instant perimeter $= 40$ cm.

Advanced Example: 3-4-5 and 5-12-13 triples appear often!

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: The angles of a quadrilateral are in the ratio $3 : 5 : 9 : 13$. Find all the angles.

Given & Required: Ratio $3 : 5 : 9 : 13$. | **Concept & Formula:** Sum $= 360^\circ$.

Detailed Solution: $3x + 5x + 9x + 13x = 360^\circ \implies 30x = 360^\circ \implies x = 12^\circ$. Angles: $36^\circ, 60^\circ, 108^\circ, 156^\circ$. **Final Answer:** $36^\circ, 60^\circ, 108^\circ, 156^\circ$

Fastest Alternative / Shortcut: $360/30 = 12$.

SOLVED EXAMPLE 2 [Beginner]

Question: In parallelogram $ABCD$, $\angle A = 75^\circ$. Find $\angle B, \angle C, \angle D$.

Given & Required: $\angle A = 75^\circ$. | **Concept & Formula:** Opposite equal, adjacent 180°.

Detailed Solution: $\angle C = 75^\circ$, $\angle B = \angle D = 180^\circ - 75^\circ = 105^\circ$. **Final Answer:** $105^\circ, 75^\circ, 105^\circ$

Fastest Alternative / Shortcut: Supplement $= 105^\circ$.

SOLVED EXAMPLE 3 [Beginner]

Question: In $\triangle ABC$, D, E are midpoints of AB, AC. If $BC = 12.6$ cm, find DE.

Given & Required: $BC = 12.6$ cm. | **Concept & Formula:** Mid-point Theorem.

Detailed Solution: $DE = \frac{1}{2}BC = 6.3$ cm. **Final Answer:** 6.3 **cm**

Fastest Alternative / Shortcut: Half of 12.6.

SOLVED EXAMPLE 4 [Beginner]

Question: If the diagonals of a rhombus are 16 cm and 12 cm, find its side length.

Given & Required: $d_1 = 16, d_2 = 12$. | **Concept & Formula:** Diagonals bisect at $90°$.

Detailed Solution: Half-diagonals are 8 and 6. Side $= \sqrt{8^2 + 6^2} = 10$ cm. **Final Answer:** 10 **cm**

Fastest Alternative / Shortcut: 6-8-10 triple.

SOLVED EXAMPLE 5 [Intermediate]

Question: If the diagonals of a parallelogram are equal, show that it is a rectangle.

Given & Required: Parallelogram $ABCD$ with $AC = BD$. | **Concept & Formula:** SSS congruence + co-interior.

Detailed Solution: In $\triangle ABC$ and $\triangle DCB$: $AB = DC, BC = CB, AC = DB \implies \triangle ABC \cong \triangle DCB$ (SSS) $\implies \angle B = \angle C$. Since $AB \parallel DC$, $\angle B + \angle C = 180° \implies \angle B = \angle C = 90°$. Hence rectangle. **Final Answer: Proved**

Fastest Alternative / Shortcut: SSS on $\triangle ABC, \triangle DCB$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Show that the diagonals of a rhombus are perpendicular to each other.

Given & Required: Rhombus $ABCD$ ($AB = BC = CD = DA$). | **Concept & Formula:** SSS on adjacent $\triangle AOB, \triangle COB$.

Detailed Solution: Diagonals bisect at O, so $AO = CO$. Also $AB = CB$ and $BO = BO$. By SSS, $\triangle AOB \cong \triangle COB \implies \angle AOB = \angle COB = 90°$ (linear pair). **Final Answer: Proved** ($90°$)

Fastest Alternative / Shortcut: Adjacent triangles by SSS.

SOLVED EXAMPLE 7 [Intermediate]

Question: Diagonal AC of a parallelogram $ABCD$ bisects $\angle A$. Show that it bisects $\angle C$ also, and $ABCD$ is a rhombus.

Given & Required: $\angle DAC = \angle BAC$. | **Concept & Formula:** Alternate interior + isosceles.

Detailed Solution: $\angle DCA = \angle BAC = \angle DAC = \angle BCA$ (alt. int.), so AC bisects $\angle C$. In $\triangle ABC$, $\angle BAC = \angle BCA \implies AB = BC$, hence all sides equal (rhombus). **Final Answer: Proved**

Fastest Alternative / Shortcut: Alt. interior gives equal base angles.

SOLVED EXAMPLE 8 [Intermediate]

Question: In $\triangle ABC$, D, E, F are midpoints of AB, BC, CA. If $AB = 8, BC = 10, CA = 12$, find the perimeter of $\triangle DEF$.

Given & Required: Sides $8, 10, 12$. | **Concept & Formula:** Mid-point Theorem.

Detailed Solution: $DE = 6, EF = 4, FD = 5$. Perimeter $= 6 + 4 + 5 = 15$ cm. **Final Answer:** 15 **cm**

Fastest Alternative / Shortcut: Half of $8 + 10 + 12 = 30/2 = 15$.

SOLVED EXAMPLE 9 [Board Standard]

Question: $ABCD$ is a rectangle and P, Q, R, S are midpoints of sides AB, BC, CD, DA. Show that $PQRS$ is a rhombus.

Given & Required: Rectangle $ABCD$ ($AC = BD$). | **Concept & Formula:** Mid-point Theorem on both diagonals.

Detailed Solution: In $\triangle ABC$, $PQ \parallel AC, PQ = \frac{1}{2}AC$; in $\triangle ADC$, $SR \parallel AC, SR = \frac{1}{2}AC \implies PQRS$ is a parallelogram. Similarly $QR = \frac{1}{2}BD$. Since $AC = BD$ (diagonals of rectangle), $PQ = QR$, so $PQRS$ is a rhombus. **Final Answer: Proved**

Fastest Alternative / Shortcut: Equal diagonals $\implies$ equal sides!

SOLVED EXAMPLE 10 [Board Standard]

Question: $ABCD$ is a rhombus and P, Q, R, S are midpoints of AB, BC, CD, DA. Show that $PQRS$ is a rectangle.

Given & Required: Rhombus $ABCD$ ($AC \perp BD$). | **Concept & Formula:** Mid-point Theorem + 90° angle.

Detailed Solution: $PQRS$ is a parallelogram (by Mid-point Theorem on AC). Since $PQ \parallel AC$ and $QR \parallel BD$, and $AC \perp BD$, we have $PQ \perp QR \implies \angle PQR = 90^\circ$. Hence rectangle. **Final Answer: Proved**

Fastest Alternative / Shortcut: Perpendicular diagonals $\implies 90^\circ$ corner!

SOLVED EXAMPLE 11 [Board Standard]

Question: $ABCD$ is a trapezium in which $AB \parallel DC$, BD is a diagonal, and E is the midpoint of AD. A line through E parallel to AB intersects BC at F. Show that F is the midpoint of BC.

Given & Required: $AB \parallel EF \parallel DC$, $AE = ED$. | **Concept & Formula:** Converse of Mid-point Theorem twice.

Detailed Solution: Let EF cut BD at G. In $\triangle DAB$, E is midpoint of AD and $EG \parallel AB \implies G$ is midpoint of BD. In $\triangle BCD$, G is midpoint of BD and $GF \parallel DC \implies F$ is midpoint of BC. **Final Answer: Proved**

Fastest Alternative / Shortcut: Apply Converse twice via diagonal BD.

SOLVED EXAMPLE 12 [Board Standard]

Question: Show that the bisectors of angles of a parallelogram form a rectangle.

Given & Required: Parallelogram $ABCD$. | **Concept & Formula:** Adjacent angles sum to 180°.

Detailed Solution: $\angle A + \angle B = 180^\circ \implies \frac{1}{2}\angle A + \frac{1}{2}\angle B = 90^\circ$. Thus in $\triangle ASB$, $\angle ASB = 90^\circ$, so VOA inside the bisector quad is 90°. Similarly all 4 angles are 90°, forming a rectangle. **Final Answer: Proved**

Fastest Alternative / Shortcut: Half of 180° is 90°.

SOLVED EXAMPLE 13 [Board Standard]

Question: In parallelogram $ABCD$, E and F are midpoints of AB and CD. Show that line segments AF and EC trisect diagonal BD.

Given & Required: $AE = \frac{1}{2}AB = \frac{1}{2}CD = FC$ and $AE \parallel FC$. | **Concept & Formula:** Parallelogram $AECF$ + Midpoint Converse.

Detailed Solution: $AECF$ is a parallelogram $\implies AF \parallel EC$. In $\triangle DQC$, F is midpoint of DC and $FP \parallel CQ \implies DP = PQ$. In $\triangle APB$, E is midpoint of AB and $EQ \parallel AP \implies PQ = QB$. Thus $DP = PQ = QB$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Prove $AECF$ is a parallelogram first.

SOLVED EXAMPLE 14 [Challenge]

Question: In $\triangle ABC$, AD is a median and E is the midpoint of AD. BE is produced to meet AC at F. Prove that $AF = \frac{1}{3}AC$.

Given & Required: $BD = DC$, $AE = ED$. | **Concept & Formula:** Draw $DG \parallel BF$ to AC.

Detailed Solution: Draw $DG \parallel EF$ (G on AC). In $\triangle ADG$, E is midpoint of AD and $EF \parallel DG \implies AF = FG$. In $\triangle BCF$, D is midpoint of BC and $DG \parallel BF \implies FG = GC$. Thus $AF = FG = GC = \frac{1}{3}AC$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Draw $DG \parallel BF$.

SOLVED EXAMPLE 15 [Challenge]

Question: Two parallel lines l and m are intersected by a transversal p. Show that the quadrilateral formed by the bisectors of interior angles is a rectangle.

Given & Required: $l \parallel m$. | **Concept & Formula:** Alt. interior + co-interior.

Detailed Solution: Equal half-alternate interior angles give parallel opposite sides (parallelogram). Half co-interior angles sum to 90°, giving a 90° corner $\implies$ rectangle. **Final Answer: Proved**

Fastest Alternative / Shortcut: Same logic as parallelogram bisectors.

SOLVED EXAMPLE 16 [Challenge]

Question: In a square $ABCD$, E is midpoint of AB and F is midpoint of BC. Find the angle between DE and AF.

Given & Required: Square $ABCD$. | **Concept & Formula:** SAS congruence $\triangle DAE \cong \triangle ABF$.

Detailed Solution: $DA = AB, \angle A = \angle B = 90^\circ, AE = BF \implies \triangle DAE \cong \triangle ABF$ (SAS) $\implies \angle ADE = \angle BAF$. Since $\angle BAF + \angle DAF = 90^\circ$, $\angle ADE + \angle DAF = 90^\circ \implies DE \perp AF$ (90°). **Final Answer:** 90°

Fastest Alternative / Shortcut: Rotating square by 90° maps $DE \to AF$!

G. Competency MCQ Mastery Section

Q1. Sum of all interior angles of a quadrilateral is:

(A) 180° (B) 270° (C) 360° (D) 540°

Answer: (C) $(4-2) \times 180^\circ = 360^\circ$. *Fast Method:* 360°. *Distractor Note:* Picking 180°.

Q2. Which quadrilateral has diagonals that are equal AND bisect each other at 90°?

(A) Parallelogram (B) Rectangle (C) Rhombus (D) Square

Answer: (D) Square has both rectangle (equal) and rhombus (90°) diagonal properties. *Fast Method:* Square = Rectangle + Rhombus. *Distractor Note:* Picking Rhombus.

Q3. Joining the midpoints of consecutive sides of a rectangle forms a:

(A) Rectangle (B) Rhombus (C) Square (D) Trapezium

Answer: (B) Equal diagonals of rectangle make all 4 midpoint sides equal. *Fast Method:* Rectangle $\to$ Rhombus. *Distractor Note:* Picking Rectangle.

Q4. In $\triangle ABC$, D, E are midpoints of AB, AC. If $DE = 4.5$ cm, then BC is:

(A) 4.5 cm (B) 9 cm (C) 2.25 cm (D) 13.5 cm

Answer: (B) $BC = 2 \cdot DE = 9$ cm. *Fast Method:* Double DE. *Distractor Note:* Halving 4.5.

Q5. Two adjacent angles of a parallelogram are $(2x+25)^\circ$ and $(3x-5)^\circ$. Value of x is:

(A) 28 (B) 32 (C) 36 (D) 40

Answer: (B) $5x + 20 = 180 \implies 5x = 160 \implies x = 32$. *Fast Method:* Adjacent sum $= 180^\circ$. *Distractor Note:* Equating them.

Q6. Diagonals of a rhombus bisect each other at:

(A) 45° (B) 60° (C) 90° (D) 180°

Answer: (C) Theorem: rhombus diagonals are perpendicular. *Fast Method:* 90°. *Distractor Note:* None.

Q7. A diagonal of a parallelogram divides it into two:

(A) Congruent triangles (B) Equilateral triangles (C) Right triangles (D) Isosceles triangles

Answer: (A) Theorem 8.1: ASA/SSS gives congruent triangles. *Fast Method:* Congruent. *Distractor Note:* Picking isosceles.

Q8. If opposite angles of a parallelogram are $(3x-2)^\circ$ and $(50-x)^\circ$, then x is:

(A) 13 (B) 12 (C) 15 (D) 37

Answer: (A) $3x - 2 = 50 - x \implies 4x = 52 \implies x = 13$. *Fast Method:* Opposite angles equal. *Distractor Note:* Adding to 180°.

Q9. Joining midpoints of consecutive sides of ANY quadrilateral always forms a:

(A) Trapezium (B) Parallelogram (C) Rhombus (D) Rectangle

Answer: (B) Varignon's theorem via Mid-point Theorem. *Fast Method:* Always a parallelogram. *Distractor Note:* Picking square.

Q10. If a quadrilateral has 3 angles $75^\circ, 90^\circ, 75^\circ$, the 4th angle is:

(A) 90° (B) 100° (C) 120° (D) 110°

Answer: (C) $360 - 240 = 120^\circ$. *Fast Method:* $360 - 240$. *Distractor Note:* Arithmetic slip.

Q11. In a square $ABCD$, if $AC = 10\sqrt{2}$ cm, each side is:

(A) 5 cm (B) 10 cm (C) 20 cm (D) $10\sqrt{2}$ cm

Answer: (B) Diagonal $= a\sqrt{2} = 10\sqrt{2} \implies a = 10$. *Fast Method:* $d/\sqrt{2}$. *Distractor Note:* Dividing by 2.

Q12. Bisectors of adjacent angles of a parallelogram intersect at:

(A) $45°$ (B) $60°$ (C) $90°$ (D) $120°$

Answer: (C) $\frac{A+B}{2} = 90°$. *Fast Method:* $90°$. *Distractor Note:* Picking $60°$.

H. Special Exam & MCQ Strategies

- To prove ANY quadrilateral is a parallelogram, show ONE pair of opposite sides is both equal and parallel.
- In any midpoint problem on a 4-sided figure, immediately draw ONE diagonal to split it into two triangles and apply the Mid-point Theorem.
- Keep the diagonal hierarchy clear: Parallelogram (bisect) $\rightarrow$ Rectangle (bisect + equal), Rhombus (bisect + $90°$), Square (all three).

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Assuming a quadrilateral is a parallelogram if one pair is parallel and the OTHER pair is equal. $\longrightarrow$ **Correction:** An isosceles trapezium has $AB \parallel CD$ and $AD = BC$! The SAME pair must be equal and parallel.

[COMMON MISTAKE] **Error:** Writing $EF = 2BC$ in the Mid-point Theorem. $\longrightarrow$ **Correction:** The segment joining midpoints is HALF the third side: $EF = \frac{1}{2}BC$.

[COMMON MISTAKE] **Error:** Saying diagonals of a rectangle intersect at $90°$. $\longrightarrow$ **Correction:** Rectangle diagonals are EQUAL, not perpendicular (unless it is also a square).

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $\sum$ angles $= 360^\circ$; Mid-point Thm: $EF \parallel BC, EF = \frac{1}{2}BC$; Rhombus Area $= \frac{1}{2}d_1 d_2$.

[QUICK REVISION] Key Theorems & Results: Parallelogram: opp. sides/angles $=$, diag bisect; Rect: diag $=$; Rhombus: diag $\perp$; Square: diag $=, \perp$.

[QUICK REVISION] Shortcut Summary: Midpoints of Quad $\rightarrow$ Pgm, Rect $\rightarrow$ Rhombus, Rhombus $\rightarrow$ Rect; Bisectors of Pgm $\rightarrow$ Rect.

[QUICK REVISION] High-Frequency Exam Patterns: Angle ratios, Rectangle/Rhombus diagonal proofs, Varignon midpoint quad, Trapezium midpoint converse.

Last-Minute Revision Checklist:

- ☐ Same pair equal and parallel
- ☐ Half of third side in Mid-point Thm
- ☐ Adjacent Pgm angles $= 180^\circ$
- ☐ Draw diagonal in quad midpoint proofs

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. In parallelogram $ABCD$, if $\angle A = 110^\circ$, then $\angle B$ is: (A) 110° (B) 70° (C) 90° (D) 55°
2. Midpoint segment DE in $\triangle ABC$ with $BC = 18$ cm is: (A) 36 cm (B) 9 cm (C) 6 cm (D) 18 cm
3. Angles of a quad are $60^\circ, 80^\circ, 100^\circ, x$. Value of x: (A) 100° (B) 120° (C) 140° (D) 80°
4. Diagonals of a rectangle are always: (A) Perpendicular (B) Equal (C) Unequal (D) Parallel

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Midpoints of consecutive sides of a rhombus form a: (A) Rhombus (B) Rectangle (C) Trapezium (D) Kite
2. Perimeter of triangle formed by joining midpoints of sides $10, 12, 14$: (A) 36 (B) 18 (C) 12 (D) 9
3. If diagonals of a rhombus are 10 cm and 24 cm, its perimeter is: (A) 52 cm (B) 26 cm (C) 68 cm (D) 48 cm
4. In parallelogram $ABCD$, $\angle A = 2\angle B$. Value of $\angle A$: (A) 60° (B) 120° (C) 90° (D) 100°

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Area of $\triangle DEF$ formed by midpoints of $\triangle ABC$ (Area 48 cm^2) is: (A) 24 cm^2 (B) 12 cm^2 (C) 16 cm^2 (D) 8 cm^2
2. In trapezium $ABCD$ ($AB \parallel DC$), $AB = 14, DC = 8$. Segment joining midpoints of AD, BC has length: (A) 11 (B) 6 (C) 22 (D) 10
3. If $ABCD$ is a square and $\triangle EDC$ is equilateral inside it, $\angle DAE$ is: (A) 15° (B) 30° (C) 45° (D) 75°
4. Bisectors of interior angles of a rectangle form a: (A) Rectangle (B) Rhombus (C) Square (D) Trapezium

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $180 - 110 = 70^\circ$. **Q2 (B):** $18/2 = 9$ cm. **Q3 (B):** $360 - 240 = 120^\circ$. **Q4 (B):** Equal and bisect each other.
Exam Std	**Q1 (B):** Perpendicular diagonals give 90° corners (Rectangle). **Q2 (B):** $(10 + 12 + 14)/2 = 18$. **Q3 (A):** Half-diags $5, 12 \implies$ side $13 \implies 4 \times 13 = 52$ cm. **Q4 (B):** $3\angle B = 180 \implies \angle B = 60 \implies \angle A = 120^\circ$.
Challenge	**Q1 (B):** 4 congruent sub-triangles: $48/4 = 12$ cm^2. **Q2 (A):** $\frac{AB+DC}{2} = \frac{14+8}{2} = 11$. **Q3 (D):** $\angle ADE = 30^\circ$, $AD = DE \implies \angle DAE = (180 - 30)/2 = 75^\circ$. **Q4 (C):** Symmetric 45°-45°-90° triangles form a Square.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Parallelogram properties & proofs. Day 2: Rectangle, rhombus, square. Day 3: Mid-point Theorem & converse.
- **How to Introduce the Concept:** Use a foldable parallelogram frame to show how pushing a rectangle tilts it into a general parallelogram while keeping opposite sides equal!
- **Difficult Areas & Remediation:** Varignon's midpoint quadrilateral proofs and drawing the construction line in Mid-point Theorem.
- **Blackboard Teaching Examples:** Draw the 4-box diagonal comparison chart (Parallelogram, Rectangle, Rhombus, Square).
- **Classroom Questions to Ask Students:** Why does $AB \parallel CD$ and $AD = BC$ NOT guarantee a parallelogram?
- **Common Student Doubts & Answers:** Students ask about the quadrilateral angle-sum proof; note it is excluded in 2026–27.
- **Homework & Revision Strategy:** Assign 3 diagonal-property proofs and 4 Mid-point Theorem problems.

[TEACHER'S TIP] Emphasize the Mid-point Theorem—it is tested in almost every Class 9 final exam.

UNIT IV: GEOMETRY TYBA COACHING CENTRE

Chapter 9: Circles

Chords, Perpendicular from Centre, Angle Subtended by Arc & Cyclic Quadrilaterals (2026–27)

A. Chapter Introduction & Examination Relevance

A circle is the most symmetric curve in geometry. Chapter 9 explores the exact relationships between chords, distances from the centre, angles subtended by arcs, and cyclic quadrilaterals.

- **What You Will Learn:** Equal chords subtend equal angles at the centre (and converse), perpendicular from centre bisects a chord (and converse), equal chords are equidistant from the centre, angle at centre $= 2\times$ angle at circumference, angles in the same segment, angle in a semicircle ($90°$), and cyclic quadrilateral properties.
- **Why This Chapter Matters:** Chapter 9 is the direct foundation for Class 10 Circles (Tangents to a Circle) and Class 11 Conic Sections.
- **Real-Life Applications:** Wheel & gear engineering, arch bridge design, satellite horizon geometry, and optical lens curvature.
- **Cross-Chapter Connections:** Heavily uses Chapter 7 (RHS/SSS congruence) and Chapter 8 (quadrilaterals); directly leads to Class 10 Chapter 10.
- **Exam & Foundation Relevance:** Unit IV carries 28/100 marks in RBSE (25/80 in CBSE). Circles is one of the highest-yield chapters for 1M angle-chasing MCQs and 3M/5M chord/cyclic proofs. (Circle through 3 non-collinear points proof is excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Chords & Perpendicular from the Centre

Theorem 9.1/9.2: Equal chords subtend equal angles at the centre (and conversely). **Theorem 9.3/9.4:** The perpendicular from the centre of a circle to a chord **bisects the chord** (and the line joining the centre to the midpoint of a chord is perpendicular to the chord). **Theorem 9.5/9.6:** Equal chords are equidistant from the centre (and conversely).

[CONCEPT MASTER – 1. Chords & Perpendicular from the Centre] Chord-Radius Triangle: For any chord of length c at distance d from centre of radius r, Pythagoras gives $r^2 = d^2 + \left(\frac{c}{2}\right)^2$!

2. Angle Subtended by an Arc (Central Angle Theorem)

Theorem 9.7: The angle subtended by an arc at the centre is **double** the angle subtended by it at any point on the remaining part of the circle: $\angle AOB = 2\angle APB$. Two vital corollaries: (1) **Angles in the same segment** of a circle are equal; (2) **Angle in a semicircle is a right angle ($90°$)**.

[CONCEPT MASTER – 2. Angle Subtended by an Arc (Central Angle Theorem)] Watch Reflex Angles! If P lies on the *minor* arc AB, then $\angle APB$ faces the *major* arc, so Reflex$(\angle AOB) = 2\angle APB$!

3. Cyclic Quadrilaterals

A quadrilateral $ABCD$ whose all four vertices lie on a circle is a **cyclic quadrilateral**. **Theorem 9.10:** The sum of either pair of opposite angles of a cyclic quadrilateral is $180°$ ($\angle A + \angle C = 180°$ and $\angle B + \angle D = 180°$). Conversely, if opposite angles sum to $180°$, the quadrilateral is cyclic.

[CONCEPT MASTER – 3. Cyclic Quadrilaterals] Exterior Angle Property: An exterior angle of a cyclic quadrilateral equals the interior opposite angle!

4. Classic Circle Theorems & Applications

Combining these results proves that a cyclic parallelogram is ALWAYS a **rectangle**, equal chords intersecting inside a circle make equal angles with the diameter through their intersection, and two intersecting circles have their line of centres as the perpendicular bisector of the common chord.

[CONCEPT MASTER – 4. Classic Circle Theorems & Applications] Rationalised 2026–27 Scope: The theorem 'There is one and only one circle passing through three non-collinear points' is excluded from the 2026–27 NCERT syllabus.

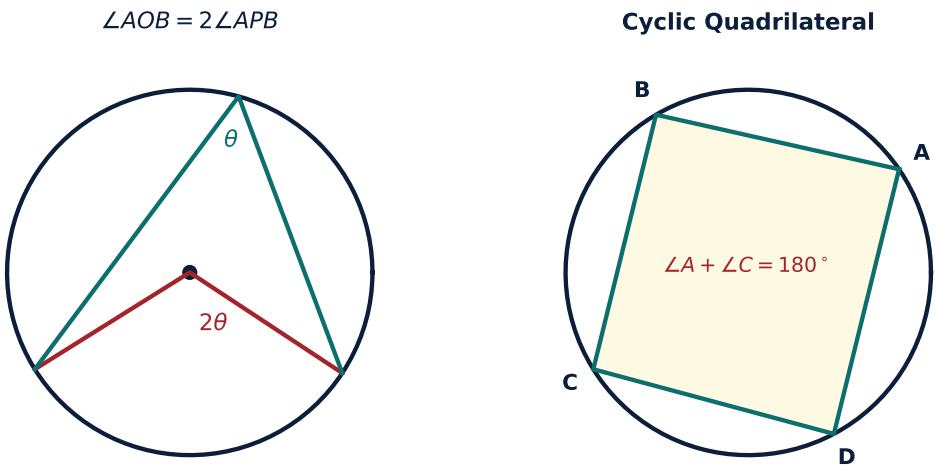


Figure 1: **Visual Model:** Central Angle Theorem ($\angle AOB = 2\angle APB$) and Cyclic Quadrilateral supplementary property ($\angle A + \angle C = 180°$).

C. Complete Formula Bank

[FORMULA BANK – 1. Chord-Radius-Distance Formula] **Expression:** $r^2 = d^2 + \left(\frac{c}{2}\right)^2$
Symbols: r: radius, d: $\perp$ distance from centre, c: chord length.
Conditions & When to Use: Perpendicular from centre bisects chord.
Solved Mini-Example: Radius 5 cm, chord 8 cm $\implies d = \sqrt{25-16} = 3$ cm.
Common Mistake: Forgetting to halve the chord c.

[FORMULA BANK – 2. Central Angle Theorem] **Expression:** $\angle AOB = 2\angle APB$
Symbols: O: centre, P: point on remaining arc.
Conditions & When to Use: Both angles subtended by the SAME arc AB.
Solved Mini-Example: If $\angle AOB = 100°$, then $\angle APB = 50°$.
Common Mistake: Halving instead of doubling when going from circumference to centre.

[FORMULA BANK – 3. Angle in a Semicircle] **Expression:** AB is diameter $\implies \angle ACB = 90°$
Symbols: C: any point on circle ($C \neq A, B$).
Conditions & When to Use: Half of $180°$ straight angle at O.
Solved Mini-Example: If AB is diameter and $\angle CAB = 35°$, then $\angle CBA = 55°$.
Common Mistake: Assuming any chord gives $90°$.

[FORMULA BANK – 4. Angles in the Same Segment] **Expression:** $\angle ACB = \angle ADB$
Symbols: C, D: points on the same arc.
Conditions & When to Use: Both equal $\frac{1}{2}\angle AOB$.
Solved Mini-Example: Used to prove cyclic points.
Common Mistake: Equating angles on opposite arcs (they sum to $180°$!).

[FORMULA BANK – 5. Cyclic Quadrilateral Sum] **Expression:** $\angle A + \angle C = 180^\circ$; $\angle B + \angle D = 180^\circ$
Symbols: A, B, C, D on a circle.
Conditions & When to Use: Opposite angles are supplementary.
Solved Mini-Example: If $\angle A = 75^\circ$, $\angle C = 105^\circ$.
Common Mistake: Setting opposite angles equal instead of supplementary.

[FORMULA BANK – 6. Two Circles Common Chord] **Expression:** $O_1O_2 \perp AB$ and O_1O_2 bisects AB
Symbols: O_1, O_2: centres, AB: common chord.
Conditions & When to Use: Intersecting circles.
Solved Mini-Example: Radii $5, 3$, distance $4 \implies$ common chord $= 6$ cm.
Common Mistake: Assuming O_1O_2 bisects the chord only when radii are equal.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Olympiad/NTSE Foundation problem structures.

Pattern 1: Chord-Radius Pythagoras & Parallel Chords Distance

- **Identification Cue:** Two parallel chords of lengths 6 cm and 8 cm in a circle of radius 5 cm; find distance between them. | **Concept & Formula:** $d = \sqrt{r^2 - (c/2)^2}$.
- **Standard Step-by-Step:** Compute d_1 and d_2 from centre; if on opposite sides of centre, distance $= d_1 + d_2$; if on same side, $|d_1 - d_2|$.
- **Fastest Valid Method:** 3-4-5 triple gives $d_1 = 4, d_2 = 3$; opposite sides $= 7$, same side $= 1$! | **Alternative:** Coordinate check.
- **Common Trap:** Giving only one case when 'same/opposite side' isn't specified.
- **Practice Drill:** Ex. 1 & Ex. 9.

Pattern 2: Central Angle & Same-Segment Angle Chasing

- **Identification Cue:** Find $\angle ADC$ or $\angle OPR$ given $\angle AOB, \angle BOC$, or $\angle PQR = 100^\circ$ on minor arc. | **Concept & Formula:** $\angle_{\text{centre}} = 2\angle_{\text{circ}}$ + isosceles $\triangle OAB$ ($OA = OB = r$).
- **Standard Step-by-Step:** Use $OA = OB = r$ for base angles, or Reflex$(\angle POR) = 2\angle PQR$ for minor-arc points.
- **Fastest Valid Method:** In $\triangle OAB$, $\angle OAB = 90^\circ - \frac{1}{2}\angle AOB = 90^\circ - \angle ACB$! | **Alternative:** Cyclic quad $180^\circ - \theta$.
- **Common Trap:** Using $\angle POR$ instead of Reflex$(\angle POR)$ when Q is on the minor arc.
- **Practice Drill:** Ex. 5 & Ex. 10.

Pattern 3: Cyclic Quadrilateral Angle & Proof Problems

- **Identification Cue:** Prove a cyclic parallelogram is a rectangle, or find $\angle BCD$ in cyclic quad $ABCD$ with diagonal angles given. | **Concept & Formula:** Same-segment angles + opposite sum $= 180^\circ$.
- **Standard Step-by-Step:** Combine $\angle A = \angle C$ (parallelogram) with $\angle A + \angle C = 180^\circ$ (cyclic) $\implies 2\angle A = 180^\circ \implies 90^\circ$.
- **Fastest Valid Method:** Use same-segment theorem first to transfer diagonal angles into one triangle. | **Alternative:** Exterior angle = interior opposite.
- **Common Trap:** Missing the same-segment transfer.
- **Practice Drill:** Ex. 7 & Ex. 11.

Pattern 4: Two-Circle Common Chord & Equal Chords Congruence

- **Identification Cue:** Two circles of radii 5 cm and 3 cm intersect at two points and distance between centres is 4 cm; find common chord length. | **Concept & Formula:** Right triangle O_1O_2A (3-4-5!).
- **Standard Step-by-Step:** Since O_1O_2 is $\perp$ bisector of AB, find altitude AM of $\triangle O_1O_2A$, then $AB = 2AM$.
- **Fastest Valid Method:** Notice $3^2 + 4^2 = 5^2$, so O_2 is the midpoint of AB itself: $AB = 2(3) = 6$ cm! | **Alternative:** Heron's formula + $\frac{1}{2}bh$ for altitude.
- **Common Trap:** Forgetting to double AM for full chord AB.
- **Practice Drill:** Ex. 12.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Radius-Chord $\angle OAB = 90^\circ - \angle ACB$] **Core Idea & Why It Works:** In any circle with centre O, chord AB, and point C on the major arc, $\angle OAB = 90^\circ - \angle ACB$ ALWAYS!

Valid Domain: All circles. | **Where It Fails:** If C is on minor arc (then $\angle OAB = \angle ACB - 90^\circ$).

Easy Example: $\angle ACB = 55^\circ \implies \angle AOB = 110^\circ \implies \angle OAB = 35^\circ = 90^\circ - 55^\circ$.

Exam-Level Example: Instant 1-mark MCQ solver.

Advanced Example: Saves 2 steps in NCERT Ex. 9.3 Q2.

[SMART SHORTCUT – 2. Minor-Arc Reflex Formula] **Core Idea & Why It Works:** If P, Q, R lie on a circle with Q on the minor arc PR, then $\angle PQR = 180^\circ - \frac{1}{2}\angle POR$, so $\angle OPR = \angle PQR - 90^\circ$.

Valid Domain: All minor-arc central angle questions. | **Where It Fails:** Major-arc Q.

Easy Example: NCERT Ex. 9.3 Q3: $\angle PQR = 100^\circ \implies \angle OPR = 100^\circ - 90^\circ = 10^\circ$!

Exam-Level Example: Solves a classic 3-mark question in 3 seconds to verify!

Advanced Example: Proof uses Reflex$(\angle POR) = 200^\circ$.

[SMART SHORTCUT – 3. Cyclic Trapezium = Isosceles Trapezium] **Core Idea & Why It Works:** If a trapezium is cyclic ($AB \parallel DC$), its non-parallel sides are ALWAYS equal ($AD = BC$) and base angles are equal.

Valid Domain: All cyclic trapeziums. | **Where It Fails:** Non-cyclic trapeziums.

Easy Example: $\angle A + \angle C = 180^\circ$ and $\angle A + \angle D = 180^\circ \implies \angle C = \angle D$.

Exam-Level Example: Instant MCQ answer.

Advanced Example: Diagonals are also equal.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: A chord of length 8 cm is at a distance of 3 cm from the centre of a circle. Find the radius.

Given & Required: $c = 8, d = 3$. | **Concept & Formula:** $r^2 = d^2 + (c/2)^2$.

Detailed Solution: $r = \sqrt{3^2 + 4^2} = 5$ cm. **Final Answer:** 5 **cm**

Fastest Alternative / Shortcut: 3-4-5 triple.

SOLVED EXAMPLE 2 [Beginner]

Question: In a circle with centre O, arc AB subtends $\angle AOB = 110^\circ$ at the centre. Find $\angle ACB$ on the major arc.

Given & Required: $\angle AOB = 110^\circ$. | **Concept & Formula:** $\angle ACB = \frac{1}{2}\angle AOB$.

Detailed Solution: $\angle ACB = \frac{110^\circ}{2} = 55^\circ$. **Final Answer:** 55°

Fastest Alternative / Shortcut: Half of central angle.

SOLVED EXAMPLE 3 [Beginner]

Question: In cyclic quadrilateral $ABCD$, $\angle A = 68^\circ$ and $\angle B = 105^\circ$. Find $\angle C$ and $\angle D$.

Given & Required: $\angle A = 68^\circ, \angle B = 105^\circ$. | **Concept & Formula:** Opposite angles $= 180^\circ$.

Detailed Solution: $\angle C = 180^\circ - 68^\circ = 112^\circ$; $\angle D = 180^\circ - 105^\circ = 75^\circ$. **Final Answer:** 112° **and** 75°

Fastest Alternative / Shortcut: Subtract from 180°.

SOLVED EXAMPLE 4 [Beginner]

Question: AB is a diameter of a circle and C is a point on the circle with $\angle CAB = 38°$. Find $\angle ABC$.

Given & Required: Diameter $AB \implies \angle C = 90°$. | **Concept & Formula:** Angle in semicircle.

Detailed Solution: $\angle ABC = 90° - 38° = 52°$. **Final Answer:** $52°$

Fastest Alternative / Shortcut: $90 - 38 = 52°$.

SOLVED EXAMPLE 5 [Intermediate]

Question: In a circle with centre O, $\angle AOB = 60°$ and $\angle BOC = 30°$ (A, B, C on circle). Find $\angle ADC$ on the major arc AC.

Given & Required: $\angle AOC = 60° + 30° = 90°$. | **Concept & Formula:** Central angle theorem.

Detailed Solution: $\angle ADC = \frac{1}{2}\angle AOC = \frac{90°}{2} = 45°$. **Final Answer:** $45°$

Fastest Alternative / Shortcut: Half of $90°$.

SOLVED EXAMPLE 6 [Intermediate]

Question: A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also on the major arc.

Given & Required: Chord $AB = r$. | **Concept & Formula:** Equilateral $\triangle OAB \implies 60°$.

Detailed Solution: $\triangle OAB$ is equilateral, so $\angle AOB = 60°$. Major arc angle $= \frac{60°}{2} = 30°$. Minor arc angle $= 180° - 30° = 150°$. **Final Answer:** $150°$ **(minor),** $30°$ **(major)**

Fastest Alternative / Shortcut: Cyclic sum $= 180°$.

SOLVED EXAMPLE 7 [Intermediate]

Question: In a circle, $\angle ABC = 69°$ and $\angle ACB = 31°$. Find $\angle BDC$ (where D is on the same segment as A).

Given & Required: $\triangle ABC$ angles $69°, 31°$. | **Concept & Formula:** Angles in same segment.

Detailed Solution: $\angle BAC = 180° - (69° + 31°) = 80°$. Thus $\angle BDC = \angle BAC = 80°$. **Final Answer:** $80°$

Fastest Alternative / Shortcut: Same segment = equal.

SOLVED EXAMPLE 8 [Intermediate]

Question: Prove that a cyclic parallelogram is a rectangle.

Given & Required: Cyclic parallelogram $ABCD$. | **Concept & Formula:** $\angle A = \angle C$ and $\angle A + \angle C = 180°$.

Detailed Solution: $\angle A = \angle C$ (opposite angles of parallelogram) and $\angle A + \angle C = 180°$ (cyclic) $\implies 2\angle A = 180° \implies \angle A = 90°$. Hence rectangle. **Final Answer: Proved**

Fastest Alternative / Shortcut: 2-line proof.

SOLVED EXAMPLE 9 [Board Standard]

Question: Two parallel chords of lengths 16 cm and 12 cm lie on opposite sides of the centre of a circle of radius 10 cm. Find the distance between them.

Given & Required: $r = 10, c_1 = 16, c_2 = 12$. | **Concept & Formula:** $d = \sqrt{r^2 - (c/2)^2}$.

Detailed Solution: $d_1 = \sqrt{10^2 - 8^2} = 6$ cm; $d_2 = \sqrt{10^2 - 6^2} = 8$ cm. Opposite sides $\implies$ Distance $= 6 + 8 = 14$ cm. **Final Answer:** 14 **cm**

Fastest Alternative / Shortcut: 6-8-10 triple twice!

SOLVED EXAMPLE 10 [Board Standard]

Question: In a circle with centre O, $\angle PQR = 100°$, where P, Q, R are points on the circle (Q on minor arc PR). Find $\angle OPR$.

Given & Required: $\angle PQR = 100°$. | **Concept & Formula:** Reflex central angle + isosceles $\triangle OPR$.

Detailed Solution: Reflex $\angle POR = 2\angle PQR = 200°$, so interior $\angle POR = 360° - 200° = 160°$. Since $OP = OR = r$, $\angle OPR = \frac{180° - 160°}{2} = 10°$. **Final Answer:** $10°$

Fastest Alternative / Shortcut: $100° - 90° = 10°$.

SOLVED EXAMPLE 11 [Board Standard]

Question: $ABCD$ is a cyclic quadrilateral whose diagonals intersect at E. If $\angle DBC = 70^\circ$ and $\angle BAC = 30^\circ$, find $\angle BCD$. Further, if $AB = BC$, find $\angle ECD$.

Given & Required: $\angle DBC = 70^\circ, \angle BAC = 30^\circ$. | **Concept & Formula:** Same segment + triangle sum.

Detailed Solution: $\angle BDC = \angle BAC = 30^\circ$ (same segment BC). In $\triangle BCD$: $\angle BCD = 180^\circ - (70^\circ + 30^\circ) = 80^\circ$. If $AB = BC$, $\angle BCA = \angle BAC = 30^\circ \implies \angle ECD = 80^\circ - 30^\circ = 50^\circ$. **Final Answer:** $\angle BCD = 80^\circ$, $\angle ECD = 50^\circ$

Fastest Alternative / Shortcut: Transfer 30° to $\angle BDC$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centres is 4 cm. Find the length of the common chord.

Given & Required: $r_1 = 5, r_2 = 3, O_1O_2 = 4$. | **Concept & Formula:** 3-4-5 right triangle.

Detailed Solution: In $\triangle O_1O_2A$, $5^2 = 4^2 + 3^2$, so $\angle O_1O_2A = 90^\circ$. Thus O_2 lies on the common chord AB and is its midpoint. Length $AB = 2(3) = 6$ cm. **Final Answer:** 6 **cm**

Fastest Alternative / Shortcut: $3^2 + 4^2 = 5^2 \implies AB = 2(3) = 6$ cm.

SOLVED EXAMPLE 13 [Board Standard]

Question: If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Given & Required: Chords $AB = CD$ meet at E. | **Concept & Formula:** Equal chords equidistant from O + RHS.

Detailed Solution: Draw $OM \perp AB, ON \perp CD \implies OM = ON$ and $AM = MB, CN = ND$. By RHS, $\triangle OME \cong \triangle ONE \implies ME = NE$. Adding/subtracting gives $AE = CE$ and $BE = DE$. **Final Answer:** **Proved**

Fastest Alternative / Shortcut: RHS on $\triangle OME, \triangle ONE$.

SOLVED EXAMPLE 14 [Challenge]

Question: If diagonals of a cyclic quadrilateral are diameters of the circle through its vertices, prove that it is a rectangle.

Given & Required: Diagonals AC, BD are diameters. | **Concept & Formula:** Angle in semicircle $= 90^\circ$.

Detailed Solution: Each of $\angle A, \angle B, \angle C, \angle D$ is an angle in a semicircle, hence $= 90^\circ$. Thus $ABCD$ is a rectangle. **Final Answer:** **Proved**

Fastest Alternative / Shortcut: 2-line proof using semicircle theorem!

SOLVED EXAMPLE 15 [Challenge]

Question: Prove that the quadrilateral formed by the internal angle bisectors of any quadrilateral is cyclic.

Given & Required: Quad $ABCD$, bisectors form $EFGH$. | **Concept & Formula:** Sum of opposite angles $= 180^\circ$.

Detailed Solution: $\angle FEH = 180^\circ - \frac{A+B}{2}$ and $\angle FGH = 180^\circ - \frac{C+D}{2}$. Adding gives $360^\circ - \frac{A+B+C+D}{2} = 360^\circ - 180^\circ = 180^\circ$. Hence cyclic. **Final Answer:** **Proved**

Fastest Alternative / Shortcut: Add opposite angles using $360^\circ/2 = 180^\circ$.

SOLVED EXAMPLE 16 [Challenge]

Question: In a circle of radius 13 cm, a chord AB has length 24 cm. Find the distance of AB from the centre.

Given & Required: $r = 13, c = 24$. | **Concept & Formula:** 5-12-13 triple.

Detailed Solution: $d = \sqrt{13^2 - 12^2} = 5$ cm. **Final Answer:** 5 **cm**

Fastest Alternative / Shortcut: 5-12-13 triple!

G. Competency MCQ Mastery Section

Q1. Angle in a semicircle is:

(A) 45° (B) 60° (C) 90° (D) 180°

Answer: (C) Half of $180^\circ = 90^\circ$. *Fast Method:* 90°. *Distractor Note:* Picking 180°.

Q2. Sum of opposite angles of a cyclic quadrilateral is:

(A) 90° (B) 180° (C) 270° (D) 360°

Answer: (B) Theorem 9.10. *Fast Method:* 180°. *Distractor Note:* Picking 360°.

Q3. The longest chord of a circle is its:

(A) Radius (B) Tangent (C) Diameter (D) Secant

Answer: (C) Diameter passes through the centre. *Fast Method:* Diameter. *Distractor Note:* Picking radius.

Q4. If a chord of length 10 cm is at distance 12 cm from the centre, radius is:

(A) 13 cm (B) 17 cm (C) $\sqrt{244}$ cm (D) 7 cm

Answer: (A) $\sqrt{5^2+12^2}=13$ cm. *Fast Method:* 5-12-13 triple. *Distractor Note:* Not halving 10.

Q5. If $\angle AOB=80^\circ$ at centre O, angle subtended by arc AB on major arc is:

(A) 160° (B) 40° (C) 80° (D) 100°

Answer: (B) $80^\circ/2=40^\circ$. *Fast Method:* Halve central angle. *Distractor Note:* Doubling to 160°.

Q6. A cyclic parallelogram is always a:

(A) Rhombus (B) Rectangle (C) Trapezium (D) Kite

Answer: (B) Equal + supplementary $=90^\circ$ corners. *Fast Method:* Rectangle. *Distractor Note:* Picking rhombus.

Q7. Angles in the same segment of a circle are:

(A) Complementary (B) Supplementary (C) Equal (D) 90°

Answer: (C) Theorem 9.8. *Fast Method:* Equal. *Distractor Note:* Picking supplementary.

Q8. In cyclic quad $ABCD$, if $\angle A-\angle C=20^\circ$, then $\angle A$ is:

(A) 80° (B) 100° (C) 110° (D) 70°

Answer: (B) $A+C=180, A-C=20 \implies A=100^\circ$. *Fast Method:* $(180+20)/2$. *Distractor Note:* Picking 80° ($\angle C$).

Q9. Perpendicular from the centre of a circle to a chord:

(A) Doubles the chord (B) Bisects the chord (C) Trisects the chord (D) Equals the radius

Answer: (B) Theorem 9.3. *Fast Method:* Bisects. *Distractor Note:* None.

Q10. Equal chords of a circle are:

(A) Parallel (B) Perpendicular (C) Equidistant from the centre (D) Diameters

Answer: (C) Theorem 9.5. *Fast Method:* Equidistant from O. *Distractor Note:* Picking parallel.

Q11. If O is centre and $\angle ACB=40^\circ$ on major arc, then $\angle OAB$ is:

(A) 40° (B) 50° (C) 80° (D) 20°

Answer: (B) $90^\circ-40^\circ=50^\circ$. *Fast Method:* $90^\circ-\angle ACB$. *Distractor Note:* Picking 80° ($\angle AOB$).

Q12. Exterior angle of a cyclic quadrilateral is 110°. Its interior opposite angle is:

(A) 70° (B) 110° (C) 90° (D) 55°

Answer: (B) Exterior angle = interior opposite angle $=110^\circ$. *Fast Method:* Equal. *Distractor Note:* Subtracting from 180°.

H. Special Exam & MCQ Strategies

- Whenever a radius and a chord appear together, draw the perpendicular from O to the chord to form a right triangle $r^2=d^2+(c/2)^2$.
- Check whether a point Q lies on the MAJOR arc (use $\angle AOB$) or MINOR arc (use Reflex $\angle AOB$) before halving!
- In cyclic quadrilateral problems with diagonals, always look for equal angles in the same segment first.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Using chord length c instead of $c/2$ in $r^2 = d^2 + (c/2)^2$. $\longrightarrow$ **Correction:** The perpendicular from centre BISECTS the chord, so the leg is $c/2$.

[COMMON MISTAKE] **Error:** Setting opposite angles of a cyclic quadrilateral equal. $\longrightarrow$ **Correction:** Opposite angles of a cyclic quadrilateral are SUPPLEMENTARY ($\angle A + \angle C = 180^\circ$).

[COMMON MISTAKE] **Error:** Halving the interior $\angle AOB$ when the point lies on the minor arc AB. $\longrightarrow$ **Correction:** Use Reflex$(\angle AOB) = 2\angle APB$ for minor-arc points.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $r^2 = d^2 + (c/2)^2$; $\angle AOB = 2\angle APB$; Cyclic: $\angle A + \angle C = 180^\circ$; Semicircle $= 90^\circ$.

[QUICK REVISION] Key Theorems & Results: Equal chords $\iff$ equal central angles $\iff$ equidistant from O; Same-segment angles equal.

[QUICK REVISION] Shortcut Summary: $\angle OAB = 90^\circ - \angle ACB$; Minor arc $\angle OPR = \angle PQR - 90^\circ$; Cyclic Pgm = Rectangle.

[QUICK REVISION] High-Frequency Exam Patterns: Parallel chords distance, Minor-arc reflex $\angle OPR$, Cyclic quad diagonals, Two-circle common chord.

Last-Minute Revision Checklist:

- ☐ Halve chord $c/2$
- ☐ Check major vs minor arc
- ☐ Cyclic opposite sum $= 180^\circ$
- ☐ Common chord $\perp$ line of centres

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Radius 5 cm, distance from centre 4 cm. Chord length is: (A) 3 cm (B) 6 cm (C) 8 cm (D) 10 cm
2. If $\angle ACB = 35^\circ$ on major arc, central $\angle AOB$ is: (A) 35° (B) 70° (C) 17.5° (D) 145°
3. In cyclic quad $ABCD$, if $\angle B = 85^\circ$, then $\angle D$ is: (A) 85° (B) 95° (C) 105° (D) 15°
4. Angle in a semicircle is always: (A) Acute (B) Obtuse (C) Right (D) Reflex

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Two parallel chords 6 cm and 8 cm on same side of centre ($r = 5$ cm). Distance between them: (A) 1 cm (B) 7 cm (C) 2 cm (D) 4 cm
2. If $\angle PQR = 115^\circ$ (Q on minor arc PR), then $\angle OPR$ is: (A) 25° (B) 35° (C) 65° (D) 50°
3. In cyclic quad $ABCD$, $\angle A = 2x + 4, \angle C = 4x - 64$. Value of x: (A) 30 (B) 40 (C) 20 (D) 45
4. Chord AB equals radius r. Angle subtended on major arc is: (A) 60° (B) 30° (C) 150° (D) 45°

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Two circles of radii 10 cm and 8 cm intersect; common chord is 12 cm. Distance between centres: (A) $8 + 2\sqrt{7}$ (B) 14 (C) $6 + 2\sqrt{7}$ (D) 10
2. If O is circumcentre of $\triangle ABC$ with $\angle B = 65^\circ, \angle C = 55^\circ$, then $\angle OBC$ is: (A) 30° (B) 60° (C) 40° (D) 25°
3. In cyclic quad $ABCD$, AB is diameter and $\angle ADC = 130^\circ$. Then $\angle BAC$ is: (A) 40° (B) 50° (C) 30° (D) 60°
4. Max number of common tangents? Wait: max number of points a line can intersect a circle at: (A) 1 (B) 2 (C) 3 (D) 4

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $c/2 = 3 \implies c = 6$ cm. **Q2 (B):** $2 \times 35 = 70^\circ$. **Q3 (B):** $180 - 85 = 95^\circ$. **Q4 (C):** 90°.
Exam Std	**Q1 (A):** $4 - 3 = 1$ cm. **Q2 (A):** $115^\circ - 90^\circ = 25^\circ$. **Q3 (B):** $6x - 60 = 180 \implies x = 40$. **Q4 (B):** $60^\circ/2 = 30^\circ$.
Challenge	**Q1 (A):** $\sqrt{10^2 - 6^2} + \sqrt{8^2 - 6^2} = 8 + \sqrt{28} = 8 + 2\sqrt{7}$. **Q2 (A):** $\angle A = 60^\circ \implies \angle BOC = 120^\circ \implies \angle OBC = 30^\circ$. **Q3 (A):** $\angle B = 50^\circ, \angle ACB = 90^\circ \implies \angle BAC = 40^\circ$. **Q4 (B):** At most 2 points (secant).

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Chords & $r^2 = d^2 + (c/2)^2$. Day 2: Central angle & same-segment theorems. Day 3: Cyclic quadrilaterals.
- **How to Introduce the Concept:** Draw a circle and ask: 'If we only know a chord is 8 cm and 3 cm from the centre, how do we find the radius?'
- **Difficult Areas & Remediation:** Minor-arc reflex central angle and 2-case parallel chords (same vs opposite side).
- **Blackboard Teaching Examples:** Draw $\angle AOB = 2\angle APB$ for both major-arc P and minor-arc P side-by-side.
- **Classroom Questions to Ask Students:** Why is a cyclic parallelogram always a rectangle?
- **Common Student Doubts & Answers:** Students ask about 3 non-collinear points proof; note it is excluded in 2026–27.
- **Homework & Revision Strategy:** Assign 4 chord-Pythagoras problems and 4 angle-chasing/cyclic proofs.

[TEACHER'S TIP] Mastery of Chapter 9 makes Class 10 Circles (Tangents) very easy.

UNIT V: MENSURATION TYBA COACHING CENTRE

Chapter 10: Heron's Formula

Semi-perimeter $s = \frac{a+b+c}{2}$ & Exact Triangle Area $\sqrt{s(s-a)(s-b)(s-c)}$ (2026–27)

A. Chapter Introduction & Examination Relevance

Calculating a triangle's area via $\frac{1}{2} \times$ base $\times$ height fails when the height is unknown. Heron of Alexandria gave a universal formula that computes the exact area of ANY triangle directly from its three side lengths a, b, c.

- **What You Will Learn:** Semi-perimeter $s = \frac{a+b+c}{2}$, Heron's Formula $\sqrt{s(s-a)(s-b)(s-c)}$, deriving equilateral triangle area $\frac{\sqrt{3}}{4}a^2$ and isosceles triangle area, ratio-of-sides perimeter problems, and real-world park/field fencing.
- **Why This Chapter Matters:** Heron's Formula is a guaranteed 3M/5M calculation question in Class 9 exams and is used in surveying, coordinate geometry, and 3D mensuration.
- **Real-Life Applications:** Irregular land-plot surveying, triangular sail/roof area estimation, park fencing & grass-turfing cost calculations.
- **Cross-Chapter Connections:** Uses square-root simplification from Chapter 1 and supports Chapter 11 (slant heights & cone bases).
- **Exam & Foundation Relevance:** Unit V carries 20/100 marks in RBSE (14/80 in CBSE). Expect 1M equilateral/perimeter MCQs and 3M/5M ratio-of-sides or park-fencing Heron calculations. (Quadrilateral area applications of Heron's formula are excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Semi-Perimeter & Heron's Formula

For a triangle with side lengths a, b, c (satisfying triangle inequality $a + b > c$), the **perimeter** is $2s = a + b + c$ and the **semi-perimeter** is $s = \frac{a+b+c}{2}$. Heron's Formula gives the area as $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$.

[CONCEPT MASTER – 1. Semi-Perimeter & Heron's Formula] Always factorize the four numbers $s, (s-a), (s-b), (s-c)$ inside the square root into prime/square pairs—NEVER multiply all four into a giant number first!

2. Special Cases: Equilateral & Isosceles Triangles

For an **equilateral triangle** ($a = b = c$), $s = \frac{3a}{2}$, so $\Delta = \sqrt{\frac{3a}{2} \cdot \frac{a}{2} \cdot \frac{a}{2} \cdot \frac{a}{2}} = \frac{\sqrt{3}}{4}a^2$. For an **isosceles triangle** with equal sides a, a and base b, $\Delta = \frac{b}{4}\sqrt{4a^2 - b^2}$.

[CONCEPT MASTER – 2. Special Cases: Equilateral & Isosceles Triangles] Remember: If you spot a Pythagorean triple (like $3, 4, 5$; $5, 12, 13$; $8, 15, 17$; $7, 24, 25$), $\frac{1}{2} \times$ base $\times$ height gives the area in 3 seconds!

3. Ratio of Sides & Missing Third Side Problems

When sides are in ratio $p : q : r$ with given perimeter P, write sides as px, qx, rx, solve $(p + q + r)x = P$ to find x, compute a, b, c, and then apply Heron's Formula. When two sides and perimeter are given, subtract from perimeter to find the third side $c = P - (a + b)$.

[CONCEPT MASTER – 3. Ratio of Sides & Missing Third Side Problems] Always check that $s > a, s > b, s > c$! If any $(s - a) \leq 0$, the given sides cannot form a triangle.

4. Altitude from Heron's Area & Costing

Once Δ is found via Heron's Formula, the altitude h_a onto side a is $h_a = \frac{2\Delta}{a}$. For fencing/turfing word problems, subtract gate width (Fencing length = Perimeter − gate) before multiplying by rate per metre.

[CONCEPT MASTER – 4. Altitude from Heron's Area & Costing] Rationalised 2026–27 Scope: Applications of Heron's formula to finding areas of quadrilaterals (splitting into 2 triangles) are excluded from the 2026–27 NCERT syllabus.

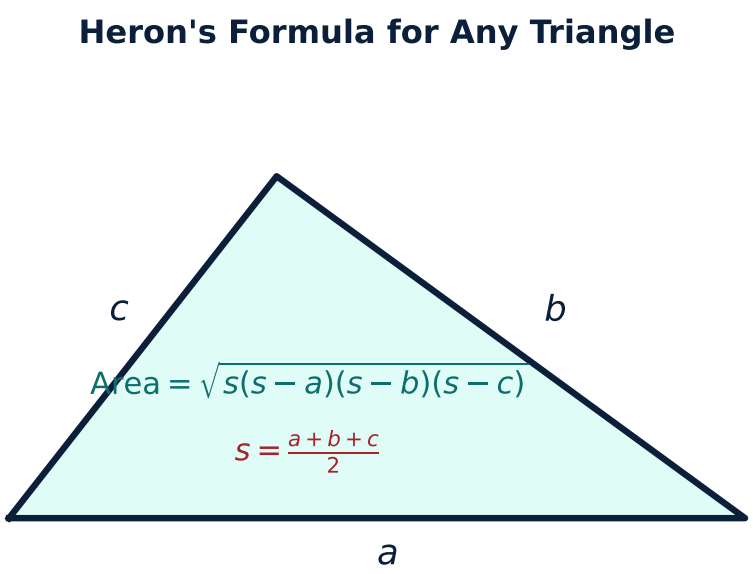


Figure 1: **Visual Model:** General triangle with sides a, b, c, semi-perimeter $s = \frac{a+b+c}{2}$, and Heron's Area formula.

C. Complete Formula Bank

[FORMULA BANK – 1. Semi-Perimeter] **Expression:** $s = \frac{a+b+c}{2}$
Symbols: a, b, c: side lengths of $\triangle ABC$.
Conditions & When to Use: Same units for a, b, c.
Solved Mini-Example: Sides $13, 14, 15 \implies s = \frac{42}{2} = 21$.
Common Mistake: Forgetting to divide $a + b + c$ by 2.

[FORMULA BANK – 2. Heron's Area Formula] **Expression:** Area $= \sqrt{s(s-a)(s-b)(s-c)}$
Symbols: s: semi-perimeter.
Conditions & When to Use: $s > a, s > b, s > c$.
Solved Mini-Example: Sides $13, 14, 15$ ($s = 21$): $\sqrt{21 \times 8 \times 7 \times 6} = 84$.
Common Mistake: Multiplying $21 \times 8 \times 7 \times 6 = 7056$ instead of pairing $7 \times 3 \times 4 \times 2 \times 7 \times 2 \times 3$.

[FORMULA BANK – 3. Equilateral Triangle Area & Height] **Expression:** Area $= \frac{\sqrt{3}}{4}a^2$; Height $h = \frac{\sqrt{3}}{2}a$
Symbols: a: side length.
Conditions & When to Use: All 3 sides equal.
Solved Mini-Example: If $a = 6$ cm, Area $= 9\sqrt{3}$ cm^2.
Common Mistake: Writing $\frac{\sqrt{3}}{2}a^2$ for area.

[FORMULA BANK – 4. Isosceles Triangle Area] **Expression:** Area $= \frac{b}{4}\sqrt{4a^2 - b^2}$
Symbols: a: each equal side, b: base.
Conditions & When to Use: $2a > b$.
Solved Mini-Example: Equal sides 5, base 6: $\frac{6}{4}\sqrt{100 - 36} = 1.5 \times 8 = 12$.
Common Mistake: Swapping a (equal side) and b (base).

[FORMULA BANK – 5. Altitude onto Side a] **Expression:** $h_a = \frac{2\Delta}{a}$
Symbols: Δ: Heron's area, a: base.
Conditions & When to Use: Shortest altitude is onto the LONGEST side!
Solved Mini-Example: Area 84, base $14 \implies h = 168/14 = 12$.
Common Mistake: Forgetting the factor of 2.

[FORMULA BANK – 6. Scaling Sides by k] **Expression:** If each side is multiplied by k, Area scales by k^2
Symbols: $k > 0$: scale factor.
Conditions & When to Use: Similar triangles.
Solved Mini-Example: Doubling all sides ($k = 2$) makes Area $4\times$ (300% increase!).
Common Mistake: Saying doubling sides doubles the area.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Olympiad/NTSE Foundation problem structures.

Pattern 1: Direct 3-Side & Missing-Side Heron Calculation

- **Identification Cue:** Find area of triangle with sides 18 cm, 10 cm, and perimeter 42 cm. | **Concept & Formula:** $c = P - (a + b)$, then Heron's formula.
- **Standard Step-by-Step:** Find $c = 42 - 28 = 14$, $s = 21$, compute $\sqrt{21(3)(11)(7)} = 21\sqrt{11}$.
- **Fastest Valid Method:** Factorize each bracket into primes before taking $\sqrt{}$. | **Alternative:** Use $\frac{1}{2}bh$ if right-angled.
- **Common Trap:** Arithmetic error in $s - a, s - b, s - c$ (check: $(s - a) + (s - b) + (s - c) = s$!).
- **Practice Drill:** Ex. 1 & Ex. 5.

Pattern 2: Ratio of Sides with Given Perimeter

- **Identification Cue:** Sides of a triangle are in the ratio $12 : 17 : 25$ and its perimeter is 540 cm. | **Concept & Formula:** Set $a = 12x, b = 17x, c = 25x$.
- **Standard Step-by-Step:** Solve $(12 + 17 + 25)x = 540 \implies x = 10$, sides $120, 170, 250$, then Heron.
- **Fastest Valid Method:** Scale $12, 17, 25$ ($s = 27$) first, then multiply area by $10^2 = 100$! | **Alternative:** Direct Heron on $120, 170, 250$.
- **Common Trap:** Forgetting to multiply by x to get sides.
- **Practice Drill:** Ex. 7 & Ex. 9.

Pattern 3: Equilateral & Isosceles Perimeter-to-Area

- **Identification Cue:** Perimeter of an isosceles triangle is 30 cm and each equal side is 12 cm; or signal board equilateral side a with perimeter 180 cm. | **Concept & Formula:** Find base b, then apply Heron or $\frac{\sqrt{3}}{4}a^2$.
- **Standard Step-by-Step:** Base $b = 30 - 24 = 6$, $s = 15$, Area $= \sqrt{15(3)(3)(9)} = 9\sqrt{15}$.
- **Fastest Valid Method:** Use $\frac{\sqrt{3}}{4}a^2$ directly for equilateral triangles. | **Alternative:** Drop altitude in isosceles to bisect base.
- **Common Trap:** Confusing equal side with base.
- **Practice Drill:** Ex. 2 & Ex. 6.

Pattern 4: Park Fencing, Turfing & Altitude Problems

- **Identification Cue:** Triangular park sides 120 m, 80 m, 50 m; leave 3 m space for gate, find fencing cost at Rs. 20/m and area for grass. | **Concept & Formula:** Fencing $= (P -$ gate$) \times$ rate; Grass $=$ Heron Area.
- **Standard Step-by-Step:** Compute $P - 3 = 247$ m for fence, then $s = 125$ for area.
- **Fastest Valid Method:** Check 3 m gate subtraction carefully. | **Alternative:** None.
- **Common Trap:** Forgetting to subtract the 3 m gate width.
- **Practice Drill:** Ex. 10 & Ex. 11.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Sanity Check: $(s-a)+(s-b)+(s-c)=s$] **Core Idea & Why It Works:** Before multiplying inside $\sqrt{s(s-a)(s-b)(s-c)}$, add the three differences: they MUST equal s!

Valid Domain: Every triangle. | **Where It Fails:** Nothing.

Easy Example: Sides $13, 14, 15$, $s = 21$: differences are $8, 7, 6$, and $8+7+6 = 21 = s$!

Exam-Level Example: Catches subtraction errors in 2 seconds before taking square root.

Advanced Example: Zero wasted time.

[SMART SHORTCUT – 2. Scale-Down Heron for Large Multiples] **Core Idea & Why It Works:** If sides are $120, 170, 250$ (all $\times 10$), compute Heron on $12, 17, 25$ and multiply the final area by $10^2 = 100$!

Valid Domain: All triangles with a common factor k in a, b, c. | **Where It Fails:** Remember to multiply by k^2, NOT k!

Easy Example: Sides $30, 40, 50$ ($k = 10$): Area$(3,4,5) = 6 \implies 6 \times 100 = 600$.

Exam-Level Example: Slashes big-number arithmetic in ratio problems.

Advanced Example: Works for fractional scales too.

[SMART SHORTCUT – 3. Percentage Area Increase When Sides Scale by k] **Core Idea & Why It Works:** If all sides are doubled ($k = 2$), new Area $= 4\Delta$, so percentage increase $= 300\%$ (NOT 100% or 200%)!

Valid Domain: Any triangle or 2D shape. | **Where It Fails:** If only one side changes.

Easy Example: Sides tripled ($k = 3$) $\implies$ Area $= 9\Delta \implies 800\%$ increase.

Exam-Level Example: Solves NCERT Exemplar percentage-increase MCQs in 2 seconds!

Advanced Example: Formula: $(k^2 - 1) \times 100\%$.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: Find the area of a triangle whose sides are 13 cm, 14 cm, and 15 cm.

Given & Required: $a = 13, b = 14, c = 15$. | **Concept & Formula:** Heron's formula.

Detailed Solution: $s = \frac{13+14+15}{2} = 21$. Area $= \sqrt{21(8)(7)(6)} = \sqrt{(7\cdot 3)(4\cdot 2)(7)(2\cdot 3)} = 7\cdot 3\cdot 2\cdot 2 = 84$ cm^2. **Final Answer:** 84 **cm^2**

Fastest Alternative / Shortcut: Remember 13-14-15 $\rightarrow$ Area 84!

SOLVED EXAMPLE 2 [Beginner]

Question: Find the area of an equilateral triangle of side 10 cm.

Given & Required: $a = 10$ cm. | **Concept & Formula:** $\frac{\sqrt{3}}{4}a^2$.

Detailed Solution: Area $= \frac{\sqrt{3}}{4}(100) = 25\sqrt{3}$ cm^2. **Final Answer:** $25\sqrt{3}$ **cm^2**

Fastest Alternative / Shortcut: Direct equilateral formula.

SOLVED EXAMPLE 3 [Beginner]

Question: Find the semi-perimeter of a triangle with sides 8 cm, 11 cm, and 13 cm.

Given & Required: $8, 11, 13$. | **Concept & Formula:** $s = \frac{a+b+c}{2}$.

Detailed Solution: $s = \frac{8+11+13}{2} = \frac{32}{2} = 16$ cm. **Final Answer:** 16 **cm**

Fastest Alternative / Shortcut: $32/2 = 16$.

SOLVED EXAMPLE 4 [Beginner]

Question: If the perimeter of an equilateral triangle is 180 cm, find its area using Heron's formula.

Given & Required: $3a = 180 \implies a = 60$. | **Concept & Formula:** Heron / equilateral.

Detailed Solution: $s = 90$, $s - a = 30$. Area $= \sqrt{90(30)(30)(30)} = 30 \cdot 30\sqrt{3} = 900\sqrt{3}$ cm^2. **Final Answer:** $900\sqrt{3}$ **cm^2**

Fastest Alternative / Shortcut: $\frac{\sqrt{3}}{4}(60^2) = 900\sqrt{3}$.

SOLVED EXAMPLE 5 [Intermediate]

Question: Find the area of a triangle two sides of which are 18 cm and 10 cm and the perimeter is 42 cm.

Given & Required: $a = 18, b = 10, P = 42$. | **Concept & Formula:** Find c first.

Detailed Solution: $c = 42 - (18 + 10) = 14$ cm. $s = 21$. Area $= \sqrt{21(3)(11)(7)} = \sqrt{7 \cdot 3 \cdot 3 \cdot 11 \cdot 7} = 21\sqrt{11}$ cm^2. **Final Answer:** $21\sqrt{11}$ **cm^2**

Fastest Alternative / Shortcut: Check $3 + 11 + 7 = 21 = s$.

SOLVED EXAMPLE 6 [Intermediate]

Question: An isosceles triangle has perimeter 30 cm and each of the equal sides is 12 cm. Find the area of the triangle.

Given & Required: $a = 12, c = 12, P = 30$. | **Concept & Formula:** Base $b = 6$ cm.

Detailed Solution: $b = 30 - 24 = 6$ cm. $s = 15$. Area $= \sqrt{15(3)(3)(9)} = 3 \cdot 3\sqrt{15} = 9\sqrt{15}$ cm^2. **Final Answer:** $9\sqrt{15}$ **cm^2**

Fastest Alternative / Shortcut: Pair 3×3 and $9 = 3^2$.

SOLVED EXAMPLE 7 [Intermediate]

Question: The sides of a triangular plot are in the ratio $3 : 5 : 7$ and its perimeter is 300 m. Find its area.

Given & Required: $3x + 5x + 7x = 300$. | **Concept & Formula:** Ratio scaling.

Detailed Solution: $15x = 300 \implies x = 20$. Sides: $60, 100, 140$ m; $s = 150$. Area $= \sqrt{150(90)(50)(10)} = 1500\sqrt{3}$ m^2. **Final Answer:** $1500\sqrt{3}$ **m^2**

Fastest Alternative / Shortcut: Area of $(3, 5, 7)$ is $\frac{15\sqrt{3}}{4} \times 20^2 = 1500\sqrt{3}$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Find the area of a right-angled triangle with hypotenuse 13 cm and one side 5 cm, and verify using Heron's formula.

Given & Required: $a = 5, c = 13$. | **Concept & Formula:** $b = 12$, $s = 15$.

Detailed Solution: Third side $b = \sqrt{13^2 - 5^2} = 12$ cm. $\frac{1}{2}(5)(12) = 30$ cm^2. Heron: $s = 15$, $\sqrt{15(10)(3)(2)} = \sqrt{900} = 30$ cm^2. **Final Answer:** 30 **cm^2**

Fastest Alternative / Shortcut: 5-12-13 triple.

SOLVED EXAMPLE 9 [Board Standard]

Question: Sides of a triangle are in the ratio $12 : 17 : 25$ and its perimeter is 540 cm. Find its area.

Given & Required: $54x = 540 \implies x = 10$. | **Concept & Formula:** Ratio + Scale-down Heron.

Detailed Solution: Sides: $120, 170, 250$ cm; $s = 270$. Area $= \sqrt{270(150)(100)(20)} = 9000$ cm^2. **Final Answer:** 9000 **cm^2**

Fastest Alternative / Shortcut: $12, 17, 25$ has area $90 \implies 90 \times 100 = 9000$.

SOLVED EXAMPLE 10 [Board Standard]

Question: A triangular park ABC has sides 120 m, 80 m, and 50 m. A gardener has to put a fence all around it (leaving 3 m for a gate) and plant grass inside. Find the cost of fencing at Rs. 20 per metre and the area to be planted.

Given & Required: $120, 80, 50$; gate 3 m. | **Concept & Formula:** Fence $= P - 3$; Area = Heron.

Detailed Solution: Perimeter $= 250$ m; fence $= 247$ m $\implies$ Cost $= 247 \times 20 =$ Rs. 4940. $s = 125$; Area $= \sqrt{125(5)(45)(75)} = 375\sqrt{15}$ m^2. **Final Answer: Rs.** 4940**;** $375\sqrt{15}$ **m^2**

Fastest Alternative / Shortcut: Subtract 3 m gate before multiplying by 20.

SOLVED EXAMPLE 11 [Board Standard]

Question: There is a slide in a park. One of its side walls is painted with 'KEEP THE PARK GREEN AND CLEAN'. If the sides of the wall are 15 m, 11 m, and 6 m, find the area painted in colour.

Given & Required: $15, 11, 6$. | **Concept & Formula:** Heron's formula.

Detailed Solution: $s = \frac{32}{2} = 16$. Area $= \sqrt{16(1)(5)(10)} = \sqrt{16 \times 25 \times 2} = 20\sqrt{2}$ m^2. **Final Answer:** $20\sqrt{2}$ **m^2**

Fastest Alternative / Shortcut: $16 = 4^2$, $5 \times 10 = 25 \times 2$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Find the shortest altitude of a triangle whose sides are 35 cm, 54 cm, and 61 cm.

Given & Required: $a = 35, b = 54, c = 61$. | **Concept & Formula:** Shortest altitude is on LONGEST side (61 cm).

Detailed Solution: $s = \frac{150}{2} = 75$. Area $= \sqrt{75(40)(21)(14)} = 420\sqrt{5}$ cm^2. Shortest altitude $h = \frac{2\Delta}{61} = \frac{840\sqrt{5}}{61} \approx 30.79$ cm.

Final Answer: $\frac{840\sqrt{5}}{61}$ **cm**

Fastest Alternative / Shortcut: Divide 2Δ by longest side 61.

SOLVED EXAMPLE 13 [Board Standard]

Question: The perimeter of a triangular field is 420 m and its sides are in the ratio $6 : 7 : 8$. Find the area of the field.

Given & Required: $21x = 420 \implies x = 20$. | **Concept & Formula:** Heron's formula.

Detailed Solution: Sides: $120, 140, 160$ m; $s = 210$. Area $= \sqrt{210(90)(70)(50)} = 2100\sqrt{15}$ m^2. **Final Answer:** $2100\sqrt{15}$ **m^2**

Fastest Alternative / Shortcut: Scale down by 10: $21\sqrt{15} \times 100$.

SOLVED EXAMPLE 14 [Challenge]

Question: If each side of a triangle is doubled, find the percentage increase in the area of the triangle.

Given & Required: Sides $2a, 2b, 2c$. | **Concept & Formula:** New $s' = 2s$.

Detailed Solution: $\Delta' = \sqrt{2s(2(s-a))(2(s-b))(2(s-c))} = 4\Delta$. Increase $= \frac{4\Delta - \Delta}{\Delta} \times 100\% = 300\%$. **Final Answer:** 300%

Fastest Alternative / Shortcut: $(2^2 - 1) \times 100 = 300\%$.

SOLVED EXAMPLE 15 [Challenge]

Question: An umbrella is made by stitching 10 triangular pieces of cloth of two different colours, each piece measuring 20 cm, 50 cm, and 50 cm. How much cloth of each colour is required?

Given & Required: 5 pieces of each colour; sides $20, 50, 50$. | **Concept & Formula:** Area of 1 piece $\times 5$.

Detailed Solution: $s = 60$. Area of 1 piece $= \sqrt{60(40)(10)(10)} = 200\sqrt{6}$ cm^2. Cloth of each colour $= 5 \times 200\sqrt{6} = 1000\sqrt{6}$ cm^2.

Final Answer: $1000\sqrt{6}$ **cm^2 each**

Fastest Alternative / Shortcut: 5 pieces per colour.

SOLVED EXAMPLE 16 [Challenge]

Question: Find the area of an equilateral triangle whose altitude is 6 cm.

Given & Required: $h = \frac{\sqrt{3}}{2}a = 6$. | **Concept & Formula:** Area $= \frac{h^2}{\sqrt{3}}$.

Detailed Solution: $a = \frac{12}{\sqrt{3}} = 4\sqrt{3}$ cm. Area $= \frac{\sqrt{3}}{4}(48) = 12\sqrt{3}$ cm^2. **Final Answer:** $12\sqrt{3}$ **cm^2**

Fastest Alternative / Shortcut: Area $= \frac{h^2}{\sqrt{3}} = \frac{36}{\sqrt{3}} = 12\sqrt{3}$!

G. Competency MCQ Mastery Section

Q1. Semi-perimeter of a triangle with sides 7 cm, 8 cm, 9 cm is:

(A) 24 cm (B) 12 cm (C) 16 cm (D) 8 cm

Answer: (B) $(7 + 8 + 9)/2 = 12$ cm. *Fast Method:* Half of perimeter. *Distractor Note:* Picking 24 (perimeter).

Q2. Area of an equilateral triangle with side $2\sqrt{3}$ cm is:

(A) $3\sqrt{3}$ cm^2 (B) $6\sqrt{3}$ cm^2 (C) $12\sqrt{3}$ cm^2 (D) $4\sqrt{3}$ cm^2

Answer: (A) $\frac{\sqrt{3}}{4}(12) = 3\sqrt{3}$. *Fast Method:* $\frac{\sqrt{3}}{4}a^2$. *Distractor Note:* Squaring $2\sqrt{3}$ as 6.

Q3. If each side of a triangle is doubled, its area becomes:

(A) 2 times (B) 3 times (C) 4 times (D) 8 times

Answer: (C) $2^2 = 4$ times. *Fast Method:* Area $\propto a^2$. *Distractor Note:* Picking 2 times.

Q4. Area of a triangle with sides 3 cm, 4 cm, 5 cm is:

(A) 6 cm^2 (B) 12 cm^2 (C) 7.5 cm^2 (D) 10 cm^2

Answer: (A) $\frac{1}{2}(3)(4) = 6$. *Fast Method:* Right triangle. *Distractor Note:* Forgetting $\frac{1}{2}$.

Q5. Perimeter of an equilateral triangle is 60 m. Its area is:

(A) $100\sqrt{3}$ m^2 (B) $200\sqrt{3}$ m^2 (C) $50\sqrt{3}$ m^2 (D) $400\sqrt{3}$ m^2

Answer: (A) $a = 20 \implies \frac{\sqrt{3}}{4}(400) = 100\sqrt{3}$. *Fast Method:* $a = 20$. *Distractor Note:* Using $a = 60$.

Q6. Area of an isosceles right triangle with equal sides 8 cm is:

(A) 64 cm^2 (B) 32 cm^2 (C) 16 cm^2 (D) $32\sqrt{2}$ cm^2

Answer: (B) $\frac{1}{2}(8)(8) = 32$. *Fast Method:* $\frac{1}{2}a^2$. *Distractor Note:* Forgetting $\frac{1}{2}$.

Q7. If $(s-a) = 5, (s-b) = 6, (s-c) = 7$, then perimeter $2s$ is:

(A) 18 (B) 36 (C) 9 (D) 27

Answer: (B) $s = 5+6+7 = 18 \implies 2s = 36$. *Fast Method:* Sum of $(s-a), (s-b), (s-c)$ is s! *Distractor Note:* Giving $s = 18$ instead of $2s = 36$.

Q8. Percentage increase in area of a triangle if each side is tripled:

(A) 300% (B) 600% (C) 800% (D) 900%

Answer: (C) $(3^2 - 1) \times 100 = 800\%$. *Fast Method:* $9\Delta - \Delta = 8\Delta$. *Distractor Note:* Picking 900%.

Q9. Altitude of an equilateral triangle of side 8 cm is:

(A) $4\sqrt{3}$ cm (B) $8\sqrt{3}$ cm (C) $2\sqrt{3}$ cm (D) $16\sqrt{3}$ cm

Answer: (A) $\frac{\sqrt{3}}{2}(8) = 4\sqrt{3}$. *Fast Method:* $\frac{\sqrt{3}}{2}a$. *Distractor Note:* Using area formula.

Q10. In Heron's formula, $s - a$ can NEVER be:

(A) Positive (B) Fractional (C) Zero or negative (D) Prime

Answer: (C) By triangle inequality, $b + c > a \implies s > a$. *Fast Method:* $s - a > 0$. *Distractor Note:* Thinking it can't be fractional.

Q11. Area of triangle with sides 15 cm, 20 cm, 25 cm is:

(A) 150 cm^2 (B) 300 cm^2 (C) 250 cm^2 (D) 187.5 cm^2

Answer: (A) $15^2 + 20^2 = 25^2 \implies \frac{1}{2}(15)(20) = 150$. *Fast Method:* 3-4-5 $\times 5$. *Distractor Note:* Multiplying 15×20.

Q12. If side of an equilateral triangle is a, its semi-perimeter is:

(A) $3a$ (B) $3a/2$ (C) $a/2$ (D) $2a$

Answer: (B) $\frac{a+a+a}{2} = \frac{3a}{2}$. *Fast Method:* $3a/2$. *Distractor Note:* Picking $3a$.

H. Special Exam & MCQ Strategies

- Factorize $s, (s-a), (s-b), (s-c)$ inside the square root into prime pairs instead of multiplying them into a huge integer.
- Always verify $(s-a) + (s-b) + (s-c) = s$ before proceeding with the square root.
- Check if the three sides form a right triangle ($a^2 + b^2 = c^2$)—if so, $\frac{1}{2}ab$ is much faster unless Heron is specifically required.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Forgetting to divide $a+b+c$ by 2 when finding s. $\longrightarrow$ **Correction:** s is the SEMI-perimeter: $s = \frac{a+b+c}{2}$.

[COMMON MISTAKE] **Error:** Saying doubling the sides increases area by 100% or 200%. $\longrightarrow$ **Correction:** Area becomes 4Δ, which is a 300% increase ($\frac{4\Delta-\Delta}{\Delta} \times 100\%$).

[COMMON MISTAKE] **Error:** Forgetting to subtract gate width in fencing problems. $\longrightarrow$ **Correction:** Fencing length $=$ Perimeter $-$ gate width.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $s = \frac{a+b+c}{2}$; $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$; Equilateral: $\frac{\sqrt{3}}{4}a^2$, $h = \frac{\sqrt{3}}{2}a$.

[QUICK REVISION] Key Theorems & Results: $(s-a)+(s-b)+(s-c) = s$; Scaling sides by k scales area by k^2; $h_a = \frac{2\Delta}{a}$.

[QUICK REVISION] Shortcut Summary: Pair primes inside $\sqrt{}$; Scale-down by GCF k (multiply area by k^2); 13-14-15 $\rightarrow$ 84.

[QUICK REVISION] High-Frequency Exam Patterns: Direct 3-side Heron, Ratio of sides + perimeter, Park fencing + turfing, % area change.

Last-Minute Revision Checklist:

- ☐ Divide by 2 for s
- ☐ Verify $(s-a)+(s-b)+(s-c) = s$
- ☐ Pair factors inside $\sqrt{}$
- ☐ Subtract gate in fencing

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Semi-perimeter of sides $5, 6, 7$ is: (A) 18 (B) 9 (C) 6 (D) 12
2. Area of equilateral triangle of side 4 cm: (A) $4\sqrt{3}$ (B) $8\sqrt{3}$ (C) $16\sqrt{3}$ (D) $2\sqrt{3}$
3. If $a = 6, b = 8, c = 10$, area is: (A) 48 (B) 24 (C) 30 (D) 40
4. Heron's formula uses: (A) Angles only (B) All 3 sides (C) 2 sides and 1 angle (D) Base and height

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Area of isosceles triangle with base 12 cm and equal sides 10 cm: (A) 48 (B) 60 (C) 24 (D) 96
2. Sides in ratio $3:4:5$ with perimeter 144 cm. Area is: (A) 864 (B) 432 (C) 1728 (D) 600
3. Two sides are 8 cm and 11 cm, perimeter 32 cm. Third side is: (A) 13 cm (B) 19 cm (C) 11 cm (D) 15 cm
4. If $(s-a) = 4, (s-b) = 5, (s-c) = 6$, then a is: (A) 11 (B) 10 (C) 9 (D) 15

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If each side is increased by 50% ($k = 1.5$), % increase in area is: (A) 125% (B) 100% (C) 50% (D) 225%
2. Altitude onto longest side of $20, 21, 29$ triangle is: (A) $420/29$ (B) $210/29$ (C) 20 (D) 21
3. Area of equilateral triangle with height $3\sqrt{3}$ cm is: (A) $9\sqrt{3}$ (B) $18\sqrt{3}$ (C) $27\sqrt{3}$ (D) $12\sqrt{3}$
4. Sides a, b, c satisfy $s = 10, s-a = 2, s-b = 3, s-c = 5$. Area is: (A) $10\sqrt{3}$ (B) 30 (C) $10\sqrt{6}$ (D) $20\sqrt{3}$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $(5+6+7)/2 = 9$. **Q2 (A):** $\frac{\sqrt{3}}{4}(16) = 4\sqrt{3}$. **Q3 (B):** $\frac{1}{2}(6)(8) = 24$. **Q4 (B):** Uses a, b, c only.
Exam Std	**Q1 (A):** $s = 16, \sqrt{16 \cdot 6 \cdot 6 \cdot 4} = 48$. **Q2 (A):** $12x = 144 \implies x = 12$; sides $36, 48, 60$; Area $= 6 \times 144 = 864$. **Q3 (A):** $32 - 19 = 13$ cm. **Q4 (A):** $s = 4+5+6 = 15 \implies a = 15-4 = 11$.
Challenge	**Q1 (A):** $(1.5^2 - 1) \times 100 = 125\%$. **Q2 (A):** $20^2 + 21^2 = 29^2 \implies \Delta = 210 \implies h = 420/29$. **Q3 (A):** $a = 6 \implies \frac{\sqrt{3}}{4}(36) = 9\sqrt{3}$. **Q4 (A):** $\sqrt{10 \cdot 2 \cdot 3 \cdot 5} = \sqrt{300} = 10\sqrt{3}$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Semi-perimeter & Heron's formula. Day 2: Ratio, isosceles, and real-world park problems.
- **How to Introduce the Concept:** Ask: 'How do you find the area of a triangular farm plot if you only have a measuring tape for the 3 boundary walls?'
- **Difficult Areas & Remediation:** Simplifying $\sqrt{s(s-a)(s-b)(s-c)}$ without multiplying into huge numbers.
- **Blackboard Teaching Examples:** Show side-by-side: WRONG ($\sqrt{7056}$) vs SMART ($\sqrt{7 \cdot 3 \cdot 4 \cdot 2 \cdot 7 \cdot 2 \cdot 3}$).
- **Classroom Questions to Ask Students:** Why must $(s-a)+(s-b)+(s-c)$ always equal s?
- **Common Student Doubts & Answers:** Students ask about quadrilateral Heron problems; clarify they are excluded in 2026–27.
- **Homework & Revision Strategy:** Assign 4 direct Heron problems, 2 ratio problems, and 2 park/umbrella problems.

[TEACHER'S TIP] Pairing prime factors inside the square root prevents 90% of calculation errors.

UNIT V: MENSURATION TYBA COACHING CENTRE

Chapter 11: Surface Areas and Volumes

Right Circular Cone, Sphere & Hemisphere: CSA, TSA & Volume Mastery (2026–27)

A. Chapter Introduction & Examination Relevance

In Class 8 you studied cubes, cuboids, and cylinders. Class 9 Chapter 11 focuses on the two classic curved 3D solids: the Right Circular Cone and the Sphere / Hemisphere.

- **What You Will Learn:** Slant height $l = \sqrt{r^2 + h^2}$, Curved Surface Area (CSA) and Total Surface Area (TSA) of a cone, Volume of a cone ($\frac{1}{3}\pi r^2 h$), Surface Area of sphere ($4\pi r^2$), CSA ($2\pi r^2$) and TSA ($3\pi r^2$) of a hemisphere, and Volume of sphere/hemisphere ($\frac{4}{3}\pi r^3$, $\frac{2}{3}\pi r^3$).
- **Why This Chapter Matters:** Mensuration carries 20/100 marks in RBSE (14/80 in CBSE) and directly feeds into Class 10 Surface Areas & Volumes (combinations of solids).
- **Real-Life Applications:** Conical tents & grain heaps, spherical domes, ice-cream cones, water storage tanks, and planetary/ball bearings.
- **Cross-Chapter Connections:** Builds on circle formulas and Pythagoras theorem; prepares directly for Class 10 Chapter 12.
- **Exam & Foundation Relevance:** Expect 1M ratio/percentage MCQs and 3M/5M conical tent (canvas area/cost), hemispherical dome painting, and right-triangle revolution problems. (Cube, cuboid, and cylinder exercises are excluded from Class 9 in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Right Circular Cone: Slant Height, CSA, TSA & Volume

A right circular cone of base radius r and vertical height h has **slant height** $l = \sqrt{r^2 + h^2}$. Its **Curved Surface Area** is $\text{CSA} = \pi r l$, **Total Surface Area** is $\text{TSA} = \pi r(l + r)$, and **Volume** is $V = \frac{1}{3}\pi r^2 h$ (one-third of a cylinder of the same r and h).

[CONCEPT MASTER – 1. Right Circular Cone: Slant Height, CSA, TSA & Volume] Remember: Canvas for a conical tent covers ONLY the CSA ($\pi r l$), NOT the floor! And when a right triangle revolves about a leg h, that leg becomes height h and the other leg becomes radius r.

2. Sphere & Hollow/Solid Hemisphere

A **sphere** of radius r has Surface Area $= 4\pi r^2$ and Volume $= \frac{4}{3}\pi r^3$. Slicing a sphere in half gives a **hemisphere**: $\text{CSA} = 2\pi r^2$, **TSA** $= 3\pi r^2$ (includes flat circular base πr^2), and Volume $= \frac{2}{3}\pi r^3$.

[CONCEPT MASTER – 2. Sphere & Hollow/Solid Hemisphere] Hemisphere Trap: A hemispherical *bowl* (open top) uses $\text{CSA} = 2\pi r^2$, whereas a solid hemisphere uses $\text{TSA} = 3\pi r^2$!

3. Unit Conversions & Capacity

Always check units before plugging into formulas: $1\ \text{m}^3 = 1000$ litres, $1000\ \text{cm}^3 = 1$ litre, and $1\ \text{m}^2 = 10000\ \text{cm}^2$. Diameter d must always be halved to $r = d/2$.

[CONCEPT MASTER – 3. Unit Conversions & Capacity] If π is not specified, use $\pi = \frac{22}{7}$; use 3.14 only when explicitly instructed.

4. Rationalised 2026–27 Scope

In the 2026–27 rationalised NCERT syllabus, standalone cuboid, cube, and cylinder exercises were moved/removed so Class 9 concentrates deeply on Cones, Spheres, and Hemispheres (with cylinder volume $\pi r^2 h$ used only as prior knowledge for comparing $\frac{1}{3}\pi r^2 h$).

[CONCEPT MASTER – 4. Rationalised 2026–27 Scope] Ratio Questions: Notice that π and constant factors cancel completely when comparing two cones or two spheres!

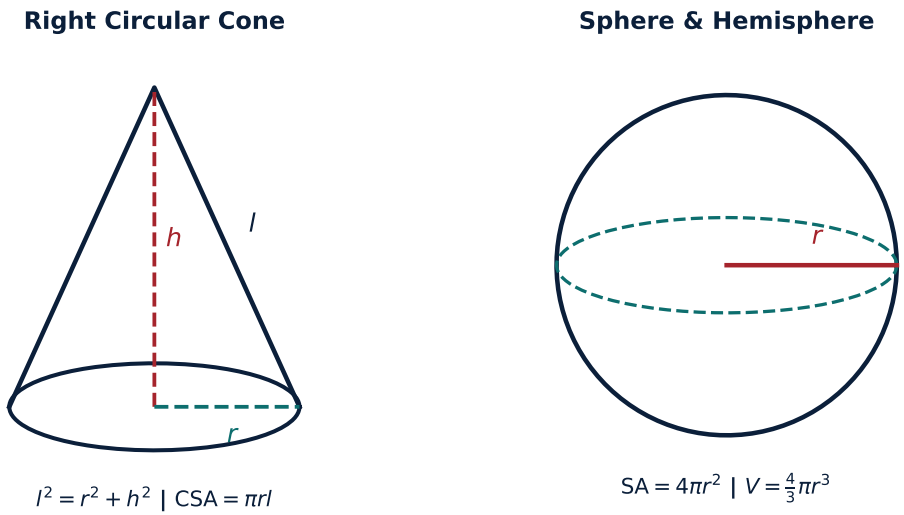


Figure 1: **Visual Model:** Right circular cone ($l^2 = r^2 + h^2$, CSA $= \pi rl$) and sphere/hemisphere (SA $= 4\pi r^2$, $V = \frac{4}{3}\pi r^3$).

C. Complete Formula Bank

[FORMULA BANK – 1. Cone Slant Height & CSA] **Expression:** $l = \sqrt{r^2 + h^2}$; CSA $= \pi rl$
Symbols: r: radius, h: height, l: slant height.
Conditions & When to Use: Always find l before CSA/TSA.
Solved Mini-Example: $r = 3, h = 4 \implies l = 5$, CSA $= 15\pi$.
Common Mistake: Using h instead of l in πrl.

[FORMULA BANK – 2. Cone TSA & Volume] **Expression:** TSA $= \pi r(l + r)$; $V = \frac{1}{3}\pi r^2 h$
Symbols: l, r, h.
Conditions & When to Use: Volume uses vertical height h, NOT l!
Solved Mini-Example: $r = 7, h = 12 \implies V = \frac{1}{3} \cdot \frac{22}{7} \cdot 49 \cdot 12 = 616$ cm^3.
Common Mistake: Using slant height l in the volume formula.

[FORMULA BANK – 3. Sphere Surface Area & Volume] **Expression:** SA $= 4\pi r^2$; $V = \frac{4}{3}\pi r^3$
Symbols: r: radius.
Conditions & When to Use: Area $= 4$ great-circle areas.
Solved Mini-Example: $r = 7 \implies$ SA $= 4 \cdot \frac{22}{7} \cdot 49 = 616$ cm^2.
Common Mistake: Using $2\pi r^2$ or $\frac{4}{3}\pi r^2$.

[FORMULA BANK – 4. Hemisphere CSA, TSA & Volume] **Expression:** CSA $= 2\pi r^2$; TSA $= 3\pi r^2$; $V = \frac{2}{3}\pi r^3$
Symbols: r: radius.
Conditions & When to Use: Open bowl $\rightarrow 2\pi r^2$; Solid $\rightarrow 3\pi r^2$.
Solved Mini-Example: $r = 10, \pi = 3.14 \implies$ TSA $= 3(3.14)(100) = 942$ cm^2.
Common Mistake: Using $2\pi r^2$ when TSA of solid hemisphere is asked.

[FORMULA BANK – 5. Hollow Hemispherical Shell Volume] **Expression:** $V = \frac{2}{3}\pi(R^3 - r^3)$
Symbols: R: outer radius, r: inner radius ($R = r + t$).
Conditions & When to Use: Thickness $t = R - r$.
Solved Mini-Example: Outer SA $= 2\pi R^2$.
Common Mistake: Writing $(R - r)^3$ instead of $R^3 - r^3$!

[FORMULA BANK – 6. Melting / Recasting Conservation] **Expression:** $V_{\text{total}} = n \cdot V_{\text{small}}$
Symbols: n: number of small cones/spheres.
Conditions & When to Use: Volume is conserved when melting/recasting.
Solved Mini-Example: 27 small spheres of radius r melt into 1 sphere of radius $R = 3r$.
Common Mistake: Equating surface areas instead of volumes.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Olympiad/NTSE Foundation problem structures.

Pattern 1: Cone Tent / Canvas / Grain Heap Problems

- **Identification Cue:** Conical tent of height 10 m and radius 24 m; find slant height and cost of canvas at Rs. 70/m^2 (or width 3 m). | **Concept & Formula:** $l = \sqrt{r^2 + h^2}$, CSA $= \pi r l$, Length $= \frac{\text{CSA}}{\text{width}}$.
- **Standard Step-by-Step:** Compute l via Pythagoras, find CSA, divide by cloth width (add stitching margin if stated).
- **Fastest Valid Method:** Spot triples: $(7, 24, 25)$, $(5, 12, 13)$, $(10, 24, 26)$, $(6, 8, 10)$! | **Alternative:** Leave π until final step.
- **Common Trap:** Including base area πr^2 for a tent canvas.
- **Practice Drill:** Ex. 5 & Ex. 9.

Pattern 2: Right Triangle Revolving Around a Side

- **Identification Cue:** Right $\triangle ABC$ with sides $5, 12, 13$ cm is revolved about side 12 cm (or 5 cm); find volume. | **Concept & Formula:** Fixed axis = height h; rotating leg = radius r.
- **Standard Step-by-Step:** Identify h (axis of rotation) and r (other leg), compute $\frac{1}{3}\pi r^2 h$.
- **Fastest Valid Method:** Ratio of volumes about 12 cm vs 5 cm is simply $5 : 12$! | **Alternative:** Leave answer in π if asked.
- **Common Trap:** Swapping r and h.
- **Practice Drill:** Ex. 10.

Pattern 3: Sphere / Hemisphere Ratio & % Change

- **Identification Cue:** Diameter of Moon is $\frac{1}{4}$ of Earth; find ratio of surface areas and volumes. Or radius doubled. | **Concept & Formula:** SA $\propto r^2$ and $V \propto r^3$.
- **Standard Step-by-Step:** Write $\frac{\text{SA}_1}{\text{SA}_2} = (\frac{r_1}{r_2})^2$ and $\frac{V_1}{V_2} = (\frac{r_1}{r_2})^3$.
- **Fastest Valid Method:** Never plug π or $\frac{4}{3}$ in ratio problems—they cancel! | **Alternative:** Plug $r_1 = 1, r_2 = 4$.
- **Common Trap:** Forgetting to cube for volume ratio.
- **Practice Drill:** Ex. 6 & Ex. 11.

Pattern 4: Hemispherical Bowl / Dome & Capacity in Litres

- **Identification Cue:** Hemispherical bowl of thickness 0.25 cm and inner radius 5 cm; or tank capacity in litres. | **Concept & Formula:** Outer radius $R = r + t$; 1000 cm^3 = 1 L.
- **Standard Step-by-Step:** Add thickness to inner radius for outer SA; compute V in cm^3 and divide by 1000 for litres.
- **Fastest Valid Method:** Multiply m^3 by 1000 for litres. | **Alternative:** Keep $22/7$ ready to cancel 7 or 21.
- **Common Trap:** Using inner radius r when outer CSA is asked.
- **Practice Drill:** Ex. 8 & Ex. 12.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Cancel π & Constants in All Ratios] **Core Idea & Why It Works:** For Spheres: $\frac{\text{SA}_1}{\text{SA}_2} = \left(\frac{r_1}{r_2}\right)^2$ and $\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3$. For Cones: $\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^2 \left(\frac{h_1}{h_2}\right)$.
Valid Domain: All ratio and percentage-change problems. | **Where It Fails:** Never!
Easy Example: Moon $d = \frac{1}{4}$ Earth $\implies$ SA ratio $= 1:16$, Volume ratio $= 1:64$.
Exam-Level Example: Solves NCERT Ex. 11.2 Q7 and Ex. 11.4 Q4 in 5 seconds!
Advanced Example: If sphere radius increases by 10% (1.1), SA increases by 21% and Volume by 33.1%.

[SMART SHORTCUT – 2. 7-Radius Magic Numbers ($r = 7$ or 3.5)] **Core Idea & Why It Works:** For $r = 7$: $\pi r^2 = 154$, $4\pi r^2 = 616$, $2\pi r^2 = 308$, $3\pi r^2 = 462$.
Valid Domain: All problems with $r = 7$ (or $r = 7k$, multiply by k^2). | **Where It Fails:** If $\pi = 3.14$ is forced.
Easy Example: Hemisphere $r = 7$ cm: CSA $= 308$ cm^2, TSA $= 462$ cm^2.
Exam-Level Example: Instant arithmetic check on dozens of NCERT questions!
Advanced Example: For $r = 3.5$ ($k = 1/2$), divide by 4: $\pi r^2 = 38.5$, $4\pi r^2 = 154$.

[SMART SHORTCUT – 3. 27 Small Spheres $\to R = 3r$] **Core Idea & Why It Works:** When n identical spheres merge into 1 big sphere, $R = n^{1/3} r$ and new SA $= n^{2/3}$(small SA).
Valid Domain: Any $n = k^3$. | **Where It Fails:** Non-spherical shapes.
Easy Example: 27 iron spheres of radius r melt into 1 sphere $\implies R = 3r$, $S' = 9S$.
Exam-Level Example: Solves NCERT Ex. 11.4 Q9 in 3 seconds!
Advanced Example: 8 spheres $\implies R = 2r$.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Find the curved surface area of a right circular cone of slant height 10 cm and base radius 7 cm.
Given & Required: $l = 10, r = 7$. | **Concept & Formula:** CSA $= \pi r l$.
Detailed Solution: CSA $= \frac{22}{7} \times 7 \times 10 = 220$ cm^2. **Final Answer:** 220 **cm^2**
Fastest Alternative / Shortcut: 7 cancels 7.

SOLVED EXAMPLE 2 [Beginner]
Question: Find the surface area of a sphere of radius 7 cm.
Given & Required: $r = 7$ cm. | **Concept & Formula:** $4\pi r^2$.
Detailed Solution: $4 \times \frac{22}{7} \times 7 \times 7 = 616$ cm^2. **Final Answer:** 616 **cm^2**
Fastest Alternative / Shortcut: 7-radius magic: 616.

SOLVED EXAMPLE 3 [Beginner]
Question: Find the total surface area of a solid hemisphere of radius 10 cm ($\pi = 3.14$).
Given & Required: $r = 10, \pi = 3.14$. | **Concept & Formula:** TSA $= 3\pi r^2$.
Detailed Solution: $3 \times 3.14 \times 100 = 942$ cm^2. **Final Answer:** 942 **cm^2**
Fastest Alternative / Shortcut: $3 \times 314 = 942$.

SOLVED EXAMPLE 4 [Beginner]
Question: Find the volume of a right circular cone with radius 6 cm and height 7 cm.
Given & Required: $r = 6, h = 7$. | **Concept & Formula:** $V = \frac{1}{3}\pi r^2 h$.
Detailed Solution: $\frac{1}{3} \times \frac{22}{7} \times 36 \times 7 = 22 \times 12 = 264$ cm^3. **Final Answer:** 264 **cm^3**
Fastest Alternative / Shortcut: 7 cancels 7, $36/3 = 12$.

SOLVED EXAMPLE 5 [Intermediate]

Question: Height of a cone is 16 cm and its base radius is 12 cm. Find its CSA and TSA ($\pi = 3.14$).

Given & Required: $h = 16, r = 12$. | **Concept & Formula:** $l = \sqrt{r^2 + h^2}$.

Detailed Solution: $l = \sqrt{12^2 + 16^2} = 20$ cm. $\text{CSA} = 3.14 \times 12 \times 20 = 753.6$ cm^2. $\text{TSA} = 3.14 \times 12 \times (20 + 12) = 1205.76$ cm^2.
Final Answer: **CSA** $= 753.6$ **cm**^{2}**, TSA** $= 1205.76$ **cm**^{2}

Fastest Alternative / Shortcut: 12-16-20 triple (3-4-5 $\times$ 4).

SOLVED EXAMPLE 6 [Intermediate]

Question: The diameter of the Moon is approximately one-fourth of the diameter of the Earth. Find the ratio of their surface areas.

Given & Required: $r_M = \frac{1}{4} r_E$. | **Concept & Formula:** $\text{SA} \propto r^2$.

Detailed Solution: $\frac{\text{SA}_M}{\text{SA}_E} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$. **Final Answer:** $1 : 16$

Fastest Alternative / Shortcut: Square the radius ratio.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find the capacity in litres of a conical vessel with radius 7 cm and slant height 25 cm.

Given & Required: $r = 7, l = 25$. | **Concept & Formula:** $h = \sqrt{l^2 - r^2}$, 1000 cm$^3 = 1$ L.

Detailed Solution: $h = \sqrt{25^2 - 7^2} = 24$ cm. $V = \frac{1}{3} \cdot \frac{22}{7} \cdot 49 \cdot 24 = 1232$ cm$^3 = 1.232$ litres. **Final Answer:** 1.232 **litres**

Fastest Alternative / Shortcut: 7-24-25 triple!

SOLVED EXAMPLE 8 [Intermediate]

Question: A hemispherical bowl is made of steel 0.25 cm thick. The inner radius of the bowl is 5 cm. Find the outer curved surface area.

Given & Required: $r = 5, t = 0.25 \implies R = 5.25$. | **Concept & Formula:** $2\pi R^2$.

Detailed Solution: $\text{CSA} = 2 \times \frac{22}{7} \times 5.25 \times 5.25 = 173.25$ cm^2. **Final Answer:** 173.25 **cm**^{2}

Fastest Alternative / Shortcut: $5.25/7 = 0.75$.

SOLVED EXAMPLE 9 [Board Standard]

Question: A conical tent is 10 m high and the radius of its base is 24 m. Find (i) slant height, (ii) cost of canvas at Rs. 70 per m^2.

Given & Required: $h = 10, r = 24$. | **Concept & Formula:** $l = \sqrt{r^2 + h^2}$, $\text{Cost} = \pi r l \times 70$.

Detailed Solution: (i) $l = \sqrt{24^2 + 10^2} = 26$ m. (ii) $\text{Cost} = \frac{22}{7} \times 24 \times 26 \times 70 = 22 \times 24 \times 26 \times 10 = \text{Rs. } 137,280$. **Final Answer:** $l = 26$ **m; Rs.** $1,37,280$

Fastest Alternative / Shortcut: Keep $70/7 = 10$ to avoid decimal π!

SOLVED EXAMPLE 10 [Board Standard]

Question: A right triangle ABC with sides 5 cm, 12 cm, and 13 cm is revolved about the side 12 cm. Find the volume of the solid so obtained.

Given & Required: Revolved about 12 cm. | **Concept & Formula:** Cone with $h = 12, r = 5$.

Detailed Solution: $V = \frac{1}{3}\pi(5)^2(12) = 100\pi$ cm^3 (≈ 314.29 cm^3). **Final Answer:** 100π **cm**^{3}

Fastest Alternative / Shortcut: $12/3 = 4$, $4 \times 25 = 100\pi$.

SOLVED EXAMPLE 11 [Board Standard]

Question: A hemispherical dome of a building needs to be painted. If the circumference of the base is 17.6 m, find the cost of painting at Rs. 5 per 100 cm^2.

Given & Required: $2\pi r = 17.6$ m. | **Concept & Formula:** $\text{CSA} = 2\pi r^2$, unit conversion.

Detailed Solution: $2 \cdot \frac{22}{7} r = 17.6 \implies r = 2.8$ m. $\text{CSA} = 2 \cdot \frac{22}{7} \cdot 2.8 \cdot 2.8 = 49.28$ m$^2 = 492800$ cm^2. $\text{Cost} = \frac{492800}{100} \times 5 = \text{Rs. } 24,640$. **Final Answer:** **Rs.** $24,640$

Fastest Alternative / Shortcut: Rs. $5/100$ cm$^2 =$ Rs. $500/$m^2!

SOLVED EXAMPLE 12 [Board Standard]

Question: A heap of wheat is in the form of a cone whose diameter is 10.5 m and height is 3 m. Find its volume and the area of canvas needed to cover it ($\sqrt{36.5625} \approx 6.05$).

Given & Required: $r = 5.25, h = 3$. | **Concept & Formula:** $V = \frac{1}{3}\pi r^2 h$, CSA $= \pi r l$.

Detailed Solution: $V = \frac{1}{3} \cdot \frac{22}{7} \cdot 5.25^2 \cdot 3 = 86.625$ m^3. $l = \sqrt{5.25^2 + 3^2} \approx 6.05$ m. CSA $= \frac{22}{7}(5.25)(6.05) = 99.825$ m^2. **Final Answer:** 86.625 **m^3**; 99.825 **m^2**

Fastest Alternative / Shortcut: $5.25/7 = 0.75$.

SOLVED EXAMPLE 13 [Board Standard]

Question: 27 solid iron spheres, each of radius r and surface area S, are melted to form a sphere with surface area S'. Find (i) radius r' of the new sphere, (ii) ratio $S : S'$.

Given & Required: $27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (r')^3$. | **Concept & Formula:** Volume conservation.

Detailed Solution: $(r')^3 = 27r^3 \implies r' = 3r$. Ratio $S : S' = 4\pi r^2 : 4\pi(3r)^2 = 1 : 9$. **Final Answer: (i)** $r' = 3r$, **(ii)** $1 : 9$

Fastest Alternative / Shortcut: $27^{1/3} = 3$, $3^2 = 9$.

SOLVED EXAMPLE 14 [Challenge]

Question: The radius and slant height of a cone are in the ratio $4 : 7$. If its CSA is 792 cm^2, find its radius.

Given & Required: $r = 4x, l = 7x$, CSA $= 792$. | **Concept & Formula:** $\pi r l = 792$.

Detailed Solution: $\frac{22}{7}(4x)(7x) = 792 \implies 88x^2 = 792 \implies x^2 = 9 \implies x = 3$. Radius $r = 4(3) = 12$ cm. **Final Answer:** 12 **cm**

Fastest Alternative / Shortcut: 7 cancels 7, $792/88 = 9$.

SOLVED EXAMPLE 15 [Challenge]

Question: A bus stop is barricaded using 50 hollow cones made of recycled cardboard. Each cone has base diameter 40 cm and height 1 m. If outer side is painted at Rs. 12/m^2, find cost ($\pi = 3.14$, $\sqrt{1.04} = 1.02$).

Given & Required: $r = 0.2$ m, $h = 1$ m, 50 cones. | **Concept & Formula:** $50 \times \pi r l \times 12$.

Detailed Solution: $l = \sqrt{0.04 + 1} = 1.02$ m. Area of 50 cones $= 50 \times 3.14 \times 0.2 \times 1.02 = 32.028$ m^2. Cost $= 32.028 \times 12 =$ Rs. 384.34. **Final Answer: Rs.** 384.34

Fastest Alternative / Shortcut: $50 \times 0.2 = 10$.

SOLVED EXAMPLE 16 [Challenge]

Question: If the surface area of a sphere is 154 cm^2, find its volume.

Given & Required: $4\pi r^2 = 154$. | **Concept & Formula:** Find r first.

Detailed Solution: $4 \cdot \frac{22}{7} r^2 = 154 \implies r^2 = \frac{49}{4} \implies r = 3.5$ cm. $V = \frac{4}{3} \cdot \frac{22}{7} \cdot \frac{7}{2} \cdot \frac{7}{2} \cdot \frac{7}{2} = \frac{539}{3} = 179\frac{2}{3}$ cm^3. **Final Answer:** 179.67 **cm^3**

Fastest Alternative / Shortcut: 7-radius magic: $r = 3.5 \implies 4\pi r^2 = 154$.

G. Competency MCQ Mastery Section

Q1. Slant height of a cone with $r = 3$ cm and $h = 4$ cm is:

(A) 5 cm (B) 7 cm (C) 12 cm (D) $\sqrt{7}$ cm

Answer: (A) $\sqrt{3^2 + 4^2} = 5$ cm. *Fast Method:* 3-4-5 triple. *Distractor Note:* Adding $3 + 4 = 7$.

Q2. Total surface area of a solid hemisphere of radius r is:

(A) $2\pi r^2$ (B) $3\pi r^2$ (C) $4\pi r^2$ (D) $\frac{2}{3}\pi r^3$

Answer: (B) Curved $2\pi r^2$ + flat $\pi r^2 = 3\pi r^2$. *Fast Method:* $3\pi r^2$. *Distractor Note:* Picking $2\pi r^2$.

Q3. Volume of a cone is what fraction of a cylinder of the same radius and height?

(A) $1/2$ (B) $1/3$ (C) $1/4$ (D) $2/3$

Answer: (B) $\frac{1}{3}\pi r^2 h$. *Fast Method:* $1/3$. *Distractor Note:* Picking $1/2$.

Q4. If radius of a sphere is doubled, its surface area becomes:

(A) 2 times (B) 4 times (C) 8 times (D) 16 times

Answer: (B) $2^2 = 4$ times. *Fast Method:* SA $\propto r^2$. *Distractor Note:* Picking 8 (that's volume!).

Q5. Volume of a sphere of radius r is:

(A) $4\pi r^2$ (B) $\frac{2}{3}\pi r^3$ (C) $\frac{4}{3}\pi r^3$ (D) $\frac{4}{3}\pi r^2$

Answer: (C) Standard formula. *Fast Method:* $\frac{4}{3}\pi r^3$. *Distractor Note:* Picking $2/3$ (hemisphere).

Q6. If volume and surface area of a sphere are numerically equal, its radius is:

(A) 1 (B) 2 (C) 3 (D) 4

Answer: (C) $\frac{4}{3}\pi r^3 = 4\pi r^2 \implies r = 3$. *Fast Method:* $r/3 = 1 \implies r = 3$. *Distractor Note:* Picking 2.

Q7. CSA of a cone of radius 7 cm and slant height 10 cm is:

(A) 220 cm^2 (B) 440 cm^2 (C) 154 cm^2 (D) 70 cm^2

Answer: (A) $\frac{22}{7}(7)(10) = 220$. *Fast Method:* 22×10. *Distractor Note:* Multiplying by 2.

Q8. Radius of a cone is halved and height is doubled; its volume:

(A) Remains same (B) Becomes half (C) Becomes double (D) Becomes 4 times

Answer: (B) $(1/2)^2 \times 2 = 1/2$. *Fast Method:* r^2h scales by $1/4 \times 2 = 1/2$. *Distractor Note:* Saying 'Remains same'.

Q9. 1 cubic metre (1 m^3) equals how many litres?

(A) 10 (B) 100 (C) 1000 (D) 10000

Answer: (C) 1 m^3 = 1000 litres. *Fast Method:* 1000 L. *Distractor Note:* Saying 100.

Q10. If 64 small spheres of radius 1 cm melt into one big sphere, the big radius is:

(A) 4 cm (B) 8 cm (C) 16 cm (D) 64 cm

Answer: (A) $64^{1/3} = 4$ cm. *Fast Method:* Cube root of 64. *Distractor Note:* Taking square root 8.

Q11. Ratio of volumes of a cone and a hemisphere of equal base radius r and equal height $h = r$ is:

(A) 1 : 1 (B) 1 : 2 (C) 1 : 3 (D) 2 : 3

Answer: (B) $\frac{1}{3}\pi r^3 : \frac{2}{3}\pi r^3 = 1 : 2$. *Fast Method:* $1/3 : 2/3 = 1 : 2$. *Distractor Note:* Picking $1 : 3$.

Q12. CSA of a hemisphere of diameter 14 cm is:

(A) 154 cm^2 (B) 308 cm^2 (C) 462 cm^2 (D) 616 cm^2

Answer: (B) $r = 7 \implies 2\pi r^2 = 308$. *Fast Method:* Halve diameter $14 \to 7$! *Distractor Note:* Using $r = 14$.

H. Special Exam & MCQ Strategies

- Always halve diameter $d \to r = d/2$ as your VERY FIRST line in every mensuration problem.
- In cost/painting problems, keep $\pi = 22/7$ unmultiplied until the very end so denominators like 7 cancel cleanly.
- Distinguish carefully between Open Bowl ($2\pi r^2$), Solid Hemisphere ($3\pi r^2$), and Conical Tent (πrl).

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Using vertical height h in Cone CSA (πrh) or slant height l in Cone Volume ($\frac{1}{3}\pi r^2 l$). $\longrightarrow$ **Correction:** Area uses slant height l (πrl); Volume uses vertical height h ($\frac{1}{3}\pi r^2 h$).

[COMMON MISTAKE] **Error:** Using $2\pi r^2$ for Total Surface Area of a solid hemisphere. $\longrightarrow$ **Correction:** Solid hemisphere TSA $= 2\pi r^2 + \pi r^2 = 3\pi r^2$.

[COMMON MISTAKE] **Error:** Forgetting to halve the given diameter d before substituting into r^2 or r^3. $\longrightarrow$ **Correction:** Write '$d = \cdots \implies r = d/2 = \ldots$' on line 1.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: Cone: $l = \sqrt{r^2 + h^2}, \text{CSA} = \pi r l, \text{TSA} = \pi r(l + r), V = \frac{1}{3}\pi r^2 h$; Sphere: $4\pi r^2, \frac{4}{3}\pi r^3$; Hemisphere: $2\pi r^2, 3\pi r^2, \frac{2}{3}\pi r^3$.

[QUICK REVISION] Key Theorems & Results: $1\ \text{m}^3 = 1000\ \text{L}$; $1000\ \text{cm}^3 = 1\ \text{L}$; Shell $V = \frac{2}{3}\pi(R^3 - r^3)$.

[QUICK REVISION] Shortcut Summary: Cancel π in ratios; $r = 7 \to \pi r^2 = 154, 4\pi r^2 = 616$; n spheres $\to R = n^{1/3} r$.

[QUICK REVISION] High-Frequency Exam Patterns: Conical tent canvas/cost, Revolving right triangle, Moon-Earth ratios, Hemispherical bowl/dome.

Last-Minute Revision Checklist:

- ☐ Halve diameter $d \to r$
- ☐ l for CSA, h for Volume
- ☐ Check $2\pi r^2$ vs $3\pi r^2$
- ☐ Convert $\text{cm}^3 \to$ litres

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. CSA of cone with $r = 7$ cm, $l = 15$ cm: (A) 330 (B) 154 (C) 484 (D) 220
2. Volume of sphere of radius 3 cm: (A) 12π (B) 36π (C) 27π (D) 9π
3. Slant height of cone with $r = 5, h = 12$: (A) 13 (B) 17 (C) 7 (D) 60
4. TSA of hemisphere of radius r: (A) $2\pi r^2$ (B) $3\pi r^2$ (C) $4\pi r^2$ (D) πr^2

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Volume of cone with $r = 6$ cm, $l = 10$ cm: (A) 96π (B) 120π (C) 144π (D) 48π
2. If two spheres have volumes in ratio $8 : 27$, ratio of their surface areas is: (A) $2 : 3$ (B) $4 : 9$ (C) $8 : 27$ (D) $16 : 81$
3. Radius of sphere whose surface area is $616\ \text{cm}^2$: (A) 7 cm (B) 14 cm (C) 3.5 cm (D) 21 cm
4. Hemispherical tank has radius 3 m. Capacity in litres ($\pi = 3.14$): (A) 56520 L (B) 28260 L (C) 113040 L (D) 56.52 L

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Right triangle $6, 8, 10$ revolved about 6 cm gives volume V_1, about 8 cm gives V_2. Ratio $V_1 : V_2$: (A) $4 : 3$ (B) $3 : 4$ (C) $16 : 9$ (D) $1 : 1$
2. If radius of a sphere increases by 20%, % increase in volume is: (A) 60% (B) 72.8% (C) 44% (D) 80%
3. Ratio of TSA of a sphere to TSA of a hemisphere of the same radius is: (A) $2 : 1$ (B) $4 : 3$ (C) $3 : 2$ (D) $1 : 1$
4. A cone and cylinder have equal base and equal volume. Ratio of their heights (cone : cylinder) is: (A) $1 : 3$ (B) $3 : 1$ (C) $1 : 1$ (D) $2 : 3$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (A):** $22 \times 15 = 330$ cm^2. **Q2 (B):** $\frac{4}{3}\pi(27) = 36\pi$. **Q3 (A):** 5-12-13 triple. **Q4 (B):** $3\pi r^2$.
Exam Std	**Q1 (A):** $h = 8 \implies \frac{1}{3}\pi(36)(8) = 96\pi$. **Q2 (B):** $r_1/r_2 = 2/3 \implies (2/3)^2 = 4:9$. **Q3 (A):** $4\pi r^2 = 616 \implies r = 7$ cm. **Q4 (A):** $\frac{2}{3}(3.14)(27) = 56.52$ m$^3 = 56520$ L.
Challenge	**Q1 (A):** $V_1 = \frac{1}{3}\pi(8^2)(6), V_2 = \frac{1}{3}\pi(6^2)(8) \implies 8:6 = 4:3$. **Q2 (B):** $(1.2^3 - 1) \times 100 = 72.8\%$. **Q3 (B):** $4\pi r^2 : 3\pi r^2 = 4:3$. **Q4 (B):** $\frac{1}{3}\pi r^2 h_1 = \pi r^2 h_2 \implies h_1 : h_2 = 3:1$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Cone $l, \text{CSA}, \text{TSA}, V$. Day 2: Sphere & Hemisphere. Day 3: Word problems, ratios, and unit conversions.
- **How to Introduce the Concept:** Fill a hollow cone with sand/water and pour it into a cylinder of the same base and height—it takes exactly 3 cones!
- **Difficult Areas & Remediation:** Confusing h vs l in cones and $2\pi r^2$ vs $3\pi r^2$ in hemispheres.
- **Blackboard Teaching Examples:** Draw the 3-4-5 right triangle inside a cone showing r, h, l clearly.
- **Classroom Questions to Ask Students:** Why does a solid hemisphere have TSA $3\pi r^2$ instead of half of $4\pi r^2$?
- **Common Student Doubts & Answers:** Students ask about cuboids and cylinders; confirm they are excluded from Class 9 2026–27.
- **Homework & Revision Strategy:** Assign 4 cone problems, 4 sphere/hemisphere problems, and 2 ratio problems.

[TEACHER'S TIP] Memorizing $r = 7 \implies \pi r^2 = 154, 4\pi r^2 = 616$ saves minutes in every test.

UNIT VI: STATISTICS TYBA COACHING CENTRE

Chapter 12: Statistics

Graphical Representation: Bar Graphs, Histograms (Equal & Varying Widths) & Frequency Polygons (2026–27)

A. Chapter Introduction & Examination Relevance

Raw numbers in a table are hard to read at a glance, whereas a well-drawn chart reveals trends immediately. Chapter 12 focuses on the three foundational statistical graphs: Bar Graphs, Histograms, and Frequency Polygons.

- **What You Will Learn:** Class limits, class marks ($\frac{\text{lower}+\text{upper}}{2}$), Bar Graphs (discrete/categorical data), Histograms for equal and **unequal** class widths (adjusted frequency), and Frequency Polygons (with and without histograms).
- **Why This Chapter Matters:** Statistics carries 10/100 marks in RBSE (10/80 in CBSE) and leads directly into Class 10 Mean/Median/Mode and Class 11 Dispersion.
- **Real-Life Applications:** Election vote-share charts, rainfall & temperature histograms, exam score distributions, and business sales dashboards.
- **Cross-Chapter Connections:** Uses Chapter 3 (Cartesian axes & scales) and feeds into Class 10 Chapter 13 and Class 11 Chapter 13.
- **Exam & Foundation Relevance:** Expect 1M MCQs on class marks/width/kink/adjusted height and 3M/5M graph-drawing questions (especially varying-width histograms and frequency polygons). (Data collection & Mean/Median/Mode are excluded from Class 9 in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Bar Graphs vs Histograms

A **Bar Graph** displays discrete or categorical data using separated bars of equal width with gaps between them (height $\propto$ frequency). A **Histogram** displays continuous grouped class intervals with **adjacent rectangles (no gaps)**. If intervals are discontinuous (inclusive, like 1–$10, 11$–20), convert them to continuous boundaries (0.5–$10.5, 10.5$–20.5) first by subtracting/adding 0.5.

[CONCEPT MASTER – 1. Bar Graphs vs Histograms] Kink (Break) Rule: Whenever the horizontal axis does not start at 0 (e.g., starts at 140 cm), draw a zig-zag **kink** near the origin 0!

2. Histograms with Varying (Unequal) Class Widths

In a histogram, the **area** of each rectangle must be proportional to its frequency. When class widths vary, adjust each bar's height using the minimum class width $w_{\min}$: Adjusted Frequency (Height) $= \frac{\text{Frequency}}{\text{Class Width}} \times w_{\min}$.

[CONCEPT MASTER – 2. Histograms with Varying (Unequal) Class Widths] Never plot raw frequencies directly when class widths are unequal—wider classes would look artificially huge!

3. Frequency Polygons

A **Frequency Polygon** is formed by joining the **class marks** (midpoints: $\frac{\text{Lower}+\text{Upper}}{2}$) at their frequencies with straight-line segments. To close the polygon on the x-axis, add one imaginary zero-frequency class before the first class and one after the last class.

[CONCEPT MASTER – 3. Frequency Polygons] Area of a frequency polygon equals the total area of the corresponding histogram!

4. Rationalised 2026–27 Scope

In the 2026–27 rationalised NCERT syllabus, Chapter 12 concentrates exclusively on Graphical Representation of Data (Bar Graphs, Histograms, and Frequency Polygons). Collection of data, frequency table construction, and Mean/Median/Mode have been moved/removed.

[CONCEPT MASTER – 4. Rationalised 2026–27 Scope] Always write the **Scale** on the top-right of every graph (e.g., x-axis: 1 cm $=$ 10 units, y-axis: 1 cm $=$ 5 students).

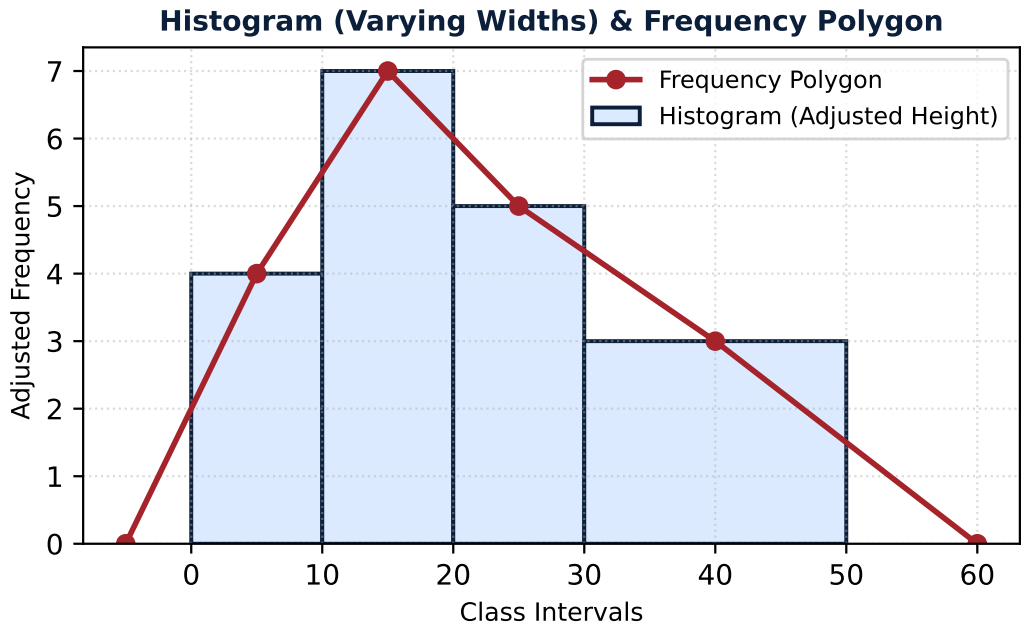


Figure 1: **Visual Model:** Continuous histogram with varying class widths (using adjusted frequency density) overlaid with a closed frequency polygon.

C. Complete Formula Bank

[FORMULA BANK – 1. Class Mark (Midpoint)] **Expression:** Class Mark $= \frac{\text{Lower Limit}+\text{Upper Limit}}{2}$
Symbols: Class interval $[a, b)$.
Conditions & When to Use: Used on x-axis for Frequency Polygons.
Solved Mini-Example: Class mark of 140–150 is 145.
Common Mistake: Subtracting limits instead of adding.

[FORMULA BANK – 2. Class Size (Width)] **Expression:** $h =$ Upper Limit $-$ Lower Limit
Symbols: Continuous boundaries.
Conditions & When to Use: Convert inclusive classes first.
Solved Mini-Example: Width of 10–25 is 15.
Common Mistake: Using 10–19 width as 9 instead of 10 (boundary 9.5–19.5).

[FORMULA BANK – 3. Adjusted Frequency (Varying Width)] **Expression:** Adjusted Height $= \left(\frac{\text{Frequency}}{\text{Class Width}}\right) \times$ Minimum Class Width
Symbols: $w_{\min}$: smallest class width in the table.
Conditions & When to Use: Mandatory whenever class widths differ.
Solved Mini-Example: Min width $= 2$; class 10–16 (width 6) with freq 9 has height $\frac{9}{6} \times 2 = 3$.
Common Mistake: Forgetting to multiply by $w_{\min}$.

[FORMULA BANK – 4. Continuous Boundary Adjustment] **Expression:** $d = \frac{\text{Lower}_{i+1} - \text{Upper}_i}{2}$; New limits: $[L - d,\ U + d]$
Symbols: Usually $d = 0.5$ for integer gap 1.
Conditions & When to Use: Inclusive intervals like 1–5, 6–10.
Solved Mini-Example: 1–5 $\rightarrow$ 0.5–5.5; 6–10 $\rightarrow$ 5.5–10.5.
Common Mistake: Drawing gaps in a histogram of inclusive classes.

[FORMULA BANK – 5. Recovering Class Limits from Class Marks] **Expression:** Lower $= m - \frac{h}{2}$, Upper $= m + \frac{h}{2}$
Symbols: m: class mark, h: uniform class width.
Conditions & When to Use: Consecutive marks m_1, m_2 give $h = m_2 - m_1$.
Solved Mini-Example: Marks $15, 25, 35 \implies h = 10$; first class is 10–20.
Common Mistake: Using $m \pm h$ instead of $m \pm h/2$.

[FORMULA BANK – 6. Range of Data] **Expression:** Range $=$ Maximum Value $-$ Minimum Value
Symbols: Raw data.
Conditions & When to Use: Measures total spread.
Solved Mini-Example: Max 95, Min 12 $\implies$ Range $= 83$.
Common Mistake: Dividing by 2.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Olympiad/NTSE Foundation problem structures.

Pattern 1: Class Mark, Class Width & Boundary Recovery

- **Identification Cue:** Class marks are $10, 20, 30, \ldots$; find the class corresponding to 20. Or convert 1–10, 11–20 to continuous. | **Concept & Formula:** $h = m_2 - m_1$, $[m - h/2, m + h/2]$.
- **Standard Step-by-Step:** Find h from difference of consecutive marks, subtract/add $h/2$ to get limits.
- **Fastest Valid Method:** Halfway between 10 and 20 is 15; between 20 and 30 is 25 $\implies$ 15–25! | **Alternative:** Formula $m \pm h/2$.
- **Common Trap:** Adding/subtracting full h instead of $h/2$.
- **Practice Drill:** Ex. 1 & Ex. 5.

Pattern 2: Drawing a Bar Graph for Categorical / Discrete Data

- **Identification Cue:** Represent causes of illness or political party seats as a bar graph. | **Concept & Formula:** Equal-width bars with equal gaps.
- **Standard Step-by-Step:** Choose clean y-scale, draw equal-width separated bars, label both axes and scale.
- **Fastest Valid Method:** Keep bar width = gap width for neat presentation. | **Alternative:** None.
- **Common Trap:** Drawing bars touching each other (that's a histogram!).
- **Practice Drill:** Ex. 2 & Ex. 6.

Pattern 3: Histogram with Equal or Varying Class Widths

- **Identification Cue:** Draw a histogram for marks 0–20, 20–30, 30–40, 40–50, 50–70, 70–100. | **Concept & Formula:** Adjusted height $= \frac{f}{w} \times w_{\min}$.
- **Standard Step-by-Step:** Add a column for Class Width w, find $w_{\min}$, compute Adjusted Frequency, and plot rectangles with no gaps.
- **Fastest Valid Method:** Check $w_{\min}$ first—usually $1, 2, 5$, or 10. | **Alternative:** Convert inclusive boundaries by ± 0.5 first if needed.
- **Common Trap:** Plotting raw frequency on unequal-width intervals.
- **Practice Drill:** Ex. 7 & Ex. 9.

Pattern 4: Frequency Polygon (With or Without Histogram)

- **Identification Cue:** Draw frequency polygons for two teams A and B on the same axes. | **Concept & Formula:** Plot (x_i, f_i) at class marks + zero endpoints.
- **Standard Step-by-Step:** Compute class marks, add one zero-freq class mark before start and one after end, join with ruler.
- **Fastest Valid Method:** Always use a ruler for straight segments (never a freehand curve!). | **Alternative:** Draw histogram first and join top midpoints.
- **Common Trap:** Forgetting to close the polygon down to the x-axis at both ends.
- **Practice Drill:** Ex. 8 & Ex. 11.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Instant Adjusted-Height Check] **Core Idea & Why It Works:** In a varying-width histogram, for any class with width $w = k \cdot w_{\min}$, simply divide its frequency by k!

Valid Domain: All unequal-width tables where widths are multiples of $w_{\min}$. | **Where It Fails:** Non-multiple widths (use full formula $\frac{f}{w} \times w_{\min}$).

Easy Example: If $w_{\min} = 10$ and a class 50–70 has width 20 ($k = 2$) and freq 18, height $= 18/2 = 9$!

Exam-Level Example: Mental check in 1 second per row.

Advanced Example: Total area remains preserved.

[SMART SHORTCUT – 2. Midway Boundary Trick for Class Marks] **Core Idea & Why It Works:** Given consecutive class marks m_1, m_2, m_3, the limits around m_2 are simply the midpoints $\frac{m_1+m_2}{2}$ and $\frac{m_2+m_3}{2}$!

Valid Domain: All uniform-width tables. | **Where It Fails:** Unequal widths.

Easy Example: Marks $104, 114, 124 \implies$ class for 114 is 109–119.

Exam-Level Example: Zero formula memorization needed!

Advanced Example: Instant 1-mark MCQ solver.

[SMART SHORTCUT – 3. Graph Checklist (4 Mandatory Items)] **Core Idea & Why It Works:** Every CBSE/RBSE graph loses 0.5–1 mark if any of these 4 is missing: (1) Axis labels, (2) Scale box, (3) Kink if not starting at 0, (4) Zero-closed ends on Frequency Polygons.

Valid Domain: All graph questions. | **Where It Fails:** Nothing.

Easy Example: Check all 4 before moving to the next question.

Exam-Level Example: Guarantees 5/5 on graph problems.

Advanced Example: Use sharp pencil and ruler.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: Find the class mark and class size of the interval 15–25.

Given & Required: Lower $= 15$, Upper $= 25$. | **Concept & Formula:** $\frac{L+U}{2}$ and $U - L$.

Detailed Solution: Class mark $= \frac{15+25}{2} = 20$. Class size $= 25 - 15 = 10$. **Final Answer:** **Mark** $= 20$**, Size** $= 10$

Fastest Alternative / Shortcut: Average for mark, difference for size.

SOLVED EXAMPLE 2 [Beginner]

Question: Convert the inclusive classes 11–20, 21–30, 31–40 into continuous exclusive classes.

Given & Required: Gap $= 21 - 20 = 1$. | **Concept & Formula:** $d = 0.5$.

Detailed Solution: 10.5–20.5, 20.5–30.5, 30.5–40.5. **Final Answer:** 10.5**–**20.5, 20.5**–**30.5, 30.5**–**40.5

Fastest Alternative / Shortcut: Subtract 0.5 from lower, add 0.5 to upper.

SOLVED EXAMPLE 3 [Beginner]

Question: In a histogram, if the minimum class width is 5 and a class 20–35 has frequency 12, find its adjusted height.

Given & Required: $w_{\min} = 5, w = 15, f = 12$. | **Concept & Formula:** $\frac{f}{w} \times w_{\min}$.

Detailed Solution: Height $= \frac{12}{15} \times 5 = 4$. **Final Answer:** 4

Fastest Alternative / Shortcut: $15 = 3 \times 5 \implies 12/3 = 4$.

SOLVED EXAMPLE 4 [Beginner]

Question: When should a kink (zig-zag break) be drawn on the horizontal axis of a histogram?

Given & Required: Axis starting at 140 instead of 0. | **Concept & Formula:** Compressed scale.

Detailed Solution: Whenever the first class interval does not start at 0, to indicate a break between 0 and the first lower limit. **Final Answer: When first lower limit** > 0

Fastest Alternative / Shortcut: Prevents huge blank gap from 0.

SOLVED EXAMPLE 5 [Intermediate]

Question: The class marks of a distribution are $26, 31, 36, 41, 46$. Find the true class limits for the class mark 36.

Given & Required: $h = 31 - 26 = 5$. | **Concept & Formula:** $m \pm h/2$.

Detailed Solution: Lower $= 36 - 2.5 = 33.5$; Upper $= 36 + 2.5 = 38.5$. Class: 33.5–38.5. **Final Answer:** 33.5**–**38.5

Fastest Alternative / Shortcut: Midpoints of $(31, 36)$ and $(36, 41)$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Explain how to close a frequency polygon for classes 0–$10, 10$–$20, 20$–$30, 30$–40.

Given & Required: Class marks $5, 15, 25, 35$. | **Concept & Formula:** Add zero-frequency endpoints.

Detailed Solution: Add $(-5, 0)$ before the first class mark 5 and $(45, 0)$ after the last class mark 35, then join all points with straight lines. **Final Answer: Join** $(-5, 0)$ **at start and** $(45, 0)$ **at end**

Fastest Alternative / Shortcut: Step back/forward by $h = 10$.

SOLVED EXAMPLE 7 [Intermediate]

Question: In a bar graph, if 1 cm represents 50 students, what bar height represents 325 students?

Given & Required: Scale 1 cm $= 50$. | **Concept & Formula:** Proportional height.

Detailed Solution: Height $= \frac{325}{50} = 6.5$ cm. **Final Answer:** 6.5 **cm**

Fastest Alternative / Shortcut: $325/50 = 6.5$.

SOLVED EXAMPLE 8 [Intermediate]

Question: What is the key visual difference between a Bar Graph and a Histogram?

Given & Required: Comparison. | **Concept & Formula:** Discrete vs Continuous.

Detailed Solution: A Bar Graph has separated bars (gaps) for discrete/categorical data; a Histogram has adjacent rectangles (no gaps) for continuous class intervals, with area $\propto$ frequency. **Final Answer: Gaps (Bar) vs No Gaps (Histogram)**

Fastest Alternative / Shortcut: Bar = gaps; Histogram = touching.

SOLVED EXAMPLE 9 [Board Standard]

Question: Compute the adjusted frequencies for a histogram of: 0–20 (7), 20–30 (10), 30–40 (10), 40–50 (20), 50–60 (20), 60–70 (15), 70–100 (8).

Given & Required: Widths: $20, 10, 10, 10, 10, 10, 30$. | **Concept & Formula:** $w_{\min} = 10$.

Detailed Solution: Adjusted heights $(\frac{f}{w} \times 10)$: 0–20: $\frac{7}{20} \times 10 = 3.5$; 20–30: 10; 30–40: 10; 40–50: 20; 50–60: 20; 60–70: 15; 70–100: $\frac{8}{30} \times 10 = 2.67$. **Final Answer:** $3.5, 10, 10, 20, 20, 15, 2.67$

Fastest Alternative / Shortcut: Divide width-20 by 2, width-30 by 3.

SOLVED EXAMPLE 10 [Board Standard]

Question: Compute adjusted heights for surname letter counts: 1–4 (6), 4–6 (30), 6–8 (44), 8–12 (16), 12–20 (4).

Given & Required: Widths: $3, 2, 2, 4, 8$; $w_{\min} = 2$. | **Concept & Formula:** $\frac{f}{w} \times 2$.

Detailed Solution: Heights: $\frac{6}{3} \times 2 = 4$; 30; 44; $\frac{16}{4} \times 2 = 8$; $\frac{4}{8} \times 2 = 1$. **Final Answer:** $4, 30, 44, 8, 1$

Fastest Alternative / Shortcut: Clean integers $4, 30, 44, 8, 1$!

SOLVED EXAMPLE 11 [Board Standard]

Question: Find the plotting coordinates for a frequency polygon of: 140–150 (5), 150–160 (10), 160–170 (20), 170–180 (9), 180–190 (6), 190–200 (2).

Given & Required: $h = 10$. | **Concept & Formula:** Class marks + zero ends.

Detailed Solution: Points: $(135, 0), (145, 5), (155, 10), (165, 20), (175, 9), (185, 6), (195, 2), (205, 0)$. **Final Answer:** $(135, 0)$ **through** $(205, 0)$

Fastest Alternative / Shortcut: Midpoints $145..195$ plus $135, 205$.

SOLVED EXAMPLE 12 [Board Standard]

Question: A leaf-length table has inclusive classes $118–126, 127–135, 136–144, \ldots$ Convert to continuous limits and state the class width.

Given & Required: Gap $= 127 - 126 = 1$. | **Concept & Formula:** $d = 0.5$.

Detailed Solution: Continuous classes: 117.5–126.5, 126.5–135.5, 135.5–144.5. Class width $= 126.5 - 117.5 = 9$. **Final Answer:** $117.5–126.5, \ldots$**; width** $= 9$

Fastest Alternative / Shortcut: $126 - 118 + 1 = 9$.

SOLVED EXAMPLE 13 [Board Standard]

Question: In a city, 100 families' monthly expenditure is grouped: 1000–1500 (24), 1500–2000 (40), 2000–2500 (33), 2500–3000 (28)... wait: which class has the maximum frequency and what is its class mark?

Given & Required: 1500–2000 has 40. | **Concept & Formula:** Modal class & midpoint.

Detailed Solution: Maximum frequency is 40 in 1500–2000; class mark $= \frac{1500+2000}{2} = 1750$. **Final Answer:** 1500**–**2000**, mark** $= 1750$

Fastest Alternative / Shortcut: Midpoint of 1500–2000.

SOLVED EXAMPLE 14 [Challenge]

Question: If the class mark of a continuous class is 25 and its class width is 10, find the class interval.

Given & Required: $m = 25, h = 10$. | **Concept & Formula:** $[m - h/2,\ m + h/2]$.

Detailed Solution: Lower $= 25 - 5 = 20$; Upper $= 25 + 5 = 30$. Interval: 20–30. **Final Answer:** 20**–**30

Fastest Alternative / Shortcut: 25 ± 5.

SOLVED EXAMPLE 15 [Challenge]

Question: In a histogram, the area of each rectangle is proportional to what quantity?

Given & Required: Histogram property. | **Concept & Formula:** Area = width × height.

Detailed Solution: Proportional to the **class frequency** (which is why unequal widths require adjusted height = frequency density $\times w_{\min}$). **Final Answer: Class frequency**

Fastest Alternative / Shortcut: Area $\propto$ Frequency.

SOLVED EXAMPLE 16 [Challenge]

Question: Why are two frequency polygons preferred over two histograms when comparing two datasets (e.g., Section A vs Section B) on the same graph?

Given & Required: Comparison of 2 distributions. | **Concept & Formula:** Visual clarity.

Detailed Solution: Overlapping histogram bars obscure each other, whereas two frequency polygon lines can be overlaid cleanly on the same axes. **Final Answer: Polygons overlay cleanly without obscuring bars**

Fastest Alternative / Shortcut: Line plots don't hide each other.

G. Competency MCQ Mastery Section

Q1. Class mark of the class interval 90–120 is:

(A) 90 (B) 105 (C) 115 (D) 120

Answer: (B) $(90+120)/2 = 105$. *Fast Method:* Average of limits. *Distractor Note:* Subtracting to get 30.

Q2. In a histogram, the rectangles have:

(A) Gaps between them (B) No gaps between them (C) Equal heights (D) No y-axis

Answer: (B) Continuous classes touch with no gaps. *Fast Method:* No gaps. *Distractor Note:* Confusing with bar graph.

Q3. Width of the class interval 10–19 (inclusive) after making it continuous is:

(A) 9 (B) 10 (C) 9.5 (D) 19

Answer: (B) $19.5 - 9.5 = 10$. *Fast Method:* $19 - 10 + 1 = 10$. *Distractor Note:* Writing $19 - 10 = 9$.

Q4. If class marks are $15, 20, 25, \ldots$, the class corresponding to 20 is:

(A) 15–20 (B) 17.5–22.5 (C) 15–25 (D) 20–25

Answer: (B) $h = 5$, $20 \pm 2.5 = 17.5$–22.5. *Fast Method:* Midpoints of $(15, 20)$ and $(20, 25)$. *Distractor Note:* Picking 15–25.

Q5. In a varying-width histogram, $w_{\min} = 5$. For class 10–25 with frequency 18, adjusted height is:

(A) 6 (B) 18 (C) 3 (D) 9

Answer: (A) $\frac{18}{15} \times 5 = 6$. *Fast Method:* $15 = 3 \times 5 \implies 18/3 = 6$. *Distractor Note:* Using raw 18.

Q6. A kink on the x-axis indicates:

(A) An error in data (B) A break in scale from 0 (C) Zero frequency (D) Equal class widths

Answer: (B) Shows compressed gap from 0 to first limit. *Fast Method:* Scale break. *Distractor Note:* Saying zero frequency.

Q7. To draw a frequency polygon directly without a histogram, we plot:

(A) (Lower limit, freq) (B) (Upper limit, freq) (C) (Class mark, freq) (D) (Width, freq)

Answer: (C) Class marks on x-axis, frequencies on y-axis. *Fast Method:* Class mark = midpoint. *Distractor Note:* Using upper limit (that's ogive!).

Q8. In a histogram, the area of each bar is proportional to:

(A) Class mark (B) Class width (C) Class frequency (D) Upper limit

Answer: (C) Area $= w \times (f/w) = f$. *Fast Method:* Area $\propto$ Frequency. *Distractor Note:* Picking class width.

Q9. Range of the data $25, 18, 20, 22, 16, 6, 17, 15, 12, 30$ is:

(A) 24 (B) 18 (C) 12 (D) 6

Answer: (A) $30 - 6 = 24$. *Fast Method:* Max − Min. *Distractor Note:* Subtracting ends $30 - 25$.

Q10. If m is the midpoint and l is the upper limit of a class, the lower limit is:

(A) $2m - l$ (B) $2m + l$ (C) $m - l$ (D) $m + l$

Answer: (A) $m = \frac{\text{lower}+l}{2} \implies \text{lower} = 2m - l$. *Fast Method:* $2m - l$. *Distractor Note:* Writing $m - l$.

Q11. Which graph is best for comparing two continuous frequency distributions on the same axes?

(A) Bar graph (B) Pie chart (C) Frequency polygon (D) Pictograph

Answer: (C) Two polygon lines overlay cleanly. *Fast Method:* Frequency polygon. *Distractor Note:* Picking bar graph.

Q12. Upper limit of the class 10–19 in continuous form is:

(A) 19 (B) 19.5 (C) 18.5 (D) 20

Answer: (B) Add 0.5 to upper limit 19. *Fast Method:* 19.5. *Distractor Note:* Picking 19.

H. Special Exam & MCQ Strategies

- Before drawing ANY histogram, check two things: (1) Are the classes continuous (no gaps)? (2) Are all class widths equal?
- Always write the Scale, Axis Labels, and Kink (if first limit > 0) on every graph.
- Join frequency polygon points with a ruler (straight segments) and close both ends down to the x-axis at zero frequency.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Plotting raw frequencies in a histogram with unequal class widths. $\longrightarrow$ **Correction:** Always compute Adjusted Frequency $= \frac{f}{\text{width}} \times w_{\min}$ first.

[COMMON MISTAKE] **Error:** Finding class interval from class marks $10, 20, 30$ as 10–30 for 20. $\longrightarrow$ **Correction:** Add and subtract HALF the width ($h/2 = 5$): 15–25.

[COMMON MISTAKE] **Error:** Leaving a frequency polygon floating in the air without closing it to the x-axis. $\longrightarrow$ **Correction:** Include the preceding and succeeding zero-frequency class marks.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: Class Mark $= \frac{L+U}{2}$; Width $= U - L$; Adjusted Height $= \frac{f}{w} \times w_{\min}$; Limits $= m \pm \frac{h}{2}$.

[QUICK REVISION] Key Theorems & Results: Bar graph = gaps; Histogram = no gaps (area $\propto f$); Frequency polygon = closed at x-axis.

[QUICK REVISION] Shortcut Summary: Width $k \cdot w_{\min} \implies$ divide f by k; Class from marks = midpoints between consecutive marks.

[QUICK REVISION] High-Frequency Exam Patterns: Class mark/limits, Inclusive-to-continuous, Varying-width histogram table, Frequency polygon.

Last-Minute Revision Checklist:

- ☐ Continuous limits (± 0.5)
- ☐ Check equal vs unequal widths
- ☐ Scale + Labels + Kink
- ☐ Close polygon at both ends

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Class mark of 20–30 is: (A) 10 (B) 25 (C) 50 (D) 20
2. A histogram is drawn for: (A) Categorical data (B) Continuous grouped data (C) Single points (D) Ungrouped names
3. Range of $12, 18, 5, 29, 14$ is: (A) 24 (B) 2 (C) 17 (D) 29
4. In a bar graph, width of all bars must be: (A) Different (B) Equal (C) Zero (D) Proportional to frequency

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Class marks are $12, 18, 24, 30$. Class for 18 is: (A) 12–24 (B) 15–21 (C) 16–20 (D) 15–24
2. Adjusted height of 30–50 (freq 16) when $w_{\min} = 5$ is: (A) 4 (B) 8 (C) 16 (D) 3.2
3. Continuous form of 5–9, 10–14 has class width: (A) 4 (B) 5 (C) 4.5 (D) 9
4. To close a frequency polygon of 10–20, 20–30, endpoints on x-axis are: (A) $(5, 0)$ and $(35, 0)$ (B) $(10, 0)$ and $(30, 0)$ (C) $(0, 0)$ and $(40, 0)$ (D) $(15, 0)$ and $(25, 0)$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If m is class mark and h is class width, upper limit is: (A) $m + h$ (B) $m + h/2$ (C) $m - h/2$ (D) $2m - h$
2. Total area of a frequency polygon compared to its histogram is: (A) Smaller (B) Larger (C) Equal (D) Double
3. In inclusive class 21–30, true lower limit is: (A) 20 (B) 20.5 (C) 21 (D) 21.5
4. If $w_{\min} = 10$ and a class 40–70 has adjusted height 6, its actual frequency is: (A) 18 (B) 2 (C) 12 (D) 24

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $(20+30)/2 = 25$. **Q2 (B):** Continuous intervals. **Q3 (A):** $29-5=24$. **Q4 (B):** Equal width, separated by equal gaps.
Exam Std	**Q1 (B):** $h=6, 18\pm3 = 15$–21. **Q2 (A):** $\frac{16}{20}\times 5 = 4$. **Q3 (B):** $9.5-4.5=5$. **Q4 (A):** Marks $15, 25 \rightarrow$ ends $5, 35$.
Challenge	**Q1 (B):** $m+h/2$. **Q2 (C):** Congruent triangles added and subtracted cancel out: equal area. **Q3 (B):** $21-0.5=20.5$. **Q4 (A):** $f = 6\times\frac{30}{10} = 18$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Bar graphs & equal-width histograms. Day 2: Varying-width histograms & frequency polygons.
- **How to Introduce the Concept:** Ask why a 30-mark wide bar (70–100) looks 3x bigger than a 10-mark bar (60–70) if you don't adjust height!
- **Difficult Areas & Remediation:** Adjusted frequency calculation and closing frequency polygons at zero-frequency endpoints.
- **Blackboard Teaching Examples:** Draw the 4-column table (Interval, Freq, Width, Adjusted Height) before plotting.
- **Classroom Questions to Ask Students:** Why must a frequency polygon be closed down to the x-axis?
- **Common Student Doubts & Answers:** Students ask about Mean, Median, Mode; clarify they were moved to Class 10 (only graphical representation is in Class 9 2026–27).
- **Homework & Revision Strategy:** Assign 1 bar graph, 1 varying-width histogram, and 1 double frequency polygon.

[TEACHER'S TIP] Remind students to bring a sharp pencil and ruler—neatness wins full marks in Statistics.

BACK MATTER: COMPLETE CLASS 9 REVISION TYBA COACHING CENTRE

Chapter BM: Master Formula Index, Shortcut Bank & 80-Mark Sample Paper

TYBA Coaching Centre, Ajmer | RBSE & CBSE Foundation (2026–27) Final Exam Readiness

1. Master Formula Index (All 12 Chapters at a Glance)

Chapter	Essential Master Formulas
01. Number Systems	$\frac{1}{\sqrt{a}\pm\sqrt{b}} = \frac{\sqrt{a}\mp\sqrt{b}}{a-b}$; $a^m a^n = a^{m+n}$; $(a^m)^n = a^{mn}$; $a^{m/n} = (\sqrt[n]{a})^m$; $0.\overline{ab} = \frac{ab}{99}$.
02. Polynomials	Factor Thm: $(x-a)$ factor $\iff p(a)=0$; $(x+y+z)^2$; $(x\pm y)^3$; $x^3\pm y^3$; $x^3+y^3+z^3 = 3xyz$ if $x+y+z=0$.
03. Coord. Geometry	$P(x,y)$: Dist from x-axis $= \lvert y\rvert$, from y-axis $= \lvert x\rvert$; $OP = \sqrt{x^2+y^2}$; Q-I $(+,+)$, Q-II $(-,+)$, Q-III $(-,-)$, Q-IV $(+,-)$.
04. Linear Equations	$ax+by+c=0$ $(a^2+b^2\neq 0)$; ∞ solutions; x-int $(-c/a, 0)$, y-int $(0,-c/b)$; Origin iff $c=0$.
05. Euclid's Geometry	7 Axioms (transitivity, $+$, $-$, coincidence, whole>part, doubles, halves) + 5 Postulates (Postulate 5: $\angle 1+\angle 2 < 180^\circ$).
06. Lines & Angles	Comp: 90°; Supp: 180°; Linear Pair $= 180^\circ$; VOA $=$; Parallel: Corr. $=$, Alt. Int. $=$, Co-int. $= 180^\circ$.
07. Triangles	Valid Congruence: SAS, ASA, AAS, SSS, RHS (Invalid: SSA, AAA); $AB=AC \iff \angle B = \angle C$; CPCTC.
08. Quadrilaterals	Sum $= 360^\circ$; Pgm opp. equal, diag bisect; Rect diag $=$; Rhombus diag $\perp$; Mid-point Thm: $EF \parallel BC, EF = \frac{1}{2}BC$.
09. Circles	$r^2 = d^2 + (c/2)^2$; $\angle AOB = 2\angle APB$; Semicircle $= 90^\circ$; Same segment $=$; Cyclic: $\angle A + \angle C = 180^\circ$.
10. Heron's Formula	$s = \frac{a+b+c}{2}$; $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$; Equilateral: $\frac{\sqrt{3}}{4}a^2$; Isosceles: $\frac{b}{4}\sqrt{4a^2-b^2}$.
11. Surface Area & Vol	Cone: $l=\sqrt{r^2+h^2}, \pi rl, \pi r(l+r), \frac{1}{3}\pi r^2 h$; Sphere: $4\pi r^2, \frac{4}{3}\pi r^3$; Hemisphere: $2\pi r^2, 3\pi r^2, \frac{2}{3}\pi r^3$.
12. Statistics	Class Mark $= \frac{L+U}{2}$; Width $= U-L$; Adjusted Height $= \frac{f}{w}\times w_{\min}$; Limits $= m \pm \frac{h}{2}$; Range $= L-S$.

2. Master Smart Shortcut Index (High-Speed Exam Techniques)

- **Ch. 1–2:** $0.A\overline{B} = \frac{AB-A}{90}$; $\frac{1}{\sqrt{n+1}+\sqrt{n}} = \sqrt{n+1}-\sqrt{n}$; $(x-1)$ factor $\iff$ sum of coeffs $=0$; Shared-variable sign rule.
- **Ch. 3–4:** Opposite-letter axis distance; Axis-name-stays-same reflection; Taxi fare $y = F + r(x-1)$.
- **Ch. 5–7:** Geometry words $\to$ Postulate; F-Z-C transversal patterns; M-bend $= \alpha+\beta$; Base angle $= 90^\circ - A/2$.
- **Ch. 8–9:** Midpoints: Quad $\to$ Pgm, Rect $\to$ Rhombus, Rhombus $\to$ Rect; $\angle OAB = 90^\circ - \angle ACB$; Minor arc $\angle OPR = \angle PQR - 90^\circ$.
- **Ch. 10–12:** Check $(s-a)+(s-b)+(s-c) = s$; Scale sides by $k \implies$ Area scales by k^2; $r=7 \to 4\pi r^2 = 616$; $w = k\cdot w_{\min} \implies f/k$.

3. Full-Length Class 9 Annual Sample Question Paper (80 Marks • 3 Hours)

General Instructions: All questions are compulsory. Section A: Q1–Q20 (1M each); Section B: Q21–Q25 (2M each); Section C: Q26–Q31 (3M each); Section D: Q32–Q35 (5M each); Section E: Q36–Q38 (4M Case-Based each).

Section A: Multiple Choice & Assertion-Reason Questions ($20 \times 1 = 20$ Marks)

1. Value of $(256)^{0.16} \times (256)^{0.09}$ is: (A) 4 (B) 16 (C) 64 (D) 256.25
2. If $x = 2 + \sqrt{3}$, then $x + \frac{1}{x}$ equals: (A) $2\sqrt{3}$ (B) 4 (C) 2 (D) $4 + 2\sqrt{3}$
3. Degree of the zero polynomial is: (A) 0 (B) 1 (C) Any natural number (D) Not defined
4. If $x + y + z = 0$, then $x^3 + y^3 + z^3$ equals: (A) 0 (B) xyz (C) $3xyz$ (D) $-3xyz$
5. Distance of the point $(-3, -4)$ from the x-axis is: (A) 3 (B) -4 (C) 4 (D) 5
6. Point $(0, -7)$ lies on: (A) Q-III (B) Q-IV (C) x-axis (D) y-axis
7. Equation $x = 7$ in two variables is written as: (A) $x + y = 7$ (B) $1x + 0y = 7$ (C) $0x + 1y = 7$ (D) $0x + 0y = 7$
8. If $(2, 0)$ is a solution of $2x + 3y = k$, then k is: (A) 4 (B) 6 (C) 5 (D) 2
9. 'The whole is greater than the part' is Euclid's: (A) Postulate 5 (B) Axiom 5 (C) Definition (D) Theorem
10. Angle between the bisectors of a linear pair is: (A) 45° (B) 60° (C) 90° (D) 180°
11. Which of the following is NOT a triangle congruence rule? (A) SAS (B) ASA (C) SSA (D) RHS
12. In $\triangle ABC$, $AB = BC$ and $\angle B = 80^\circ$. Then $\angle A$ equals: (A) 80° (B) 40° (C) 50° (D) 100°
13. Joining midpoints of consecutive sides of a rectangle forms a: (A) Rectangle (B) Rhombus (C) Square (D) Trapezium
14. If chord length is 10 cm and distance from centre is 12 cm, radius is: (A) 13 cm (B) 17 cm (C) 7 cm (D) 24 cm
15. In cyclic quadrilateral $ABCD$, if $\angle A - \angle C = 20^\circ$, then $\angle A$ is: (A) 80° (B) 100° (C) 110° (D) 70°
16. If each side of a triangle is doubled, its area becomes: (A) 2 times (B) 3 times (C) 4 times (D) 8 times
17. Total surface area of a solid hemisphere of radius r is: (A) $2\pi r^2$ (B) $3\pi r^2$ (C) $4\pi r^2$ (D) $\frac{2}{3}\pi r^3$
18. Class mark of the class interval 90–120 is: (A) 90 (B) 105 (C) 115 (D) 120
19. **Assertion (A):** A cyclic parallelogram is a rectangle. **Reason (R):** Opposite angles of a cyclic quad sum to 180° and opposite angles of a parallelogram are equal. (A) Both true, R explains A (B) Both true, R doesn't explain A (C) A true, R false (D) A false, R true.
20. **Assertion (A):** In a varying-width histogram, raw frequencies cannot be plotted directly. **Reason (R):** Area of each rectangle must be proportional to its frequency. (A) Both true, R explains A (B) Both true, R doesn't explain A (C) A true, R false (D) A false, R true.

Section B: Very Short Answer Questions ($5 \times 2 = 10$ Marks)

21. Express $0.4\overline{7}$ in the form $\frac{p}{q}$.
22. Find k if $(x - 1)$ is a factor of $p(x) = 2x^2 + kx + \sqrt{2}$.
23. Find the coordinates where $3x + 4y = 12$ meets the x-axis and y-axis.
24. In line A–B–C–D, if $AC = BD$, prove $AB = CD$ using Euclid's Axiom.
25. Find the curved surface area of a cone of slant height 10 cm and radius 7 cm.

Section C: Short Answer Questions ($6 \times 3 = 18$ Marks)

26. Find a and b if $\frac{3+\sqrt{7}}{3-\sqrt{7}} = a + b\sqrt{7}$.
27. Factorise $x^3 - 3x^2 - 9x - 5$.
28. If $PQ \parallel ST$, $\angle PQR = 110^\circ$, and $\angle RST = 130^\circ$, find $\angle QRS$.
29. BE and CF are two equal altitudes of $\triangle ABC$. Using RHS, prove $\triangle ABC$ is isosceles.
30. Two parallel chords of lengths 16 cm and 12 cm lie on opposite sides of the centre ($r = 10$ cm). Find the distance between them.

31. Sides of a triangle are in the ratio $12:17:25$ and its perimeter is 540 cm. Find its area.

Section D: Long Answer Questions ($4 \times 5 = 20$ Marks)

32. Verify that $x^3+y^3+z^3-3xyz = \frac{1}{2}(x+y+z)[(x-y)^2+(y-z)^2+(z-x)^2]$, and evaluate $(-12)^3+7^3+5^3$.
33. $ABCD$ is a rectangle and P, Q, R, S are midpoints of sides AB, BC, CD, DA. Prove $PQRS$ is a rhombus.
34. A conical tent is 10 m high and radius 24 m. Find (i) slant height, (ii) cost of canvas at Rs. $70/\text{m}^2$.
35. Compute adjusted frequencies and describe the histogram for: 1–4 (6), 4–6 (30), 6–8 (44), 8–12 (16), 12–20 (4).

Section E: Case-Based Questions ($3 \times 4 = 12$ Marks)

36. **Grid Park Layout (Coord. Geometry):** A rectangular park has vertices $A(2,1), B(6,1), C(6,4)$. Find: (i) 4th vertex D (1M), (ii) area of park (2M), (iii) distance of B from x-axis (1M).
37. **Circular Stage (Circles):** In a circular stage with centre O, $\angle PQR = 100^\circ$ (Q on minor arc PR). Find: (i) Reflex $\angle POR$ (1M), (ii) interior $\angle POR$ (1M), (iii) $\angle OPR$ (2M).
38. **Iron Spheres (Mensuration):** 27 solid iron spheres of radius r and surface area S melt into one large sphere of radius r' and area S'. Find: (i) r' in terms of r (2M), (ii) ratio $S:S'$ (2M).

4. Sample Paper Answer Key (Quick Verification)

Section	Verified Answers
Section A	1(A), 2(B), 3(D), 4(C), 5(C), 6(D), 7(B), 8(A), 9(B), 10(C), 11(C), 12(C), 13(B), 14(A), 15(B), 16(C), 17(B), 18(B), 19(A), 20(A).
Section B	21. $43/90$; 22. $-(2+\sqrt{2})$; 23. $(4,0),(0,3)$; 24. Subtract BC (Axiom 3); 25. 220 cm^2.
Section C	26. $a=8, b=3$; 27. $(x+1)^2(x-5)$; 28. 60°; 29. $\triangle BEC \cong \triangle CFB$ (RHS); 30. 14 cm; 31. 9000 cm^2.
Section D	32. -1260; 33. $AC=BD \implies PQ=QR$; 34. $l=26$ m, Rs. $1,37,280$; 35. Heights: $4, 30, 44, 8, 1$.
Section E	36. (i) $D(2,4)$, (ii) 12 sq. units, (iii) 1 unit; 37. (i) 200°, (ii) 160°, (iii) 10°; 38. (i) $r'=3r$, (ii) $1:9$.