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CLASS 8 MATHEMATICS

THE ULTIMATE COMPLETE MASTER TEXTBOOK

Dual-Board Edition: Rationalised NCERT / RBSE & CBSE Foundation (2026–27)

13 Complete Chapters • 12-Section Mastery Architecture • 100% English Edition

Concept Derivations • 13 Custom Vector Diagrams • Formula Banks • Smart Shortcuts • 208 Solved Examples • 156 Competency MCQs • 3-Tier Chapter Tests • Full 80-Mark Sample Paper

1. Edition Scope, Copyright & How to Use This Textbook

This master textbook is engineered for Class 8 students preparing for **RBSE Class 8 Board Examinations (2026–27)**, **CBSE Class 8 NCERT Curriculum**, and foundational **Sainik School / JNVST / NTSE / Olympiad** mathematics. Every chapter follows a uniform **12-Section Mastery Architecture (Sections A–L)** so students build deep conceptual clarity alongside rapid exam-hall execution.

- **Sections A–C (Concept & Formula Core):** Exam relevance, intuitive derivations, custom vector figures, and complete formula banks with boundary conditions.
- **Sections D–F (Problem Classification & Worked Mastery):** High-frequency problem patterns, mathematically validated shortcuts, and 16 graded solved examples per chapter (Beginner to Challenge).
- **Sections G–J (MCQ Mastery, Strategy & Quick Revision):** 12 competency MCQs with distractor analysis, exam-hall tactics, common error fixes, and a dedicated 1-page revision sheet.
- **Sections K–L (3-Tier Assessment & Faculty Guide):** Basic, Exam Standard, and Challenge tests with separate answer keys plus TYBA faculty classroom notes.

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2. Official Class 8 Syllabus & Unit Weightage (2026–27)

Rationalised NCERT & RBSE Class 8 Mathematics organize the 13 core chapters across 6 foundational units (80 Marks Theory + 20 Marks Internal Assessment = 100 Marks).

Unit	Unit Name & Included Chapters	Weightage
Unit I	**Number System** (Ch. 1 Rational Numbers, Ch. 5 Squares & Roots, Ch. 6 Cubes & Roots, Ch. 10 Exponents & Powers)	**20%**
Unit II	**Algebra** (Ch. 2 Linear Equations, Ch. 8 Algebraic Expressions, Ch. 12 Factorisation)	**25%**
Unit III	**Ratio & Proportion** (Ch. 7 Comparing Quantities, Ch. 11 Direct & Inverse Proportions)	**15%**
Unit IV	**Geometry** (Ch. 3 Understanding Quadrilaterals)	**12%**
Unit V	**Mensuration** (Ch. 9 Mensuration: Polygons, Cube, Cuboid, Cylinder)	**16%**
Unit VI	**Data Handling & Graphs** (Ch. 4 Data Handling, Ch. 13 Introduction to Graphs)	**12%**
Total	**All 6 Units / 13 Rationalised NCERT Chapters**	**100%**

3. Complete 13-Chapter Table of Contents & Rationalised Scope

Ch.	Chapter Title	Core Focus & Rationalised Exclusions (2026–27)
01	Rational Numbers	Closure, commutativity, associativity, distributivity, 0 & 1 identities. *(Excl: Number-line insertion)*
02	Linear Equations in One Variable	Solving equations with variables on both sides & reducing to simpler form. *(Excl: Word-problem exercises)*
03	Understanding Quadrilaterals	Polygons, angle sums, trapezium, kite, parallelogram, rectangle, rhombus, square. *(Full core retained)*
04	Data Handling	Bar/double-bar graphs, pie charts (central angles), basic probability. *(Excl: Grouped tally tables)*
05	Squares and Square Roots	Square properties, Pythagorean triplets, prime factorisation & long-division roots. *(Full core retained)*
06	Cubes and Cube Roots	Hardy-Ramanujan numbers, cube unit digits, prime factorisation cube roots. *(Full core retained)*
07	Comparing Quantities	Percentages, discount, GST/tax, Compound Interest (annual & half-yearly). *(Excl: Ratio-increase basics)*
08	Algebraic Expressions & Identities	Terms, addition/subtraction, multiplication of polynomials, standard identities. *(Excl: Identity proofs exercise)*
09	Mensuration	Trapezium, polygon areas; Cube, Cuboid & Cylinder TSA, LSA, Volume. *(Excl: General quad splits)*
10	Exponents and Powers	Negative exponents, laws of exponents, scientific standard form $k \times 10^n$. *(Full chapter retained)*
11	Direct and Inverse Proportions	$x/y = k$ vs $xy = k$, time-work-speed-pipes problems. *(Full chapter retained)*
12	Factorisation	Common factor, regrouping, identities, middle-term split, polynomial division. *(Full chapter retained)*
13	Introduction to Graphs	Line graphs, coordinates (x, y), linear graphs (cost, interest, distance). *(Excl: Bar/pie recap)*

UNIT I: NUMBER SYSTEM TYBA COACHING CENTRE

Chapter 1: Rational Numbers

Closure, Commutativity, Associativity, Distributivity & Identities (2026–27)

A. Chapter Introduction & Examination Relevance

Whole numbers and integers cannot express measurements like $\frac{3}{4}$ km or $-\frac{5}{2}^{\circ}$C. Rational Numbers (p/q, $q \neq 0$) complete the four basic arithmetic operations and obey clean algebraic properties.

- **What You Will Learn:** Closure, Commutativity, Associativity, Distributivity of multiplication over addition/subtraction, Additive Identity (0), Multiplicative Identity (1), Additive Inverse ($-a/b$), and Multiplicative Inverse / Reciprocal (b/a).
- **Why This Chapter Matters:** Properties of rational numbers simplify complex fraction calculations and form the foundation for Class 9 Real Numbers and Class 8 Algebraic Expressions.
- **Real-Life Applications:** Recipe scaling, financial shares, carpentry measurements in fractions, and algebraic equation simplification.
- **Cross-Chapter Connections:** Directly supports Chapter 2 (Linear Equations) and Chapter 8 (Algebraic Expressions).
- **Exam & Foundation Relevance:** Unit I carries 20% weightage. Expect 1M MCQs naming properties (e.g. $a \times b = b \times a$) and 2M/3M simplification problems using distributivity. (Number-line insertion is excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Closure, Commutativity & Associativity

A **rational number** has the form $\frac{p}{q}$ ($p, q \in \mathbb{Z}, q \neq 0$). Rational numbers are **closed** under addition, subtraction, and multiplication (not division, since division by 0 is undefined). Addition and multiplication are **commutative** ($a + b = b + a$, $ab = ba$) and **associative** ($(a + b) + c = a + (b + c)$), whereas subtraction and division are NOT.

[CONCEPT MASTER – 1. Closure, Commutativity & Associativity] Remember: 0 has NO reciprocal because $\frac{1}{0}$ is undefined!

2. Role of Zero (0) and One (1)

0 is the **additive identity** ($a + 0 = 0 + a = a$). 1 is the **multiplicative identity** ($a \times 1 = 1 \times a = a$). The **additive inverse** (negative) of $\frac{a}{b}$ is $-\frac{a}{b}$ (sum $= 0$). The **multiplicative inverse** (reciprocal) of $\frac{a}{b}$ ($a \neq 0$) is $\frac{b}{a}$ (product $= 1$).

[CONCEPT MASTER – 2. Role of Zero (0) and One (1)] Only TWO rational numbers are equal to their own reciprocals: 1 and -1!

3. Distributivity of Multiplication over Addition/Subtraction

For all rational numbers a, b, c: $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$. Factoring out a common fraction a turns long multi-term fraction products into 2-step mental math.

[CONCEPT MASTER – 3. Distributivity of Multiplication over Addition/Subtraction] Spot the common factor first! E.g., $\frac{2}{5} \times (-\frac{3}{7}) - \frac{1}{6} \times \frac{3}{2} + \frac{1}{14} \times \frac{2}{5}$ groups the two $\frac{2}{5}$ terms via commutativity and distributivity.

4. Rationalised 2026–27 Scope

In the 2026–27 rationalised NCERT syllabus, Chapter 1 focuses strictly on the algebraic properties of rational numbers (closure, commutativity, associativity, distributivity, identities 0 and 1). Representing rational numbers on the number line and inserting rationals between two rationals are covered in Class 7/9.

[CONCEPT MASTER – 4. Rationalised 2026–27 Scope] Sign Rule: $-\left(-\frac{a}{b}\right) = \frac{a}{b}$ always!

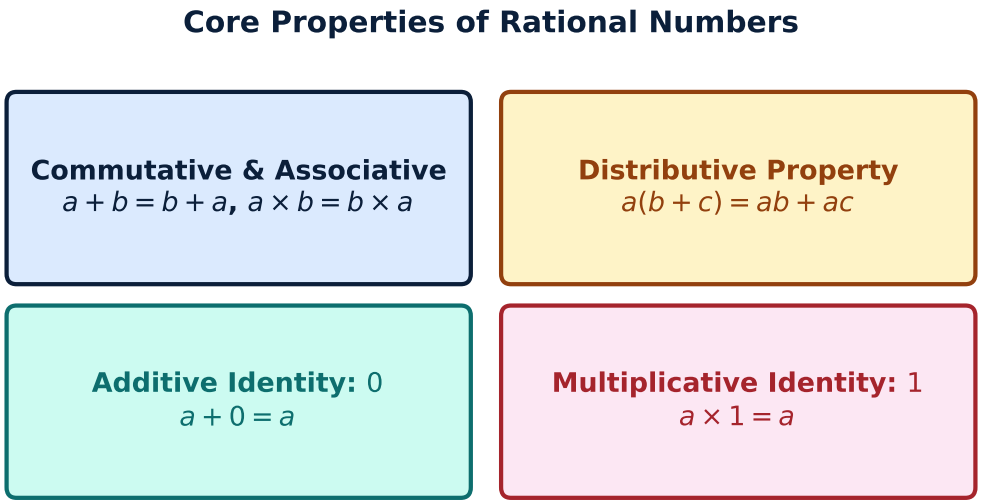


Figure 1: **Visual Model:** Core algebraic properties of Rational Numbers ($\mathbb{Q}$): Commutativity, Associativity, Distributivity, and Identities 0 and 1.

C. Complete Formula Bank

[FORMULA BANK – 1. Commutative Laws] **Expression:** $a + b = b + a$; $a \times b = b \times a$
Symbols: $a, b \in \mathbb{Q}$.
Conditions & When to Use: Fails for $-$ and $\div$.
Solved Mini-Example: $\frac{2}{3} + \frac{1}{5} = \frac{1}{5} + \frac{2}{3}$.
Common Mistake: Assuming $a - b = b - a$.

[FORMULA BANK – 2. Associative Laws] **Expression:** $(a + b) + c = a + (b + c)$; $(ab)c = a(bc)$
Symbols: $a, b, c \in \mathbb{Q}$.
Conditions & When to Use: Fails for $-$ and $\div$.
Solved Mini-Example: Used to regroup 3-term products.
Common Mistake: Regrouping division $(a \div b) \div c$.

[FORMULA BANK – 3. Distributive Law] **Expression:** $a(b \pm c) = ab \pm ac$
Symbols: $a, b, c \in \mathbb{Q}$.
Conditions & When to Use: Universal for all rationals.
Solved Mini-Example: $\frac{7}{5} \times (-\frac{3}{12}) + \frac{7}{5} \times \frac{5}{12} = \frac{7}{5}(\frac{2}{12}) = \frac{7}{30}$.
Common Mistake: Forgetting to multiply a by c too.

[FORMULA BANK – 4. Additive Identity & Inverse] **Expression:** $a + 0 = a$; $a + (-a) = 0$
Symbols: 0 is identity; $-a$ is inverse.
Conditions & When to Use: Always exists for every $a \in \mathbb{Q}$.
Solved Mini-Example: Additive inverse of $-\frac{5}{9}$ is $+\frac{5}{9}$.
Common Mistake: Flipping numerator and denominator for additive inverse.

[FORMULA BANK – 5. Multiplicative Identity & Inverse] **Expression:** $a \times 1 = a$; $\frac{a}{b} \times \frac{b}{a} = 1$ $(a, b \neq 0)$
Symbols: 1 is identity; $\frac{b}{a}$ is reciprocal.
Conditions & When to Use: Exists for all $a \neq 0$.
Solved Mini-Example: Reciprocal of $-\frac{3}{4}$ is $-\frac{4}{3}$ (sign stays!).
Common Mistake: Changing the minus sign when taking a reciprocal.

[FORMULA BANK – 6. Double Negative] **Expression:** $-(-x) = x$
Symbols: $x \in \mathbb{Q}$.
Conditions & When to Use: All rationals.
Solved Mini-Example: $-(-\frac{11}{15}) = \frac{11}{15}$.
Common Mistake: Leaving a minus sign.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Sainik/Navodaya/Olympiad Foundation problem structures.

Pattern 1: Naming the Property Used

- **Identification Cue:** Name the property under multiplication used in $-\frac{4}{5} \times 1 = 1 \times (-\frac{4}{5}) = -\frac{4}{5}$. | **Concept & Formula:** Identity, Commutativity, Associativity, Distributivity.
- **Standard Step-by-Step:** Compare the equation structure against the 4 core templates.
- **Fastest Valid Method:** $\times 1 \rightarrow$ Multiplicative Identity; $a \times b = b \times a \rightarrow$ Commutativity; $a \times \frac{1}{a} = 1 \rightarrow$ Multiplicative Inverse. | **Alternative:** Test structure with a, b, c.
- **Common Trap:** Confusing Multiplicative Identity ($\times 1$) with Multiplicative Inverse ($= 1$).
- **Practice Drill:** Ex. 1 & Ex. 5.

Pattern 2: Simplifying Multi-Term Expressions via Distributivity

- **Identification Cue:** Find $\frac{2}{5} \times (-\frac{3}{7}) - \frac{1}{6} \times \frac{3}{2} + \frac{1}{14} \times \frac{2}{5}$ using appropriate properties. | **Concept & Formula:** Commutativity + Distributivity.
- **Standard Step-by-Step:** Group terms sharing a common factor, pull out the factor, simplify inside brackets first.
- **Fastest Valid Method:** Circle the repeated fraction first! | **Alternative:** Direct multiplication (slower and loses property marks).
- **Common Trap:** Dropping negative signs when rearranging terms.
- **Practice Drill:** Ex. 9 & Ex. 10.

Pattern 3: Additive vs Multiplicative Inverse Verification

- **Identification Cue:** Is 0.3 the multiplicative inverse of $3\frac{1}{3}$? Why or why not? | **Concept & Formula:** Product $= 1 \implies$ Yes; Sum $= 0 \implies$ Additive inverse.
- **Standard Step-by-Step:** Convert mixed/decimal to improper fraction and multiply; check if product is $+1$.
- **Fastest Valid Method:** $0.3 = 3/10$, $3\frac{1}{3} = 10/3$, product $= 1 \implies$ Yes! | **Alternative:** Convert to decimals.
- **Common Trap:** Getting -1 instead of $+1$ (e.g., $-1\frac{1}{8}$ and $8/9$ give -1, so NOT inverse!).
- **Practice Drill:** Ex. 6 & Ex. 7.

Pattern 4: Verifying $-(-x) = x$ and Distributivity

- **Identification Cue:** Verify $a \times (b + c) = ab + ac$ for $a = -1/2, b = 2/3, c = -4/5$. | **Concept & Formula:** LHS = RHS.
- **Standard Step-by-Step:** Compute LHS and RHS in two separate columns and show equality.
- **Fastest Valid Method:** Reduce fractions at every step. | **Alternative:** Common denominator.
- **Common Trap:** Sign slip on $(-a)(-c)$.
- **Practice Drill:** Ex. 8.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Identity vs Inverse 1-Word Check] **Core Idea & Why It Works:** Does the expression output the SAME number ($a \times 1 = a$, $a + 0 = a$)? That is **Identity**. Does it output 1 or 0 ($a \times \frac{1}{a} = 1$, $a + (-a) = 0$)? That is **Inverse**.
Valid Domain: All property MCQs. | **Where It Fails:** Nothing.
Easy Example: $-\frac{19}{29} \times \frac{29}{-19} = 1 \implies$ Multiplicative Inverse.
Exam-Level Example: Eliminates confusion in NCERT Ex. 1.1 Q1.
Advanced Example: Zero confusion.

[SMART SHORTCUT – 2. Sign-Preserving Reciprocal] **Core Idea & Why It Works:** Multiplicative inverse flips numerator and denominator ONLY—it NEVER flips the $+$ or $-$ sign!
Valid Domain: All non-zero rationals. | **Where It Fails:** 0 has no reciprocal.
Easy Example: Reciprocal of $-\frac{7}{11}$ is $-\frac{11}{7}$.
Exam-Level Example: Because $(-\frac{7}{11})(-\frac{11}{7}) = +1$.
Advanced Example: Additive inverse flips ONLY the sign.

[SMART SHORTCUT – 3. Common-Factor Extraction] **Core Idea & Why It Works:** In $ab + ac + d$, always factor $a(b + c) + d$ instead of multiplying ab and ac separately.
Valid Domain: All NCERT Ex. 1.1 simplification problems. | **Where It Fails:** When no factor repeats.
Easy Example: $\frac{9}{16} \times \frac{4}{12} + \frac{9}{16} \times (-\frac{3}{9}) = \frac{9}{16}(\frac{1}{3} - \frac{1}{3}) = 0$!
Exam-Level Example: Often cancels to 0 or 1 inside brackets!
Advanced Example: Saves 70% arithmetic.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Write the additive inverse of (i) $\frac{2}{8}$, (ii) $-\frac{5}{9}$, (iii) $\frac{-6}{-5}$.
Given & Required: Three rationals. | **Concept & Formula:** Flip overall sign.
Detailed Solution: (i) $-\frac{2}{8} = -\frac{1}{4}$; (ii) $\frac{5}{9}$; (iii) $\frac{-6}{-5} = \frac{6}{5}$, so additive inverse is $-\frac{6}{5}$. **Final Answer:** $-1/4,\ 5/9,\ -6/5$
Fastest Alternative / Shortcut: Simplify $\frac{-6}{-5} = +\frac{6}{5}$ first!

SOLVED EXAMPLE 2 [Beginner]
Question: Write the multiplicative inverse of (i) -13, (ii) $-\frac{13}{19}$, (iii) $-1 \times (-\frac{2}{5})$.
Given & Required: Three rationals. | **Concept & Formula:** Flip fraction, keep sign.
Detailed Solution: (i) $-\frac{1}{13}$; (ii) $-\frac{19}{13}$; (iii) Product $= \frac{2}{5}$, reciprocal $= \frac{5}{2}$. **Final Answer:** $-1/13,\ -19/13,\ 5/2$
Fastest Alternative / Shortcut: Simplify product before flipping.

SOLVED EXAMPLE 3 [Beginner]
Question: Name the property used in: $\frac{-1}{3} \times (6 \times \frac{4}{3}) = (\frac{-1}{3} \times 6) \times \frac{4}{3}$.
Given & Required: Grouping of 3 factors. | **Concept & Formula:** Associativity.
Detailed Solution: Associative property of multiplication. **Final Answer: Associativity of multiplication**
Fastest Alternative / Shortcut: Brackets shifted = Associative.

SOLVED EXAMPLE 4 [Beginner]
Question: Verify $-(-x) = x$ for $x = -\frac{13}{17}$.
Given & Required: $x = -13/17$. | **Concept & Formula:** Double negative.
Detailed Solution: $-x = -(-\frac{13}{17}) = \frac{13}{17}$. Thus $-(-x) = -(\frac{13}{17}) = -\frac{13}{17} = x$. **Final Answer: Verified**
Fastest Alternative / Shortcut: Two negatives make positive.

SOLVED EXAMPLE 5 [Intermediate]
Question: Is $\frac{8}{9}$ the multiplicative inverse of $-1\frac{1}{8}$? Why or why not?
Given & Required: $8/9$ and $-9/8$. | **Concept & Formula:** Product must be $+1$.
Detailed Solution: $\frac{8}{9} \times (-\frac{9}{8}) = -1 \neq 1$. Hence No. **Final Answer: No (product is -1, not 1)**
Fastest Alternative / Shortcut: Opposite signs never multiply to $+1$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Is 0.3 the multiplicative inverse of $3\frac{1}{3}$?
Given & Required: $0.3 = 3/10$, $3\frac{1}{3} = 10/3$. | **Concept & Formula:** Product $= 1$.
Detailed Solution: $\frac{3}{10} \times \frac{10}{3} = 1$. Yes! **Final Answer: Yes (product is 1)**
Fastest Alternative / Shortcut: $0.3 \times 3.333\cdots = 1$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Multiply $\frac{6}{13}$ by the reciprocal of $-\frac{7}{16}$.
Given & Required: $6/13$ and $-7/16$. | **Concept & Formula:** Reciprocal is $-16/7$.
Detailed Solution: $\frac{6}{13} \times \left(-\frac{16}{7}\right) = -\frac{96}{91}$. **Final Answer:** $-\frac{96}{91}$
Fastest Alternative / Shortcut: Keep minus sign in reciprocal.

SOLVED EXAMPLE 8 [Intermediate]

Question: Name the property under multiplication: $-\frac{4}{5} \times 1 = 1 \times \left(-\frac{4}{5}\right) = -\frac{4}{5}$.
Given & Required: Multiplying by 1. | **Concept & Formula:** Identity.
Detailed Solution: 1 is the multiplicative identity. **Final Answer: Multiplicative Identity (1)**
Fastest Alternative / Shortcut: Same number returned $\Longrightarrow$ Identity.

SOLVED EXAMPLE 9 [Board Standard]

Question: Using appropriate properties, find: $-\frac{2}{3} \times \frac{3}{5} + \frac{5}{2} - \frac{3}{5} \times \frac{1}{6}$.
Given & Required: Common factor $3/5$. | **Concept & Formula:** Commutativity + Distributivity.
Detailed Solution: $\frac{3}{5}\left(-\frac{2}{3} - \frac{1}{6}\right) + \frac{5}{2} = \frac{3}{5}\left(-\frac{5}{6}\right) + \frac{5}{2} = -\frac{1}{2} + \frac{5}{2} = 2$. **Final Answer:** 2
Fastest Alternative / Shortcut: Group the two $3/5$ terms.

SOLVED EXAMPLE 10 [Board Standard]

Question: Using appropriate properties, find: $\frac{2}{5} \times \left(-\frac{3}{7}\right) - \frac{1}{6} \times \frac{3}{2} + \frac{1}{14} \times \frac{2}{5}$.
Given & Required: Common factor $2/5$. | **Concept & Formula:** Distributivity.
Detailed Solution: $\frac{2}{5}\left(-\frac{3}{7} + \frac{1}{14}\right) - \frac{1}{4} = \frac{2}{5}\left(-\frac{5}{14}\right) - \frac{1}{4} = -\frac{1}{7} - \frac{1}{4} = -\frac{11}{28}$. **Final Answer:** $-\frac{11}{28}$
Fastest Alternative / Shortcut: $-\frac{3}{7} + \frac{1}{14} = -\frac{5}{14}$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Find $\frac{7}{5} \times \left(-\frac{3}{12}\right) + \frac{7}{5} \times \frac{5}{12}$ using distributivity.
Given & Required: Common factor $7/5$. | **Concept & Formula:** $a(b + c)$.
Detailed Solution: $\frac{7}{5}\left(-\frac{3}{12} + \frac{5}{12}\right) = \frac{7}{5} \times \frac{2}{12} = \frac{7}{5} \times \frac{1}{6} = \frac{7}{30}$. **Final Answer:** $\frac{7}{30}$
Fastest Alternative / Shortcut: Add $-3 + 5 = 2$ over 12.

SOLVED EXAMPLE 12 [Board Standard]

Question: Verify $x \times (y + z) = x \times y + x \times z$ for $x = -\frac{1}{2}, y = \frac{2}{3}, z = -\frac{4}{5}$.
Given & Required: Three rationals. | **Concept & Formula:** Distributivity verification.
Detailed Solution: LHS $= -\frac{1}{2}(\frac{10-12}{15}) = -\frac{1}{2}(-\frac{2}{15}) = \frac{1}{15}$. RHS $= -\frac{1}{3} + \frac{2}{5} = \frac{-5+6}{15} = \frac{1}{15}$. Verified. **Final Answer: LHS = RHS** $= \frac{1}{15}$
Fastest Alternative / Shortcut: Both equal $1/15$.

SOLVED EXAMPLE 13 [Board Standard]

Question: What should be added to $-\frac{5}{7}$ to get $-\frac{2}{3}$?
Given & Required: $-\frac{5}{7} + x = -\frac{2}{3}$. | **Concept & Formula:** Transpose.
Detailed Solution: $x = -\frac{2}{3} + \frac{5}{7} = \frac{-14+15}{21} = \frac{1}{21}$. **Final Answer:** $\frac{1}{21}$
Fastest Alternative / Shortcut: Subtract $-5/7$ from $-2/3$.

SOLVED EXAMPLE 14 [Challenge]

Question: The product of two rational numbers is $-\frac{16}{9}$. If one of them is $-\frac{4}{3}$, find the other.
Given & Required: $x \cdot (-4/3) = -16/9$. | **Concept & Formula:** Multiply by reciprocal.
Detailed Solution: $x = -\frac{16}{9} \times \left(-\frac{3}{4}\right) = \frac{4}{3}$. **Final Answer:** $\frac{4}{3}$
Fastest Alternative / Shortcut: Two negatives give positive.

SOLVED EXAMPLE 15 [Challenge]

Question: Divide the sum of $\frac{65}{12}$ and $\frac{8}{3}$ by their difference.
Given & Required: Sum / Difference. | **Concept & Formula:** LCM $= 12$.
Detailed Solution: Sum $= \frac{65+32}{12} = \frac{97}{12}$; Difference $= \frac{65-32}{12} = \frac{33}{12}$. Ratio $= \frac{97}{33}$. **Final Answer:** $\frac{97}{33}$
Fastest Alternative / Shortcut: Denominator 12 cancels!

SOLVED EXAMPLE 16 [Challenge]

Question: Find a rational number that is equal to its negative (additive inverse).
Given & Required: $x = -x$. | **Concept & Formula:** $2x = 0$.
Detailed Solution: $x = 0$ is the only rational number equal to its own negative. **Final Answer:** 0
Fastest Alternative / Shortcut: $0 = -0$.

G. Competency MCQ Mastery Section

Q1. The additive identity for rational numbers is:

(A) 1 (B) -1 (C) 0 (D) None

Answer: (C) $a + 0 = a$. *Fast Method:* 0. *Distractor Note:* Picking 1.

Q2. Which rational number does NOT have a reciprocal?

(A) 1 (B) -1 (C) 0 (D) $1/2$

Answer: (C) $1/0$ is undefined. *Fast Method:* 0. *Distractor Note:* Picking -1.

Q3. The rational numbers equal to their own reciprocals are:

(A) $0, 1$ (B) $1, -1$ (C) $0, -1$ (D) 1 only

Answer: (B) $1/1 = 1$ and $1/(-1) = -1$. *Fast Method:* 1 and -1. *Distractor Note:* Including 0.

Q4. Multiplicative inverse of $-\frac{3}{8}$ is:

(A) $\frac{3}{8}$ (B) $\frac{8}{3}$ (C) $-\frac{8}{3}$ (D) 1

Answer: (C) Flip fraction, keep minus sign. *Fast Method:* $-8/3$. *Distractor Note:* Dropping minus sign.

Q5. Additive inverse of $\frac{-7}{-9}$ is:

(A) $\frac{7}{9}$ (B) $-\frac{7}{9}$ (C) $\frac{9}{7}$ (D) $-\frac{9}{7}$

Answer: (B) $\frac{-7}{-9} = +\frac{7}{9}$, so additive inverse is $-\frac{7}{9}$. *Fast Method:* Cancel two negatives first! *Distractor Note:* Picking $+7/9$.

Q6. Rational numbers are NOT closed under:

(A) Addition (B) Subtraction (C) Multiplication (D) Division

Answer: (D) Division by 0 is not defined. *Fast Method:* Division by 0. *Distractor Note:* Picking subtraction.

Q7. $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \cdot \frac{c}{d} + \frac{a}{b} \cdot \frac{e}{f}$ is:

(A) Closure (B) Commutativity (C) Associativity (D) Distributivity

Answer: (D) Multiplication distributes over addition. *Fast Method:* Distributive property. *Distractor Note:* Picking Associativity.

Q8. Sum of a rational number and its additive inverse is always:

(A) 1 (B) 0 (C) -1 (D) The number itself

Answer: (B) $a + (-a) = 0$. *Fast Method:* 0. *Distractor Note:* Picking 1.

Q9. Product of a non-zero rational and its reciprocal is:

(A) 0 (B) 1 (C) -1 (D) a^2

Answer: (B) $a \times \frac{1}{a} = 1$. *Fast Method:* 1. *Distractor Note:* Picking 0.

Q10. Reciprocal of a negative rational number is always:

(A) Positive (B) Negative (C) Zero (D) 1

Answer: (B) Sign does not change in reciprocal. *Fast Method:* Negative. *Distractor Note:* Picking Positive.

Q11. Value of $(-\frac{5}{7}) \times 0$ is:

(A) $-5/7$ (B) 1 (C) 0 (D) $-7/5$

Answer: (C) Multiplying by 0 gives 0. *Fast Method:* 0. *Distractor Note:* Confusing with $+0$.

Q12. If $x = \frac{4}{5}$, then $-(-x)$ equals:

(A) $-4/5$ (B) $5/4$ (C) $4/5$ (D) $-5/4$

Answer: (C) $-(-x) = x$. *Fast Method:* $4/5$. *Distractor Note:* Leaving minus sign.

H. Special Exam & MCQ Strategies

- Before finding additive or multiplicative inverse, simplify double signs like $\frac{-a}{-b} = +\frac{a}{b}$.
- In multi-term fraction problems, look for a common fraction to factor out via Distributivity before doing any multiplication.
- Remember the two identity numbers: 0 for Addition, 1 for Multiplication.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Changing the minus sign when taking the multiplicative inverse (e.g., $-3/5 \to +5/3$). $\longrightarrow$ **Correction:** Keep the sign! $(-3/5) \times (-5/3) = +1$.

[COMMON MISTAKE] **Error:** Claiming rational numbers are closed under division. $\longrightarrow$ **Correction:** Division by 0 is undefined, so $\mathbb{Q}$ is NOT closed under division.

[COMMON MISTAKE] **Error:** Writing additive inverse of $\frac{-3}{-4}$ as $\frac{3}{4}$. $\longrightarrow$ **Correction:** $\frac{-3}{-4} = +\frac{3}{4}$, so its additive inverse is $-\frac{3}{4}$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: Commutative $(+, \times)$; Associative $(+, \times)$; Distributive $a(b \pm c) = ab \pm ac$; $-(-x) = x$.

[QUICK REVISION] Key Theorems & Results: Additive identity: 0; Multiplicative identity: 1; Own reciprocals: $1, -1$; No reciprocal: 0.

[QUICK REVISION] Shortcut Summary: Identity returns same a; Inverse returns 0 or 1; Reciprocal keeps sign.

[QUICK REVISION] High-Frequency Exam Patterns: Property naming, Distributivity simplification, Inverse verification, Product/quotient word problems.

Last-Minute Revision Checklist:

- ☐ Simplify $\frac{-a}{-b}$ first
- ☐ Keep sign in reciprocal
- ☐ 0 has no reciprocal
- ☐ Factor out common fraction

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Additive inverse of $\frac{5}{7}$ is: (A) $7/5$ (B) $-5/7$ (C) $-7/5$ (D) 0
2. Multiplicative identity for rationals is: (A) 0 (B) 1 (C) -1 (D) 10
3. Reciprocal of -1 is: (A) 1 (B) 0 (C) -1 (D) Undefined
4. $a + b = b + a$ is called: (A) Closure (B) Commutativity (C) Associativity (D) Distributivity

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Product of $\frac{4}{9}$ and reciprocal of $-\frac{2}{3}$ is: (A) $-8/27$ (B) $-2/3$ (C) $2/3$ (D) $8/27$
2. Value of $\frac{3}{5} \times \frac{1}{2} + \frac{3}{5} \times \frac{1}{3}$ is: (A) $1/2$ (B) $3/10$ (C) $1/5$ (D) $3/5$
3. What should be added to $-\frac{3}{4}$ to get 1? (A) $1/4$ (B) $7/4$ (C) $-7/4$ (D) $4/3$
4. Which operation is NOT associative for rationals? (A) Addition (B) Multiplication (C) Subtraction (D) All of these

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If x is the additive inverse of $\frac{2}{3}$ and y is the reciprocal of $-\frac{4}{9}$, then $x + y$ is: (A) $-35/12$ (B) $19/12$ (C) $-19/12$ (D) $35/12$
2. Sum of a number and its reciprocal is 2. The number is: (A) 0 (B) 1 (C) -1 (D) 2
3. Which property is used in $\frac{1}{2} \times (-\frac{3}{4} + \frac{5}{6}) = -\frac{3}{8} + \frac{5}{12}$? (A) Associativity (B) Commutativity (C) Distributivity (D) Closure
4. Product of two rationals is $-\frac{5}{6}$. If one is $\frac{10}{3}$, the other is: (A) $-1/4$ (B) $1/4$ (C) -4 (D) $-25/9$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** Flip sign to $-5/7$. **Q2 (B):** $a \times 1 = a$. **Q3 (C):** $(-1)(-1) = 1$. **Q4 (B):** Commutative property.
Exam Std	**Q1 (B):** $\frac{4}{9} \times (-\frac{3}{2}) = -\frac{2}{3}$. **Q2 (A):** $\frac{3}{5}(\frac{5}{6}) = \frac{1}{2}$. **Q3 (B):** $1 - (-3/4) = 7/4$. **Q4 (C):** Subtraction is not associative.
Challenge	**Q1 (A):** $-2/3 + (-9/4) = -35/12$. **Q2 (B):** $1+1 = 2$. **Q3 (C):** Distributive law. **Q4 (A):** $(-5/6) \times (3/10) = -1/4$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Closure, commutativity, associativity. Day 2: Identities $0, 1$, inverses, distributivity.
- **How to Introduce the Concept:** Ask: 'Is $10 - 4$ equal to $4 - 10$?' Show why addition/multiplication commute while subtraction/division don't!
- **Difficult Areas & Remediation:** Distinguishing additive vs multiplicative inverse and grouping terms for distributivity.
- **Blackboard Teaching Examples:** Draw the 4-card summary grid of properties on the board.
- **Classroom Questions to Ask Students:** Why does 0 have no reciprocal?
- **Common Student Doubts & Answers:** Students ask about number-line insertion; clarify it is covered in Class 7/9 and excluded from Class 8 2026–27.
- **Homework & Revision Strategy:** Assign 4 property-identification questions and 4 distributivity simplifications.

[TEACHER'S TIP] Emphasize that simplifying fractions before multiplying prevents large-number errors.

UNIT II: ALGEBRA TYBA COACHING CENTRE

Chapter 2: Linear Equations in One Variable

Transposition, Variables on Both Sides, Cross-Multiplication & Simplification (2026–27)

A. Chapter Introduction & Examination Relevance

An algebraic equation is a mathematical balance scale: whatever you do to one side, you must do to the other. Chapter 2 builds complete fluency in solving linear equations with variables on both sides and fractional denominators.

- **What You Will Learn:** Linear expressions vs equations, transposition method, solving equations with variables on both sides, clearing denominators via LCM, cross-multiplication, and verifying roots.
- **Why This Chapter Matters:** Linear equation solving is used in EVERY chapter of Class 8 (Quadrilaterals, Comparing Quantities, Mensuration, Proportions) and launches Class 9–10 Two-Variable Equations.
- **Real-Life Applications:** Finding unknown angles in polygons, age/number puzzles, geometry perimeter problems, and break-even pricing.
- **Cross-Chapter Connections:** Directly powers Chapter 3 (polygon angle equations), Chapter 7 (discount/CI), and Chapter 9 (missing height/radius).
- **Exam & Foundation Relevance:** Unit II carries 25% weightage. Expect 1M transposition MCQs and 2M/3M multi-bracket fractional equations. (Standalone word-problem exercises were rationalised out in 2026–27, though algebraic solving remains core.)

B. Complete Concept Explanation & Derivations

1. Linear Equations & the Transposition Rule

A **linear equation in one variable** has the highest power of the variable equal to 1 (e.g., $2x - 3 = 7$). Moving a term from one side of $=$ to the other is called **transposition**: $+$ becomes $-$, $-$ becomes $+$, $\times$ becomes $\div$, and $\div$ becomes $\times$.

[CONCEPT MASTER – 1. Linear Equations & the Transposition Rule] Always move variable terms to the LHS and constant numbers to the RHS in ONE clean step!

2. Equations with Variables on Both Sides

For equations like $5x + \frac{7}{2} = \frac{3}{2}x - 14$, collect all x-terms on the LHS ($5x - \frac{3}{2}x$) and all constants on the RHS ($-14 - \frac{7}{2}$), combine like terms, and divide by the coefficient of x.

[CONCEPT MASTER – 2. Equations with Variables on Both Sides] Check Your Answer in 5 Seconds: Plug your final value of x back into both LHS and RHS—if LHS $=$ RHS, you are 100% sure it's right!

3. Clearing Denominators via LCM & Expanding Brackets

When fractions appear on both sides (e.g., $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$), find the LCM of all denominators (60) and multiply EVERY term on both sides by the LCM to eliminate fractions immediately. When brackets like $3(t - 3) = 5(2t + 1)$ appear, expand carefully before transposing.

[CONCEPT MASTER – 3. Clearing Denominators via LCM & Expanding Brackets] Watch the Minus Sign Before a Fraction! In $m - \frac{m-1}{2} = 1 - \frac{m-2}{3}$, the minus sign applies to the ENTIRE numerator: $-(m - 1) = -m + 1$!

4. Cross-Multiplication & Rationalised 2026–27 Scope

For equations of the form $\frac{ax+b}{cx+d} = \frac{p}{q}$, cross-multiply to get $q(ax + b) = p(cx + d)$, expand, and solve as usual (checking $cx + d \neq 0$).

[CONCEPT MASTER – 4. Cross-Multiplication & Rationalised 2026–27 Scope] Rationalised 2026–27 Scope: Standalone age/digit word-problem exercises were rationalised out of NCERT Class 8 Ch. 2, focusing mastery on algebraic equation solving.

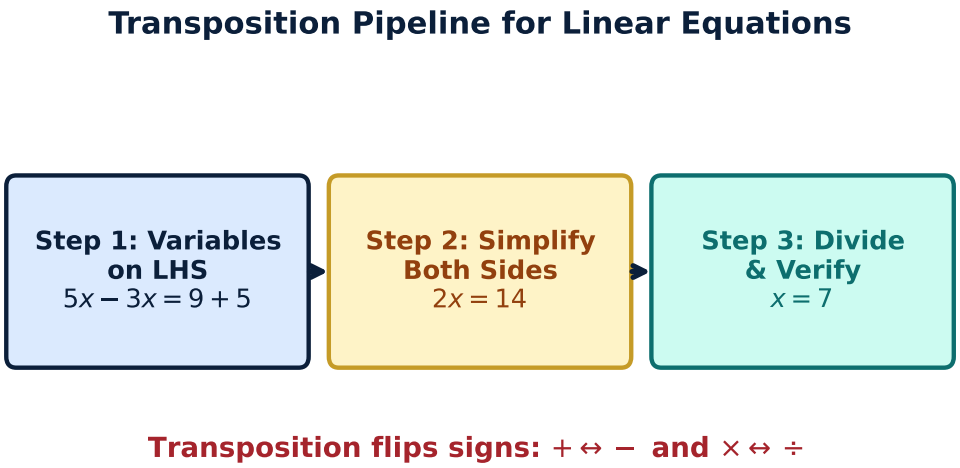


Figure 1: **Visual Model:** Three-step transposition pipeline for solving linear equations with variables on both sides.

C. Complete Formula Bank

[FORMULA BANK – 1. Standard Form] **Expression:** $ax + b = 0 \implies x = -\frac{b}{a}$ $(a \neq 0)$
Symbols: $a, b \in \mathbb{Q}$, x: variable.
Conditions & When to Use: Unique solution.
Solved Mini-Example: $3x + 12 = 0 \implies x = -4$.
Common Mistake: Writing $x = b/a$ without flipping sign.

[FORMULA BANK – 2. Transposition Rules] **Expression:** $+ \leftrightarrow -$; $\times \leftrightarrow \div$ across $=$
Symbols: Any algebraic term.
Conditions & When to Use: Do not divide by 0.
Solved Mini-Example: $4z + 3 = 6 + 2z \implies 2z = 3 \implies z = 3/2$.
Common Mistake: Forgetting to flip sign when moving across $=$.

[FORMULA BANK – 3. LCM Denominator Clearing] **Expression:** Multiply EVERY term by $\mathrm{LCM}(d_1, d_2, \dots)$
Symbols: d_i: denominators.
Conditions & When to Use: Do not skip integer terms without denominators!
Solved Mini-Example: In $\frac{x}{2} - 1 = \frac{x}{3}$, multiply by 6: $3x - 6 = 2x \implies x = 6$.
Common Mistake: Multiplying fraction terms by LCM but forgetting the integer -1.

[FORMULA BANK – 4. Cross-Multiplication] **Expression:** $\frac{A}{B} = \frac{C}{D} \implies A \cdot D = B \cdot C$
Symbols: $B \neq 0, D \neq 0$.
Conditions & When to Use: Single fraction on LHS and single fraction on RHS.
Solved Mini-Example: $\frac{2x-1}{3} = \frac{x+2}{2} \implies 4x - 2 = 3x + 6 \implies x = 8$.
Common Mistake: Cross-multiplying before combining two separate fractions on one side.

[FORMULA BANK – 5. Negative Bracket Expansion] **Expression:** $-k(ax - b) = -kax + kb$
Symbols: k: multiplier.
Conditions & When to Use: Negative $\times$ negative $=$ positive.
Solved Mini-Example: $-2(3x - 5) = -6x + 10$.
Common Mistake: Writing $-6x - 10$.

[FORMULA BANK – 6. Solution Verification] **Expression:** $\text{LHS}(x_0) = \text{RHS}(x_0)$
Symbols: x_0: candidate root.
Conditions & When to Use: Always check denominator $\neq 0$.
Solved Mini-Example: Check $x = 6$ in $\frac{x}{2} - 1 = \frac{x}{3}$: $3 - 1 = 2$ and $6/3 = 2$.
Common Mistake: Skipping verification on 3-mark questions.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Sainik/Navodaya/Olympiad Foundation problem structures.

Pattern 1: Variables on Both Sides (Integer Coefficients)

- **Identification Cue:** Solve $3x = 2x + 18$ or $8x + 4 = 3(x - 1) + 7$. | **Concept & Formula:** Expand brackets → transpose.
- **Standard Step-by-Step:** Expand all brackets first, move x-terms left and numbers right, divide.
- **Fastest Valid Method:** Combine constants on RHS before transposing. | **Alternative:** Move x to the side with larger coefficient.
- **Common Trap:** Expanding $3(x - 1)$ as $3x - 1$.
- **Practice Drill:** Ex. 1 & Ex. 5.

Pattern 2: Fractional Equations via LCM

- **Identification Cue:** Solve $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$ or $\frac{n}{2} - \frac{3n}{4} + \frac{5n}{6} = 21$. | **Concept & Formula:** Multiply every term by LCM.
- **Standard Step-by-Step:** Find LCM of all denominators, multiply every term, then solve integer equation.
- **Fastest Valid Method:** Write LCM × above each term mentally to avoid missing constants. | **Alternative:** Combine LHS fractions and RHS fractions separately.
- **Common Trap:** Forgetting to multiply the constant term (21) by LCM (12).
- **Practice Drill:** Ex. 6 & Ex. 9.

Pattern 3: Negative Numerator Grouping ($m - \frac{m-1}{2}$)

- **Identification Cue:** Solve $m - \frac{m-1}{2} = 1 - \frac{m-2}{3}$. | **Concept & Formula:** Put parentheses around every multi-term numerator!
- **Standard Step-by-Step:** Write $m - \frac{(m-1)}{2} = 1 - \frac{(m-2)}{3}$, multiply by 6: $6m - 3(m - 1) = 6 - 2(m - 2)$.
- **Fastest Valid Method:** ALWAYS wrap numerators like $(m - 1)$ in brackets before clearing denominators. | **Alternative:** Split fraction $\frac{m}{2} - \frac{1}{2}$.
- **Common Trap:** Writing $-3m - 3$ instead of $-3m + 3$!
- **Practice Drill:** Ex. 10 & Ex. 11.

Pattern 4: Decimal & Cross-Multiplication Equations

- **Identification Cue:** Solve $0.25(4f - 3) = 0.05(10f - 9)$ or $\frac{3y+4}{2-6y} = -\frac{2}{5}$. | **Concept & Formula:** Clear decimals ($imes 100$) or cross-multiply.
- **Standard Step-by-Step:** Multiply decimals by 100 to get integers $25(4f - 3) = 5(10f - 9)$, then solve.
- **Fastest Valid Method:** Divide both sides by 5: $5(4f - 3) = 10f - 9$! | **Alternative:** Expand decimals directly.
- **Common Trap:** Misplacing decimal point in $0.05 \times 9 = 0.45$.
- **Practice Drill:** Ex. 12 & Ex. 13.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Multiply Decimals by 100 First] **Core Idea & Why It Works:** In decimal linear equations like $0.25(4f - 3) = 0.05(10f - 9)$, multiply both sides by 100 (or 10) to turn everything into clean integers!
Valid Domain: All decimal equations. | **Where It Fails:** Nothing.
Easy Example: $25(4f - 3) = 5(10f - 9) \implies 5(4f - 3) = 10f - 9 \implies 10f = 6 \implies f = 0.6$.
Exam-Level Example: Eliminates decimal arithmetic errors in NCERT Ex. 2.2 Q10.
Advanced Example: Zero decimal mistakes.

[SMART SHORTCUT – 2. Numerator-Bracket Shield] **Core Idea & Why It Works:** Whenever you see $-\frac{ax+b}{c}$, immediately draw parentheses $-\frac{(ax+b)}{c}$ before taking LCM.
Valid Domain: All fractional linear equations. | **Where It Fails:** Nothing.
Easy Example: $6m - 3(m-1) = 6m - 3m + 3$.
Exam-Level Example: Prevents the #1 sign error in Class 8 Algebra.
Advanced Example: Essential for Class 9 and 10 rational equations too.

[SMART SHORTCUT – 3. 5-Second Root Check] **Core Idea & Why It Works:** Substitute your integer/fraction answer back into LHS and RHS separately to verify equality.
Valid Domain: All equations. | **Where It Fails:** Nothing.
Easy Example: In $5t - 3 = 3t - 5$, $t = -1$: LHS $= -8$, RHS $= -8$. Verified!
Exam-Level Example: Guarantees 100% accuracy in exams.
Advanced Example: Catches transposition sign slips instantly.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Solve $3x = 2x + 18$.
Given & Required: Variable on both sides. | **Concept & Formula:** Transpose $2x$.
Detailed Solution: $3x - 2x = 18 \implies x = 18$. Check: $54 = 36 + 18$. **Final Answer:** $x = 18$
Fastest Alternative / Shortcut: Subtract $2x$.

SOLVED EXAMPLE 2 [Beginner]
Question: Solve $5t - 3 = 3t - 5$.
Given & Required: Variable on both sides. | **Concept & Formula:** Transpose.
Detailed Solution: $5t - 3t = -5 + 3 \implies 2t = -2 \implies t = -1$. **Final Answer:** $t = -1$
Fastest Alternative / Shortcut: Check $t = -1$: $-8 = -8$.

SOLVED EXAMPLE 3 [Beginner]
Question: Solve $4z + 3 = 6 + 2z$.
Given & Required: Linear equation. | **Concept & Formula:** Transpose.
Detailed Solution: $4z - 2z = 6 - 3 \implies 2z = 3 \implies z = \frac{3}{2}$. **Final Answer:** $z = \frac{3}{2}$
Fastest Alternative / Shortcut: Check: $6 + 3 = 6 + 3$.

SOLVED EXAMPLE 4 [Beginner]
Question: Solve $2x - 1 = 14 - x$.
Given & Required: Linear equation. | **Concept & Formula:** Transpose.
Detailed Solution: $2x + x = 14 + 1 \implies 3x = 15 \implies x = 5$. **Final Answer:** $x = 5$
Fastest Alternative / Shortcut: Check: $9 = 9$.

SOLVED EXAMPLE 5 [Intermediate]
Question: Solve $8x + 4 = 3(x - 1) + 7$.
Given & Required: Bracket on RHS. | **Concept & Formula:** Expand then transpose.
Detailed Solution: $8x + 4 = 3x - 3 + 7 = 3x + 4 \implies 5x = 0 \implies x = 0$. **Final Answer:** $x = 0$
Fastest Alternative / Shortcut: $x = 0$ is a valid solution!

SOLVED EXAMPLE 6 [Intermediate]

Question: Solve $x = \frac{4}{5}(x + 10)$.

Given & Required: Fraction multiplier. | **Concept & Formula:** Multiply by 5.

Detailed Solution: $5x = 4(x + 10) = 4x + 40 \implies x = 40$. **Final Answer:** $x = 40$

Fastest Alternative / Shortcut: Check: $\frac{4}{5}(50) = 40$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Solve $\frac{2x}{3} + 1 = \frac{7x}{15} + 3$.

Given & Required: Denominators $3, 15$. | **Concept & Formula:** LCM $= 15$.

Detailed Solution: Multiply by 15: $10x + 15 = 7x + 45 \implies 3x = 30 \implies x = 10$. **Final Answer:** $x = 10$

Fastest Alternative / Shortcut: Check: $\frac{20}{3} + 1 = \frac{70}{15} + 3 = \frac{23}{3}$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Solve $2y + \frac{5}{3} = \frac{26}{3} - y$.

Given & Required: Denominator 3. | **Concept & Formula:** Transpose y.

Detailed Solution: $3y = \frac{26}{3} - \frac{5}{3} = \frac{21}{3} = 7 \implies y = \frac{7}{3}$. **Final Answer:** $y = \frac{7}{3}$

Fastest Alternative / Shortcut: Same denominator 3 subtracts cleanly!

SOLVED EXAMPLE 9 [Board Standard]

Question: Solve $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$.

Given & Required: Denominators $2, 5, 3, 4$. | **Concept & Formula:** LCM $= 60$.

Detailed Solution: Multiply by 60: $30x - 12 = 20x + 15 \implies 10x = 27 \implies x = \frac{27}{10}$. **Final Answer:** $x = \frac{27}{10}$

Fastest Alternative / Shortcut: Or group $\frac{x}{2} - \frac{x}{3} = \frac{1}{4} + \frac{1}{5} \implies \frac{x}{6} = \frac{9}{20}$!

SOLVED EXAMPLE 10 [Board Standard]

Question: Solve $\frac{n}{2} - \frac{3n}{4} + \frac{5n}{6} = 21$.

Given & Required: LCM of $2, 4, 6$ is 12. | **Concept & Formula:** Clear denominators.

Detailed Solution: Multiply by 12: $6n - 9n + 10n = 252 \implies 7n = 252 \implies n = 36$. **Final Answer:** $n = 36$

Fastest Alternative / Shortcut: $252/7 = 36$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Solve $m - \frac{m-1}{2} = 1 - \frac{m-2}{3}$.

Given & Required: Multi-term numerators. | **Concept & Formula:** Bracket shield + LCM 6.

Detailed Solution: $6m - 3(m - 1) = 6 - 2(m - 2) \implies 6m - 3m + 3 = 6 - 2m + 4 \implies 3m + 3 = 10 - 2m \implies 5m = 7 \implies m = \frac{7}{5}$. **Final Answer:** $m = \frac{7}{5}$

Fastest Alternative / Shortcut: Check $m = 7/5$: LHS $= 7/5 - 1/5 = 6/5$; RHS $= 1 + 1/5 = 6/5$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Solve $3(5z - 7) - 2(9z - 11) = 4(8z - 13) - 17$.

Given & Required: Multiple brackets. | **Concept & Formula:** Expand carefully.

Detailed Solution: $15z - 21 - 18z + 22 = 32z - 52 - 17 \implies -3z + 1 = 32z - 69 \implies 70 = 35z \implies z = 2$. **Final Answer:** $z = 2$

Fastest Alternative / Shortcut: Watch $-2(-11) = +22$.

SOLVED EXAMPLE 13 [Board Standard]

Question: Solve $0.25(4f-3)=0.05(10f-9)$.

Given & Required: Decimals. | **Concept & Formula:** Multiply by 100.

Detailed Solution: $25(4f-3)=5(10f-9) \implies 5(4f-3)=10f-9 \implies 20f-15=10f-9 \implies 10f=6 \implies f=0.6$.

Final Answer: $f=0.6$ $(\frac{3}{5})$

Fastest Alternative / Shortcut: Cancel 5 after $\times 100$.

SOLVED EXAMPLE 14 [Challenge]

Question: Solve $\frac{3t-2}{4}-\frac{2t+3}{3}=\frac{2}{3}-t$.

Given & Required: LCM $=12$. | **Concept & Formula:** Bracket shield + LCM.

Detailed Solution: $3(3t-2)-4(2t+3)=8-12t \implies 9t-6-8t-12=8-12t \implies t-18=8-12t \implies 13t=26 \implies t=2$.

Final Answer: $t=2$

Fastest Alternative / Shortcut: Check $t=2$: $4/4-7/3=2/3-2=-4/3$.

SOLVED EXAMPLE 15 [Challenge]

Question: Solve $\frac{7y+4}{y+2}=-\frac{4}{3}$.

Given & Required: Rational equation. | **Concept & Formula:** Cross-multiplication.

Detailed Solution: $3(7y+4)=-4(y+2) \implies 21y+12=-4y-8 \implies 25y=-20 \implies y=-\frac{4}{5}$. **Final Answer:** $y=-\frac{4}{5}$

Fastest Alternative / Shortcut: Denominator $y+2\neq 0$ holds.

SOLVED EXAMPLE 16 [Challenge]

Question: If $\frac{x-a}{b}+\frac{x-b}{a}=2$ $(a,b\neq 0)$, find x in terms of a and b.

Given & Required: Literal linear equation. | **Concept & Formula:** Rewrite $2=1+1$.

Detailed Solution: $\left(\frac{x-a}{b}-1\right)+\left(\frac{x-b}{a}-1\right)=0 \implies (x-a-b)\left(\frac{1}{b}+\frac{1}{a}\right)=0 \implies x=a+b$. **Final Answer:** $x=a+b$

Fastest Alternative / Shortcut: Plug $x=a+b$: $\frac{b}{b}+\frac{a}{a}=2$!

G. Competency MCQ Mastery Section

Q1. Solution of $2x-3=7$ is:

(A) 2 (B) 5 (C) 4 (D) 10

Answer: (B) $2x=10 \implies x=5$. *Fast Method:* Mental. *Distractor Note:* Subtracting 3 instead of adding.

Q2. Highest power of the variable in a linear equation is:

(A) 0 (B) 1 (C) 2 (D) 3

Answer: (B) Linear means degree 1. *Fast Method:* 1. *Distractor Note:* Picking 2 (quadratic).

Q3. If $3x=2x+15$, then x equals:

(A) 3 (B) 5 (C) 15 (D) -15

Answer: (C) $3x-2x=15$. *Fast Method:* 15. *Distractor Note:* Adding $3+2=5$.

Q4. If $\frac{x}{3}+1=\frac{7}{15}$, then x is:

(A) $-8/5$ (B) $8/5$ (C) $-24/15$ (D) Both A and C

Answer: (A) $\frac{x}{3}=-\frac{8}{15} \implies x=-\frac{8}{5}$. *Fast Method:* Multiply by 15. *Distractor Note:* Sign error.

Q5. Which of the following is NOT a linear equation in one variable?

(A) $3x+2=0$ (B) $x^2+1=5$ (C) $5y-7=2y$ (D) $\frac{x}{2}=4$

Answer: (B) x^2 has degree 2. *Fast Method:* Degree must be 1. *Distractor Note:* None.

Q6. Solution of $5x-2(2x-7)=2(3x-1)+\frac{7}{2}$ is:

(A) $5/2$ (B) $3/2$ (C) $2/5$ (D) $-5/2$

Answer: (A) $x + 14 = 6x + 3/2 \implies 5x = 25/2 \implies x = 5/2$. *Fast Method:* Expand carefully. *Distractor Note:* Sign error on $-2(-7)$.

Q7. If $0.5x + 0.7 = 0.25x + 1.2$, then x is:

(A) 1 (B) 2 (C) 0.5 (D) 4

Answer: (B) $0.25x = 0.5 \implies x = 2$. *Fast Method:* Multiply by 100. *Distractor Note:* Decimal slip.

Q8. If $x = 3$ is a solution of $4x + k = 19$, then k is:

(A) 7 (B) 12 (C) -7 (D) 5

Answer: (A) $12 + k = 19 \implies k = 7$. *Fast Method:* Plug $x = 3$. *Distractor Note:* Arithmetic slip.

Q9. Root of $\frac{2x-1}{3} = 5$ is:

(A) 7 (B) 8 (C) 6 (D) 4

Answer: (B) $2x - 1 = 15 \implies x = 8$. *Fast Method:* $15 + 1 = 16, 16/2 = 8$. *Distractor Note:* Subtracting 1 instead of adding.

Q10. Number of solutions of a linear equation in one variable is:

(A) 0 (B) 1 (C) 2 (D) Infinitely many

Answer: (B) Unique solution $x = -b/a$. *Fast Method:* 1 unique solution. *Distractor Note:* Confusing with 2-variable linear eq.

Q11. If $\frac{x}{5} = 10$, then x is:

(A) 2 (B) 5 (C) 50 (D) 15

Answer: (C) $x = 10 \times 5 = 50$. *Fast Method:* Multiply by 5. *Distractor Note:* Dividing $10/5 = 2$.

Q12. If $7x - 9 = 16$, then $7x$ equals:

(A) 7 (B) 25 (C) -25 (D) $25/7$

Answer: (B) $7x = 16 + 9 = 25$. *Fast Method:* Read question: asks for $7x$, not x! *Distractor Note:* Picking $25/7$ without reading.

H. Special Exam & MCQ Strategies

- Wrap multi-term numerators like $(m - 1)$ in parentheses BEFORE clearing denominators.
- Multiply decimal equations by 10 or 100 first to convert all coefficients into integers.
- Always plug your final value of x back into the original equation to verify LHS = RHS.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Expanding $-2(3x - 5)$ as $-6x - 10$. $\longrightarrow$ **Correction:** Negative times negative is positive: $-2(-5) = +10$.

[COMMON MISTAKE] **Error:** Multiplying fraction terms by LCM but forgetting to multiply standalone integers on the RHS. $\longrightarrow$ **Correction:** Every single term on both sides must be multiplied by the LCM.

[COMMON MISTAKE] **Error:** Writing $-\frac{m-1}{2}$ with LCM 6 as $-3m - 3$. $\longrightarrow$ **Correction:** It is $-3(m - 1) = -3m + 3$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $ax + b = 0 \implies x = -b/a$; Transposition: $+ \leftrightarrow -$, $\times \leftrightarrow \div$; Cross-mult: $\frac{A}{B} = \frac{C}{D} \implies AD = BC$.

[QUICK REVISION] Key Theorems & Results: Unique solution for $a \neq 0$; LCM clears all denominators.

[QUICK REVISION] Shortcut Summary: Multiply decimals by 100; Bracket shield on $-\frac{ax+b}{c}$; 5-sec LHS=RHS check.

[QUICK REVISION] High-Frequency Exam Patterns: Both-sides x, LCM fractional equations, Negative numerator brackets, Decimal/cross-multiplication.

Last-Minute Revision Checklist:

- ☐ Flip sign on transpose
- ☐ Bracket numerators
- ☐ Multiply ALL terms by LCM
- ☐ Verify LHS = RHS

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Solve $4x - 7 = 9$: (A) 4 (B) $1/2$ (C) 16 (D) 2
2. Solve $7x = 5x + 12$: (A) 2 (B) 6 (C) 12 (D) 4
3. Solve $\frac{y}{4} = 9$: (A) 36 (B) $9/4$ (C) 13 (D) 5
4. Degree of $3x + 5 = 0$ is: (A) 0 (B) 1 (C) 2 (D) 3

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Solve $5x + 9 = 5 + 3x$: (A) 2 (B) -2 (C) 7 (D) -7
2. Solve $\frac{x-5}{3} = \frac{x-3}{5}$: (A) 8 (B) -8 (C) 2 (D) 4
3. Solve $0.3x + 0.4 = 0.28x + 1.16$: (A) 38 (B) 3.8 (C) 380 (D) 19
4. Solve $3(t - 3) = 5(2t + 1)$: (A) 2 (B) -2 (C) $14/7$ (D) $-14/13$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Solve $x - \frac{x-1}{2} = 1 - \frac{x-2}{3}$: (A) $5/7$ (B) $7/5$ (C) $1/5$ (D) $-7/5$
2. If $\frac{2x+1}{3x-2} = \frac{9}{10}$, then x is: (A) 4 (B) $28/7$ (C) Both A and B (D) 3
3. Solve $\frac{x}{2} + \frac{x}{3} + \frac{x}{6} = 15$: (A) 15 (B) 30 (C) 10 (D) 45
4. If $5x - 3 = 3x + 7$, then $2x + 1$ equals: (A) 5 (B) 10 (C) 11 (D) 9

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (A):** $4x = 16 \implies x = 4$. **Q2 (B):** $2x = 12 \implies x = 6$. **Q3 (A):** $y = 9 \times 4 = 36$. **Q4 (B):** Linear = degree 1.
Exam Std	**Q1 (B):** $2x = -4 \implies x = -2$. **Q2 (A):** $5x - 25 = 3x - 9 \implies 2x = 16 \implies x = 8$. **Q3 (A):** $2x = 76 \implies x = 38$. **Q4 (B):** $3t - 9 = 10t + 5 \implies -7t = 14 \implies t = -2$.
Challenge	**Q1 (B):** $6x-3x+3 = 6-2x+4 \implies 5x = 7 \implies x = 7/5$. **Q2 (A):** $20x+10 = 27x-18 \implies 7x = 28 \implies x = 4$. **Q3 (A):** $(\frac{3+2+1}{6})x = x = 15$. **Q4 (C):** $2x = 10 \implies 2x + 1 = 11$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Transposition & both-sides x. Day 2: LCM fractions, bracket shield, decimals, cross-multiplication.
- **How to Introduce the Concept:** Use a physical weighing balance analogy: removing 3 kg from left requires removing 3 kg from right.
- **Difficult Areas & Remediation:** Negative signs before fractional numerators and forgetting to multiply standalone constants by LCM.
- **Blackboard Teaching Examples:** Draw parentheses around $(m-1)$ in colored chalk before multiplying by 6.
- **Classroom Questions to Ask Students:** Why can't $ax+b=0$ have $a=0$?
- **Common Student Doubts & Answers:** Students ask about word problems; explain they were rationalised out in NCERT 2026–27, while Ch. 3, 7, 9 apply these equations.
- **Homework & Revision Strategy:** Assign 6 both-side equations and 4 fractional/decimal equations with mandatory verification.

[TEACHER'S TIP] Require a 1-line 'Check: LHS = RHS' at the end of homework problems—it builds self-correction.

UNIT IV: GEOMETRY TYBA COACHING CENTRE

Chapter 3: Understanding Quadrilaterals

Polygons, Interior & Exterior Angle Sums, Trapezium, Kite, Parallelogram, Rectangle, Rhombus & Square (2026–27)

A. Chapter Introduction & Examination Relevance

Polygons and four-sided figures surround us—from tiles and window frames to kites and bridges. Chapter 3 classifies polygons, derives their interior and exterior angle sums, and maps out the exact properties of special quadrilaterals.

- **What You Will Learn:** Convex vs concave polygons, regular polygons, interior angle sum $(n-2)\times 180^\circ$, exterior angle sum $= 360^\circ$, trapezium, kite, parallelogram (opposite sides/angles, diagonals), rhombus, rectangle, and square.
- **Why This Chapter Matters:** Geometry carries 12% in Class 8 and expands to 28/100 marks in Class 9 (Quadrilaterals) and Class 10.
- **Real-Life Applications:** Floor tiling/tessellations, architectural bracing, kite aerodynamics, and mechanical linkages.
- **Cross-Chapter Connections:** Uses Linear Equations from Chapter 2 to solve polygon angle equations and prepares directly for Class 9 Chapter 8.
- **Exam & Foundation Relevance:** Expect 1M MCQs on exterior angles/regular polygon sides $(360^\circ/e)$ and 2M/3M parallelogram diagonal/angle algebra problems.

B. Complete Concept Explanation & Derivations

1. Polygons, Diagonals & Interior Angle Sum

A **polygon** is a simple closed curve made of line segments. In a **convex** polygon, all diagonals lie inside and every interior angle is $< 180^\circ$; in a **concave** polygon, at least one interior angle is $> 180^\circ$. A **regular** polygon is both equilateral (equal sides) and equiangular (equal angles). For any n-sided polygon, the **sum of interior angles** is $(n-2)\times 180^\circ$, and the number of diagonals is $\frac{n(n-3)}{2}$.

[CONCEPT MASTER – 1. Polygons, Diagonals & Interior Angle Sum] For $n=4$ (quadrilateral): $(4-2)\times 180^\circ = 360^\circ$; for $n=5$ (pentagon): 540°; for $n=6$ (hexagon): 720°.

2. Sum of Exterior Angles (360°)

For ANY convex polygon (regardless of the number of sides n!), the **sum of all exterior angles** taken in order is ALWAYS 360°. Therefore, for a regular n-gon: each exterior angle $e = \frac{360^\circ}{n}$, number of sides $n = \frac{360^\circ}{e}$, and each interior angle $i = 180^\circ - e$.

[CONCEPT MASTER – 2. Sum of Exterior Angles (360°)] Golden Rule: Always find the EXTERIOR angle first—it is $360^\circ/n$, whereas the interior formula is much slower!

3. Trapezium, Kite & Parallelogram

Trapezium: Exactly one pair of parallel sides. **Kite:** Two pairs of equal consecutive sides; diagonals are perpendicular, and one diagonal bisects the other. **Parallelogram:** Both pairs of opposite sides parallel; opposite sides equal, opposite angles equal, adjacent angles supplementary (180°), and diagonals bisect each other.

[CONCEPT MASTER – 3. Trapezium, Kite & Parallelogram] Is a square a kite? No, in strict Class 8 definitions a kite has distinct adjacent pairs, whereas a rhombus/square has all 4 sides equal (though a square satisfies kite's side condition too).

4. Special Parallelograms: Rhombus, Rectangle & Square

Rhombus: Parallelogram with all sides equal; diagonals are **perpendicular bisectors**. **Rectangle:** Parallelogram with all angles 90°; diagonals are **equal** and bisect each other. **Square:** All sides equal, all angles 90°, diagonals **equal AND perpendicular bisectors**.

[CONCEPT MASTER – 4. Special Parallelograms: Rhombus, Rectangle & Square] Square is a Rectangle, a Rhombus, AND a Parallelogram—it inherits ALL their properties!

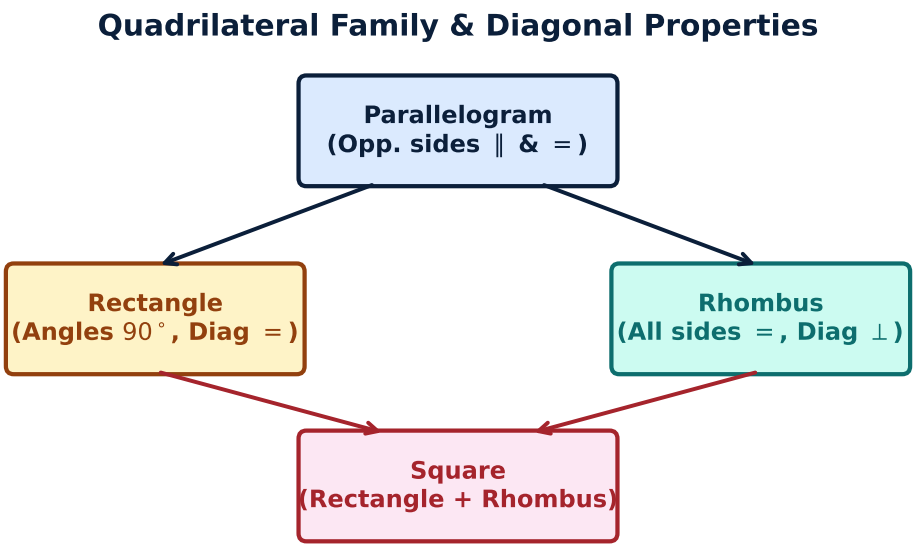


Figure 1: **Visual Model:** Hierarchy of special parallelograms showing how Rectangle, Rhombus, and Square inherit and refine Parallelogram properties.

C. Complete Formula Bank

[FORMULA BANK – 1. Interior Angle Sum of n-gon] **Expression:** $S = (n-2) \times 180^\circ$
Symbols: n: number of sides ($n \geq 3$).
Conditions & When to Use: All simple polygons.
Solved Mini-Example: Octagon ($n = 8$): $6 \times 180^\circ = 1080^\circ$.
Common Mistake: Using $n \times 180^\circ$.

[FORMULA BANK – 2. Exterior Angle Sum & Regular n-gon] **Expression:** $\sum e = 360^\circ$; $e = \frac{360^\circ}{n}$; $n = \frac{360^\circ}{e}$
Symbols: e: each exterior angle of a regular polygon.
Conditions & When to Use: Convex polygons.
Solved Mini-Example: If $e = 24^\circ$, $n = \frac{360}{24} = 15$ sides.
Common Mistake: Dividing 180° instead of 360° by e.

[FORMULA BANK – 3. Interior + Exterior Pair] **Expression:** $i + e = 180^\circ$
Symbols: i: interior angle, e: exterior angle.
Conditions & When to Use: Linear pair at each vertex.
Solved Mini-Example: If $i = 165^\circ$, $e = 15^\circ \implies n = 360/15 = 24$.
Common Mistake: Dividing 360° by i instead of e!

[FORMULA BANK – 4. Number of Diagonals] **Expression:** $D = \frac{n(n-3)}{2}$
Symbols: n: number of sides.
Conditions & When to Use: $n \geq 3$.
Solved Mini-Example: Hexagon ($n = 6$): $\frac{6 \times 3}{2} = 9$ diagonals.
Common Mistake: Using $n(n-1)/2$ (that includes sides too).

[FORMULA BANK – 5. Parallelogram Angle & Side Rules] **Expression:** $\angle A = \angle C, \angle A + \angle B = 180^\circ, OA = OC, OB = OD$
Symbols: O: intersection of diagonals.
Conditions & When to Use: Any parallelogram.
Solved Mini-Example: If $OA = 3x - 1$ and $OC = 2x + 4$, then $3x - 1 = 2x + 4 \implies x = 5$.
Common Mistake: Equating adjacent angles.

[FORMULA BANK – 6. Regular Polygon Minimum Interior Angle] **Expression:** $i_{\min} = 60^\circ$ (Equilateral triangle); $e_{\max} = 120^\circ$
Symbols: $n = 3$ is smallest polygon.
Conditions & When to Use: Regular polygons.
Solved Mini-Example: Can a regular polygon have $i = 50^\circ$? No, $50^\circ < 60^\circ$.
Common Mistake: Testing $360/(180 - 50)$ when $i < 60^\circ$.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Sainik/Navodaya/Olympiad Foundation problem structures.

Pattern 1: Polygon Angle Sum & Regular Polygon Sides ($n = 360/e$)

- **Identification Cue:** How many sides does a regular polygon have if each interior angle is 165°? Or exterior angle is 24°? | **Concept & Formula:** $e = 180^\circ - i$, then $n = 360^\circ/e$.
- **Standard Step-by-Step:** Convert interior angle i to exterior angle $e = 180^\circ - i$, then divide 360° by e.
- **Fastest Valid Method:** Check if e divides 360° cleanly—if NOT, such a regular polygon is IMPOSSIBLE! | **Alternative:** Solve $\frac{(n-2)180}{n} = i$ (slower).
- **Common Trap:** Dividing 360° by 165° directly.
- **Practice Drill:** Ex. 2 & Ex. 5.

Pattern 2: Missing Angle x in Convex Polygons

- **Identification Cue:** Find x in a pentagon/quadrilateral with 3–4 given interior or exterior angles. | **Concept & Formula:** Interior sum $(n - 2)180^\circ$ or Exterior sum 360°.
- **Standard Step-by-Step:** Set up linear equation summing all angles to 360° (quad/exterior) or 540° (pentagon).
- **Fastest Valid Method:** If exterior angles are given, sum $= 360^\circ$ regardless of n! | **Alternative:** Convert linear pairs at vertices.
- **Common Trap:** Adding a 90° right-angle symbol as unknown.
- **Practice Drill:** Ex. 1 & Ex. 6.

Pattern 3: Parallelogram Angles, Sides & Diagonals Algebra

- **Identification Cue:** In parallelogram $GUNS$, $GU = 3y - 1, SN = 26$, $GS = 3x, UN = 18$; or diagonals $OR = 20, OT = x + y, OE = y + 7$. | **Concept & Formula:** Opposite sides equal; diagonals bisect each other.
- **Standard Step-by-Step:** Equate opposite sides/angles or half-diagonals and solve for x, y.
- **Fastest Valid Method:** Solve the 1-variable half-diagonal first ($y + 7 = 20 \implies y = 13$), then plug into $x + y$! | **Alternative:** Co-interior $= 180^\circ$.
- **Common Trap:** Equating the two DIFFERENT diagonals ($AC = BD$) in a general parallelogram.
- **Practice Drill:** Ex. 7 & Ex. 9.

Pattern 4: Special Quadrilateral Identification & Properties

- **Identification Cue:** Name the quadrilateral whose diagonals bisect at 90° or are equal. | **Concept & Formula:** Property table of Parallelogram, Rectangle, Rhombus, Square.
- **Standard Step-by-Step:** Match side, angle, and diagonal conditions to the exact quadrilateral.
- **Fastest Valid Method:** Equal diag $\rightarrow$ Rect/Square; $\perp$ diag $\rightarrow$ Rhombus/Square/Kite. | **Alternative:** Draw quick sketch.
- **Common Trap:** Saying Square when only equal diagonals are given (Rectangle too!).

- **Practice Drill:** Ex. 4 & Ex. 11.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Exterior-First Rule ($n = 360°/e$)] **Core Idea & Why It Works:** Never use $\frac{(n-2)180}{n} = i$ to find n! Just do $e = 180° - i$, then $n = \frac{360°}{e}$.
Valid Domain: All regular polygons. | **Where It Fails:** Nothing.
Easy Example: $i = 156° \implies e = 24° \implies n = \frac{360}{24} = 15$ sides!
Exam-Level Example: Saves 4 algebraic steps in NCERT Ex. 3.2 Q4.
Advanced Example: Impossible test: if $360/e$ is not an integer, the polygon cannot exist!

[SMART SHORTCUT – 2. Diagonal Property 3-Word Key] **Core Idea & Why It Works: Parallelogram:** Bisect. **Rectangle:** Bisect + Equal. **Rhombus:** Bisect + $90°$. **Square:** Bisect + Equal + $90°$.
Valid Domain: All special parallelograms. | **Where It Fails:** Kite (only ONE diagonal bisects the other) & Trapezium (need not bisect).
Easy Example: Equal AND $\perp$ bisectors $\implies$ Square.
Exam-Level Example: Answers all NCERT Ex. 3.4 Q4 questions in 2 seconds!
Advanced Example: Zero confusion.

[SMART SHORTCUT – 3. Adjacent Parallelogram Ratio] **Core Idea & Why It Works:** If two adjacent angles of a parallelogram are in ratio $a : b$, they are $\frac{a}{a+b} \times 180°$ and $\frac{b}{a+b} \times 180°$.
Valid Domain: All parallelograms. | **Where It Fails:** Opposite angles (those are equal!).
Easy Example: Ratio $3 : 2 \implies 108°$ and $72°$.
Exam-Level Example: Instant 1-mark MCQ solver.
Advanced Example: Opposite angles copy these two.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Find the sum of interior angles of a polygon with 7 sides (heptagon).
Given & Required: $n = 7$. | **Concept & Formula:** $(n-2) \times 180°$.
Detailed Solution: $(7-2) \times 180° = 5 \times 180° = 900°$. **Final Answer:** $900°$
Fastest Alternative / Shortcut: $5 \times 180 = 900°$.

SOLVED EXAMPLE 2 [Beginner]
Question: Find the number of sides of a regular polygon whose each exterior angle is $45°$.
Given & Required: $e = 45°$. | **Concept & Formula:** $n = 360°/e$.
Detailed Solution: $n = \frac{360°}{45°} = 8$ sides (regular octagon). **Final Answer:** 8 **sides**
Fastest Alternative / Shortcut: $360/45 = 8$.

SOLVED EXAMPLE 3 [Beginner]
Question: In a quadrilateral, three angles are $50°, 130°, 120°$. Find the fourth angle x.
Given & Required: Sum $= 360°$. | **Concept & Formula:** Angle sum of quad.
Detailed Solution: $x = 360° - (50 + 130 + 120)° = 360° - 300° = 60°$. **Final Answer:** $60°$
Fastest Alternative / Shortcut: $360 - 300 = 60°$.

SOLVED EXAMPLE 4 [Beginner]

Question: How many diagonals does a regular hexagon ($n = 6$) have?
Given & Required: $n = 6$. | **Concept & Formula:** $\frac{n(n-3)}{2}$.
Detailed Solution: $\frac{6(6-3)}{2} = 9$ diagonals. **Final Answer:** 9
Fastest Alternative / Shortcut: $6 \times 3/2 = 9$.

SOLVED EXAMPLE 5 [Intermediate]

Question: How many sides does a regular polygon have if each of its interior angles is $165°$?
Given & Required: $i = 165°$. | **Concept & Formula:** Exterior-first rule.
Detailed Solution: $e = 180° - 165° = 15°$. $n = \frac{360°}{15°} = 24$ sides. **Final Answer:** 24 **sides**
Fastest Alternative / Shortcut: $360/15 = 24$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Is it possible to have a regular polygon with each exterior angle equal to $22°$?
Given & Required: $e = 22°$. | **Concept & Formula:** $n = 360/e$ must be an integer.
Detailed Solution: $\frac{360}{22} = 16.36\ldots$ (not a whole number). Hence, No. **Final Answer: No (22 does not divide 360)**
Fastest Alternative / Shortcut: Divisibility test of 360.

SOLVED EXAMPLE 7 [Intermediate]

Question: In parallelogram $RUNS$, diagonals intersect at O, with $OR = 20, OT = x + y, OE = y + 7, OS = 16$. Find x and y.
Given & Required: Diagonals bisect each other. | **Concept & Formula:** $y + 7 = 20, x + y = 16$.
Detailed Solution: $y + 7 = 20 \implies y = 13$. Then $x + 13 = 16 \implies x = 3$. **Final Answer:** $x = 3, y = 13$
Fastest Alternative / Shortcut: Solve y first, then x.

SOLVED EXAMPLE 8 [Intermediate]

Question: Two adjacent angles of a parallelogram have equal measure. Find the measure of each angle of the parallelogram.
Given & Required: $x + x = 180°$. | **Concept & Formula:** Adjacent sum $= 180°$.
Detailed Solution: $2x = 180° \implies x = 90°$. All four angles are $90°$ (it is a rectangle). **Final Answer:** $90°$ **each**
Fastest Alternative / Shortcut: $180/2 = 90°$.

SOLVED EXAMPLE 9 [Board Standard]

Question: Find x in a pentagon whose interior angles are $30°, x, x, (180° - 70°), (180° - 60°)$.
Given & Required: $n = 5 \implies$ sum $= 540°$. | **Concept & Formula:** Pentagon angle sum.
Detailed Solution: Base angles are $110°$ and $120°$. $30 + 110 + 120 + 2x = 540 \implies 2x = 280 \implies x = 140°$. **Final Answer:** $x = 140°$
Fastest Alternative / Shortcut: Convert exterior angles to interior first.

SOLVED EXAMPLE 10 [Board Standard]

Question: In a parallelogram $RING$, if $m\angle R = 70°$, find all the other angles.
Given & Required: $\angle R = 70°$. | **Concept & Formula:** Opposite $=$, adjacent $180°$.
Detailed Solution: $\angle N = \angle R = 70°$; $\angle I = \angle G = 180° - 70° = 110°$. **Final Answer:** $110°, 70°, 110°$
Fastest Alternative / Shortcut: Supplement $= 110°$.

SOLVED EXAMPLE 11 [Board Standard]

Question: In parallelogram $HOPE$, exterior angle at P is $70°$, $\angle EHP = 40°$. Find $x = \angle E$, $y = \angle HPO$, $z = \angle OHP$.
Given & Required: $\angle HOP = 180° - 70° = 110°$. | **Concept & Formula:** Opposite + alternate interior.
Detailed Solution: $x = \angle HOP = 110°$. $y = \angle EHP = 40°$ (alt. int.). Since $70°$ is exterior at P, $40° + z = 70° \implies z = 30°$.
Final Answer: $x = 110°, y = 40°, z = 30°$
Fastest Alternative / Shortcut: Check $110 + 40 + 30 = 180°$ in $\triangle HOP$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Two adjacent angles of a parallelogram are in the ratio $3:2$. Find all four angles of the parallelogram.
Given & Required: $3x + 2x = 180°$. | **Concept & Formula:** Adjacent $= 180°$.
Detailed Solution: $5x = 180° \implies x = 36°$. Angles: $108°, 72°, 108°, 72°$. **Final Answer:** $108°, 72°, 108°, 72°$
Fastest Alternative / Shortcut: $180/5 = 36°$.

SOLVED EXAMPLE 13 [Board Standard]

Question: In rectangle $RENT$, diagonals meet at O. If $OR = 2x + 4$ and $OT = 3x + 1$, find x.
Given & Required: Rectangle diagonals are equal and bisect. | **Concept & Formula:** $OR = OT$.
Detailed Solution: $2x + 4 = 3x + 1 \implies x = 3$. **Final Answer:** $x = 3$
Fastest Alternative / Shortcut: Half-diagonals of rectangle are equal!

SOLVED EXAMPLE 14 [Challenge]

Question: What is the minimum interior angle possible for a regular polygon, and what is the maximum exterior angle possible?
Given & Required: Smallest $n = 3$. | **Concept & Formula:** Equilateral triangle.
Detailed Solution: At $n = 3$, interior angle is minimum $(60°)$ and exterior angle is maximum $(120°)$. **Final Answer: Min int** $= 60°$**; Max ext** $= 120°$
Fastest Alternative / Shortcut: $n = 3$ gives extremes.

SOLVED EXAMPLE 15 [Challenge]

Question: In a kite $ABCD$ ($AB = AD, CB = CD$), $\angle B = 110°$ and $\angle C = 60°$. Find $\angle A$.
Given & Required: $\angle D = \angle B = 110°$. | **Concept & Formula:** Kite has 1 pair of equal opposite angles.
Detailed Solution: $\angle A = 360° - (110 + 110 + 60)° = 80°$. **Final Answer:** $80°$
Fastest Alternative / Shortcut: Opposite angles between unequal sides are equal!

SOLVED EXAMPLE 16 [Challenge]

Question: Find the number of sides of a regular polygon whose interior angle is 5 times its exterior angle.
Given & Required: $i = 5e$. | **Concept & Formula:** $i + e = 180°$.
Detailed Solution: $5e + e = 180° \implies e = 30° \implies n = \frac{360°}{30°} = 12$ sides. **Final Answer:** 12 **sides**
Fastest Alternative / Shortcut: $6e = 180 \implies e = 30 \implies n = 12$.

G. Competency MCQ Mastery Section

Q1. Sum of all exterior angles of a 10-sided convex polygon is:

(A) $1440°$ (B) $360°$ (C) $180°$ (D) $1800°$

Answer: (B) Always $360°$ for any n. *Fast Method:* $360°$. *Distractor Note:* Computing $(10 - 2) \times 180$ (interior sum).

Q2. Each exterior angle of a regular nonagon ($n = 9$) is:

(A) $40°$ (B) $45°$ (C) $36°$ (D) $140°$

Answer: (A) $360°/9 = 40°$. *Fast Method:* $360/9$. *Distractor Note:* Picking $140°$ (interior).

Q3. Number of diagonals in a pentagon ($n = 5$) is:

(A) 2 (B) 5 (C) 10 (D) 4

Answer: (B) $\frac{5(5-3)}{2} = 5$. *Fast Method:* 5. *Distractor Note:* Picking 10.

Q4. Which quadrilateral has equal diagonals that bisect at $90°$?

(A) Parallelogram (B) Rectangle (C) Rhombus (D) Square

Answer: (D) Square. *Fast Method:* Square. *Distractor Note:* Picking Rhombus.

Q5. Sum of interior angles of a hexagon ($n = 6$) is:

(A) $360°$ (B) $540°$ (C) $720°$ (D) $900°$

Answer: (C) $(6-2) \times 180° = 720°$. *Fast Method:* 4×180. *Distractor Note:* Picking $540°$ (pentagon).

Q6. If each interior angle of a regular polygon is $120°$, number of sides is:

(A) 4 (B) 5 (C) 6 (D) 8

Answer: (C) $e = 60° \implies n = 360/60 = 6$. *Fast Method:* Exterior-first. *Distractor Note:* Dividing $360/120 = 3$.

Q7. A quadrilateral with exactly one pair of parallel sides is a:

(A) Kite (B) Trapezium (C) Rhombus (D) Parallelogram

Answer: (B) Definition of trapezium. *Fast Method:* Trapezium. *Distractor Note:* Picking Kite.

Q8. In a parallelogram, adjacent angles are always:

(A) Equal (B) Complementary (C) Supplementary (D) Right angles

Answer: (C) Sum $= 180°$. *Fast Method:* $180°$. *Distractor Note:* Picking Equal.

Q9. If one angle of a parallelogram is $90°$, it must be a:

(A) Square (B) Rhombus (C) Rectangle (D) Kite

Answer: (C) All 4 angles become $90°$ (Rectangle; sides need not be equal). *Fast Method:* Rectangle. *Distractor Note:* Picking Square.

Q10. Maximum exterior angle possible for a regular polygon is:

(A) $60°$ (B) $90°$ (C) $120°$ (D) $180°$

Answer: (C) Equilateral triangle ($n = 3$) has $e = 120°$. *Fast Method:* $360/3 = 120°$. *Distractor Note:* Picking $60°$.

Q11. Diagonals of a kite:

(A) Are equal (B) Bisect each other (C) Are perpendicular to each other (D) Are parallel

Answer: (C) Diagonals of a kite meet at $90°$. *Fast Method:* $90°$. *Distractor Note:* Saying both bisect.

Q12. Four angles of a quadrilateral are $x, x+10, x+20, x+30$. Value of x is:

(A) $60°$ (B) $75°$ (C) $80°$ (D) $90°$

Answer: (B) $4x + 60 = 360 \implies 4x = 300 \implies x = 75°$. *Fast Method:* $300/4 = 75°$. *Distractor Note:* Arithmetic slip.

H. Special Exam & MCQ Strategies

- For any regular polygon question, convert interior angle to exterior angle ($e = 180° - i$) FIRST, then use $n = 360°/e$.
- In parallelogram problems, check whether the two given expressions are OPPOSITE (set equal) or ADJACENT (sum to $180°$).
- Keep the 4-box diagonal summary in mind: Bisect (Pgm), +Equal (Rect), $+\perp$ (Rhombus), +Equal+$\perp$ (Square).

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Dividing $360°$ by the INTERIOR angle to find number of sides n. $\longrightarrow$ **Correction:** Divide $360°$ by the EXTERIOR angle $e = 180° - i$.

[COMMON MISTAKE] **Error:** Saying sum of exterior angles of a hexagon is greater than that of a triangle. $\longrightarrow$ **Correction:** Exterior angle sum is ALWAYS $360°$ for every convex polygon!

[COMMON MISTAKE] **Error:** Equating the two diagonals of a general parallelogram. $\longrightarrow$ **Correction:** Diagonals bisect each other ($OA = OC, OB = OD$), but $AC = BD$ holds ONLY for Rectangles and Squares.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: Int sum $= (n-2)180^\circ$; Ext sum $= 360^\circ$; $e = 360^\circ/n$; $i + e = 180^\circ$; Diagonals $= \frac{n(n-3)}{2}$.

[QUICK REVISION] Key Theorems & Results: Pgm: opp $=$, adj 180°, diag bisect; Rect: diag $=$; Rhombus: diag $\perp$; Square: diag $=, \perp$.

[QUICK REVISION] Shortcut Summary: Exterior-first $n = 360/(180 - i)$; Min $i = 60^\circ$, Max $e = 120^\circ$; Ratio $a : b$ of 180°.

[QUICK REVISION] High-Frequency Exam Patterns: Regular polygon sides, Polygon missing angle, Parallelogram algebra, Diagonal identification.

Last-Minute Revision Checklist:

- ☐ $e = 180^\circ - i$ first
- ☐ Ext sum always 360°
- ☐ Opposite $=$ vs Adjacent 180°
- ☐ Rect diag $=$, Rhombus diag $\perp$

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Interior angle sum of a pentagon ($n = 5$) is: (A) 360° (B) 540° (C) 720° (D) 180°
2. Each exterior angle of a regular decagon ($n = 10$) is: (A) 36° (B) 144° (C) 40° (D) 30°
3. A parallelogram with all sides equal and one angle 90° is a: (A) Rhombus (B) Rectangle (C) Square (D) Kite
4. Number of diagonals in a quadrilateral is: (A) 1 (B) 2 (C) 3 (D) 4

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Number of sides of a regular polygon with interior angle 135°: (A) 6 (B) 8 (C) 10 (D) 12
2. In parallelogram $ABCD$, $\angle A = 3x - 10, \angle B = 2x + 40$. Value of x: (A) 30 (B) 50 (C) 20 (D) 40
3. Diagonals of a rhombus are 12 cm and 16 cm. Its side is: (A) 10 cm (B) 20 cm (C) 14 cm (D) 8 cm
4. Which regular polygon has exterior angle equal to interior angle? (A) Triangle (B) Square (C) Pentagon (D) Hexagon

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. A polygon has 20 diagonals. Number of sides is: (A) 6 (B) 7 (C) 8 (D) 10
2. Interior angle of a regular polygon is 108° more than its exterior angle. Number of sides is: (A) 8 (B) 10 (C) 12 (D) 6
3. If $ABCD$ is a square, $\angle BAC$ equals: (A) 30° (B) 45° (C) 60° (D) 90°
4. Sum of all interior angles of a polygon is 1440°. Number of sides is: (A) 8 (B) 9 (C) 10 (D) 12

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $3 \times 180 = 540^\circ$. **Q2 (A):** $360/10 = 36^\circ$. **Q3 (C):** Square. **Q4 (B):** $4(1)/2 = 2$.
Exam Std	**Q1 (B):** $e = 45^\circ \implies 360/45 = 8$. **Q2 (A):** $5x + 30 = 180 \implies x = 30$. **Q3 (A):** $\sqrt{6^2 + 8^2} = 10$ cm. **Q4 (B):** $e = i = 90^\circ \implies n = 4$ (Square).
Challenge	**Q1 (C):** $n(n-3) = 40 \implies 8 \times 5 = 40 \implies n = 8$. **Q2 (B):** $i + e = 180, i - e = 108 \implies 2e = 72 \implies e = 36 \implies n = 10$. **Q3 (B):** Diagonal bisects 90° corner $\rightarrow 45^\circ$. **Q4 (C):** $n - 2 = 1440/180 = 8 \implies n = 10$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Polygons, interior & exterior angle sums. Day 2: Trapezium, kite, parallelogram. Day 3: Rectangle, rhombus, square.
- **How to Introduce the Concept:** Walk around a polygon drawn on the floor—at every corner you turn by the exterior angle, completing 1 full 360° turn!
- **Difficult Areas & Remediation:** Dividing 360° by interior angle instead of exterior angle, and diagonal properties.
- **Blackboard Teaching Examples:** Draw the 4-box hierarchy from Parallelogram $\rightarrow$ Rectangle/Rhombus $\rightarrow$ Square.
- **Classroom Questions to Ask Students:** Why is every square a rectangle, but not every rectangle a square?
- **Common Student Doubts & Answers:** Students confuse kite vs rhombus; show a diamond kite with unequal top/bottom side lengths.
- **Homework & Revision Strategy:** Assign 4 regular-polygon problems and 4 parallelogram algebra problems.

[TEACHER'S TIP] The Exterior-First shortcut ($n = 360/(180 - i)$) saves huge time in both Class 8 and competitive exams.

UNIT VI: DATA HANDLING & GRAPHS TYBA COACHING CENTRE

Chapter 4: Data Handling

Pie Charts (Circle Graphs), Central Angles, Outcomes & Basic Probability (2026–27)

A. Chapter Introduction & Examination Relevance

Data is everywhere—from school marks and cricket scores to budget allocations. Chapter 4 teaches how to read and construct Pie Charts (Circle Graphs) and introduces the mathematical measure of Chance: Probability.

- **What You Will Learn:** Pie charts (central angle $= \frac{\text{value}}{\text{total}} \times 360°$), reading values from given pie charts, random experiments, equally likely outcomes, and probability $P(E) = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$.
- **Why This Chapter Matters:** Pie charts appear in competitive Data Interpretation (DI), and Probability launches directly into Class 9/10/11 Probability.
- **Real-Life Applications:** Household/government budget charts, election vote-share graphics, weather chance percentages, and board-game dice/spinner odds.
- **Cross-Chapter Connections:** Uses percentage/fraction skills (Ch. 1 & 7) and prepares for Class 9 Statistics and Class 10 Probability.
- **Exam & Foundation Relevance:** Unit VI carries 12% weightage. Expect 1M MCQs on central angles/dice/cards probability and 3M/5M pie-chart construction or reading problems. (Grouped tally tables & histograms moved to Class 9 in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Circle Graphs (Pie Charts) & Central Angles

A **Pie Chart** shows the relationship between a whole ($360°$) and its parts. The central angle of each sector is proportional to the fraction of the total: $\theta = \left(\frac{\text{Item Value}}{\text{Total Value}}\right) \times 360°$. Conversely, given a sector's central angle θ, its value is $\frac{\theta}{360°} \times$ Total.

[CONCEPT MASTER – 1. Circle Graphs (Pie Charts) & Central Angles] Check Your Table: The sum of all central angles in a pie chart MUST equal $360°$!

2. Random Experiments & Equally Likely Outcomes

A **random experiment** has unpredictable outcomes (tossing a coin, rolling a die, spinning a wheel). Outcomes are **equally likely** when each has the same chance of occurring. An **event** is a collection of some outcomes of the experiment.

[CONCEPT MASTER – 2. Random Experiments & Equally Likely Outcomes] Standard Totals: 1 Coin $= 2$ outcomes (H, T); 2 Coins $= 4$; 1 Die $= 6$ (1..6); Standard Deck $= 52$ cards.

3. Probability of an Event

When outcomes are equally likely, Probability $P(E) = \frac{\text{Number of outcomes making } E}{\text{Total number of outcomes}}$. Probability always lies between 0 and 1: impossible event has $P = 0$, sure event has $P = 1$, and $P(\text{not } E) = 1 - P(E)$.

[CONCEPT MASTER – 3. Probability of an Event] Warning: 1 is NEITHER prime NOR composite! So prime numbers on a die are $\{2, 3, 5\}$ (3 outcomes, NOT 4!).

4. Rationalised 2026–27 Scope

In the 2026–27 rationalised NCERT syllabus, Chapter 4 focuses cleanly on Pie Charts (reading and drawing) and Chance/Probability, while grouped frequency distributions and histograms are covered in Class 9.

[CONCEPT MASTER – 4. Rationalised 2026–27 Scope] Percentage to Degrees: Since $100\% = 360°$, every 1% equals $3.6°$!

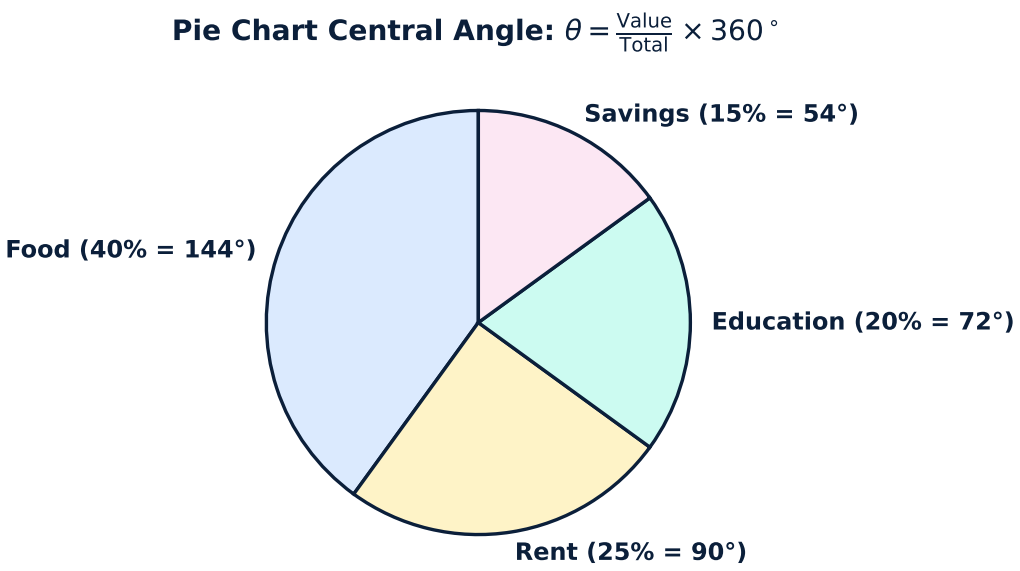


Figure 1: **Visual Model:** Pie chart illustrating sector central angles $\theta = \frac{\text{Category Value}}{\text{Total}} \times 360^\circ$.

C. Complete Formula Bank

[FORMULA BANK – 1. Pie Chart Central Angle] **Expression:** $\theta = \frac{\text{Value}}{\text{Total}} \times 360^\circ$
Symbols: θ: sector angle in degrees.
Conditions & When to Use: $\sum \theta_i = 360^\circ$.
Solved Mini-Example: Fraction $\frac{1}{4}$ gives $\frac{1}{4} \times 360^\circ = 90^\circ$.
Common Mistake: Multiplying by 100 instead of 360°.

[FORMULA BANK – 2. Percentage to Central Angle] **Expression:** $\theta = p\% \times 3.6^\circ$
Symbols: p: percentage.
Conditions & When to Use: $\sum p_i = 100\%$.
Solved Mini-Example: $25\% \implies 25 \times 3.6 = 90^\circ$; $15\% \implies 54^\circ$.
Common Mistake: Writing $25\% = 25^\circ$.

[FORMULA BANK – 3. Value from Central Angle] **Expression:** $\text{Value} = \frac{\theta}{360^\circ} \times \text{Total}$
Symbols: θ: given sector angle.
Conditions & When to Use: If total is unknown, use $\frac{\text{Value}_1}{\theta_1} = \frac{\text{Value}_2}{\theta_2}$.
Solved Mini-Example: If 90° represents Rs. 3000, Total $= 3000 \times 4 =$ Rs. 12000.
Common Mistake: Dividing by 100 instead of 360°.

[FORMULA BANK – 4. Probability Formula] **Expression:** $P(E) = \frac{\text{Favourable Outcomes}}{\text{Total Outcomes}}$
Symbols: $0 \leq P(E) \leq 1$.
Conditions & When to Use: Equally likely outcomes.
Solved Mini-Example: Prime on a die $\{2, 3, 5\}$: $\frac{3}{6} = \frac{1}{2}$.
Common Mistake: Including 1 as a prime number.

[FORMULA BANK – 5. Complementary Probability] **Expression:** $P(\text{not } E) = 1 - P(E)$
Symbols: E: any event.
Conditions & When to Use: Always valid.
Solved Mini-Example: If $P(\text{win}) = 0.35$, $P(\text{not win}) = 0.65$.
Common Mistake: Subtracting from 100 instead of 1.

[FORMULA BANK – 6. Probability Bounds] **Expression:** $0 \leq P(E) \leq 1$
Symbols: $P(\text{impossible}) = 0$, $P(\text{sure}) = 1$.
Conditions & When to Use: Never negative, never > 1.
Solved Mini-Example: Getting 7 on a standard die has $P = 0$.
Common Mistake: Writing a percentage like 50 without % sign.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Sainik/Navodaya/Olympiad Foundation problem structures.

Pattern 1: Constructing a Pie Chart Table (Central Angles)

- **Identification Cue:** Draw a pie chart for monthly expenditure or favourite colours (Blue 18, Green 9, Red 6, Yellow 3). | **Concept & Formula:** $\theta = \frac{\text{Value}}{\text{Total}} \times 360^\circ$.
- **Standard Step-by-Step:** Sum all values ($T = 36$), compute $\frac{v}{T} \times 360^\circ$ for each row, check sum $= 360^\circ$, draw sectors with protractor.
- **Fastest Valid Method:** Compute $360^\circ/T$ first ($360/36 = 10^\circ$ per unit), then multiply each value by 10°! | **Alternative:** Reduce fraction v/T first.
- **Common Trap:** Sum of angles not equaling 360° due to rounding.
- **Practice Drill:** Ex. 1 & Ex. 5.

Pattern 2: Reading Values from a Given Pie Chart

- **Identification Cue:** In a pie chart of 540 marks, Maths has 90°, Hindi 70°, Science 80°; find marks in Maths or which subject scored 105. | **Concept & Formula:** Value $= \frac{\theta}{360^\circ} \times$ Total; $\theta = \frac{\text{Value}}{\text{Total}} \times 360^\circ$.
- **Standard Step-by-Step:** Multiply $\frac{\theta}{360}$ by Total; for reverse, compute $\frac{\text{Marks}}{540} \times 360^\circ$.
- **Fastest Valid Method:** Compute $1^\circ = \frac{540}{360} = 1.5$ marks! Then Maths $= 90 \times 1.5 = 135$, and $105/1.5 = 70^\circ$ (Hindi)! | **Alternative:** Proportional ratios.
- **Common Trap:** Confusing degrees ($^\circ$) with marks or percentages.
- **Practice Drill:** Ex. 6 & Ex. 9.

Pattern 3: Die, Spinner & Number-Slip Probability

- **Identification Cue:** Numbers 1 to 10 on slips, or a die is thrown; find $P(\text{prime})$, $P(> 4)$, $P(\text{1-digit})$. | **Concept & Formula:** Count favourable / total.
- **Standard Step-by-Step:** List all outcomes in S, list outcomes in E, divide $n(E)/n(S)$ and reduce fraction.
- **Fastest Valid Method:** List E explicitly so you don't miscount 1 (not prime) or boundary $< vs \leq$. | **Alternative:** Use $1 - P(E')$.
- **Common Trap:** Counting 1 as prime or reading '< 6' as '≤ 6'.
- **Practice Drill:** Ex. 3 & Ex. 7.

Pattern 4: 52-Card Deck & Bag-of-Balls Probability

- **Identification Cue:** 1 ball drawn from 4 red, 2 yellow, 3 blue; or 1 card from 52 cards (Ace, Red King, Face card). | **Concept & Formula:** $P(E) = n(E)/n(S)$.
- **Standard Step-by-Step:** Sum total items for $n(S)$, count matching items for $n(E)$, simplify.
- **Fastest Valid Method:** Know 52-card counts: 4 Aces, 12 Face cards, 26 Red, 13 per suit. | **Alternative:** Complement.

- **Common Trap:** Using 13 instead of 52 or 4 instead of 2 for 'Red King'.
- **Practice Drill:** Ex. 8 & Ex. 11.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Unit-Angle / Unit-Value Multiplier] **Core Idea & Why It Works:** In ANY Pie Chart, compute $k = \frac{360^\circ}{\text{Total}}$ (degrees per item) or $m = \frac{\text{Total}}{360^\circ}$ (value per degree) ONCE—then multiply every row by that single number!
Valid Domain: All pie chart tables and reading questions. | **Where It Fails:** Nothing.
Easy Example: Total $= 72 \implies 360/72 = 5^\circ$ per item! Multiply every item by 5°.
Exam-Level Example: Slashes calculation time in half for NCERT Ex. 4.1.
Advanced Example: Total $= 540 \implies 1.5$ marks per degree.

[SMART SHORTCUT – 2. 1% = 3.6° Converter] **Core Idea & Why It Works:** To convert percentage $p\%$ to Pie Chart degrees, multiply $p \times 3.6^\circ$ (or $p \times \frac{18}{5}$).
Valid Domain: All percentage pie charts. | **Where It Fails:** Nothing.
Easy Example: $10\% = 36^\circ$, $20\% = 72^\circ$, $25\% = 90^\circ$, $50\% = 180^\circ$.
Exam-Level Example: Instant mental conversion.
Advanced Example: Reverse: divide degrees by 3.6 to get %.

[SMART SHORTCUT – 3. Prime-on-Die Trap Alert] **Core Idea & Why It Works:** On a die $\{1,2,3,4,5,6\}$: Primes $= \{2,3,5\}$ $(3/6 = 1/2)$; Composites $= \{4,6\}$ $(2/6 = 1/3)$; 1 is NEITHER!
Valid Domain: All die probability questions. | **Where It Fails:** Nothing.
Easy Example: P(non-prime) $= \{1,4,6\} = 3/6 = 1/2$.
Exam-Level Example: Prevents the #1 MCQ error in Class 8 Probability.
Advanced Example: Applies to 1–10 slips too: primes $\{2,3,5,7\} = 4/10 = 2/5$.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: In a pie chart, a family spends 20% of its income on rent. What is the central angle of the rent sector?
Given & Required: 20%. | **Concept & Formula:** $p\% \times 3.6^\circ$.
Detailed Solution: $\frac{20}{100} \times 360^\circ = 72^\circ$. **Final Answer:** 72°
Fastest Alternative / Shortcut: $20 \times 3.6 = 72^\circ$.

SOLVED EXAMPLE 2 [Beginner]
Question: A die is thrown once. Find the probability of getting (i) an even number, (ii) a number greater than 4.
Given & Required: $S = \{1,2,3,4,5,6\}$. | **Concept & Formula:** $n(E)/6$.
Detailed Solution: (i) $\{2,4,6\} \implies \frac{3}{6} = \frac{1}{2}$; (ii) $\{5,6\} \implies \frac{2}{6} = \frac{1}{3}$. **Final Answer:** $\frac{1}{2}$ **and** $\frac{1}{3}$
Fastest Alternative / Shortcut: Strict > 4 excludes 4.

SOLVED EXAMPLE 3 [Beginner]
Question: A bag has 4 red balls and 2 yellow balls. A ball is drawn without looking. Find the probability of getting a red ball.
Given & Required: Total $= 6$. | **Concept & Formula:** $n(E)/n(S)$.
Detailed Solution: $P(\text{Red}) = \frac{4}{6} = \frac{2}{3}$. **Final Answer:** $\frac{2}{3}$
Fastest Alternative / Shortcut: Reduce fraction $4/6 = 2/3$.

SOLVED EXAMPLE 4 [Beginner]

Question: In a pie chart, a sector of $90°$ represents 15 students. Find the total number of students.

Given & Required: $90° \leftrightarrow 15$. | **Concept & Formula:** $90° = \frac{1}{4}$ of $360°$.

Detailed Solution: Total $= 15 \times \frac{360}{90} = 15 \times 4 = 60$ students. **Final Answer:** 60 **students**

Fastest Alternative / Shortcut: $90°$ is a quarter circle!

SOLVED EXAMPLE 5 [Intermediate]

Question: Find the central angles for a pie chart of favourite colours among 36 people: Blue (18), Green (9), Red (6), Yellow (3).

Given & Required: Total $= 36$. | **Concept & Formula:** $360/36 = 10°$ per person.

Detailed Solution: Blue: $180°$, Green: $90°$, Red: $60°$, Yellow: $30°$. Sum $= 360°$. **Final Answer:** $180°, 90°, 60°, 30°$

Fastest Alternative / Shortcut: Multiply each by $10°$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Numbers 1 to 10 are written on ten separate slips, mixed in a box, and one is chosen. Find P(getting 6), P(less than 6), and P(1-digit number).

Given & Required: $n(S) = 10$. | **Concept & Formula:** Count favourable.

Detailed Solution: (i) $\frac{1}{10}$; (ii) $\{1,2,3,4,5\} \implies \frac{5}{10} = \frac{1}{2}$; (iii) $\{1..9\} \implies \frac{9}{10}$. **Final Answer:** $\frac{1}{10}, \frac{1}{2}, \frac{9}{10}$

Fastest Alternative / Shortcut: 10 is the only 2-digit number.

SOLVED EXAMPLE 7 [Intermediate]

Question: A spinning wheel has 3 green sectors, 1 blue sector, and 1 red sector. Find the probability of getting (i) a green sector, (ii) a non-blue sector.

Given & Required: Total $= 5$ sectors. | **Concept & Formula:** Counts / 5.

Detailed Solution: (i) $\frac{3}{5}$; (ii) Non-blue $= 3 + 1 = 4 \implies \frac{4}{5}$. **Final Answer:** $\frac{3}{5}$ **and** $\frac{4}{5}$

Fastest Alternative / Shortcut: $1 - 1/5 = 4/5$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Two coins are tossed together. Find the probability of getting at least one head.

Given & Required: $S = \{$HH, HT, TH, TT$\}$. | **Concept & Formula:** $n(S) = 4$.

Detailed Solution: At least 1 head: $\{$HH, HT, TH$\} \implies \frac{3}{4}$. **Final Answer:** $\frac{3}{4}$

Fastest Alternative / Shortcut: $1 - P(\text{TT}) = 1 - 1/4 = 3/4$.

SOLVED EXAMPLE 9 [Board Standard]

Question: In a pie chart of a student's total 540 marks: Maths ($90°$), Social Science ($65°$), Science ($80°$), Hindi ($70°$), English ($55°$). (i) In which subject did the student score 105 marks? (ii) How many more marks were obtained in Maths than in Hindi?

Given & Required: Total $= 540 \implies 1° = 1.5$ marks. | **Concept & Formula:** Unit-value multiplier.

Detailed Solution: (i) 105 marks $= \frac{105}{540} \times 360° = 70° \implies$ Hindi. (ii) Maths $= 90 \times 1.5 = 135$, Hindi $= 105$, difference $= 30$ marks. **Final Answer: (i) Hindi, (ii)** 30 **marks**

Fastest Alternative / Shortcut: $(90° - 70°) \times 1.5 = 20 \times 1.5 = 30$!

SOLVED EXAMPLE 10 [Board Standard]

Question: Find the central angles for a pie chart of 720 students' mode of transport: Bus (300), Cycle (180), Walk (120), Car (120).

Given & Required: Total $= 720 \implies 360/720 = 0.5°$ per student. | **Concept & Formula:** Multiply by $0.5°$.

Detailed Solution: Bus: $150°$, Cycle: $90°$, Walk: $60°$, Car: $60°$. Sum $= 360°$. **Final Answer:** $150°, 90°, 60°, 60°$

Fastest Alternative / Shortcut: Half of each student count!

SOLVED EXAMPLE 11 [Board Standard]

Question: A die is thrown. Find the probability of getting (i) a prime number, (ii) a number not prime, (iii) a multiple of 3.

Given & Required: $S = \{1, 2, 3, 4, 5, 6\}$. | **Concept & Formula:** Primes $= \{2, 3, 5\}$.

Detailed Solution: (i) $\frac{3}{6} = \frac{1}{2}$; (ii) $\{1, 4, 6\} \implies \frac{3}{6} = \frac{1}{2}$; (iii) $\{3, 6\} \implies \frac{2}{6} = \frac{1}{3}$. **Final Answer:** $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{3}$

Fastest Alternative / Shortcut: 1 is non-prime!

SOLVED EXAMPLE 12 [Board Standard]

Question: One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting (i) an Ace, (ii) a red Queen, (iii) a black card.

Given & Required: $n(S) = 52$. | **Concept & Formula:** Standard deck counts.

Detailed Solution: (i) $\frac{4}{52} = \frac{1}{13}$; (ii) $\frac{2}{52} = \frac{1}{26}$; (iii) $\frac{26}{52} = \frac{1}{2}$. **Final Answer:** $\frac{1}{13}$, $\frac{1}{26}$, $\frac{1}{2}$

Fastest Alternative / Shortcut: 2 red Queens (Hearts, Diamonds).

SOLVED EXAMPLE 13 [Board Standard]

Question: In a survey of 360 people on favourite season: Summer (90), Rainy (120), Winter (150). Find the central angles and percentage of Winter.

Given & Required: Total $= 360$. | **Concept & Formula:** 1 person $= 1^\circ$!

Detailed Solution: Central angles: Summer 90°, Rainy 120°, Winter 150°. Winter $\% = \frac{150}{360} \times 100 = 41.67\%$. **Final Answer:** $90^\circ, 120^\circ, 150^\circ$; 41.67%

Fastest Alternative / Shortcut: Total 360 means value = angle!

SOLVED EXAMPLE 14 [Challenge]

Question: In a pie chart, the expenditure on Food is 144° and on Rent is 90°. If Rent is Rs. 4500, find the expenditure on Food and the total income.

Given & Required: $90^\circ \leftrightarrow 4500$. | **Concept & Formula:** Ratio Value/$\theta = 50$ Rs/deg.

Detailed Solution: $1^\circ = \frac{4500}{90} =$ Rs. 50. Food $= 144 \times 50 =$ Rs. 7200. Total $= 360 \times 50 =$ Rs. $18,000$. **Final Answer: Food Rs.** 7200**, Total Rs.** $18,000$

Fastest Alternative / Shortcut: Find Rs. per degree (50) first!

SOLVED EXAMPLE 15 [Challenge]

Question: A letter is chosen at random from the word MATHEMATICS. Find the probability that it is a vowel.

Given & Required: 11 letters: A, E, A, I are 4 vowels. | **Concept & Formula:** $n(E)/n(S)$.

Detailed Solution: $P(\text{vowel}) = \frac{4}{11}$. **Final Answer:** $\frac{4}{11}$

Fastest Alternative / Shortcut: Count all 11 letters including repeats.

SOLVED EXAMPLE 16 [Challenge]

Question: If the probability of winning a game is 0.68, what is the probability of losing it?

Given & Required: $P(\text{win}) = 0.68$. | **Concept & Formula:** $1 - P(E)$.

Detailed Solution: $P(\text{lose}) = 1 - 0.68 = 0.32$. **Final Answer:** 0.32

Fastest Alternative / Shortcut: $1 - 0.68 = 0.32$.

G. Competency MCQ Mastery Section

Q1. Sum of all central angles in a pie chart is:

(A) 180° (B) 270° (C) 360° (D) 100°

Answer: (C) Full circle $= 360^\circ$. *Fast Method:* 360°. *Distractor Note:* Picking 100°.

Q2. Central angle for 25% in a pie chart is:

(A) 25° (B) 45° (C) 90° (D) 75°

Answer: (C) $25 \times 3.6 = 90°$. *Fast Method:* Quarter of $360°$. *Distractor Note:* Picking $25°$.

Q3. Probability of getting a prime number on throwing a die is:

(A) $1/6$ (B) $1/3$ (C) $1/2$ (D) $2/3$

Answer: (C) Primes $\{2, 3, 5\}$ give $3/6 = 1/2$. *Fast Method:* $3/6 = 1/2$. *Distractor Note:* Including 1 gives $2/3$.

Q4. Which cannot be the probability of an event?

(A) 0 (B) 1 (C) $2/3$ (D) $5/4$

Answer: (D) Probability cannot exceed 1. *Fast Method:* $5/4 > 1$. *Distractor Note:* Thinking 0 or 1 is invalid.

Q5. In a pie chart of 720 students, 40 students correspond to:

(A) $20°$ (B) $40°$ (C) $80°$ (D) $10°$

Answer: (A) $\frac{40}{720} \times 360 = 20°$. *Fast Method:* Half of 40. *Distractor Note:* Picking $40°$.

Q6. Probability of getting a 7 on throwing a standard die is:

(A) $1/6$ (B) $1/7$ (C) 0 (D) 1

Answer: (C) Impossible event. *Fast Method:* 0. *Distractor Note:* Picking $1/6$.

Q7. A letter is chosen from APPLE. Probability of getting P is:

(A) $1/5$ (B) $2/5$ (C) $1/4$ (D) $3/5$

Answer: (B) 2 P's out of 5 letters. *Fast Method:* $2/5$. *Distractor Note:* Counting unique letters (4).

Q8. If a pie chart sector is 72°, what percentage of the total is it?

(A) 10% (B) 15% (C) 20% (D) 25%

Answer: (C) $72/3.6 = 20\%$. *Fast Method:* $72/360 = 1/5 = 20\%$. *Distractor Note:* Picking 15%.

Q9. When a coin is tossed once, probability of getting a head is:

(A) 0 (B) $1/2$ (C) 1 (D) $1/4$

Answer: (B) $1/2$. *Fast Method:* $1/2$. *Distractor Note:* None.

Q10. Probability of drawing a face card (J, Q, K) from 52 cards is:

(A) $1/13$ (B) $3/13$ (C) $4/13$ (D) $1/4$

Answer: (B) $12/52 = 3/13$. *Fast Method:* 3 per 13. *Distractor Note:* Picking $4/13$ (including Ace).

Q11. If $P(E) = 0.09$, then $P(\text{not } E)$ is:

(A) 0.1 (B) 0.91 (C) 0.99 (D) 0.01

Answer: (B) $1 - 0.09 = 0.91$. *Fast Method:* $1 - 0.09$. *Distractor Note:* Writing 0.99.

Q12. In a pie chart, total angle of two equal sectors representing half the data is:

(A) $90°$ (B) $180°$ (C) $270°$ (D) $360°$

Answer: (B) Half of $360° = 180°$. *Fast Method:* $180°$. *Distractor Note:* Picking $90°$.

H. Special Exam & MCQ Strategies

- Compute the unit multiplier ($360°$/Total or Total/$360°$) first in every pie chart problem.
- Always verify that your pie chart central angles sum to exactly $360°$ before drawing with a protractor.
- For probability, list the favourable outcomes explicitly (remembering 1 is NOT prime) and reduce fractions to lowest terms.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Multiplying by 100 instead of 360° when computing pie chart angles. $\longrightarrow$ **Correction:** Central angle uses 360°: $\frac{\text{Value}}{\text{Total}} \times 360^\circ$.

[COMMON MISTAKE] **Error:** Counting 1 as a prime number on a die ($4/6$ instead of $3/6$). $\longrightarrow$ **Correction:** 1 is neither prime nor composite; primes on a die are $2, 3, 5$ ($3/6 = 1/2$).

[COMMON MISTAKE] **Error:** Writing probability > 1 or < 0. $\longrightarrow$ **Correction:** Probability is always between 0 and 1.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $\theta = \frac{\text{Value}}{\text{Total}} \times 360°$; Value $= \frac{\theta}{360°} \times$ Total; $P(E) = \frac{n(E)}{n(S)}$; $P(E') = 1 - P(E)$.

[QUICK REVISION] Key Theorems & Results: $\sum \theta_i = 360°$; $1\% = 3.6°$; $0 \le P(E) \le 1$.

[QUICK REVISION] Shortcut Summary: Unit multiplier $360/T$; Primes on die $= \{2, 3, 5\} = 1/2$; Face cards $= 12/52 = 3/13$.

[QUICK REVISION] High-Frequency Exam Patterns: Pie chart table, Reading values from pie chart, Die/slip probability, 52-card probability.

Last-Minute Revision Checklist:

- ☐ Sum of angles $= 360°$
- ☐ $1\% = 3.6°$
- ☐ 1 is NOT prime
- ☐ Reduce $P(E)$ fraction

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Central angle of 50% in a pie chart is: (A) $90°$ (B) $180°$ (C) $50°$ (D) $360°$
2. Probability of getting a 5 on a die is: (A) $5/6$ (B) $1/6$ (C) $1/5$ (D) 0
3. Probability of a sure event is: (A) 0 (B) 1 (C) $1/2$ (D) 100
4. In a pie chart of 36 items, 9 items equal: (A) $45°$ (B) $90°$ (C) $120°$ (D) $60°$

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. In a pie chart, $108°$ represents what percentage? (A) 25% (B) 30% (C) 36% (D) 20%
2. A bag has 3 red, 5 white, 2 black balls. P(not white) is: (A) $1/2$ (B) $3/10$ (C) $1/5$ (D) $2/5$
3. In a pie chart of Rs. 7200, $120°$ represents: (A) Rs. 1200 (B) Rs. 2400 (C) Rs. 3600 (D) Rs. 1800
4. Probability of getting a composite number on a die is: (A) $1/2$ (B) $1/3$ (C) $2/3$ (D) $1/6$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If $144°$ sector in a pie chart equals Rs. 3200, total amount is: (A) Rs. 6400 (B) Rs. 8000 (C) Rs. 9000 (D) Rs. 7200
2. A number is chosen from 1 to 20. P(multiple of 3 or 5) is: (A) $9/20$ (B) $1/2$ (C) $2/5$ (D) $11/20$
3. Two dice are thrown. P(doublet) is: (A) $1/6$ (B) $1/12$ (C) $1/36$ (D) $5/36$
4. If $P(A) : P(\text{not } A) = 3 : 5$, then $P(A)$ is: (A) $3/5$ (B) $3/8$ (C) $5/8$ (D) $2/5$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** Half of $360° = 180°$. **Q2 (B):** $1/6$. **Q3 (B):** 1. **Q4 (B):** $\frac{9}{36} \times 360 = 90°$.
Exam Std	**Q1 (B):** $108/3.6 = 30\%$. **Q2 (A):** $(3+2)/10 = 5/10 = 1/2$. **Q3 (B):** $\frac{120}{360} \times 7200 = 2400$. **Q4 (B):** $\{4, 6\} = 2/6 = 1/3$.
Challenge	**Q1 (B):** $3200 \times \frac{360}{144} = 3200 \times 2.5 = 8000$. **Q2 (A):** $\{3, 5, 6, 9, 10, 12, 15, 18, 20\} = 9/20$. **Q3 (A):** 6 doublets out of $36 = 1/6$. **Q4 (B):** $3/(3+5) = 3/8$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Pie charts (central angles & protractor drawing). Day 2: Outcomes & probability.
- **How to Introduce the Concept:** Ask how a 24-hour day (8 hr sleep, 6 hr school, 4 hr study, 6 hr play/other) maps onto a clock face ($360°$)!
- **Difficult Areas & Remediation:** Converting between degrees, percentages, and values, and remembering 1 is not prime.
- **Blackboard Teaching Examples:** Draw a 4-column Pie Chart table (Item, Value, Fraction, Central Angle) and verify sum $= 360°$.
- **Classroom Questions to Ask Students:** Why can't the probability of an event ever be 1.2 or -0.3?
- **Common Student Doubts & Answers:** Students ask about grouped histograms; clarify they are now in Class 9 Statistics.
- **Homework & Revision Strategy:** Assign 1 pie-chart construction, 1 pie-chart reading, and 5 probability problems.

[TEACHER'S TIP] The $1\% = 3.6°$ rule makes pie-chart calculations twice as fast.

UNIT I: NUMBER SYSTEM TYBA COACHING CENTRE

Chapter 5: Squares and Square Roots

Perfect Squares, Pythagorean Triplets, Prime Factorisation & Long-Division Square Roots (2026–27)

A. Chapter Introduction & Examination Relevance

Square numbers and square roots appear whenever we compute areas, diagonals, or distances. Chapter 5 teaches how to spot perfect squares instantly, generate Pythagorean triplets, and extract exact square roots of integers and decimals.

- **What You Will Learn:** Properties of square numbers (unit digits, zeros, odd-sum pattern), Pythagorean triplets $(2m, m^2-1, m^2+1)$, square roots by repeated subtraction, prime factorisation, and long-division method (including decimals).
- **Why This Chapter Matters:** Square roots are essential for Class 8 Mensuration (Ch. 9), Class 9 Heron's Formula & Circles, and Class 10 Quadratic Equations & Pythagoras Theorem.
- **Real-Life Applications:** Finding side length of a square field from its area, diagonal bracing in construction, and estimating distances.
- **Cross-Chapter Connections:** Directly prepares for Chapter 6 (Cubes), Chapter 9 (Mensuration), and Class 9 Chapter 1 & 10.
- **Exam & Foundation Relevance:** Unit I carries 20% weightage. Expect 1M unit-digit/triplet MCQs and 3M/5M prime-factorisation ('smallest number to multiply/divide') and long-division ('least number to add/subtract') problems.

B. Complete Concept Explanation & Derivations

1. Properties of Square Numbers & Unit Digits

A number ending in **2, 3, 7, or 8** is NEVER a perfect square! A perfect square ending in 0 must have an **even** number of trailing zeros. Between n^2 and $(n+1)^2$, there are exactly $2n$ non-square numbers. Every square n^2 equals the sum of the first n odd natural numbers: $1+3+5+\cdots+(2n-1)=n^2$.

[CONCEPT MASTER – 1. Properties of Square Numbers & Unit Digits] Unit Digit Rule: Squares ending in 1 come from $1, 9$; ending in 4 from $2, 8$; ending in 5 from 5; ending in 6 from $4, 6$; ending in 9 from $3, 7$.

2. Pythagorean Triplets

For any natural number $m > 1$, the triple $(2m,\ m^2-1,\ m^2+1)$ is a **Pythagorean triplet** because $(2m)^2+(m^2-1)^2=(m^2+1)^2$. Given one member of a triplet, test $2m=k$ first (when k is even), or $m^2 \pm 1 = k$.

[CONCEPT MASTER – 2. Pythagorean Triplets] If $m=3$, $(6,8,10)$ is a triplet. Watch out: some triplets like $(5,12,13)$ come from $m^2-1=12$? Wait: $2m=12 \implies m=6$ gives $(12,35,37)$, while $m^2-1=5$? No, $(5,12,13)$ is primitive (3-4-5 and 5-12-13 are worth memorizing!).

3. Square Roots by Prime Factorisation

Factorize the number into primes and pair identical primes; take one prime from each pair to get $\sqrt{N}$. To make a non-square number a perfect square by **multiplying** or **dividing**, identify the unpaired prime factor(s).

[CONCEPT MASTER – 3. Square Roots by Prime Factorisation] Rule: Multiply by the missing prime to complete the pair; divide by the extra unpaired prime to remove it!

4. Long-Division Square Root (Integers & Decimals)

Pair digits from right to left for the integer part, and from left to right (after the decimal point) for the decimal part. This method finds exact roots of large squares, decimal roots (like $\sqrt{42.25}=6.5$), and the **least number to subtract** (the remainder!) or **least number to add** (next square minus given number) to get a perfect square.

[CONCEPT MASTER – 4. Long-Division Square Root (Integers & Decimals)] Add vs Subtract: Remainder R is what you SUBTRACT; $(\text{quotient}+1)^2 - N$ is what you ADD!

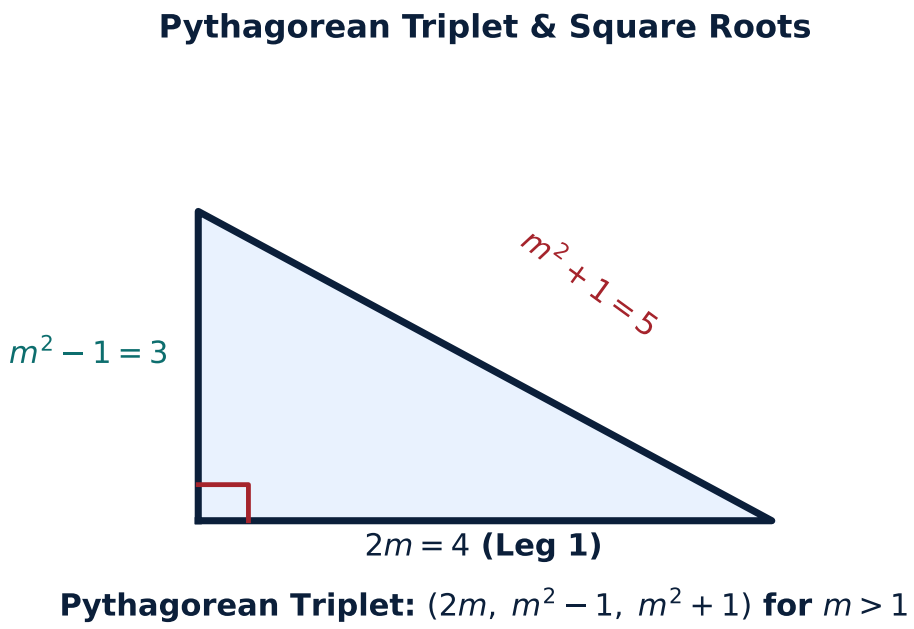


Figure 1: **Visual Model:** Pythagorean Triplet $(2m, m^2-1, m^2+1)$ right triangle linking square numbers to geometry.

C. Complete Formula Bank

[FORMULA BANK – 1. Non-Square Numbers Between n^2 and $(n+1)^2$] **Expression:** Count $= 2n$
Symbols: $n \in \mathbb{N}$.
Conditions & When to Use: Excludes n^2 and $(n+1)^2$.
Solved Mini-Example: Between 12^2 and 13^2, there are $2(12) = 24$ non-square numbers.
Common Mistake: Writing $2n+1$.

[FORMULA BANK – 2. Sum of First n Odd Numbers] **Expression:** $1+3+5+\cdots+(2n-1) = n^2$
Symbols: n: count of odd numbers.
Conditions & When to Use: Must start from 1 without skipping.
Solved Mini-Example: $1+3+5+7+9 = 5^2 = 25$.
Common Mistake: Using the last odd number as n instead of counting terms.

[FORMULA BANK – 3. Pythagorean Triplet Generator] **Expression:** $(2m,\ m^2-1,\ m^2+1)$ for $m > 1$
Symbols: $m \in \mathbb{N}, m > 1$.
Conditions & When to Use: Generates a valid right-triangle triple.
Solved Mini-Example: For $m = 4$: $(8, 15, 17)$.
Common Mistake: Setting $2m$ to an odd number.

[FORMULA BANK – 4. Square Ending in 5] **Expression:** $(a5)^2 = a(a+1) \mid 25$
Symbols: a: tens/leading digits.
Conditions & When to Use: Any integer ending in 5.
Solved Mini-Example: $75^2 = 7 \times 8 \mid 25 = 5625$.
Common Mistake: Adding 25 instead of appending 25.

[FORMULA BANK – 5. Digits in Square Root] **Expression:** If N has d digits, $\sqrt{N}$ has $\frac{d}{2}$ (even d) or $\frac{d+1}{2}$ (odd d) digits
Symbols: d: number of digits in N.
Conditions & When to Use: Equal to number of bars in long division.
Solved Mini-Example: $\sqrt{27225}$ (5 digits) has $\frac{5+1}{2} = 3$ digits.
Common Mistake: Dividing odd d by 2 without $+1$.

[FORMULA BANK – 6. Product & Quotient of Roots] **Expression:** $\sqrt{ab} = \sqrt{a}\sqrt{b}$; $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
Symbols: $a \geq 0, b > 0$.
Conditions & When to Use: Multiplication/division only (NOT $+, -$!).
Solved Mini-Example: $\sqrt{2.56} = \frac{\sqrt{256}}{10} = 1.6$.
Common Mistake: Writing $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Sainik/Navodaya/Olympiad Foundation problem structures.

Pattern 1: Unit Digit & Perfect Square Possibility

- **Identification Cue:** Which of $1057, 23453, 7928, 222222$ are obviously not perfect squares? | **Concept & Formula:** Ending in $2, 3, 7, 8$ or odd trailing zeros.
- **Standard Step-by-Step:** Check last digit and count trailing zeros.
- **Fastest Valid Method:** $2, 3, 7, 8$ or odd zeros $\implies$ NEVER a perfect square! | **Alternative:** Factorize.
- **Common Trap:** Thinking a number ending in $1, 4, 5, 6, 9$ MUST be a perfect square (e.g., 19 ends in 9!).
- **Practice Drill:** Ex. 1 & Ex. 2.

Pattern 2: Smallest Number to Multiply / Divide (Prime Factorisation)

- **Identification Cue:** Find the smallest whole number by which 252 should be multiplied (or divided) to get a perfect square. | **Concept & Formula:** Pair primes; spot lonely prime.
- **Standard Step-by-Step:** Write prime factorisation ($252 = 2^2 \times 3^2 \times 7$), identify unpaired factor (7), multiply/divide by 7.
- **Fastest Valid Method:** Group pairs with underlines while factoring. | **Alternative:** None.
- **Common Trap:** Forgetting to also find $\sqrt{\text{new number}}$ when the question asks for both!
- **Practice Drill:** Ex. 6 & Ex. 7.

Pattern 3: Long Division: Least Number to Subtract / Add

- **Identification Cue:** Find least number to subtract from 1989 (or add to 1750) to make it a perfect square. | **Concept & Formula:** Subtract $=$ Remainder R; Add $= (Q+1)^2 - N$.
- **Standard Step-by-Step:** Run long division on N to get quotient Q and remainder R.
- **Fastest Valid Method:** Subtract R; Add $(Q+1)^2 - N$! | **Alternative:** Test squares near $\sqrt{N}$.
- **Common Trap:** Subtracting R when 'added' is asked!
- **Practice Drill:** Ex. 10 & Ex. 11.

Pattern 4: Decimal Square Roots & Smallest Square Divisible by a, b, c

- **Identification Cue:** Find $\sqrt{42.25}$ or smallest square number divisible by $4, 9, 10$. | **Concept & Formula:** Long division with decimal bars, or $\text{LCM}(4, 9, 10) \times$ unpaired.
- **Standard Step-by-Step:** For LCM problem: find $\text{LCM} = 180 = 2^2 \cdot 3^2 \cdot 5$, multiply by 5 to get 900.
- **Fastest Valid Method:** Decimal root: ignore decimal, take root, halve decimal places ($42.25 \to 65 \to 6.5$). | **Alternative:** Long division from decimal point outward.
- **Common Trap:** Stopping at $\text{LCM} = 180$ without making it a square.
- **Practice Drill:** Ex. 8 & Ex. 12.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Instant Square of Numbers Ending in 5] **Core Idea & Why It Works:** For any number $a5$, multiply $a \times (a+1)$ and append 25.

Valid Domain: All numbers ending in 5 (and $5.5^2 = 30.25$!). | **Where It Fails:** Numbers not ending in 5.

Easy Example: $85^2 = 8 \times 9 \mid 25 = 7225$; $105^2 = 10 \times 11 \mid 25 = 11025$.

Exam-Level Example: Helps bracket $\sqrt{N}$ in 2 seconds!

Advanced Example: Reverse: $\sqrt{4225} = 65$ since $42 = 6 \times 7$!

[SMART SHORTCUT – 2. Add vs Subtract Long-Division Key] **Core Idea & Why It Works:** After long division of N gives quotient Q and remainder R: Least to SUBTRACT $= R$ (gives Q^2); Least to ADD $= (Q+1)^2 - N$ (gives $(Q+1)^2$).

Valid Domain: All non-square integers. | **Where It Fails:** If $R = 0$ (already a square).

Easy Example: $N = 1750$: $Q = 41$ ($41^2 = 1681, R = 69$). Next square is $42^2 = 1764 \implies$ Add $1764 - 1750 = 14$!

Exam-Level Example: Zero confusion in NCERT Ex. 5.4 Q4 & Q5.

Advanced Example: Difference between consecutive squares is $2Q + 1$.

[SMART SHORTCUT – 3. 2-Digit Square Estimation Trick] **Core Idea & Why It Works:** For a 3- or 4-digit perfect square, find the last digit from unit digit pairs and the tens digit from the first 1–2 digits.

Valid Domain: All perfect squares up to 10000. | **Where It Fails:** Non-perfect squares.

Easy Example: $\sqrt{5776}$: ends in $6 \to 4/6$; 57 is between $7^2 = 49$ and $8^2 = 64 \to 7$; $7 \times 8 = 56 < 57 \implies$ larger $6 \implies 76$!

Exam-Level Example: Instant MCQ solver!

Advanced Example: Verifies long-division answers in 3 seconds.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: What will be the unit digit of the squares of: (i) 81, (ii) 272, (iii) 799, (iv) 3853?

Given & Required: Four numbers. | **Concept & Formula:** Square unit digit only.

Detailed Solution: (i) $1^2 \to 1$; (ii) $2^2 \to 4$; (iii) $9^2 \to 1$; (iv) $3^2 \to 9$. **Final Answer:** $1, 4, 1, 9$

Fastest Alternative / Shortcut: Square the last digit only.

SOLVED EXAMPLE 2 [Beginner]

Question: How many numbers lie between squares of 12 and 13?

Given & Required: $n = 12$. | **Concept & Formula:** $2n$.

Detailed Solution: $2 \times 12 = 24$ non-square numbers. **Final Answer:** 24

Fastest Alternative / Shortcut: $2n$.

SOLVED EXAMPLE 3 [Beginner]

Question: Without adding, find the sum $1 + 3 + 5 + 7 + 9 + 11 + 13$.

Given & Required: 7 odd numbers from 1. | **Concept & Formula:** n^2.

Detailed Solution: $n = 7 \implies 7^2 = 49$. **Final Answer:** 49

Fastest Alternative / Shortcut: Count terms $= 7$, square it.

SOLVED EXAMPLE 4 [Beginner]

Question: Write a Pythagorean triplet whose one member is 6.

Given & Required: $2m = 6$. | **Concept & Formula:** $(2m, m^2 - 1, m^2 + 1)$.

Detailed Solution: $m = 3 \implies (6,\ 3^2 - 1,\ 3^2 + 1) = (6, 8, 10)$. **Final Answer:** $(6, 8, 10)$

Fastest Alternative / Shortcut: Set $2m = 6$.

SOLVED EXAMPLE 5 [Intermediate]

Question: Find the square root of 7744 by prime factorisation.

Given & Required: 7744. | **Concept & Formula:** Pair prime factors.

Detailed Solution: $7744 = 2^6 \times 11^2 = (2^3 \times 11)^2$. Thus $\sqrt{7744} = 8 \times 11 = 88$. **Final Answer:** 88

Fastest Alternative / Shortcut: Check $88^2 = 7744$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Find the smallest whole number by which 252 should be multiplied to get a perfect square, and find the square root of the square number so obtained.

Given & Required: $252 = 2^2 \times 3^2 \times 7$. | **Concept & Formula:** Unpaired prime 7.

Detailed Solution: Multiply by 7: $252 \times 7 = 1764$. $\sqrt{1764} = 2 \times 3 \times 7 = 42$. **Final Answer: Multiply by 7;** $\sqrt{1764} = 42$

Fastest Alternative / Shortcut: Lonely prime is 7.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find the smallest whole number by which 2925 should be divided to get a perfect square.

Given & Required: $2925 = 3^2 \times 5^2 \times 13$. | **Concept & Formula:** Unpaired prime 13.

Detailed Solution: Divide by 13 to get 225, whose square root is $3 \times 5 = 15$. **Final Answer: Divide by** 13**;** $\sqrt{225} = 15$

Fastest Alternative / Shortcut: Remove lonely 13.

SOLVED EXAMPLE 8 [Intermediate]

Question: Find the square root of 42.25 by long division.

Given & Required: 42.25. | **Concept & Formula:** Pair from decimal point.

Detailed Solution: $6^2 = 36$, remainder 6; bring down .25 $\rightarrow$ 625; double $6 \rightarrow 12$, $125 \times 5 = 625$. Root $= 6.5$. **Final Answer:** 6.5

Fastest Alternative / Shortcut: $6 \times 7 = 42 \implies 6.5^2 = 42.25$!

SOLVED EXAMPLE 9 [Board Standard]

Question: Find the smallest square number that is divisible by each of the numbers 4, 9, and 10.

Given & Required: 4, 9, 10. | **Concept & Formula:** LCM + complete pairs.

Detailed Solution: $\text{LCM}(4, 9, 10) = 180 = 2^2 \times 3^2 \times 5$. Factor 5 is unpaired, so multiply by 5: $180 \times 5 = 900$. **Final Answer:** 900

Fastest Alternative / Shortcut: $900 = 30^2$ is divisible by 4, 9, 10.

SOLVED EXAMPLE 10 [Board Standard]

Question: Find the least number which must be subtracted from 1989 so as to get a perfect square. Also find the square root of the perfect square.

Given & Required: $N = 1989$. | **Concept & Formula:** Long division remainder R.

Detailed Solution: Long division of 1989: $44^2 = 1936$, remainder $R = 1989 - 1936 = 53$. Subtract 53 to get 1936 ($\sqrt{1936} = 44$). **Final Answer: Subtract** 53**;** $\sqrt{1936} = 44$

Fastest Alternative / Shortcut: $44^2 = 1936, 45^2 = 2025$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Find the least number which must be added to 1750 so as to get a perfect square.

Given & Required: $N = 1750$. | **Concept & Formula:** $(Q+1)^2 - N$.

Detailed Solution: Long division gives $Q = 41$ ($41^2 = 1681 < 1750$). Next square is $42^2 = 1764$. Least number to add $= 1764 - 1750 = 14$. **Final Answer:** **Add** 14; $\sqrt{1764} = 42$

Fastest Alternative / Shortcut: $(41+1)^2 - 1750 = 14$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Find the greatest 4-digit number which is a perfect square.

Given & Required: Greatest 4-digit $= 9999$. | **Concept & Formula:** Subtract remainder from 9999.

Detailed Solution: Long division of 9999: $Q = 99$, remainder $R = 198$. Greatest 4-digit square $= 9999 - 198 = 9801$ (99^2). **Final Answer:** 9801

Fastest Alternative / Shortcut: $99^2 = 9801$ directly!

SOLVED EXAMPLE 13 [Board Standard]

Question: A gardener has 1000 plants. He wants to plant these so that the number of rows and columns remain same. Find the minimum number of plants he needs more.

Given & Required: $N = 1000$. | **Concept & Formula:** Add to next square.

Detailed Solution: $31^2 = 961 < 1000 < 32^2 = 1024$. Needs $1024 - 1000 = 24$ more plants. **Final Answer:** 24 **plants**

Fastest Alternative / Shortcut: $32^2 = 1024$.

SOLVED EXAMPLE 14 [Challenge]

Question: Find the least 4-digit number which is a perfect square.

Given & Required: Least 4-digit $= 1000$. | **Concept & Formula:** Next square above 1000.

Detailed Solution: $31^2 = 961$, so $32^2 = 1024$ is the least 4-digit perfect square. **Final Answer:** 1024

Fastest Alternative / Shortcut: $32^2 = 2^{10} = 1024$.

SOLVED EXAMPLE 15 [Challenge]

Question: In a right triangle ABC ($\angle B = 90°$), if $AB = 6$ cm and $BC = 8$ cm, find AC; if $AC = 13$ cm and $BC = 5$ cm, find AB.

Given & Required: Right triangles. | **Concept & Formula:** $AC^2 = AB^2 + BC^2$.

Detailed Solution: (i) $AC = \sqrt{36+64} = 10$ cm; (ii) $AB = \sqrt{169-25} = 12$ cm. **Final Answer:** 10 **cm and** 12 **cm**

Fastest Alternative / Shortcut: 6-8-10 and 5-12-13 triples.

SOLVED EXAMPLE 16 [Challenge]

Question: Evaluate $\sqrt{248 + \sqrt{52 + \sqrt{144}}}$.

Given & Required: Nested square roots. | **Concept & Formula:** Evaluate innermost first.

Detailed Solution: $\sqrt{144} = 12 \to \sqrt{52+12} = \sqrt{64} = 8 \to \sqrt{248+8} = \sqrt{256} = 16$. **Final Answer:** 16

Fastest Alternative / Shortcut: Work inside-out: $12 \to 8 \to 16$.

G. Competency MCQ Mastery Section

Q1. Which of the following CANNOT be the unit digit of a perfect square?

(A) 1 (B) 4 (C) 7 (D) 9

Answer: (C) Squares never end in $2, 3, 7, 8$. *Fast Method:* 2-3-7-8 rule. *Distractor Note:* None.

Q2. How many natural numbers lie between 9^2 and 10^2?

(A) 17 (B) 18 (C) 19 (D) 20

Answer: (B) $2n = 2(9) = 18$. *Fast Method:* $2n$. *Distractor Note:* Writing $2n + 1 = 19$.

Q3. Square root of 0.0009 is:

(A) 0.3 (B) 0.03 (C) 0.003 (D) 0.09

Answer: (B) 4 decimal places halve to 2: 0.03. *Fast Method:* Halve decimal places. *Distractor Note:* Picking 0.003.

Q4. Sum of first 10 odd natural numbers is:

(A) 50 (B) 100 (C) 110 (D) 81

Answer: (B) $10^2 = 100$. *Fast Method:* n^2. *Distractor Note:* Multiplying $10 \times 2 = 20$.

Q5. Which is a Pythagorean triplet?

(A) $(2, 3, 4)$ (B) $(5, 12, 13)$ (C) $(4, 5, 6)$ (D) $(6, 7, 8)$

Answer: (B) $5^2 + 12^2 = 25 + 144 = 169 = 13^2$. *Fast Method:* 5-12-13. *Distractor Note:* Picking consecutive numbers.

Q6. Number of digits in the square root of 390625 (6 digits) is:

(A) 2 (B) 3 (C) 4 (D) 6

Answer: (B) $6/2 = 3$ digits. *Fast Method:* $d/2$. *Distractor Note:* Picking 4.

Q7. Value of 65^2 is:

(A) 4225 (B) 3625 (C) 4025 (D) 4925

Answer: (A) $6 \times 7 \mid 25 = 4225$. *Fast Method:* $a(a+1)|25$. *Distractor Note:* Doing $6^2|25 = 3625$.

Q8. Smallest number to multiply 180 to get a perfect square is:

(A) 2 (B) 3 (C) 5 (D) 6

Answer: (C) $180 = 2^2 \cdot 3^2 \cdot 5 \implies$ multiply by 5. *Fast Method:* Lonely factor 5. *Distractor Note:* Multiplying by 2.

Q9. If a square has area 2304 m^2, its side is:

(A) 42 m (B) 48 m (C) 52 m (D) 58 m

Answer: (B) $48^2 = 2304$. *Fast Method:* Between 40^2 and 50^2, closer to 50, ends in 8. *Distractor Note:* Picking 42 ($42^2 = 1764$).

Q10. Least number to subtract from 50 to get a perfect square is:

(A) 1 (B) 14 (C) 2 (D) 7

Answer: (A) $50 - 1 = 49 = 7^2$. *Fast Method:* Nearest square below 50 is 49. *Distractor Note:* Picking 14 (that's to ADD for 64).

Q11. Value of $\sqrt{169} - \sqrt{144}$ is:

(A) 1 (B) 5 (C) 25 (D) $\sqrt{25}$

Answer: (A) $13 - 12 = 1$. *Fast Method:* Never subtract inside roots! *Distractor Note:* Writing $\sqrt{169 - 144} = 5$!

Q12. A perfect square can NEVER have:

(A) 2 trailing zeros (B) 3 trailing zeros (C) 4 trailing zeros (D) 6 trailing zeros

Answer: (B) Trailing zeros must be even. *Fast Method:* Odd zeros impossible. *Distractor Note:* Picking 2.

H. Special Exam & MCQ Strategies

- Memorize squares from 1^2 to 30^2—it cuts Class 8, 9, and 10 calculation time in half.
- Always distinguish 'least to subtract' (remainder R) from 'least to add' (next square $(Q+1)^2 - N$).
- When pairing decimal digits for long division, start at the decimal point and go LEFT for integer part and RIGHT for decimal part.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Computing $\sqrt{169-144} = 13-12 = 1$ or $\sqrt{169}-\sqrt{144} = \sqrt{25} = 5$. $\longrightarrow$ **Correction:** Square roots do NOT distribute over $+$ or $-$! Evaluate each root separately first.

[COMMON MISTAKE] **Error:** Subtracting remainder R when asked 'least number to be ADDED'. $\longrightarrow$ **Correction:** For ADDED, compute $(Q+1)^2 - N$.

[COMMON MISTAKE] **Error:** Writing $2n+1$ for non-squares between n^2 and $(n+1)^2$. $\longrightarrow$ **Correction:** The exact count is $2n$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: Non-squares between $n^2, (n+1)^2 = 2n$; $\sum_{k=1}^{n}(2k-1) = n^2$; Triplet: $(2m, m^2-1, m^2+1)$.

[QUICK REVISION] Key Theorems & Results: Never ends in $2, 3, 7, 8$ or odd zeros; Digits in $\sqrt{N}$: $d/2$ or $(d+1)/2$.

[QUICK REVISION] Shortcut Summary: $(a5)^2 = a(a+1)|25$; Subtract $= R$; Add $= (Q+1)^2 - N$; 2-digit root estimation.

[QUICK REVISION] High-Frequency Exam Patterns: Unit digits, Multiply/divide to make square, Long-division add/subtract, LCM square divisible by a, b, c.

Last-Minute Revision Checklist:

- ☐ 2-3-7-8 impossibility check
- ☐ Pair primes for multiply/divide
- ☐ Subtract R vs Add $(Q+1)^2 - N$
- ☐ Halve decimal places in $\sqrt{\ }$

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Square of 19 is: (A) 289 (B) 361 (C) 324 (D) 441
2. Square root of 625 is: (A) 15 (B) 25 (C) 35 (D) 45
3. Which cannot be a perfect square? (A) 1024 (B) 1296 (C) 2028 (D) 2401
4. Non-square numbers between 15^2 and 16^2: (A) 15 (B) 30 (C) 31 (D) 29

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Smallest number to divide 392 to make it a perfect square: (A) 2 (B) 7 (C) 14 (D) 4
2. Pythagorean triplet with smallest member 8 is: (A) $(8, 15, 17)$ (B) $(8, 10, 12)$ (C) $(8, 63, 65)$ (D) Both A and C
3. Least number to add to 525 to get a perfect square: (A) 4 (B) 41 (C) 14 (D) 5
4. Value of $\sqrt{12.25}$ is: (A) 3.5 (B) 4.5 (C) 2.5 (D) 3.25

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Smallest square number divisible by $8, 15, 20$ is: (A) 120 (B) 900 (C) 3600 (D) 1800
2. If $\sqrt{1 + \frac{x}{144}} = \frac{13}{12}$, then x is: (A) 1 (B) 13 (C) 25 (D) 27
3. Value of $\sqrt{10 + \sqrt{25 + \sqrt{108 + \sqrt{154 + \sqrt{225}}}}}$ is: (A) 4 (B) 6 (C) 8 (D) 10
4. If 99^2 is subtracted from 100^2, result is: (A) 1 (B) 199 (C) 99 (D) 200

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $19^2 = 361$. **Q2 (B):** $2 \times 3 \vert 25 = 625 \implies 25$. **Q3 (C):** Ends in 8. **Q4 (B):** $2(15) = 30$.
Exam Std	**Q1 (A):** $392 = 2 \times 14^2 \implies$ divide by 2. **Q2 (A):** $2m = 8 \implies m = 4 \implies (8, 15, 17)$. **Q3 (A):** $23^2 = 529 \implies 529 - 525 = 4$. **Q4 (A):** $3 \times 4 = 12 \implies 3.5$.
Challenge	**Q1 (C):** $\text{LCM} = 120 = 2^3 \cdot 3 \cdot 5 \implies \times 30 = 3600$. **Q2 (C):** $1 + \frac{x}{144} = \frac{169}{144} \implies x = 25$. **Q3 (A):** $15 \to 13 \to 11 \to 6 \to 4$. **Q4 (B):** $100 + 99 = 199$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Square properties & triplets. Day 2: Prime factorisation roots. Day 3: Long division & decimals.
- **How to Introduce the Concept:** Show $1 = 1^2$, $1 + 3 = 2^2$, $1 + 3 + 5 = 3^2$ using square grids so students see why odd numbers build squares!
- **Difficult Areas & Remediation:** Long-division bar pairing for decimals and 'least to add' vs 'least to subtract'.
- **Blackboard Teaching Examples:** Walk through $\sqrt{1989}$ and $\sqrt{42.25}$ step by step with vertical bar dividers.
- **Classroom Questions to Ask Students:** Why can't a number ending in $2, 3, 7,$ or 8 ever be a perfect square?
- **Common Student Doubts & Answers:** Students ask why trailing zeros must be even; show $(10)^2 = 100$ doubles the zero count.
- **Homework & Revision Strategy:** Assign 3 prime-factorisation problems, 3 long-division problems, and 2 LCM-square problems.

[TEACHER'S TIP] Have students memorize 1^2–25^2—it pays off in every math class through Class 12.

UNIT I: NUMBER SYSTEM TYBA COACHING CENTRE

Chapter 6: Cubes and Cube Roots

Perfect Cubes, Hardy–Ramanujan Numbers, Unit Digits & Prime-Factor Cube Roots (2026–27)

A. Chapter Introduction & Examination Relevance

Just as squares measure 2D area, cubes measure 3D volume. Chapter 6 explores cube patterns, the famous Hardy–Ramanujan number 1729, and extracting cube roots by prime factorisation and 3-digit estimation.

- **What You Will Learn:** Cubes of numbers 1–10, unit digits of cubes, consecutive odd-number cube sums, Hardy–Ramanujan numbers, prime factorisation (triplets of primes) for cube roots, and smallest multiply/divide to form a perfect cube.
- **Why This Chapter Matters:** Cube roots are used directly in Class 8–10 Mensuration (finding radius/side from sphere/cube volume) and Class 11–12 algebra.
- **Real-Life Applications:** Packaging cube-box dimensions from volume, spherical tank radius calculations, and number theory.
- **Cross-Chapter Connections:** Extends Chapter 5 (pairs of primes) to triplets of primes, supporting Chapter 9 (Cube volume) and Class 9 Chapter 11 (Sphere volume).
- **Exam & Foundation Relevance:** Unit I carries 20% weightage. Expect 1M unit-digit/cube-root estimation MCQs and 2M/3M prime-factorisation ('smallest multiply/divide') and cuboid-stacking problems.

B. Complete Concept Explanation & Derivations

1. Perfect Cubes & Consecutive Odd Sums

A **perfect cube** is $n^3 = n \times n \times n$. Unlike squares, cubes CAN end in ANY digit 0–9, and a perfect cube ending in 0 must have a **multiple of** 3 trailing zeros ($3, 6, 9, \ldots$). Also, $1^3 = 1$, $2^3 = 3 + 5$, $3^3 = 7 + 9 + 11$, $4^3 = 13 + 15 + 17 + 19$: the n-th cube is the sum of n consecutive odd numbers!

[CONCEPT MASTER – 1. Perfect Cubes & Consecutive Odd Sums] Hardy–Ramanujan Number 1729: Smallest number expressible as the sum of two cubes in two different ways: $1729 = 1^3 + 12^3 = 9^3 + 10^3$!

2. Unit Digits of Cubes ($2 \leftrightarrow 8$ and $3 \leftrightarrow 7$)

For $0, 1, 4, 5, 6, 9$, the cube ends in the SAME digit! Only four digits swap: $2^3 \to 8$ and $8^3 \to 2$; $3^3 \to 7$ and $7^3 \to 3$. Thus every digit 0–9 maps uniquely—making cube-root unit digits 100% unambiguous!

[CONCEPT MASTER – 2. Unit Digits of Cubes ($2 \leftrightarrow 8$ and $3 \leftrightarrow 7$)] Negative Cubes: $(-n)^3 = -(n^3)$ and $\sqrt[3]{-N} = -\sqrt[3]{N}$. Unlike square roots, cube roots of negative numbers ARE real!

3. Cube Roots by Prime Factorisation (Triplets)

To find $\sqrt[3]{N}$, factorize N into primes and group identical primes into **triplets** (groups of 3). Take one prime from each triplet and multiply. To make a number a perfect cube by multiplying or dividing, complete incomplete triplets (multiply) or remove extra primes (divide).

[CONCEPT MASTER – 3. Cube Roots by Prime Factorisation (Triplets)] Warning: For 2×2, you need ONE more 2 ($\times 2$) to complete a triplet; for a single 2, you need TWO more 2's ($\times 4$) to complete the triplet!

4. Instant Estimation of Cube Roots (Up to 6 Digits)

Split a 4- to 6-digit perfect cube into two groups from the right: the last 3 digits give the **unit digit** of $\sqrt[3]{N}$ (via $2 \leftrightarrow 8, 3 \leftrightarrow 7$), and the leading 1–3 digits give the **tens digit** (largest cube $\leq$ leading group).

[CONCEPT MASTER – 4. Instant Estimation of Cube Roots (Up to 6 Digits)] Example: $\sqrt[3]{17576} \to 17 \mid 576$. Ends in $6 \to 6$; $2^3 = 8 \leq 17 < 27 = 3^3 \to 2$. Answer: 26!

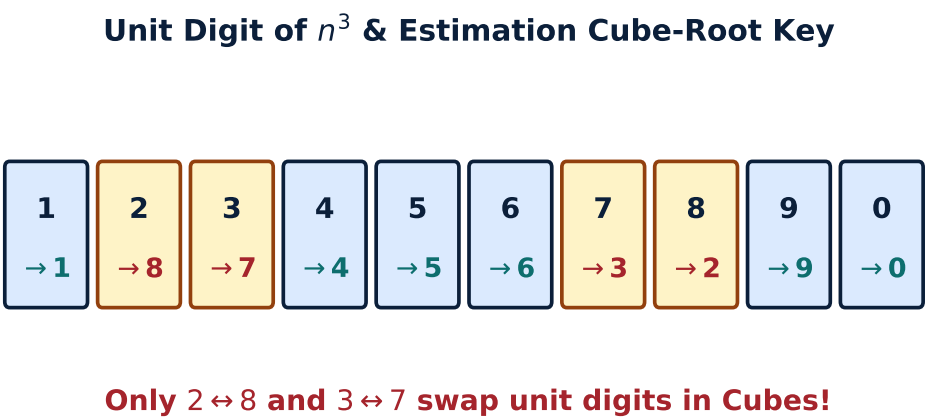


Figure 1: **Visual Model:** Unit-digit mapping for n^3 showing the unique $2 \leftrightarrow 8$ and $3 \leftrightarrow 7$ swaps.

C. Complete Formula Bank

[FORMULA BANK – 1. Cubes 1 to 10] **Expression:** $1, 8, 27, 64, 125, 216, 343, 512, 729, 1000$
Symbols: 1^3 to 10^3.
Conditions & When to Use: Memorize all 10!
Solved Mini-Example: $7^3 = 343, 8^3 = 512, 9^3 = 729$.
Common Mistake: Confusing $7^3 = 343$ with 49×3.

[FORMULA BANK – 2. Cube Root of Product/Quotient] **Expression:** $\sqrt[3]{ab} = \sqrt[3]{a} \cdot \sqrt[3]{b}$; $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$
Symbols: $a, b \in \mathbb{R}, b \neq 0$.
Conditions & When to Use: Works for negative numbers too!
Solved Mini-Example: $\sqrt[3]{-27 \times 125} = -3 \times 5 = -15$.
Common Mistake: Saying $\sqrt[3]{-8}$ is not real.

[FORMULA BANK – 3. Trailing Zeros in Cubes] **Expression:** $(k \times 10^m)^3 = k^3 \times 10^{3m}$
Symbols: $3m$ trailing zeros.
Conditions & When to Use: Multiple of 3 zeros.
Solved Mini-Example: $20^3 = 8000$ (3 zeros); $100^3 = 1000000$ (6 zeros).
Common Mistake: Writing $20^3 = 800$.

[FORMULA BANK – 4. Unit Digit Swap Key] **Expression:** $2 \leftrightarrow 8,\ 3 \leftrightarrow 7$; all others $(0, 1, 4, 5, 6, 9)$ unchanged
Symbols: Last digit of n vs n^3.
Conditions & When to Use: 1-to-1 bijection!
Solved Mini-Example: Cube root of 19683 ends in 7 (since $3 \leftrightarrow 7$).
Common Mistake: Forgetting $3 \leftrightarrow 7$ swap.

[FORMULA BANK – 5. Cuboids Needed to Form a Cube] **Expression:** $n = \frac{L^3}{l \cdot b \cdot h}$ where L completes prime triplets
Symbols: l, b, h: cuboid edges.
Conditions & When to Use: Used in Parikshit plasticine cuboid problem.
Solved Mini-Example: Cuboid $5 \times 2 \times 5 = 5^2 \cdot 2$ needs $5 \cdot 2^2 = 20$ cuboids!
Common Mistake: Using LCM instead of completing prime triplets.

[FORMULA BANK – 6. Decimal Cube Root] **Expression:** Divide decimal places by 3
Symbols: $3k$ decimal places $\to$ k decimal places.
Conditions & When to Use: Must be a multiple of 3 decimal places.
Solved Mini-Example: $\sqrt[3]{0.000027} = 0.03$ $(6/3 = 2$ places).
Common Mistake: Dividing decimal places by 2 instead of 3.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Sainik/Navodaya/Olympiad Foundation problem structures.

Pattern 1: Unit Digit & Perfect Cube Check

- **Identification Cue:** Is 243 or 10000 a perfect cube? What is the unit digit of $(53)^3$? | **Concept & Formula:** Triplets of primes & $2 \leftrightarrow 8, 3 \leftrightarrow 7$.
- **Standard Step-by-Step:** Factorize into primes; if all exponents are multiples of 3, it is a perfect cube.
- **Fastest Valid Method:** Check trailing zeros (must be $3, 6, 9$) first! | **Alternative:** Unit digit check.
- **Common Trap:** Thinking a number ending in 00 (2 zeros) is a perfect cube.
- **Practice Drill:** Ex. 1 & Ex. 2.

Pattern 2: Smallest Number to Multiply / Divide to Make a Cube

- **Identification Cue:** Find smallest number by which 243 (or 675, 704) must be multiplied/divided to obtain a perfect cube. | **Concept & Formula:** Complete or remove incomplete prime triplets.
- **Standard Step-by-Step:** Write prime factorisation in exponent form; for multiply, add needed factors to reach 3; for divide, divide by extra remainder factors.
- **Fastest Valid Method:** Group 3's; count how many more are needed to reach 3! | **Alternative:** None.
- **Common Trap:** Multiplying by 3 when 3^1 needs $3^2 = 9$ to reach 3^3!
- **Practice Drill:** Ex. 5 & Ex. 6.

Pattern 3: Parikshit's Cuboid-Stacking Problem

- **Identification Cue:** Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How many such cuboids will he need to form a cube? | **Concept & Formula:** Product $5 \times 2 \times 5 = 5^2 \cdot 2$ needs $5 \cdot 2^2 = 20$.
- **Standard Step-by-Step:** Multiply the given dimensions in prime form and multiply by missing primes to make every exponent a multiple of 3.
- **Fastest Valid Method:** Factor $l \cdot b \cdot h = 2^1 \cdot 5^2 \implies$ missing $2^2 \cdot 5^1 = 20$. | **Alternative:** Find cube side $\text{LCM}(5, 2, 5) = 10 \implies \frac{10^3}{50} = 20$.
- **Common Trap:** Answering 10 (the side length) instead of 20 (number of cuboids).
- **Practice Drill:** Ex. 9.

Pattern 4: Cube Root by Prime Factorisation & Estimation

- **Identification Cue:** Find $\sqrt[3]{13824}$ by prime factorisation or $\sqrt[3]{17576}$ by estimation. | **Concept & Formula:** Triplets vs 3-digit right-split.
- **Standard Step-by-Step:** Factorize and take 1 prime per triplet; for estimation, split 3 digits from right.
- **Fastest Valid Method:** Estimation takes 5 seconds and checks prime factorisation! | **Alternative:** Use negative sign outside for $\sqrt[3]{-N}$.
- **Common Trap:** Pairing in 2's instead of triplets of 3's.
- **Practice Drill:** Ex. 7 & Ex. 10.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. 5-Second Cube Root Estimation] **Core Idea & Why It Works:** Split 3 digits from right: Left | Right_3. Unit digit comes from Right_3 ($2 \leftrightarrow 8, 3 \leftrightarrow 7$); Tens digit is largest k with $k^3 \leq$ Left.
Valid Domain: All perfect cubes up to $1,000,000$. | **Where It Fails:** Non-perfect cubes.
Easy Example: $\sqrt[3]{32768} \to 32$ | 768. Ends in $8 \to 2$; $3^3 = 27 \leq 32 < 64 \to 3$. Root $= 32$!
Exam-Level Example: Solves NCERT Ex. 6.2 Q3 in 5 seconds!
Advanced Example: No ambiguity because unit digits are 1-to-1!

[SMART SHORTCUT – 2. Exponent Mod-3 Rule for Perfect Cubes] **Core Idea & Why It Works:** A number $N = p_1^{a_1} p_2^{a_2} \cdots$ is a perfect cube iff EVERY exponent a_i is divisible by 3.
Valid Domain: All integers. | **Where It Fails:** Nothing.
Easy Example: $2^6 \times 3^3 \times 5^3$ is a cube, with cube root $2^2 \times 3 \times 5 = 60$.
Exam-Level Example: Fastest way to handle multiply/divide questions.
Advanced Example: Divide each exponent by 3 to get $\sqrt[3]{N}$.

[SMART SHORTCUT – 3. Negative Cube Root Direct Pull-Out] **Core Idea & Why It Works:** $\sqrt[3]{-N} = -\sqrt[3]{N}$ ALWAYS.
Valid Domain: All negative numbers. | **Where It Fails:** Does NOT work for square roots!
Easy Example: $\sqrt[3]{-216} = -6$.
Exam-Level Example: Instant sign handling.
Advanced Example: $\sqrt[3]{-1000} = -10$.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Find the unit digit of the cube of: (i) 3331, (ii) 8888, (iii) 149, (iv) 1005.
Given & Required: Four numbers. | **Concept & Formula:** $2 \leftrightarrow 8, 3 \leftrightarrow 7$; others unchanged.
Detailed Solution: (i) 1; (ii) $8 \to 2$; (iii) 9; (iv) 5. **Final Answer:** $1, 2, 9, 5$
Fastest Alternative / Shortcut: Cube only the last digit.

SOLVED EXAMPLE 2 [Beginner]
Question: Find the cube of -7 and $\frac{3}{5}$.
Given & Required: -7 and $3/5$. | **Concept & Formula:** $(-n)^3 = -n^3$.
Detailed Solution: $(-7)^3 = -343$; $(\frac{3}{5})^3 = \frac{27}{125}$. **Final Answer:** -343 **and** $\frac{27}{125}$
Fastest Alternative / Shortcut: Odd power keeps minus.

SOLVED EXAMPLE 3 [Beginner]
Question: Is 216 a perfect cube? Find $\sqrt[3]{216}$.
Given & Required: 216. | **Concept & Formula:** $2^3 \times 3^3$.
Detailed Solution: $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 6^3$. Yes, $\sqrt[3]{216} = 6$. **Final Answer: Yes,** 6
Fastest Alternative / Shortcut: $6^3 = 216$.

SOLVED EXAMPLE 4 [Beginner]
Question: Find $\sqrt[3]{0.008}$.
Given & Required: $0.008 = 8/1000$. | **Concept & Formula:** Divide decimal places by 3.
Detailed Solution: $\sqrt[3]{\frac{8}{1000}} = \frac{2}{10} = 0.2$. **Final Answer:** 0.2
Fastest Alternative / Shortcut: 3 decimal places $\to$ 1 place.

SOLVED EXAMPLE 5 [Intermediate]

Question: Find the smallest number by which 243 must be multiplied to obtain a perfect cube.
Given & Required: $243 = 3^5$. | **Concept & Formula:** Complete triplet 3^6.
Detailed Solution: $243 = (3 \times 3 \times 3) \times 3 \times 3$. Needs one more 3 to make 3^6. Multiply by 3 to get 729 (9^3). **Final Answer:** 3
Fastest Alternative / Shortcut: $3^5 \times 3 = 3^6$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Find the smallest number by which 675 must be multiplied to obtain a perfect cube.
Given & Required: $675 = 3^3 \times 5^2$. | **Concept & Formula:** Missing factor for 5^2.
Detailed Solution: 5^2 needs one 5 to become 5^3. Multiply by 5 to get 3375 (15^3). **Final Answer:** 5
Fastest Alternative / Shortcut: Complete $5^2 \to 5^3$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find the smallest number by which 704 must be divided to obtain a perfect cube.
Given & Required: $704 = 2^6 \times 11$. | **Concept & Formula:** Extra factor 11.
Detailed Solution: $704 = (2^3)(2^3) \times 11$. Divide by 11 to get 64 (4^3). **Final Answer:** 11
Fastest Alternative / Shortcut: Remove lonely 11.

SOLVED EXAMPLE 8 [Intermediate]

Question: Find $\sqrt[3]{13824}$ by prime factorisation.
Given & Required: 13824. | **Concept & Formula:** Triplets of primes.
Detailed Solution: $13824 = 2^9 \times 3^3 \implies \sqrt[3]{13824} = 2^3 \times 3 = 24$. **Final Answer:** 24
Fastest Alternative / Shortcut: Est: $13|824 \to 2$ and $4 \to 24$!

SOLVED EXAMPLE 9 [Board Standard]

Question: Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How many such cuboids will he need to form a cube?
Given & Required: Volume of 1 cuboid $= 5 \times 2 \times 5$. | **Concept & Formula:** Complete triplets.
Detailed Solution: Volume $= 5^2 \times 2^1$. To make a cube, we need $5^1 \times 2^2 = 20$ such cuboids (cube of side 10 cm). **Final Answer:** 20 **cuboids**
Fastest Alternative / Shortcut: $10^3/50 = 20$.

SOLVED EXAMPLE 10 [Board Standard]

Question: Find the cube root of 17576 through estimation.
Given & Required: $17 \mid 576$. | **Concept & Formula:** 3-digit right split.
Detailed Solution: Group 1 (576) ends in $6 \implies$ unit digit $= 6$. Group 2 (17): $2^3 = 8 \leq 17 < 27 = 3^3 \implies$ tens digit $= 2$. Thus $\sqrt[3]{17576} = 26$. **Final Answer:** 26
Fastest Alternative / Shortcut: Check $26^3 = 17576$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Evaluate $\sqrt[3]{-2744}$.
Given & Required: -2744. | **Concept & Formula:** $-\sqrt[3]{2744}$.
Detailed Solution: $2744 = 2^3 \times 7^3 \implies \sqrt[3]{2744} = 14$. Thus $\sqrt[3]{-2744} = -14$. **Final Answer:** -14
Fastest Alternative / Shortcut: Est: $2|744 \to 1$ and $4 \to 14$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Express 7^3 as the sum of consecutive odd numbers.

Given & Required: $n = 7$. | **Concept & Formula:** 7 consecutive odd numbers.

Detailed Solution: Average is $7^2 = 49$. The 7 odd numbers centred at 49 are $43 + 45 + 47 + 49 + 51 + 53 + 55 = 343 = 7^3$.
Final Answer: $43 + 45 + 47 + 49 + 51 + 53 + 55$

Fastest Alternative / Shortcut: First odd $= n(n-1) + 1 = 43$!

SOLVED EXAMPLE 13 [Board Standard]

Question: Find the side of a cube whose volume is 4913 cm^3.

Given & Required: $V = a^3 = 4913$. | **Concept & Formula:** $a = \sqrt[3]{4913}$.

Detailed Solution: $4913 = 17^3 \implies a = 17$ cm. **Final Answer:** 17 **cm**

Fastest Alternative / Shortcut: Est: $4|913 \to 1$ and 7 $(3 \leftrightarrow 7) \to 17$!

SOLVED EXAMPLE 14 [Challenge]

Question: If the volume of a cube is 9261 cm^3, find its total surface area.

Given & Required: $a^3 = 9261$. | **Concept & Formula:** Find a, then $6a^2$.

Detailed Solution: $9261 = 3^3 \times 7^3 \implies a = 21$ cm. TSA $= 6(21)^2 = 6(441) = 2646$ cm^2. **Final Answer:** 2646 **cm**^{2}

Fastest Alternative / Shortcut: Est: $9|261 \to 21$.

SOLVED EXAMPLE 15 [Challenge]

Question: Evaluate $\sqrt[3]{27} + \sqrt[3]{0.008} + \sqrt[3]{0.064}$.

Given & Required: Three cube roots. | **Concept & Formula:** Decimal cube roots.

Detailed Solution: $3 + 0.2 + 0.4 = 3.6$. **Final Answer:** 3.6

Fastest Alternative / Shortcut: Add cleanly.

SOLVED EXAMPLE 16 [Challenge]

Question: What is the smallest Hardy–Ramanujan number, and write its two cube-sum forms?

Given & Required: Taxicab number. | **Concept & Formula:** $a^3 + b^3 = c^3 + d^3$.

Detailed Solution: $1729 = 1^3 + 12^3 = 9^3 + 10^3$. **Final Answer:** $1729 = 1^3 + 12^3 = 9^3 + 10^3$

Fastest Alternative / Shortcut: $1 + 1728 = 729 + 1000 = 1729$.

G. Competency MCQ Mastery Section

Q1. Unit digit of the cube of 47 is:

(A) 7 (B) 3 (C) 9 (D) 1

Answer: (B) $7^3 = 343 \to 3$. *Fast Method:* $3 \leftrightarrow 7$ swap. *Distractor Note:* Squaring $7^2 \to 9$.

Q2. Cube root of -125 is:

(A) 5 (B) -5 (C) -25 (D) Not real

Answer: (B) $(-5)^3 = -125$. *Fast Method:* -5. *Distractor Note:* Saying 'Not real'.

Q3. Which of the following is NOT a perfect cube?

(A) 216 (B) 1000 (C) 100 (D) 729

Answer: (C) 100 has 2 zeros (not a multiple of 3). *Fast Method:* 2 zeros $\neq$ cube. *Distractor Note:* None.

Q4. Smallest number to multiply 32 to make it a perfect cube is:

(A) 2 (B) 4 (C) 8 (D) 16

Answer: (A) $32 = 2^5 \implies \times 2 = 2^6 = 64$. *Fast Method:* $2^5 \to 2^6$. *Distractor Note:* Dividing by 4.

Q5. Value of $\sqrt[3]{8000}$ is:

(A) 20 (B) 200 (C) 40 (D) 80

Answer: (A) $20^3 = 8000$. *Fast Method:* 3 zeros → 1 zero. *Distractor Note:* Picking 200.

Q6. Smallest Hardy–Ramanujan number is:

(A) 1728 (B) 1729 (C) 4104 (D) 13832

Answer: (B) $1^3 + 12^3 = 9^3 + 10^3 = 1729$. *Fast Method:* 1729. *Distractor Note:* Picking 1728 (12^3).

Q7. If a cube has side $3x$, its volume becomes how many times?

(A) 3 (B) 9 (C) 27 (D) 81

Answer: (C) $3^3 = 27$ times. *Fast Method:* Volume $\propto a^3$. *Distractor Note:* Picking 9.

Q8. Value of $\sqrt[3]{0.000064}$ is:

(A) 0.4 (B) 0.04 (C) 0.004 (D) 0.08

Answer: (B) 6 decimal places / 3 = 2 places: 0.04. *Fast Method:* $6/3 = 2$ places. *Distractor Note:* Picking 0.004 or 0.08.

Q9. Sum of $1 + 3 + 5 + 7$? Wait: $7 + 9 + 11$ equals:

(A) 2^3 (B) 3^3 (C) 4^3 (D) 5^3

Answer: (B) $7 + 9 + 11 = 27 = 3^3$. *Fast Method:* 3rd group of odd numbers. *Distractor Note:* Counting 7.

Q10. If $m^3 = 343$, then m^2 is:

(A) 7 (B) 14 (C) 49 (D) 21

Answer: (C) $m = 7 \implies m^2 = 49$. *Fast Method:* Read question: asks m^2! *Distractor Note:* Picking 7.

Q11. By estimation, $\sqrt[3]{46656}$ is:

(A) 26 (B) 36 (C) 46 (D) 34

Answer: (B) $46|656 \to 3^3 = 27 \leq 46 < 64 \to 3$, ends in $6 \to 36$. *Fast Method:* 5-sec estimation. *Distractor Note:* Picking 46 ($4^3 = 64 > 46$).

Q12. Number of zeros in $(500)^3$ is:

(A) 3 (B) 5 (C) 6 (D) 9

Answer: (C) $2 \times 3 = 6$ zeros (125000000). *Fast Method:* $2 \times 3 = 6$. *Distractor Note:* Picking 3.

H. Special Exam & MCQ Strategies

- Memorize cubes 1^3 to 10^3 $(1, 8, 27, 64, 125, 216, 343, 512, 729, 1000)$—you need them for instant estimation.
- Always verify prime-factorisation cube roots using the 5-second estimation trick.
- In multiply/divide questions, check whether an unpaired prime needs ONE factor (from p^2) or TWO factors (from p^1).

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Multiplying p^1 by p instead of p^2 to make a perfect cube. ⟶ **Correction:** A triplet needs 3 identical primes, so p^1 needs p^2!

[COMMON MISTAKE] **Error:** Grouping primes in pairs of 2 instead of triplets of 3. ⟶ **Correction:** Square roots use pairs (2); Cube roots use triplets (3).

[COMMON MISTAKE] **Error:** Dividing decimal places by 2 in $\sqrt[3]{0.000027}$. ⟶ **Correction:** Divide decimal places by 3: $6/3 = 2$ places (0.03).

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $1^3..10^3$: $1, 8, 27, 64, 125, 216, 343, 512, 729, 1000$; $\sqrt[3]{-N} = -\sqrt[3]{N}$; Hardy–Ramanujan 1729.

[QUICK REVISION] Key Theorems & Results: Trailing zeros in cubes = multiple of 3; $2 \leftrightarrow 8, 3 \leftrightarrow 7$ unit digits.

[QUICK REVISION] Shortcut Summary: Left | Right-3 estimation; Exponent mod-3 check; Decimal places $\div 3$.

[QUICK REVISION] High-Frequency Exam Patterns: Unit digits, Multiply/divide to make cube, Parikshit cuboid stacking, Estimation cube roots.

Last-Minute Revision Checklist:

- ☐ Triplets of 3 primes
- ☐ $2 \leftrightarrow 8$ and $3 \leftrightarrow 7$
- ☐ Zeros multiple of 3
- ☐ Negative cube roots are real

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Cube of 9 is: (A) 81 (B) 243 (C) 729 (D) 512
2. Cube root of 1000 is: (A) 10 (B) 100 (C) 50 (D) 30
3. Unit digit of 23^3 is: (A) 3 (B) 7 (C) 9 (D) 1
4. $\sqrt[3]{-64}$ equals: (A) 4 (B) -4 (C) -8 (D) 8

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Smallest number to multiply 72 to get a perfect cube: (A) 2 (B) 3 (C) 6 (D) 9
2. Smallest number to divide 135 to get a perfect cube: (A) 3 (B) 5 (C) 9 (D) 15
3. By estimation, $\sqrt[3]{12167}$ is: (A) 23 (B) 27 (C) 33 (D) 17
4. Number of cuboids of sides $3, 3, 2$ cm to form a cube: (A) 6 (B) 12 (C) 18 (D) 36

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Value of $\sqrt[3]{-125 \times 512}$ is: (A) -40 (B) 40 (C) -20 (D) -80
2. If $\sqrt[3]{x} = 0.4$, then x is: (A) 0.16 (B) 0.064 (C) 0.64 (D) 0.016
3. Sum of cubes of first 4 natural numbers $1^3 + 2^3 + 3^3 + 4^3$ is: (A) 100 (B) 64 (C) 90 (D) 120
4. Volume of a cube is 13.824 m^3. Its side is: (A) 2.4 m (B) 2.6 m (C) 3.4 m (D) 1.4 m

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (C):** $9^3 = 729$. **Q2 (A):** $10^3 = 1000$. **Q3 (B):** $3^3 = 27 \to 7$. **Q4 (B):** $(-4)^3 = -64$.
Exam Std	**Q1 (B):** $72 = 2^3 \cdot 3^2 \implies \times 3 = 216$. **Q2 (B):** $135 = 3^3 \cdot 5 \implies \div 5 = 27$. **Q3 (A):** $12\|167 \to 2$ and 3 $(7 \to 3) \to 23$. **Q4 (B):** $3^2 \cdot 2 \to \times(3 \cdot 2^2) = 12$.
Challenge	**Q1 (A):** $-5 \times 8 = -40$. **Q2 (B):** $0.4^3 = 0.064$. **Q3 (A):** $1 + 8 + 27 + 64 = 100$ $(= (1 + 2 + 3 + 4)^2!)$. **Q4 (A):** $\sqrt[3]{13824}/10 = 2.4$ m.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Cube patterns, unit digits, 1729 story. Day 2: Prime-factor cube roots & 5-sec estimation.
- **How to Introduce the Concept:** Tell the story of Srinivasa Ramanujan instantly recognizing taxi number 1729 from his hospital bed!
- **Difficult Areas & Remediation:** Completing p^1 with p^2 (not p) and the $2 \leftrightarrow 8, 3 \leftrightarrow 7$ unit-digit swaps.
- **Blackboard Teaching Examples:** Write the 0–9 cube unit-digit chart and highlight the 4 swapping digits in gold.
- **Classroom Questions to Ask Students:** Why does $\sqrt[3]{-8}$ exist in real numbers while $\sqrt{-8}$ does not?
- **Common Student Doubts & Answers:** Students confuse pairs (squares) with triplets (cubes); drill both side-by-side.
- **Homework & Revision Strategy:** Assign 4 prime-factorisation cube roots, 2 multiply/divide problems, and 4 estimation problems.

[TEACHER'S TIP] The 5-second estimation trick builds huge student confidence.

UNIT III: RATIO & PROPORTION TYBA COACHING CENTRE

Chapter 7: Comparing Quantities

Discounts, Sales Tax / GST, Compound Interest (Annual & Half-Yearly) & Depreciation (2026–27)

A. Chapter Introduction & Examination Relevance

Every shopping bill, bank savings account, and vehicle depreciation involves percentages. Chapter 7 masters real-world commercial mathematics: Discounts, GST/Tax, and Compound Interest.

- **What You Will Learn:** Marked Price (MP), Discount & Discount%, Successive Discounts, Sales Tax / VAT / GST on Selling Price, Simple Interest vs Compound Interest, Annual vs Half-Yearly compounding, and Population Growth / Machine Depreciation.
- **Why This Chapter Matters:** Comparing Quantities carries 15% weightage in Class 8 and is one of the highest-frequency topics in Sainik/Navodaya/SSC/Banking aptitude tests.
- **Real-Life Applications:** Retail festival discounts, GST invoice billing, bank FDs, loan EMIs, and car/bike depreciation.
- **Cross-Chapter Connections:** Uses exponents from Chapter 10 and percentage ratios from earlier classes.
- **Exam & Foundation Relevance:** Expect 1M discount/GST MCQs and 3M/5M Compound Interest (annually vs half-yearly) and population/depreciation problems.

B. Complete Concept Explanation & Derivations

1. Marked Price, Discount & GST (Tax)

Discount is always calculated on the **Marked Price (MP)**: Discount $=$ MP $-$ SP, and Discount% $= \frac{\text{Discount}}{\text{MP}} \times 100$. Conversely, SP $=$ MP $\left(1 - \frac{d}{100}\right)$. **Sales Tax / VAT / GST** is ALWAYS added on top of the **Selling Price (SP)**: Bill Amount $=$ SP $\left(1 + \frac{\text{Tax\%}}{100}\right)$.

[CONCEPT MASTER – 1. Marked Price, Discount & GST (Tax)] Order Matters: Apply Discount FIRST on MP to get SP, then add GST on SP to get the final bill!

2. Compound Interest (Annual Compounding)

In Simple Interest, interest is earned only on the original Principal P. In **Compound Interest (CI)**, each year's interest is added to the principal to earn interest in the next period. For Principal P, Rate $R\%$ p.a., and n years: $A = P\left(1 + \frac{R}{100}\right)^n$ and CI $= A - P$.

[CONCEPT MASTER – 2. Compound Interest (Annual Compounding)] For $n = 1$ year (compounded annually), Simple Interest and Compound Interest are ALWAYS equal!

3. Half-Yearly Compounding & Fractional Time

When interest is compounded **half-yearly**, rate is halved ($R/2\%$) and time is doubled ($2n$ half-years): $A = P\left(1 + \frac{R}{200}\right)^{2n}$. For fractional years like $2\frac{1}{2}$ years compounded annually, use $A = P\left(1 + \frac{R}{100}\right)^2\left(1 + \frac{R/2}{100}\right)$.

[CONCEPT MASTER – 3. Half-Yearly Compounding & Fractional Time] Remember the Half-Yearly Mantra: **Half the Rate, Double the Time!**

4. Growth & Depreciation Applications

The CI formula models any percentage growth (population, bacteria) with $+$ ($P(1 + R/100)^n$) and any percentage decay/depreciation (machinery, car value) with $-$: $V = P\left(1 - \frac{R}{100}\right)^n$.

[CONCEPT MASTER – 4. Growth & Depreciation Applications] Rationalised 2026–27 Scope: Basic ratio-increase warmups were trimmed in 2026–27, keeping focus on Discounts, GST, CI, and Growth/Depreciation.

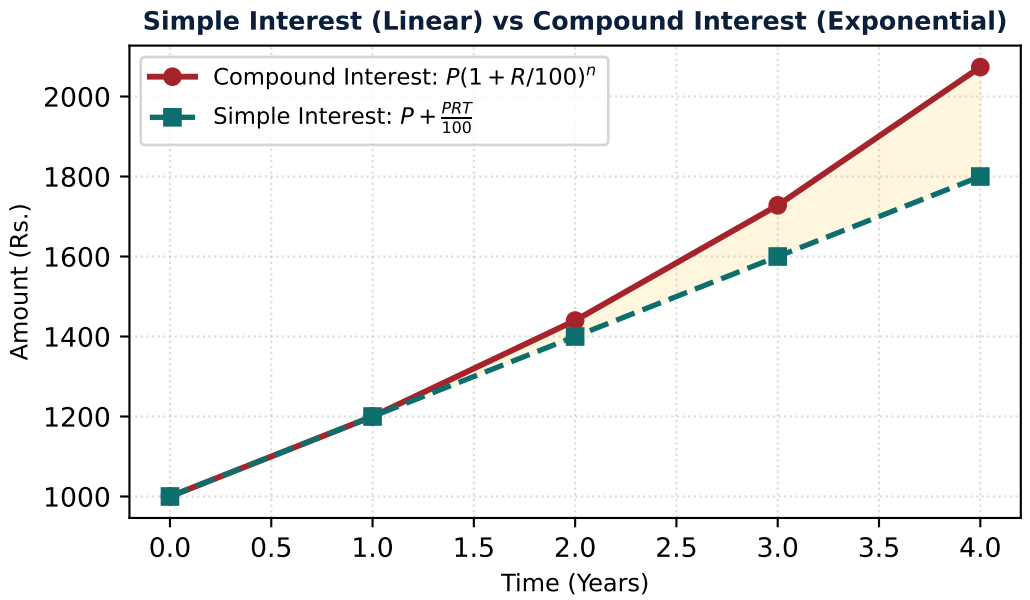


Figure 1: **Visual Model:** Exponential growth of Compound Interest $P(1+R/100)^n$ outpacing linear Simple Interest $P+PRT/100$.

C. Complete Formula Bank

[FORMULA BANK – 1. Discount & Selling Price] **Expression:** Discount $=$ MP $-$ SP; SP $=$ MP $\left(1-\frac{d}{100}\right)$
Symbols: MP: marked price, d: discount%.
Conditions & When to Use: Discount is always on MP.
Solved Mini-Example: MP Rs. 840, $d=15\% \implies$ SP $=840\times0.85=$ Rs. 714.
Common Mistake: Computing discount% on SP instead of MP.

[FORMULA BANK – 2. GST / Sales Tax Billing] **Expression:** Bill $=$ SP $\left(1+\frac{t}{100}\right)$
Symbols: t: tax/GST%.
Conditions & When to Use: Tax is added on SP (after discount).
Solved Mini-Example: SP Rs. 1000, GST $18\% \implies$ Bill $=1180$.
Common Mistake: Subtracting GST when finding original price instead of dividing by $(1+t/100)$!

[FORMULA BANK – 3. Compound Interest (Annually)] **Expression:** $A=P\left(1+\frac{R}{100}\right)^n$; CI $=A-P$
Symbols: P: principal, R: rate% p.a., n: years.
Conditions & When to Use: Annually compounded.
Solved Mini-Example: $P=10000, R=10\%, n=2 \implies A=10000(1.1)^2=12100$, CI $=2100$.
Common Mistake: Reporting Amount A when Compound Interest CI is asked!

[FORMULA BANK – 4. Half-Yearly Compounding] **Expression:** $A=P\left(1+\frac{R}{200}\right)^{2n}$
Symbols: $R/2$: half-yearly rate, $2n$: half-years.
Conditions & When to Use: Compounded every 6 months.
Solved Mini-Example: 10% p.a. for 1.5 yrs $\implies$ 5% for 3 periods.
Common Mistake: Halving R without doubling n.

[FORMULA BANK – 5. Depreciation Formula] **Expression:** $V = P\left(1 - \frac{R}{100}\right)^n$
Symbols: V: depreciated value after n years.
Conditions & When to Use: Value decreases by $R\%$ every year.
Solved Mini-Example: Scooter Rs. 42000 depreciates 8% for 1 yr: $42000(0.92) = 38640$.
Common Mistake: Using $+$ instead of $-$ for depreciation.

[FORMULA BANK – 6. 2-Year CI − SI Difference] **Expression:** $\mathrm{CI}_2 - \mathrm{SI}_2 = P\left(\frac{R}{100}\right)^2$
Symbols: P: principal, R: rate% p.a.
Conditions & When to Use: Exact for $n = 2$ years (annual).
Solved Mini-Example: $P = 10000, R = 10\% \implies \mathrm{CI} - \mathrm{SI} = 10000(0.01) = \text{Rs. } 100$.
Common Mistake: Applying this to $n = 3$ years.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Sainik/Navodaya/Olympiad Foundation problem structures.

Pattern 1: Discount% & Finding MP from SP

- **Identification Cue:** An item marked at Rs. 840 is sold for Rs. 714; or after 20% discount SP is Rs. 1600, find MP. | **Concept & Formula:** $\mathrm{SP} = \mathrm{MP}(1 - d/100)$.
- **Standard Step-by-Step:** To find MP from SP, divide SP by $(1 - d/100)$ (never add $d\%$ of SP!).
- **Fastest Valid Method:** $d = 20\% \implies 80\%$ of $\mathrm{MP} = 1600 \implies \mathrm{MP} = 2000$. | **Alternative:** Unitary method $(100 \to 80)$.
- **Common Trap:** Adding 20% of 1600 (320) to get 1920 (WRONG!).
- **Practice Drill:** Ex. 1 & Ex. 5.

Pattern 2: GST / VAT Forward & Reverse Price

- **Identification Cue:** I purchased a hair-dryer for Rs. 5400 including 8% VAT. Find price before VAT. | **Concept & Formula:** $\mathrm{SP}(1 + t/100) = \mathrm{Bill}$.
- **Standard Step-by-Step:** Set $\mathrm{SP} \times \frac{108}{100} = 5400 \implies \mathrm{SP} = 5400 \times \frac{100}{108} = 5000$.
- **Fastest Valid Method:** $108\% = 5400 \implies 1\% = 50 \implies 100\% = 5000$! | **Alternative:** Algebraic $x + 0.08x = 5400$.
- **Common Trap:** Subtracting 8% of 5400 (432) to get 4968 (WRONG!).
- **Practice Drill:** Ex. 6 & Ex. 7.

Pattern 3: Compound Interest (Annual, Half-Yearly & Fractional)

- **Identification Cue:** Find CI on Rs. 12600 for 2 years at 10% p.a. compounded annually; or Rs. 80000 for 1.5 years at 10% p.a. half-yearly. | **Concept & Formula:** $A = P(1 + R/100)^n$ and $A = P(1 + R/200)^{2n}$.
- **Standard Step-by-Step:** Identify P, R, n, adjust for half-yearly if stated, compute A, then subtract P for CI.
- **Fastest Valid Method:** For $n = 2$ or 3 with 10%, $(1.1)^2 = 1.21$ and $(1.1)^3 = 1.331$! | **Alternative:** Year-by-year SI.
- **Common Trap:** Stopping at Amount A when CI is asked.
- **Practice Drill:** Ex. 9 & Ex. 10.

Pattern 4: Population Growth / Depreciation (Past & Future)

- **Identification Cue:** Population in 2003 was 54000 at 5% p.a. growth; find population in 2001 (past) and 2005 (future). | **Concept & Formula:** Future: $P(1 + R/100)^n$; Past: $P_{\text{past}}(1 + R/100)^n = P_{\text{now}}$.
- **Standard Step-by-Step:** For past value, divide by $(1 + R/100)^n$; for future value, multiply by $(1 + R/100)^n$.
- **Fastest Valid Method:** Think chronologically: Past $\xrightarrow{\times(1+R/100)^n}$ Present $\xrightarrow{\times(1+R/100)^n}$ Future. | **Alternative:** Use $(1 - R/100)^n$ for depreciation.
- **Common Trap:** Multiplying by $(1 - 5/100)^2$ to go backward in time (WRONG!).
- **Practice Drill:** Ex. 12 & Ex. 13.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Reverse-Percentage Unitary Key ($108\% \to 100\%$)] **Core Idea & Why It Works:** When tax $t\%$ is ALREADY included in Bill B, NEVER subtract $t\%$ of B! Instead, set $(100+t)\% = B$ and compute $\frac{B}{100+t} \times 100$.
Valid Domain: All 'before-GST/VAT' and 'original MP before discount' problems. | **Where It Fails:** Nothing.
Easy Example: Rs. 5400 incl. 8% VAT: $108\% = 5400 \implies 100\% = 5000$.
Exam-Level Example: Prevents the #1 error in NCERT Ex. 7.2.
Advanced Example: For discount $d\%$, set $(100-d)\% = \text{SP}$.

[SMART SHORTCUT – 2. 2-Year Effective CI Rate ($2R + \frac{R^2}{100}\%$)] **Core Idea & Why It Works:** For $n = 2$ years at $R\%$ p.a., total CI percentage is simply $2R + \frac{R^2}{100}\%$.
Valid Domain: All 2-year annual CI (or 1-year half-yearly) problems, plus successive discounts ($-d_1 - d_2 + \frac{d_1 d_2}{100}$). | **Where It Fails:** $n \neq 2$.
Easy Example: $R = 10\% \implies 20 + 1 = 21\%$; $R = 5\% \implies 10 + 0.25 = 10.25\%$.
Exam-Level Example: Instant CI check in 3 seconds!
Advanced Example: Successive discounts 20% and $10\% = 20 + 10 - 2 = 28\%$.

[SMART SHORTCUT – 3. Power Table for Common CI Factors] **Core Idea & Why It Works:** Memorize: $5\% \to \frac{21}{20}$ $(441/400, 9261/8000)$; $10\% \to \frac{11}{10}$ $(1.21, 1.331)$; $4\% \to \frac{26}{25}$ $(676/625)$.
Valid Domain: Most textbook & exam CI questions. | **Where It Fails:** Non-standard rates.
Easy Example: $P = 8000, R = 5\%, n = 3 \implies 8000 \times \frac{9261}{8000} = 9261$ instantly!
Exam-Level Example: Slashes fraction arithmetic by 80%.
Advanced Example: Notice $8000 = 20^3$ cancels!

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: An item marked at Rs. 840 is sold for Rs. 714. What is the discount and discount%?
Given & Required: $\text{MP} = 840, \text{SP} = 714$. | **Concept & Formula:** $\frac{\text{Discount}}{\text{MP}} \times 100$.
Detailed Solution: Discount $= 840 - 714 =$ Rs. 126. Discount% $= \frac{126}{840} \times 100 = 15\%$. **Final Answer: Rs.** 126**,** 15%
Fastest Alternative / Shortcut: $126/84 = 1.5$.

SOLVED EXAMPLE 2 [Beginner]
Question: The cost of a pair of shoes was Rs. 450. Sales tax charged was 5%. Find the bill amount.
Given & Required: $\text{SP} = 450, t = 5\%$. | **Concept & Formula:** Add tax on SP.
Detailed Solution: Tax $= 5\%$ of $450 =$ Rs. 22.50. Bill $= 450 + 22.50 =$ Rs. 472.50. **Final Answer: Rs.** 472.50
Fastest Alternative / Shortcut: $450 \times 1.05 = 472.50$.

SOLVED EXAMPLE 3 [Beginner]
Question: Find the Compound Interest on Rs. $10,000$ for 2 years at 10% per annum compounded annually.
Given & Required: $P = 10000, R = 10, n = 2$. | **Concept & Formula:** $A = P(1 + R/100)^n$.
Detailed Solution: $A = 10000(1.1)^2 = 12100$. CI $= 12100 - 10000 =$ Rs. 2100. **Final Answer: Rs.** 2100
Fastest Alternative / Shortcut: 21% of $10000 = 2100$.

SOLVED EXAMPLE 4 [Beginner]

Question: A scooter was bought at Rs. $42,000$. Its value depreciated at 8% p.a. Find its value after 1 year.

Given & Required: $P = 42000, R = 8\%, n = 1$. | **Concept & Formula:** Depreciation.

Detailed Solution: $V = 42000\left(1 - \frac{8}{100}\right) = 42000 \times 0.92 = \text{Rs. } 38,640$. **Final Answer: Rs.** $38,640$

Fastest Alternative / Shortcut: $42000 - 3360 = 38640$.

SOLVED EXAMPLE 5 [Intermediate]

Question: A frock is sold for Rs. 220 after a discount of 20%. Find its Marked Price.

Given & Required: SP $= 220, d = 20\%$. | **Concept & Formula:** 80% of MP $= 220$.

Detailed Solution: MP $= \frac{220 \times 100}{80} = \text{Rs. } 275$. **Final Answer: Rs.** 275

Fastest Alternative / Shortcut: $220/0.8 = 275$.

SOLVED EXAMPLE 6 [Intermediate]

Question: I purchased a hair-dryer for Rs. $5,400$ including 8% VAT. Find the price before VAT was added.

Given & Required: $108\% = 5400$. | **Concept & Formula:** Reverse-percentage.

Detailed Solution: Original price $= \frac{5400 \times 100}{108} = \text{Rs. } 5000$. **Final Answer: Rs.** 5000

Fastest Alternative / Shortcut: $5400/108 = 50 \implies 5000$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find the Compound Interest on Rs. $12,600$ for 2 years at 10% p.a. compounded annually.

Given & Required: $P = 12600, R = 10, n = 2$. | **Concept & Formula:** $A = P(11/10)^2$.

Detailed Solution: $A = 12600 \times \frac{121}{100} = 126 \times 121 = 15246$. CI $= 15246 - 12600 = \text{Rs. } 2646$. **Final Answer: Rs.** 2646

Fastest Alternative / Shortcut: 21% of $12600 = 2646$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Fabina borrows Rs. $12,500$ at 12% p.a. for 3 years at SI, and Radha borrows the same at 10% p.a. for 3 years at CI. Who pays more interest and by how much?

Given & Required: Fabina SI vs Radha CI. | **Concept & Formula:** Compare SI and CI.

Detailed Solution: Fabina SI $= \frac{12500 \times 12 \times 3}{100} = \text{Rs. } 4500$. Radha $A = 12500(1.1)^3 = 16637.50 \implies$ CI $= 4137.50$. Fabina pays Rs. 362.50 more. **Final Answer: Fabina by Rs.** 362.50

Fastest Alternative / Shortcut: 36% vs $33.1\% = 2.9\%$ of $12500 = 362.50$.

SOLVED EXAMPLE 9 [Board Standard]

Question: Find the amount and CI on Rs. $80,000$ for $1\frac{1}{2}$ years at 10% p.a. compounded half-yearly.

Given & Required: $P = 80000, R_{\text{hy}} = 5\%, n_{\text{hy}} = 3$. | **Concept & Formula:** Half rate, double time.

Detailed Solution: $A = 80000\left(1 + \frac{5}{100}\right)^3 = 80000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = 10 \times 9261 = \text{Rs. } 92,610$. CI $= \text{Rs. } 12,610$. **Final Answer: Amount Rs.** $92,610$**; CI Rs.** $12,610$

Fastest Alternative / Shortcut: $80000/8000 = 10$.

SOLVED EXAMPLE 10 [Board Standard]

Question: Find the amount on Rs. $10,000$ for 1 year at 8% p.a. compounded half-yearly vs annually, and find the difference.

Given & Required: $P = 10000, R = 8\%$. | **Concept & Formula:** Annual vs Half-yearly.

Detailed Solution: Annually: $10000(1.08) = 10800$. Half-yearly (4% for 2 periods): $10000(1.04)^2 = 10816$. Difference $= \text{Rs. } 16$. **Final Answer: Rs.** 16 **more half-yearly**

Fastest Alternative / Shortcut: $(0.04)^2 \times 10000 = 16$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Kamala borrowed Rs. $26,400$ from a bank at 15% p.a. compounded yearly. What amount will she pay at the end of 2 years and 4 months?

Given & Required: $n = 2\frac{1}{3}$ years. | **Concept & Formula:** Fractional time CI.

Detailed Solution: For 2 yrs: $26400\left(\frac{23}{20}\right)^2 = 34914$. For next 4 months (1/3 yr) at 15% ($= 5\%$): $34914 \times 1.05 =$ Rs. $36,659.70$. **Final Answer: Rs.** $36,659.70$

Fastest Alternative / Shortcut: $15\% \times \frac{1}{3} = 5\%$ for the extra 4 months.

SOLVED EXAMPLE 12 [Board Standard]

Question: The population of a city was $54,000$ in 2003 and increased at 5% p.a. Find the population in (i) 2001, (ii) 2005.

Given & Required: $P_{2003} = 54000, R = 5\%$. | **Concept & Formula:** Past vs Future.

Detailed Solution: (i) $P_{2001}\left(\frac{21}{20}\right)^2 = 54000 \implies P_{2001} = 54000 \times \frac{400}{441} \approx 48,980$. (ii) $P_{2005} = 54000\left(\frac{21}{20}\right)^2 = 59,535$. **Final Answer: (i)** $48,980$ **(approx), (ii)** $59,535$

Fastest Alternative / Shortcut: Divide for past, multiply for future.

SOLVED EXAMPLE 13 [Board Standard]

Question: In a laboratory, the count of bacteria increased at 2.5% per hour. Find the bacteria at the end of 2 hours if the count was initially $5,06,000$.

Given & Required: $P = 506000, R = 2.5\%, n = 2$. | **Concept & Formula:** $1 + 2.5/100 = 41/40$.

Detailed Solution: $A = 506000\left(\frac{41}{40}\right)^2 = 506000 \times \frac{1681}{1600} = 531,616.25 \approx 5,31,616$. **Final Answer:** $5,31,616$

Fastest Alternative / Shortcut: $2.5/100 = 1/40$.

SOLVED EXAMPLE 14 [Challenge]

Question: What single discount is equivalent to two successive discounts of 20% and 10%?

Given & Required: $d_1 = 20\%, d_2 = 10\%$. | **Concept & Formula:** $d_1 + d_2 - \frac{d_1 d_2}{100}$.

Detailed Solution: $100 \xrightarrow{-20\%} 80 \xrightarrow{-10\%} 72 \implies 100 - 72 = 28\%$. **Final Answer:** 28%

Fastest Alternative / Shortcut: $20 + 10 - 2 = 28\%$ (NOT 30%!).

SOLVED EXAMPLE 15 [Challenge]

Question: If the difference between CI and SI on a sum at 10% p.a. for 2 years is Rs. 50, find the sum P.

Given & Required: $\text{CI} - \text{SI} = 50, R = 10, n = 2$. | **Concept & Formula:** $P(R/100)^2 = 50$.

Detailed Solution: $P(0.1)^2 = 50 \implies 0.01P = 50 \implies P =$ Rs. 5000. **Final Answer: Rs.** 5000

Fastest Alternative / Shortcut: 1% of $P = 50 \implies P = 5000$.

SOLVED EXAMPLE 16 [Challenge]

Question: In how many years will Rs. 8000 amount to Rs. 9261 at 5% p.a. compounded annually?

Given & Required: $P = 8000, A = 9261, R = 5\%$. | **Concept & Formula:** $(21/20)^n = 9261/8000$.

Detailed Solution: $\left(\frac{21}{20}\right)^n = \left(\frac{21}{20}\right)^3 \implies n = 3$ years. **Final Answer:** 3 **years**

Fastest Alternative / Shortcut: Cube recognition: $20^3 = 8000, 21^3 = 9261$.

G. Competency MCQ Mastery Section

Q1. Discount is always calculated on:

(A) Cost Price (B) Selling Price (C) Marked Price (D) Profit

Answer: (C) Discount is a reduction on Marked Price. *Fast Method:* MP. *Distractor Note:* Picking SP.

Q2. If 10% GST is included in a bill of Rs. 1100, the price before GST is:

(A) Rs. 990 (B) Rs. 1000 (C) Rs. 1010 (D) Rs. 1210

Answer: (B) $110\% = 1100 \implies 100\% = 1000$. *Fast Method:* $1100/1.1 = 1000$. *Distractor Note:* Subtracting 10% of 1100 gives 990.

Q3. When compounded half-yearly, the annual rate $R\%$ and time n years become:

(A) $2R, n/2$ (B) $R/2, 2n$ (C) $R/4, 4n$ (D) $R, 2n$

Answer: (B) Half the rate, double the time. *Fast Method:* $R/2, 2n$. *Distractor Note:* Halving both.

Q4. CI on Rs. 1000 at 10% p.a. for 2 years (annual) is:

(A) Rs. 200 (B) Rs. 210 (C) Rs. 1210 (D) Rs. 110

Answer: (B) $1210 - 1000 = 210$. *Fast Method:* 21% of 1000. *Distractor Note:* Picking 1210 (Amount) or 200 (SI).

Q5. Two successive discounts of 10% and 10% equal a single discount of:

(A) 20% (B) 19% (C) 21% (D) 18%

Answer: (B) $10 + 10 - 1 = 19\%$. *Fast Method:* $100 \to 90 \to 81$. *Distractor Note:* Adding $10 + 10 = 20\%$.

Q6. Formula for depreciated value after n years at $R\%$ p.a. is:

(A) $P(1 + R/100)^n$ (B) $P(1 - R/100)^n$ (C) $P(1 - nR/100)$ (D) $P(R/100)^n$

Answer: (B) Depreciation uses minus sign. *Fast Method:* Minus for decay. *Distractor Note:* Picking $+$.

Q7. For 1 year compounded annually, CI $-$ SI is always:

(A) R (B) $P/100$ (C) 0 (D) $PR/100$

Answer: (C) First year CI $=$ SI. *Fast Method:* Equal for 1st period. *Distractor Note:* Overthinking.

Q8. If MP $=$ Rs. 500 and discount $= 12\%$, SP is:

(A) Rs. 440 (B) Rs. 488 (C) Rs. 460 (D) Rs. 560

Answer: (A) $500 - 60 = 440$. *Fast Method:* 500×0.88. *Distractor Note:* Subtracting 12 directly.

Q9. Difference between CI and SI on Rs. 1000 at 10% p.a. for 2 years is:

(A) Rs. 5 (B) Rs. 10 (C) Rs. 20 (D) Rs. 100

Answer: (B) $1000(0.1)^2 = 10$. *Fast Method:* 10% of $100 = 10$. *Distractor Note:* Picking 100.

Q10. If Rs. 4000 becomes Rs. 4410 in 2 years compounded annually, rate R is:

(A) 5% (B) 10% (C) 4% (D) 8%

Answer: (A) $441/400 = (21/20)^2 \implies R = 5\%$. *Fast Method:* $21/20 = 1 + 1/20 = 1 + 5\%$. *Distractor Note:* Picking 10%.

Q11. If 5% GST is added to an article costing Rs. 800, bill amount is:

(A) Rs. 805 (B) Rs. 840 (C) Rs. 760 (D) Rs. 850

Answer: (B) $800 + 40 = 840$. *Fast Method:* 5% of $800 = 40$. *Distractor Note:* Adding Rs. 5.

Q12. In $A = P(1 + R/100)^n$, if n is in years and compounded quarterly, periods become:

(A) $2n$ (B) $3n$ (C) $4n$ (D) $n/4$

Answer: (C) 4 quarters per year $\implies 4n$. *Fast Method:* $4n$ periods, $R/4$ rate. *Distractor Note:* Picking $n/4$.

H. Special Exam & MCQ Strategies

- Always read whether the question asks for **Amount (A)** or **Compound Interest (CI $= A - P$)**!
- For reverse tax/discount problems ('including $t\%$ GST'), set $(100 + t)\%$ = Bill and NEVER subtract $t\%$ directly from the bill.
- For half-yearly compounding, write '$R_{\text{new}} = R/2$, $n_{\text{new}} = 2n$' at the top before plugging into the formula.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Subtracting 8% of Rs. 5400 when finding price before 8% VAT. $\longrightarrow$ **Correction:** VAT was added to the original price (100%), so $5400 = 108\%$; divide by 1.08 to get 5000.

[COMMON MISTAKE] **Error:** Writing Amount A as the final answer when Compound Interest is asked. $\longrightarrow$ **Correction:** Always finish with $\text{CI} = A - P$.

[COMMON MISTAKE] **Error:** Doubling n for half-yearly compounding without halving R. $\longrightarrow$ **Correction:** Half the rate ($R/2$) AND double the time ($2n$).

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: SP $=$ MP$(1-d/100)$; Bill $=$ SP$(1+t/100)$; $A=P(1+R/100)^n$; Half-yearly: $P(1+R/200)^{2n}$; Deprec: $P(1-R/100)^n$.

[QUICK REVISION] Key Theorems & Results: CI $= A-P$; 2-yr CI $-$ SI $= P(R/100)^2$; Successive discounts: $d_1+d_2-\frac{d_1 d_2}{100}$.

[QUICK REVISION] Shortcut Summary: Reverse GST: $\frac{\text{Bill}}{100+t}\times 100$; 2-yr CI% $= 2R+R^2/100$; $5\% \to 21/20, 10\% \to 11/10$.

[QUICK REVISION] High-Frequency Exam Patterns: Discount & MP, Reverse GST/VAT, Annual/Half-yearly CI, Population/Depreciation.

Last-Minute Revision Checklist:

- ☐ Amount vs CI check
- ☐ Reverse GST divides by $(100+t)$
- ☐ Half rate & double time
- ☐ Minus sign for depreciation

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. MP is Rs. 200, discount is 10%. SP is: (A) Rs. 190 (B) Rs. 180 (C) Rs. 220 (D) Rs. 170
2. SI on Rs. 5000 at 8% p.a. for 2 years is: (A) Rs. 400 (B) Rs. 800 (C) Rs. 832 (D) Rs. 5800
3. CI on Rs. 5000 at 10% p.a. for 1 year (annual) is: (A) Rs. 500 (B) Rs. 550 (C) Rs. 5500 (D) Rs. 1000
4. Value of Rs. 10,000 depreciating 10% in 1 year: (A) Rs. 11000 (B) Rs. 9000 (C) Rs. 1000 (D) Rs. 9900

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. An article is sold for Rs. 1260 after 10% discount. MP is: (A) Rs. 1386 (B) Rs. 1400 (C) Rs. 1134 (D) Rs. 1360
2. Amount on Rs. 10,000 for 1 year at 10% p.a. compounded half-yearly: (A) Rs. 11000 (B) Rs. 11025 (C) Rs. 11050 (D) Rs. 12100
3. Price before 12% GST if bill is Rs. 2240: (A) Rs. 2000 (B) Rs. 1971.20 (C) Rs. 2120 (D) Rs. 2200
4. Population 20,000 grows at 10% p.a. After 2 years it is: (A) 24000 (B) 24200 (C) 22000 (D) 24400

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Difference between CI and SI on Rs. 15,000 at 8% p.a. for 2 years: (A) Rs. 96 (B) Rs. 120 (C) Rs. 80 (D) Rs. 64
2. A sum doubles in 4 years at CI. In how many years will it become 8 times? (A) 8 (B) 12 (C) 16 (D) 32
3. Machine bought 2 years ago depreciates 10% p.a. Current value Rs. 8100. Original price: (A) Rs. 9720 (B) Rs. 10000 (C) Rs. 9000 (D) Rs. 10100
4. Successive discounts of 20%, 20% equal single discount of: (A) 40% (B) 36% (C) 32% (D) 38%

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $200-20=180$. **Q2 (B):** $5000\times8\times2/100=800$. **Q3 (A):** 10% of $5000=500$. **Q4 (B):** $10000\times0.9=9000$.
Exam Std	**Q1 (B):** $1260/0.9=1400$. **Q2 (B):** $10000(1.05)^2=11025$. **Q3 (A):** $2240/1.12=2000$. **Q4 (B):** $20000(1.21)=24200$.
Challenge	**Q1 (A):** $15000\times(0.08)^2=15000\times0.0064=96$. **Q2 (B):** $2^3=8\implies3\times4=12$ years. **Q3 (B):** $8100/0.81=10000$. **Q4 (B):** $20+20-4=36\%$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Discount & GST (forward + reverse). Day 2: CI annual vs half-yearly. Day 3: Growth & depreciation.
- **How to Introduce the Concept:** Compare SI vs CI on Rs. 1000 at 20% over 4 years on the board—students see CI curve bending upward!
- **Difficult Areas & Remediation:** Reverse GST/discount ($108\%\to100\%$) and half-yearly rate/time adjustment.
- **Blackboard Teaching Examples:** Draw the timeline: Past $\xrightarrow{\times(1+R/100)^n}$ Present $\xrightarrow{\times(1+R/100)^n}$ Future.
- **Classroom Questions to Ask Students:** Why is subtracting 8% from Rs. 5400 NOT the price before 8% VAT?
- **Common Student Doubts & Answers:** Students confuse Amount with CI; circle 'Amount' vs 'Compound Interest' in the question before solving.
- **Homework & Revision Strategy:** Assign 3 discount/GST problems, 4 CI problems (2 annual, 2 half-yearly), and 2 depreciation problems.

[TEACHER'S TIP] The 2-year CI percentage shortcut ($2R+R^2/100$) is a superpower for competitive exams.

UNIT II: ALGEBRA TYBA COACHING CENTRE

Chapter 8: Algebraic Expressions and Identities

Terms, Like Terms, Polynomial Multiplication & Standard Algebraic Identities (2026–27)

A. Chapter Introduction & Examination Relevance

Algebraic expressions allow us to generalize arithmetic rules using variables and constants. Chapter 8 builds fluency in adding, subtracting, and multiplying polynomials—from monomials to binomials—and applying standard identities to simplify complex products effortlessly.

- **What You Will Learn:** Terms, factors, and coefficients; monomials, binomials, trinomials, and polynomials; addition and subtraction of like terms; multiplication of monomial by polynomial and binomial by binomial; and standard identities $(a+b)^2$, $(a-b)^2$, $(a+b)(a-b)$, and $(x+a)(x+b)$.
- **Why This Chapter Matters:** Algebra carries 25% of Class 8 marks and forms the backbone of Chapter 12 (Factorisation), Class 9 Polynomials, Class 10 Quadratic Equations, and all higher secondary mathematics.
- **Real-Life Applications:** Calculating variable areas and volumes, modelling revenue and cost expressions, and mental arithmetic via algebraic identities (e.g., 103^2 or 98×102).
- **Cross-Chapter Connections:** Directly leads into Chapter 12 (Factorisation) and Class 9 Chapter 2 (Polynomials).
- **Exam & Foundation Relevance:** Unit II (Algebra) carries 25% weightage. Expect 1M coefficient/degree MCQs, 2M–3M polynomial subtraction/multiplication questions, and 4M–5M identity evaluation and 'show that' problems.

B. Complete Concept Explanation & Derivations

1. Terms, Factors, Coefficients & Polynomial Types

An algebraic expression is built from **terms** separated by $+$ or $-$ signs. Each term is a product of **factors**; the numerical factor is its **numerical coefficient**. **Like terms** have the exact same variable factors with identical exponents (e.g., $7xy^2$ and $-3y^2x$). Expressions with 1, 2, or 3 unlike terms are called **monomials**, **binomials**, and **trinomials**, respectively.

[CONCEPT MASTER – 1. Terms, Factors, Coefficients & Polynomial Types] Golden Rule: Only LIKE terms can be added or subtracted! $3x^2 + 5x^2 = 8x^2$, but $3x^2 + 5x$ CANNOT be combined.

2. Addition & Subtraction of Algebraic Expressions

To add expressions, align like terms in columns and add their coefficients. To subtract expression B from expression A ($A - B$), **reverse the sign of every term in** B before adding like terms. Always enclose subtracted expressions in brackets first to prevent sign errors.

[CONCEPT MASTER – 2. Addition & Subtraction of Algebraic Expressions] 'Subtract P from Q' means $Q - P$ (NOT $P - Q$). Distribute the minus sign to EVERY term inside P!

3. Multiplication of Monomials & Polynomials

When multiplying monomials: (1) multiply numerical coefficients, and (2) add exponents of identical variables ($a^m \times a^n = a^{m+n}$). To multiply a monomial by a polynomial, use the distributive law: $a(b+c) = ab+ac$. To multiply two binomials $(a+b)(c+d)$, multiply every term of the first by every term of the second: $a(c+d)+b(c+d) = ac+ad+bc+bd$.

[CONCEPT MASTER – 3. Multiplication of Monomials & Polynomials] Check term count before collecting like terms: (2 terms) $\times$ (3 terms) $= 6$ raw products!

4. Standard Algebraic Identities

An **identity** is an equality true for ALL values of its variables. The four fundamental identities are: (I) $(a+b)^2 = a^2 + 2ab + b^2$, (II) $(a-b)^2 = a^2 - 2ab + b^2$, (III) $(a+b)(a-b) = a^2 - b^2$, and (IV) $(x+a)(x+b) = x^2 + (a+b)x + ab$.

[CONCEPT MASTER – 4. Standard Algebraic Identities] Never forget the middle term $2ab$! $(a+b)^2 \neq a^2+b^2$ and $(a-b)^2 \neq a^2-b^2$.

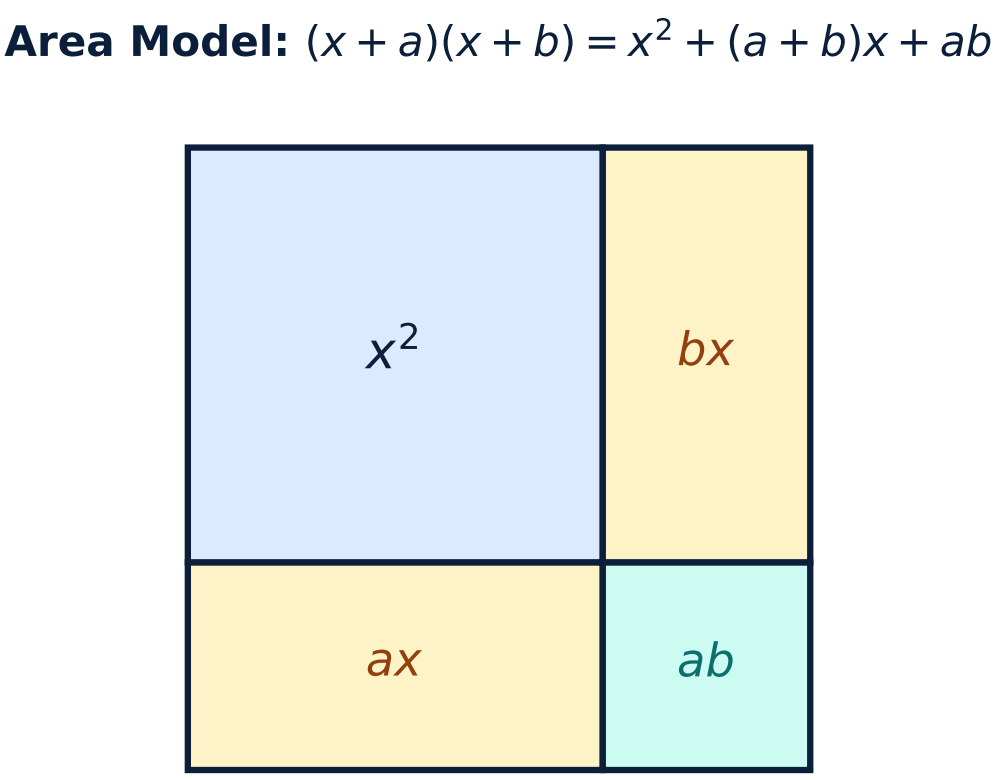


Figure 1: **Visual Model:** Area-model decomposition of $(a+b)^2 = a^2 + 2ab + b^2$ showing geometric proof of the identity.

C. Complete Formula Bank

[FORMULA BANK – 1. Square of a Sum] **Expression:** $(a+b)^2 = a^2 + 2ab + b^2$
Symbols: a, b: any algebraic terms.
Conditions & When to Use: Used for squaring sums or numbers like $103^2 = (100+3)^2$.
Solved Mini-Example: $(2x+3)^2 = 4x^2 + 12x + 9$.
Common Mistake: Omitting $+2ab$ and writing $4x^2 + 9$.

[FORMULA BANK – 2. Square of a Difference] **Expression:** $(a-b)^2 = a^2 - 2ab + b^2$
Symbols: a, b: any algebraic terms.
Conditions & When to Use: Used for squaring differences or numbers like $98^2 = (100-2)^2$.
Solved Mini-Example: $(3p-5)^2 = 9p^2 - 30p + 25$.
Common Mistake: Writing $-b^2$ instead of $+b^2$ at the end.

[FORMULA BANK – 3. Difference of Squares] **Expression:** $(a+b)(a-b) = a^2 - b^2$
Symbols: a, b: matching terms with opposite signs.
Conditions & When to Use: Fastest way to compute $a^2 - b^2$ or $(100+k)(100-k)$.
Solved Mini-Example: $51^2 - 49^2 = (51+49)(51-49) = 100 \times 2 = 200$.
Common Mistake: Computing 51^2 and 49^2 separately.

[FORMULA BANK – 4. Product of Two Linear Binomials] **Expression:** $(x+a)(x+b) = x^2 + (a+b)x + ab$
Symbols: x: common term; a, b: constants (with signs).
Conditions & When to Use: Used for $(x+4)(x-7)$ or 103×104.
Solved Mini-Example: $(x+5)(x-2) = x^2 + 3x - 10$.
Common Mistake: Adding instead of multiplying a and b for the constant term.

[FORMULA BANK – 5. Distributive Law (Monomial × Polynomial)] **Expression:** $a(b+c+d) = ab + ac + ad$
Symbols: a: monomial; $b+c+d$: polynomial.
Conditions & When to Use: Multiply a by EVERY term inside brackets.
Solved Mini-Example: $-3x(2x - 5y + 1) = -6x^2 + 15xy - 3x$.
Common Mistake: Forgetting to flip signs when a is negative.

[FORMULA BANK – 6. Sum & Difference Identity Link] **Expression:** $(a+b)^2 - (a-b)^2 = 4ab$
Symbols: a, b: any algebraic terms.
Conditions & When to Use: Instant verification for 'show that' problems.
Solved Mini-Example: $(3x+7)^2 - (3x-7)^2 = 4(3x)(7) = 84x$.
Common Mistake: Cancelling $+2ab$ and $-2ab$ instead of adding them.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Sainik/Navodaya/Olympiad Foundation problem structures.

Pattern 1: Subtracting & Simplifying Polynomials

- **Identification Cue:** Subtract $4a - 7ab + 3b + 12$ from $12a - 9ab + 5b - 3$. | **Concept & Formula:** Reverse signs of subtracted expression; combine like terms.
- **Standard Step-by-Step:** Write $(12a - 9ab + 5b - 3) - (4a - 7ab + 3b + 12)$ and distribute $-$ sign.
- **Fastest Valid Method:** Column alignment with flipped signs written below in pencil. | **Alternative:** Test with $a = 1, b = 1$ to verify.
- **Common Trap:** Subtracting the first from the second instead of 'from' expression minus first!
- **Practice Drill:** Ex. 3 & Ex. 5.

Pattern 2: Binomial × Binomial / Trinomial Multiplication

- **Identification Cue:** Multiply $(2x+5)$ and $(4x-3)$ or $(a+7)(a^2+3a+5)$. | **Concept & Formula:** Distributive property + exponent addition.
- **Standard Step-by-Step:** Expand $2x(4x-3) + 5(4x-3) = 8x^2 - 6x + 20x - 15 = 8x^2 + 14x - 15$.
- **Fastest Valid Method:** FOIL / Identity $(x+a)(x+b)$ when leading coefficients are 1. | **Alternative:** Box/Grid multiplication.
- **Common Trap:** Writing $2x \times 4x = 8x$ instead of $8x^2$.
- **Practice Drill:** Ex. 6 & Ex. 8.

Pattern 3: Numerical Evaluation Using Identities

- **Identification Cue:** Evaluate 103^2, 99^2, 103×97, or $52^2 - 48^2$ without direct multiplication. | **Concept & Formula:** Rewrite near $10, 50, 100, 1000$ and apply Identities I–IV.
- **Standard Step-by-Step:** Express $103 = 100 + 3$ and expand $(100+3)^2 = 10000 + 600 + 9 = 10609$.
- **Fastest Valid Method:** Pick the nearest multiple of 10 or 100 as a. | **Alternative:** For $a^2 - b^2$, always use $(a+b)(a-b)$.
- **Common Trap:** Writing $98^2 = (100-2)^2 = 10000 - 400 - 4$ (last term MUST be $+4$!).
- **Practice Drill:** Ex. 9 & Ex. 11.

Pattern 4: Show-That & Value-Finding Identity Proofs

- **Identification Cue:** Show that $(3x+7)^2 - 84x = (3x-7)^2$ or find $x^2 + \frac{1}{x^2}$ given $x + \frac{1}{x}$. | **Concept & Formula:** Expand LHS and RHS separately, or square both sides.
- **Standard Step-by-Step:** Expand $(3x+7)^2 = 9x^2 + 42x + 49$; subtract $84x \to 9x^2 - 42x + 49 = (3x-7)^2$.
- **Fastest Valid Method:** Use $(a+b)^2 - 4ab = (a-b)^2$ directly! | **Alternative:** Substitute $x = 1$ for quick MCQ check.
- **Common Trap:** Forgetting the $2 \cdot x \cdot \frac{1}{x} = 2$ middle term when squaring $x + \frac{1}{x}$.
- **Practice Drill:** Ex. 13 & Ex. 15.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. 1-Second Numerical Substitution Check] **Core Idea & Why It Works:** To verify ANY polynomial addition, subtraction, or multiplication in MCQs, plug in $x = 1$ (or $x = 1, y = 2$) into both the question and options!

Valid Domain: All algebraic simplifications and identities. | **Where It Fails:** If two options happen to tie at $x = 1$ (then test $x = 2$).

Easy Example: $(x+2)(x+3)$ at $x = 1$ gives $3 \times 4 = 12$; $x^2 + 5x + 6$ gives $1 + 5 + 6 = 12$!

Exam-Level Example: Catches sign errors in 3 seconds!

Advanced Example: Works even for 3-variable polynomials.

[SMART SHORTCUT – 2. Instant $a^2 - b^2$ Difference Trick] **Core Idea & Why It Works:** Whenever you see $A^2 - B^2$, NEVER square A and B—immediately compute $(A+B)(A-B)$.

Valid Domain: All differences of two squares, including decimals. | **Where It Fails:** Sums of squares $A^2 + B^2$.

Easy Example: $1.02^2 - 0.98^2 = (1.02 + 0.98)(1.02 - 0.98) = 2 \times 0.04 = 0.08$.

Exam-Level Example: Essential for NCERT Ex. 8.4 / 9.5 Q6 & Q7.

Advanced Example: Also solves $97 \times 103 = (100-3)(100+3) = 10000 - 9 = 9991$.

[SMART SHORTCUT – 3. Reciprocal Squaring Shortcut] **Core Idea & Why It Works:** If $x + \frac{1}{x} = k$, then $x^2 + \frac{1}{x^2} = k^2 - 2$. If $x - \frac{1}{x} = k$, then $x^2 + \frac{1}{x^2} = k^2 + 2$.

Valid Domain: All reciprocal expressions. | **Where It Fails:** When coefficients of x and $1/x$ differ (then use $k^2 \mp 2ab$).

Easy Example: $x + \frac{1}{x} = 5 \implies x^2 + \frac{1}{x^2} = 25 - 2 = 23$.

Exam-Level Example: High-frequency Olympiad & school exam MCQ!

Advanced Example: Apply twice to get $x^4 + \frac{1}{x^4} = 23^2 - 2 = 527$.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: Identify the terms and their numerical coefficients in $5xyz^2 - 3zy$.

Given & Required: Expression $5xyz^2 - 3zy$. | **Concept & Formula:** Terms & coefficients.

Detailed Solution: Terms are $5xyz^2$ (coefficient 5) and $-3zy$ (coefficient -3). **Final Answer:** $5xyz^2$ **(coeff.** 5**),** $-3zy$ **(coeff.** -3**)**

Fastest Alternative / Shortcut: Include the minus sign in the coefficient!

SOLVED EXAMPLE 2 [Beginner]

Question: Add: $7x^2 - 4x + 5$ and $9x - 10$.

Given & Required: Two polynomials. | **Concept & Formula:** Combine like terms.

Detailed Solution: $7x^2 + (-4x + 9x) + (5 - 10) = 7x^2 + 5x - 5$. **Final Answer:** $7x^2 + 5x - 5$

Fastest Alternative / Shortcut: Group x^2, x, and constants.

SOLVED EXAMPLE 3 [Beginner]

Question: Find the product of the monomials $-4p$ and $7pq$.

Given & Required: $-4p$ and $7pq$. | **Concept & Formula:** Multiply coefficients, add powers.

Detailed Solution: $(-4 \times 7)(p \times p \times q) = -28p^2q$. **Final Answer:** $-28p^2q$

Fastest Alternative / Shortcut: Coeff $\rightarrow -28$; $p \cdot p \rightarrow p^2$.

SOLVED EXAMPLE 4 [Beginner]

Question: Simplify $3x(4x-5)+3$ and find its value for $x=3$.

Given & Required: $x=3$. | **Concept & Formula:** Distributive law.

Detailed Solution: $3x(4x-5)+3 = 12x^2-15x+3$. At $x=3$: $12(9)-15(3)+3 = 108-45+3 = 66$. **Final Answer:** $12x^2-15x+3$; **value** $= 66$

Fastest Alternative / Shortcut: Check: $3(3)(12-5)+3 = 9(7)+3 = 66$.

SOLVED EXAMPLE 5 [Intermediate]

Question: Subtract $4p^2q-3pq+5pq^2-8p+7q-10$ from $18-3p-11q+5pq-2pq^2+5p^2q$.

Given & Required: Two 6-term expressions. | **Concept & Formula:** Flip signs of first expression.

Detailed Solution: $(5p^2q-4p^2q)+(5pq+3pq)+(-2pq^2-5pq^2)+(-3p+8p)+(-11q-7q)+(18+10) = p^2q+8pq-7pq^2+5p-18q+28$. **Final Answer:** $p^2q+8pq-7pq^2+5p-18q+28$

Fastest Alternative / Shortcut: Align like terms vertically.

SOLVED EXAMPLE 6 [Intermediate]

Question: Multiply the binomials $(2pq+3q^2)$ and $(3pq-2q^2)$.

Given & Required: Two binomials. | **Concept & Formula:** Distributive multiplication.

Detailed Solution: $2pq(3pq-2q^2)+3q^2(3pq-2q^2) = 6p^2q^2-4pq^3+9pq^3-6q^4 = 6p^2q^2+5pq^3-6q^4$. **Final Answer:** $6p^2q^2+5pq^3-6q^4$

Fastest Alternative / Shortcut: Middle terms: $-4pq^3+9pq^3 = +5pq^3$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find the volume of a rectangular box with length $5a$, breadth $3a^2$, and height $7a^4$.

Given & Required: $l=5a, b=3a^2, h=7a^4$. | **Concept & Formula:** $V = lbh$.

Detailed Solution: $V = 5a \times 3a^2 \times 7a^4 = (5\times3\times7)a^{1+2+4} = 105a^7$. **Final Answer:** $105a^7$ **cubic units**

Fastest Alternative / Shortcut: Add exponents: $1+2+4=7$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Using $(x+a)(x+b) = x^2+(a+b)x+ab$, find $(2x+5y)(2x+3y)$.

Given & Required: $X=2x, a=5y, b=3y$. | **Concept & Formula:** Identity IV.

Detailed Solution: $(2x)^2+(5y+3y)(2x)+(5y)(3y) = 4x^2+16xy+15y^2$. **Final Answer:** $4x^2+16xy+15y^2$

Fastest Alternative / Shortcut: $(5+3)\times 2 = 16$.

SOLVED EXAMPLE 9 [Board Standard]

Question: Using identities, evaluate: (i) 99^2, (ii) 103×104.

Given & Required: 99^2 and 103×104. | **Concept & Formula:** Identities II and IV.

Detailed Solution: (i) $(100-1)^2 = 10000-200+1 = 9801$. (ii) $(100+3)(100+4) = 10000+700+12 = 10712$. **Final Answer:** **(i)** 9801, **(ii)** 10712

Fastest Alternative / Shortcut: Base 100.

SOLVED EXAMPLE 10 [Board Standard]

Question: Using $a^2 - b^2 = (a+b)(a-b)$, find: (i) $51^2 - 49^2$, (ii) $(1.02)^2 - (0.98)^2$.

Given & Required: Differences of squares. | **Concept & Formula:** Identity III.

Detailed Solution: (i) $(51+49)(51-49) = 100 \times 2 = 200$. (ii) $(1.02+0.98)(1.02-0.98) = 2 \times 0.04 = 0.08$. **Final Answer: (i)** 200, **(ii)** 0.08

Fastest Alternative / Shortcut: Never square directly!

SOLVED EXAMPLE 11 [Board Standard]

Question: Simplify $(a+b)(c-d) + (a-b)(c+d) + 2(ac+bd)$.

Given & Required: Three product terms. | **Concept & Formula:** Expand and collect like terms.

Detailed Solution: $(ac - ad + bc - bd) + (ac + ad - bc - bd) + 2ac + 2bd = 4ac$. **Final Answer:** $4ac$

Fastest Alternative / Shortcut: All ad, bc, bd terms cancel!

SOLVED EXAMPLE 12 [Board Standard]

Question: Show that $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$.

Given & Required: LHS expression. | **Concept & Formula:** Identities I & II.

Detailed Solution: $(16p^2q^2 + 24pq^2 + 9q^2) - (16p^2q^2 - 24pq^2 + 9q^2) = 48pq^2 = \text{RHS}$. **Final Answer: Proved** $(48pq^2)$

Fastest Alternative / Shortcut: $4ab = 4(4pq)(3q) = 48pq^2$.

SOLVED EXAMPLE 13 [Board Standard]

Question: Simplify $(x^2 - 5)(x + 5) + 25$.

Given & Required: Product plus 25. | **Concept & Formula:** Distributive law.

Detailed Solution: $x^3 + 5x^2 - 5x - 25 + 25 = x^3 + 5x^2 - 5x$. **Final Answer:** $x^3 + 5x^2 - 5x$

Fastest Alternative / Shortcut: -25 and $+25$ cancel.

SOLVED EXAMPLE 14 [Challenge]

Question: If $x + \frac{1}{x} = 6$, find the value of $x^2 + \frac{1}{x^2}$ and $x^4 + \frac{1}{x^4}$.

Given & Required: $x + \frac{1}{x} = 6$. | **Concept & Formula:** Square both sides twice.

Detailed Solution: $(x + \frac{1}{x})^2 = 36 \implies x^2 + 2 + \frac{1}{x^2} = 36 \implies x^2 + \frac{1}{x^2} = 34$. Squaring again: $x^4 + 2 + \frac{1}{x^4} = 1156 \implies x^4 + \frac{1}{x^4} = 1154$. **Final Answer:** 34 **and** 1154

Fastest Alternative / Shortcut: $6^2 - 2 = 34$; $34^2 - 2 = 1154$.

SOLVED EXAMPLE 15 [Challenge]

Question: If $a + b = 10$ and $ab = 21$, find the value of $a^2 + b^2$ and $(a-b)^2$.

Given & Required: $a + b = 10, ab = 21$. | **Concept & Formula:** Identities I & II.

Detailed Solution: $a^2 + b^2 = (a+b)^2 - 2ab = 100 - 42 = 58$. $(a-b)^2 = (a+b)^2 - 4ab = 100 - 84 = 16$. **Final Answer:** 58 **and** 16

Fastest Alternative / Shortcut: $a = 7, b = 3$ satisfies both!

SOLVED EXAMPLE 16 [Challenge]

Question: Show that $(a-b)(a+b) + (b-c)(b+c) + (c-a)(c+a) = 0$.

Given & Required: Cyclic difference of squares. | **Concept & Formula:** Identity III.

Detailed Solution: LHS $= (a^2 - b^2) + (b^2 - c^2) + (c^2 - a^2) = 0 = \text{RHS}$. **Final Answer: Proved** (0)

Fastest Alternative / Shortcut: Telescoping cancellation.

G. Competency MCQ Mastery Section

Q1. Which of the following is a binomial?

(A) $7xy$ (B) $3x^2 - 5x + 2$ (C) $4a - 7b$ (D) $2x + 3y - z$

Answer: (C) A binomial has exactly two unlike terms. *Fast Method:* Count $+$ and $-$ separated terms. *Distractor Note:* Picking A (monomial).

Q2. The coefficient of x^2 in $-5x^2y + 3x - 7$ is:

(A) -5 (B) $-5y$ (C) 5 (D) 3

Answer: (B) Factor multiplying x^2 is $-5y$. *Fast Method:* Cover x^2, read what remains. *Distractor Note:* Missing y in $-5y$.

Q3. Product of $6a^2b$ and $-2ab^3$ is:

(A) $-12a^3b^4$ (B) $12a^2b^3$ (C) $-12a^2b^3$ (D) $-4a^3b^4$

Answer: (A) $6 \times (-2) = -12$, $a^{2+1}b^{1+3} = a^3b^4$. *Fast Method:* Add powers. *Distractor Note:* Multiplying powers.

Q4. Value of $(3x - 4)^2$ is:

(A) $9x^2 - 16$ (B) $9x^2 + 16$ (C) $9x^2 - 24x + 16$ (D) $9x^2 - 12x + 16$

Answer: (C) $9x^2 - 2(3x)(4) + 16 = 9x^2 - 24x + 16$. *Fast Method:* Middle term is $-24x$. *Distractor Note:* Forgetting 2 in $2ab$ (picking D).

Q5. Using identities, 97×103 equals:

(A) 9991 (B) 9999 (C) 10009 (D) 9981

Answer: (A) $(100 - 3)(100 + 3) = 10000 - 9 = 9991$. *Fast Method:* $100^2 - 3^2$. *Distractor Note:* Adding 9 instead of subtracting.

Q6. Subtracting $-5x^2$ from $3x^2$ gives:

(A) $-2x^2$ (B) $2x^2$ (C) $8x^2$ (D) $-8x^2$

Answer: (C) $3x^2 - (-5x^2) = 3x^2 + 5x^2 = 8x^2$. *Fast Method:* Double minus becomes plus. *Distractor Note:* Doing $3 - 5 = -2$.

Q7. If $x - \frac{1}{x} = 4$, then $x^2 + \frac{1}{x^2}$ is:

(A) 14 (B) 16 (C) 18 (D) 20

Answer: (C) $4^2 + 2 = 18$. *Fast Method:* $k^2 + 2$. *Distractor Note:* Subtracting 2 to get 14.

Q8. Which pair consists of LIKE terms?

(A) $3x^2y, 3xy^2$ (B) $-4ab, 7ba$ (C) $5p, 5p^2$ (D) $2xy, 2x$

Answer: (B) $ab = ba$, same variables and powers. *Fast Method:* Order of multiplication doesn't matter. *Distractor Note:* Picking A ($x^2y \neq xy^2$).

Q9. Value of $78^2 - 22^2$ is:

(A) 5600 (B) 560 (C) 6000 (D) 4600

Answer: (A) $(78 + 22)(78 - 22) = 100 \times 56 = 5600$. *Fast Method:* 100×56. *Distractor Note:* Arithmetic error.

Q10. $(x + 4)(x - 9)$ equals:

(A) $x^2 - 5x - 36$ (B) $x^2 + 5x - 36$ (C) $x^2 - 13x - 36$ (D) $x^2 - 5x + 36$

Answer: (A) $4 - 9 = -5$; $4 \times (-9) = -36$. *Fast Method:* Sum $= -5$, product $= -36$. *Distractor Note:* Sign of constant term.

Q11. Value of $(a + b)^2 - (a - b)^2$ is:

(A) $2(a^2 + b^2)$ (B) $4ab$ (C) $2ab$ (D) 0

Answer: (B) $2ab - (-2ab) = 4ab$. *Fast Method:* Standard identity. *Distractor Note:* Picking A (that's for $+$ between squares).

Q12. If $a^2 + b^2 = 25$ and $ab = 12$, then $(a + b)^2$ is:

(A) 37 (B) 49 (C) 1 (D) 13

Answer: (B) $25 + 2(12) = 49$. *Fast Method:* $a^2 + b^2 + 2ab$. *Distractor Note:* Forgetting factor 2 on ab.

H. Special Exam & MCQ Strategies

- Always enclose any subtracted expression in parentheses first: $A - (B)$ before changing signs.
- For 'show that' identity problems, simplify LHS and RHS independently to the same polynomial if direct transformation isn't obvious.
- In MCQs, plug in $x = 1, y = 2$ to check algebraic equivalence in seconds.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Writing $(a+b)^2 = a^2 + b^2$ or $(a-b)^2 = a^2 - b^2$. $\longrightarrow$ **Correction:** Always include the middle term $\pm 2ab$, and note that $(-b)^2 = +b^2$!

[COMMON MISTAKE] **Error:** Subtracting A from B as $A - B$. $\longrightarrow$ **Correction:** 'Subtract A from B' is always $B - A$.

[COMMON MISTAKE] **Error:** Multiplying $x^2 \times x^3$ to get x^6. $\longrightarrow$ **Correction:** When bases are the same, ADD exponents: $x^{2+3} = x^5$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $(a\pm b)^2 = a^2 \pm 2ab + b^2$; $(a+b)(a-b) = a^2 - b^2$; $(x+a)(x+b) = x^2 + (a+b)x + ab$.

[QUICK REVISION] Key Theorems & Results: $(a+b)^2 - (a-b)^2 = 4ab$; $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$.

[QUICK REVISION] Shortcut Summary: $x \pm \frac{1}{x} = k \implies x^2 + \frac{1}{x^2} = k^2 \mp 2$; plug $x = 1$ to check expansions.

[QUICK REVISION] High-Frequency Exam Patterns: Polynomial subtraction, Binomial products, Numerical squares near 100, Difference of squares, Show-that proofs.

Last-Minute Revision Checklist:

- ☐ Like terms only for $+, -$
- ☐ Include $\pm 2ab$
- ☐ $+b^2$ at end of $(a-b)^2$
- ☐ Add exponents when multiplying

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Degree/type of $5x - 7y$ is: (A) Monomial (B) Binomial (C) Trinomial (D) Constant
2. Add $3a + 2b$ and $5a - 4b$: (A) $8a - 2b$ (B) $8a + 6b$ (C) $2a - 2b$ (D) $8a + 2b$
3. Product of $3x$ and $-4y$ is: (A) $-12xy$ (B) $12xy$ (C) $-xy$ (D) $-7xy$
4. $(x+2)^2$ equals: (A) $x^2 + 4$ (B) $x^2 + 2x + 4$ (C) $x^2 + 4x + 4$ (D) $x^2 + 4x + 2$

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Value of 102^2 using identity is: (A) 10404 (B) 10004 (C) 10204 (D) 10400
2. Value of $63^2 - 37^2$ is: (A) 2600 (B) 260 (C) 1000 (D) 3600
3. Simplify $(a-b)(a+b) + b^2$: (A) a^2 (B) $a^2 + 2b^2$ (C) $2a^2$ (D) b^2
4. Subtract $2x - 3$ from $5x + 7$: (A) $3x + 10$ (B) $3x + 4$ (C) $-3x - 10$ (D) $7x + 4$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If $x + \frac{1}{x} = 7$, then $x^2 + \frac{1}{x^2}$ is: (A) 49 (B) 47 (C) 51 (D) 45
2. If $a - b = 5$ and $ab = 6$, then $a^2 + b^2$ is: (A) 13 (B) 37 (C) 31 (D) 25
3. Value of $(2x+3y)^2 - (2x-3y)^2$ is: (A) $12xy$ (B) $24xy$ (C) $8x^2 + 18y^2$ (D) $6xy$
4. Coefficient of x in $(x+3)(x-5)(x+2)$? Wait: in $(x+3)(x-5)$ it is: (A) -2 (B) 2 (C) -15 (D) 8

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** Two unlike terms. **Q2 (A):** $(3+5)a + (2-4)b = 8a - 2b$. **Q3 (A):** $3 \times (-4)xy = -12xy$. **Q4 (C):** $x^2 + 2(x)(2) + 2^2 = x^2 + 4x + 4$.
Exam Std	**Q1 (A):** $(100+2)^2 = 10000 + 400 + 4 = 10404$. **Q2 (A):** $(63+37)(63-37) = 100 \times 26 = 2600$. **Q3 (A):** $a^2 - b^2 + b^2 = a^2$. **Q4 (A):** $(5x+7) - (2x-3) = 3x + 10$.
Challenge	**Q1 (B):** $7^2 - 2 = 47$. **Q2 (B):** $a^2 + b^2 = (a-b)^2 + 2ab = 25 + 12 = 37$. **Q3 (B):** $4(2x)(3y) = 24xy$. **Q4 (A):** $x^2 + (3-5)x - 15 \implies -2$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Terms & polynomial addition/subtraction. Day 2: Monomial & binomial multiplication. Day 3: Four standard identities & numerical shortcuts.
- **How to Introduce the Concept:** Ask students to compute 103×97 mentally; then show $(100+3)(100-3) = 10000-9 = 9991$ in 2 seconds!
- **Difficult Areas & Remediation:** Sign distribution during subtraction and remembering the $2ab$ middle term in $(a \pm b)^2$.
- **Blackboard Teaching Examples:** Draw the 2×2 area square for $(a+b)^2 = a^2 + ab + ab + b^2$ so students visually see why $2ab$ cannot vanish.
- **Classroom Questions to Ask Students:** Why is $(x+2)^2$ NOT equal to $x^2 + 4$? Test with $x = 3$!
- **Common Student Doubts & Answers:** Students confuse $x + x = 2x$ with $x \times x = x^2$. Reinforce side-by-side comparison.
- **Homework & Revision Strategy:** Assign 4 subtraction/multiplication drills and 4 identity-based numerical evaluations.

[TEACHER'S TIP] Make students chant: 'First square, plus/minus twice the product, PLUS second square!'

UNIT V: MENSURATION TYBA COACHING CENTRE

Chapter 9: Mensuration

Area of Trapezium & Polygons; Surface Area & Volume of Cube, Cuboid and Cylinder (2026–27)

A. Chapter Introduction & Examination Relevance

Mensuration connects geometry to real-world measurement of land, packaging, tanks, and construction. Chapter 9 covers areas of trapeziums and general polygons in 2D, and total surface area (TSA), lateral/curved surface area (LSA/CSA), and volume of cubes, cuboids, and right circular cylinders in 3D.

- **What You Will Learn:** Area of trapezium, rhombus, and general polygons (triangulation); TSA, LSA, and volume of cuboid, cube, and cylinder; and unit conversions ($1\ \text{m}^3 = 1000\ \text{L}$, $1\ \text{L} = 1000\ \text{cm}^3$).
- **Why This Chapter Matters:** Mensuration carries 16% weightage in Class 8 and expands directly into Class 9 (Cone, Sphere, Hemisphere) and Class 10 (Combinations of Solids).
- **Real-Life Applications:** Painting walls and ceilings, designing water tanks and cylindrical pipes, calculating road-roller leveling area, and packaging box design.
- **Cross-Chapter Connections:** Uses square roots from Chapter 5 and algebraic substitution from Chapter 8; directly precedes Class 9 Chapter 11.
- **Exam & Foundation Relevance:** Unit V carries 16% weightage. Expect 2M–3M trapezium/rhombus area questions and 4M–5M word problems on painting rooms, cylindrical tanks, road rollers, and melting/recasting solids.

B. Complete Concept Explanation & Derivations

1. Area of Trapezium, Rhombus & General Polygons

A **trapezium** has one pair of parallel sides a and b separated by perpendicular height h: Area $= \frac{1}{2}(a+b)h$. A **rhombus** has perpendicular diagonals d_1, d_2: Area $= \frac{1}{2}d_1 d_2$. Any general polygon is split into triangles and trapeziums whose areas are summed.

[CONCEPT MASTER – 1. Area of Trapezium, Rhombus & General Polygons] Remember: h in a trapezium MUST be the perpendicular distance between the parallel sides, not the slanted side!

2. Surface Area of Cuboid, Cube & Cylinder

Lateral Surface Area (LSA) is the area of the vertical walls (excluding top and bottom); **Total Surface Area (TSA)** includes top and bottom. For a cuboid (l, b, h): LSA $= 2(l+b)h$, TSA $= 2(lb+bh+hl)$. For a cube of side a: LSA $= 4a^2$, TSA $= 6a^2$. For a cylinder (r, h): CSA $= 2\pi rh$, TSA $= 2\pi r(r+h)$.

[CONCEPT MASTER – 2. Surface Area of Cuboid, Cube & Cylinder] Open-top tank or painting 4 walls + ceiling? Use LSA + one base (e.g., $2(l+b)h + lb$ or $2\pi rh + \pi r^2$), NOT full TSA!

3. Volume & Capacity Conversions

Volume is the space occupied by a 3D solid: Cuboid $= lbh$, Cube $= a^3$, Cylinder $= \pi r^2 h$. Notice the universal prism rule: Volume = Base Area $\times$ Height! **Capacity** is the liquid volume a container holds: $1\ \text{m}^3 = 1000$ litres and 1 litre $= 1000\ \text{cm}^3$.

[CONCEPT MASTER – 3. Volume & Capacity Conversions] Always convert all dimensions to the SAME unit (all cm or all m) BEFORE multiplying!

4. Road Roller, Hollow Pipe & Scaling Laws

In 1 complete revolution, a cylindrical road roller covers an area equal to its **CSA** ($2\pi rh$). When a solid is melted and recast into another shape, **total volume remains constant**. If every edge of a cube is doubled ($2a$), its surface area becomes $4\times$ and its volume becomes $8\times$.

[CONCEPT MASTER – 4. Road Roller, Hollow Pipe & Scaling Laws] Road Roller Formula: Area leveled in N revolutions $= N \times 2\pi rh$.

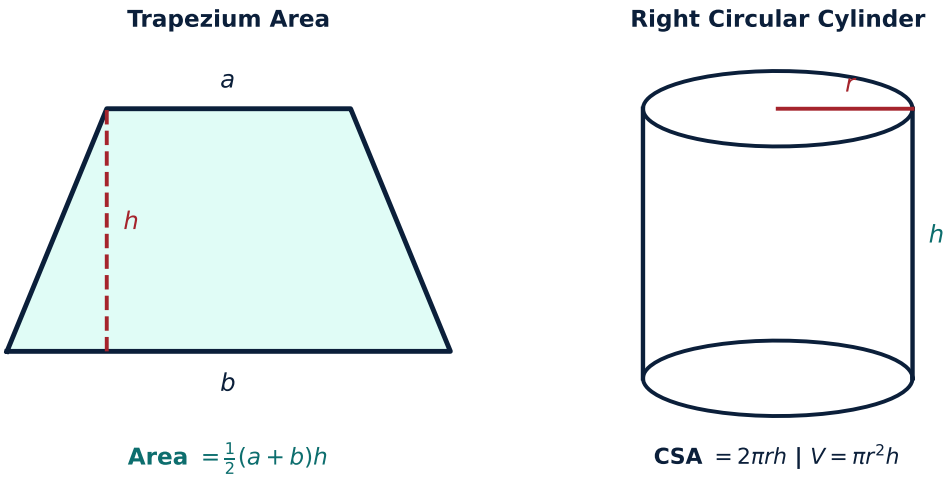


Figure 1: **Visual Model:** 2D Trapezium area formula alongside 3D Cuboid, Cube, and Cylinder surface area & volume models.

C. Complete Formula Bank

[FORMULA BANK – 1. Area of a Trapezium] **Expression:** $A = \frac{1}{2}(a+b) \times h$
Symbols: a, b: parallel sides; h: perpendicular height.
Conditions & When to Use: Used for trapeziums and polygon strips.
Solved Mini-Example: $a = 10, b = 6, h = 4 \implies A = \frac{1}{2}(16)(4) = 32$ cm^2.
Common Mistake: Using a non-parallel slanted side as h.

[FORMULA BANK – 2. Area of a Rhombus] **Expression:** $A = \frac{1}{2}d_1 d_2$
Symbols: d_1, d_2: lengths of the two diagonals.
Conditions & When to Use: Diagonals bisect at 90°.
Solved Mini-Example: $d_1 = 10, d_2 = 8.2 \implies A = 41$ cm^2.
Common Mistake: Forgetting the $\frac{1}{2}$ factor.

[FORMULA BANK – 3. Cuboid: LSA, TSA & Volume] **Expression:** LSA $= 2(l+b)h$; TSA $= 2(lb+bh+hl)$; $V = lbh$
Symbols: l: length, b: breadth, h: height.
Conditions & When to Use: All three in identical units.
Solved Mini-Example: $l = 5, b = 4, h = 3 \implies$ LSA $= 54$, TSA $= 94$, $V = 60$.
Common Mistake: Using TSA when the box has no lid.

[FORMULA BANK – 4. Cube: LSA, TSA & Volume] **Expression:** LSA $= 4a^2$; TSA $= 6a^2$; $V = a^3$
Symbols: a: edge length of the cube.
Conditions & When to Use: All 6 faces are congruent squares.
Solved Mini-Example: $a = 5$ cm $\implies$ TSA $= 150$ cm^2, $V = 125$ cm^3.
Common Mistake: Confusing $6a^2$ with a^3.

[FORMULA BANK – 5. Cylinder: CSA, TSA & Volume] **Expression:** CSA $= 2\pi rh$; TSA $= 2\pi r(r+h)$; $V = \pi r^2 h$
Symbols: r: radius $(d/2)$, h: height, $\pi \approx \frac{22}{7}$.
Conditions & When to Use: Right circular cylinder.
Solved Mini-Example: $r = 7, h = 10 \implies$ CSA $= 440, V = 1540$.
Common Mistake: Using diameter d directly instead of radius $r = d/2$!

[FORMULA BANK – 6. Capacity & Unit Conversion] **Expression:** $1\ \text{m}^3 = 1000$ L; 1 L $= 1000\ \text{cm}^3$; $1\ \text{m}^2 = 10000\ \text{cm}^2$
Symbols: L: litres.
Conditions & When to Use: Essential for tank capacity problems.
Solved Mini-Example: $1.5\ \text{m}^3 = 1500$ L.
Common Mistake: Writing $1\ \text{m}^2 = 100\ \text{cm}^2$ instead of $10000\ \text{cm}^2$.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Sainik/Navodaya/Olympiad Foundation problem structures.

Pattern 1: Trapezium Missing Side or Height

- **Identification Cue:** Area of trapezium is $34\ \text{cm}^2$, height 4 cm, one parallel side 10 cm; find the other. | **Concept & Formula:** $A = \frac{1}{2}(a+b)h$.
- **Standard Step-by-Step:** Substitute $34 = \frac{1}{2}(10+b)(4) = 2(10+b) \implies 17 = 10+b \implies b = 7$ cm.
- **Fastest Valid Method:** Compute $\frac{2A}{h} - a = \frac{68}{4} - 10 = 7$ cm. | **Alternative:** None.
- **Common Trap:** Forgetting to multiply A by 2 before dividing by h.
- **Practice Drill:** Ex. 2 & Ex. 5.

Pattern 2: Open Box / Room Painting / Cylindrical Label

- **Identification Cue:** Paint a suitcase, open tank, or paste a label on a cylinder leaving 2 cm margins. | **Concept & Formula:** Select exact faces: 4 walls + ceiling, or CSA with reduced height.
- **Standard Step-by-Step:** Subtract top/bottom margins from cylinder height for label CSA: $h_{\text{label}} = h - 4$.
- **Fastest Valid Method:** Draw a quick sketch and tick only the faces being painted! | **Alternative:** Compute TSA minus excluded face.
- **Common Trap:** Blindly applying full TSA to an open tank or label.
- **Practice Drill:** Ex. 7 & Ex. 9.

Pattern 3: Road Roller Revolutions

- **Identification Cue:** A road roller of diameter 84 cm and length 1 m takes 750 revolutions to level a road. | **Concept & Formula:** Area $= N \times 2\pi rh$.
- **Standard Step-by-Step:** Convert $r = 42$ cm $= 0.42$ m, $h = 1$ m; Area $= 750 \times 2 \times \frac{22}{7} \times 0.42 \times 1 = 1980\ \text{m}^2$.
- **Fastest Valid Method:** Cancel 7 with $0.42 \to 0.06$ immediately! | **Alternative:** Compute in cm^2 and divide by 10000.
- **Common Trap:** Using diameter 84 as radius or mixing cm and m!
- **Practice Drill:** Ex. 10.

Pattern 4: Tank Capacity, Pumping Rate & Paper Rolling

- **Identification Cue:** Water pours into a cuboidal reservoir at 60 L/min; or a rectangular sheet is rolled into a cylinder. | **Concept & Formula:** Volume $= lbh$ in L, Time $= \frac{\text{Capacity}}{\text{Rate}}$.
- **Standard Step-by-Step:** Find volume in m^3, multiply by 1000 for litres, divide by rate.
- **Fastest Valid Method:** Convert m^3 to litres first, then divide by $60 \times$ rate for hours. | **Alternative:** Keep in m^3 and convert rate.
- **Common Trap:** Dividing m^3 directly by litres/min without $\times 1000$!
- **Practice Drill:** Ex. 12 & Ex. 14.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Instant $r = 7k$ Cylinder Trick] **Core Idea & Why It Works:** When radius r is a multiple of 7 (like $7, 14, 3.5, 0.42$), use $\pi = \frac{22}{7}$ so the 7 cancels cleanly!

Valid Domain: All circle/cylinder problems with $r = 7k$. | **Where It Fails:** When $\pi = 3.14$ is explicitly mandated.

Easy Example: $r = 7, h = 10 \implies$ CSA $= 2 \times 22 \times 10 = 440$; $V = 22 \times 7 \times 10 = 1540$.

Exam-Level Example: Memorize $r = 7 \implies$ Circumference $= 44$, Area $= 154$!

Advanced Example: If $r = 14$ (double 7), Area $= 4 \times 154 = 616$!

[SMART SHORTCUT – 2. Scaling Power Law ($k \to k^2 \to k^3$)] **Core Idea & Why It Works:** If all linear dimensions are multiplied by k: Perimeter $\times k$, Surface Area $\times k^2$, Volume $\times k^3$.

Valid Domain: Cubes, cuboids (all sides scaled by k), spheres. | **Where It Fails:** If only ONE dimension (e.g., only height) is changed.

Easy Example: Cube edge doubled ($k = 2$): TSA becomes $2^2 = 4\times$; Volume becomes $2^3 = 8\times$!

Exam-Level Example: Solves NCERT Ex. 9.3 Q4 in 2 seconds!

Advanced Example: If cylinder radius is doubled and height is halved: $V_{\text{new}} = \pi(2r)^2(h/2) = 2V$.

[SMART SHORTCUT – 3. Missing Trapezium Side Formula] **Core Idea & Why It Works:** Given area A, height h, and one parallel side a, the other parallel side is $b = \frac{2A}{h} - a$.

Valid Domain: All trapeziums. | **Where It Fails:** None.

Easy Example: $A = 480, h = 15, a = 20 \implies b = \frac{960}{15} - 20 = 64 - 20 = 44$ m.

Exam-Level Example: Saves 3 algebra lines.

Advanced Example: Also $h = \frac{2A}{a+b}$.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: Find the area of a trapezium whose parallel sides are 12 cm and 8 cm and height is 5 cm.

Given & Required: $a = 12, b = 8, h = 5$. | **Concept & Formula:** $A = \frac{1}{2}(a + b)h$.

Detailed Solution: $A = \frac{1}{2}(12 + 8)(5) = \frac{1}{2}(20)(5) = 50$ cm^2. **Final Answer:** 50 **cm^2**

Fastest Alternative / Shortcut: Average parallel side $10 \times 5 = 50$.

SOLVED EXAMPLE 2 [Beginner]

Question: Find the area of a rhombus whose diagonals are 7.5 cm and 12 cm.

Given & Required: $d_1 = 7.5, d_2 = 12$. | **Concept & Formula:** $A = \frac{1}{2}d_1d_2$.

Detailed Solution: $A = \frac{1}{2} \times 7.5 \times 12 = 7.5 \times 6 = 45$ cm^2. **Final Answer:** 45 **cm^2**

Fastest Alternative / Shortcut: Halve 12 to 6 first.

SOLVED EXAMPLE 3 [Beginner]

Question: Find the side of a cube whose total surface area is 600 cm^2.

Given & Required: TSA $= 600$ cm^2. | **Concept & Formula:** $6a^2 = 600$.

Detailed Solution: $a^2 = 100 \implies a = 10$ cm. **Final Answer:** 10 **cm**

Fastest Alternative / Shortcut: $600/6 = 100 \implies 10$.

SOLVED EXAMPLE 4 [Beginner]

Question: Find the volume of a cuboid of dimensions 10 cm $\times$ 8 cm $\times$ 5 cm.

Given & Required: $l = 10, b = 8, h = 5$. | **Concept & Formula:** $V = lbh$.

Detailed Solution: $V = 10 \times 8 \times 5 = 400$ cm^3. **Final Answer:** 400 **cm^3**

Fastest Alternative / Shortcut: $8 \times 5 = 40 \times 10 = 400$.

SOLVED EXAMPLE 5 [Intermediate]

Question: The area of a trapezium is 34 cm^2 and the length of one parallel side is 10 cm and its height is 4 cm. Find the length of the other parallel side.

Given & Required: $A = 34, a = 10, h = 4$. | **Concept & Formula:** $b = \frac{2A}{h} - a$.

Detailed Solution: $34 = \frac{1}{2}(10 + b)(4) = 2(10 + b) \implies 17 = 10 + b \implies b = 7$ cm. **Final Answer:** 7 **cm**

Fastest Alternative / Shortcut: $68/4 - 10 = 7$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Find the height of a cuboid whose base area is 180 cm^2 and volume is 900 cm^3.

Given & Required: Base Area $= 180, V = 900$. | **Concept & Formula:** $V =$ Base Area $\times h$.

Detailed Solution: $h = \frac{900}{180} = 5$ cm. **Final Answer:** 5 **cm**

Fastest Alternative / Shortcut: $900/180 = 5$.

SOLVED EXAMPLE 7 [Intermediate]

Question: A closed cylindrical tank of radius 7 m and height 3 m is made from a sheet of metal. How much sheet of metal is required?

Given & Required: $r = 7$ m, $h = 3$ m. | **Concept & Formula:** TSA $= 2\pi r(r + h)$.

Detailed Solution: TSA $= 2 \times \frac{22}{7} \times 7 \times (7 + 3) = 44 \times 10 = 440$ m^2. **Final Answer:** 440 **m^2**

Fastest Alternative / Shortcut: $44 \times (7 + 3)$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Find the height of a cylinder whose radius is 7 cm and total surface area is 968 cm^2.

Given & Required: $r = 7$, TSA $= 968$. | **Concept & Formula:** $2\pi r(r + h) = 968$.

Detailed Solution: $2 \times \frac{22}{7} \times 7(7 + h) = 968 \implies 44(7 + h) = 968 \implies 7 + h = 22 \implies h = 15$ cm. **Final Answer:** 15 **cm**

Fastest Alternative / Shortcut: $968/44 - 7 = 22 - 7 = 15$.

SOLVED EXAMPLE 9 [Board Standard]

Question: Rukhsar painted the outside of a cabinet of measure 1 m $\times$ 2 m $\times$ 1.5 m. How much surface area did she cover if she painted all except the bottom of the cabinet?

Given & Required: $l = 2, b = 1, h = 1.5$ m. | **Concept & Formula:** Area $= 2(l + b)h + lb$.

Detailed Solution: $2(2 + 1)(1.5) + (2)(1) = 9 + 2 = 11$ m^2. **Final Answer:** 11 **m^2**

Fastest Alternative / Shortcut: 5 faces: TSA minus bottom lb.

SOLVED EXAMPLE 10 [Board Standard]

Question: A road roller takes 750 complete revolutions to move once over to level a road. Find the area of the road if the diameter of a road roller is 84 cm and length is 1 m.

Given & Required: $d = 84$ cm $\implies r = 0.42$ m, $h = 1$ m, $N = 750$. | **Concept & Formula:** Area $= N \times 2\pi rh$.

Detailed Solution: CSA $= 2 \times \frac{22}{7} \times 0.42 \times 1 = 2.64$ m^2. Total area $= 750 \times 2.64 = 1980$ m^2. **Final Answer:** 1980 **m^2**

Fastest Alternative / Shortcut: 750×2.64.

SOLVED EXAMPLE 11 [Board Standard]

Question: A company packages milk powder in a cylindrical container of base diameter 14 cm and height 20 cm. A label is placed around the surface 2 cm from top and bottom. Find the area of the label.

Given & Required: $r = 7$ cm, $h_{\text{label}} = 20 - 2 - 2 = 16$ cm. | **Concept & Formula:** CSA $= 2\pi r h_{\text{label}}$.

Detailed Solution: $2 \times \frac{22}{7} \times 7 \times 16 = 44 \times 16 = 704$ cm^2. **Final Answer:** 704 **cm^2**

Fastest Alternative / Shortcut: $44 \times 16 = 704$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Water is pouring into a cuboidal reservoir at the rate of 60 litres per minute. If the volume of reservoir is 108 m^3, find the number of hours it will take to fill it.

Given & Required: $V = 108$ m$^3 = 108000$ L; Rate $= 60$ L/min. | **Concept & Formula:** Time $= \frac{\text{Capacity}}{\text{Rate per hour}}$.

Detailed Solution: Rate/hr $= 60 \times 60 = 3600$ L/hr. Time $= \frac{108000}{3600} = 30$ hours. **Final Answer:** 30 **hours**

Fastest Alternative / Shortcut: $108000/3600 = 30$.

SOLVED EXAMPLE 13 [Board Standard]

Question: A rectangular piece of paper 11 cm $\times$ 4 cm is folded without overlapping to make a cylinder of height 4 cm. Find the volume of the cylinder.

Given & Required: $2\pi r = 11$ cm, $h = 4$ cm. | **Concept & Formula:** Find r from circumference, then $\pi r^2 h$.

Detailed Solution: $2 \times \frac{22}{7} \times r = 11 \implies r = \frac{7}{4}$ cm. $V = \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 4 = \frac{77}{2} = 38.5$ cm^3. **Final Answer:** 38.5 **cm^3**

Fastest Alternative / Shortcut: Keep $r = 7/4$ as fraction so 7 cancels!

SOLVED EXAMPLE 14 [Challenge]

Question: If each edge of a cube is doubled, (i) how many times will its surface area increase? (ii) how many times will its volume increase?

Given & Required: New edge $= 2a$. | **Concept & Formula:** TSA $\propto a^2, V \propto a^3$.

Detailed Solution: (i) TSA$_{\text{new}} = 6(2a)^2 = 4(6a^2) \implies 4$ times. (ii) $V_{\text{new}} = (2a)^3 = 8a^3 \implies 8$ times. **Final Answer:** 4 **times; 8 times**

Fastest Alternative / Shortcut: $2^2 = 4$, $2^3 = 8$.

SOLVED EXAMPLE 15 [Challenge]

Question: A cuboid is of dimensions 60 cm $\times$ 54 cm $\times$ 30 cm. How many small cubes with side 6 cm can be placed in the given cuboid?

Given & Required: $60, 54, 30$ (all divisible by 6). | **Concept & Formula:** Count $= \frac{V_{\text{cuboid}}}{V_{\text{cube}}}$.

Detailed Solution: $\frac{60 \times 54 \times 30}{6 \times 6 \times 6} = 10 \times 9 \times 5 = 450$ cubes. **Final Answer:** 450 **cubes**

Fastest Alternative / Shortcut: Divide each dimension by 6 first: $10 \times 9 \times 5$!

SOLVED EXAMPLE 16 [Challenge]

Question: Find the area of a trapezium whose parallel sides are 25 cm and 13 cm and non-parallel sides are 10 cm each (isosceles trapezium).

Given & Required: $a = 25, b = 13$, slant $= 10$. | **Concept & Formula:** Drop perpendiculars to find h via Pythagoras.

Detailed Solution: Base projection on each side $= \frac{25-13}{2} = 6$ cm. By Pythagoras, $h = \sqrt{10^2 - 6^2} = 8$ cm. Area $= \frac{1}{2}(25 + 13)(8) = 152$ cm^2. **Final Answer:** 152 **cm^2**

Fastest Alternative / Shortcut: 6-8-10 triple gives $h = 8$!

G. Competency MCQ Mastery Section

Q1. Area of a trapezium with parallel sides 8 cm, 12 cm and height 6 cm is:

(A) 48 cm^2 (B) 60 cm^2 (C) 120 cm^2 (D) 30 cm^2

Answer: (B) $\frac{1}{2}(8 + 12)(6) = 60$ cm^2. *Fast Method:* $10 \times 6 = 60$. *Distractor Note:* Forgetting $\frac{1}{2}$ (picking C).

Q2. Area of a rhombus with diagonals 16 cm and 12 cm is:

(A) 192 cm^2 (B) 96 cm^2 (C) 48 cm^2 (D) 56 cm^2

Answer: (B) $\frac{1}{2}(16)(12) = 96$ cm^2. *Fast Method:* 8×12. *Distractor Note:* Picking 192.

Q3. Total surface area of a cube of edge 4 cm is:

(A) 64 cm^2 (B) 96 cm^2 (C) 48 cm^2 (D) 16 cm^2

Answer: (B) $6(4)^2 = 96$ cm^2. *Fast Method:* 6×16. *Distractor Note:* Computing volume 64.

Q4. Curved surface area of a cylinder of radius 7 cm and height 10 cm is:

(A) 440 cm^2 (B) 220 cm^2 (C) 1540 cm^2 (D) 748 cm^2

Answer: (A) $2 \times \frac{22}{7} \times 7 \times 10 = 440$ cm^2. *Fast Method:* 44×10. *Distractor Note:* Computing volume 1540.

Q5. 1 m^3 is equal to:

(A) 100 L (B) 1000 L (C) 10000 L (D) 10 L

Answer: (B) 1 $\text{m}^3 = 1000$ litres. *Fast Method:* Standard capacity conversion. *Distractor Note:* Picking 100.

Q6. If the edge of a cube is tripled, its volume becomes:

(A) 3 times (B) 9 times (C) 27 times (D) 6 times

Answer: (C) $3^3 = 27$ times. *Fast Method:* k^3. *Distractor Note:* Picking 9 (that's surface area!).

Q7. Volume of a cylinder with radius r and height h is:

(A) $2\pi rh$ (B) $\pi r^2 h$ (C) $2\pi r^2 h$ (D) $\frac{1}{3}\pi r^2 h$

Answer: (B) Base area $\pi r^2 \times h$. *Fast Method:* Base $\times$ height. *Distractor Note:* Picking cone formula D.

Q8. Lateral surface area of a cuboid of $l = 6, b = 4, h = 5$ is:

(A) 100 (B) 148 (C) 120 (D) 50

Answer: (A) $2(6+4)(5) = 100$. *Fast Method:* Perimeter $\times$ height. *Distractor Note:* Computing TSA 148.

Q9. How many 2 cm cubes fit inside a 6 cm $\times$ 4 cm $\times$ 2 cm cuboid?

(A) 6 (B) 12 (C) 24 (D) 8

Answer: (A) $\frac{6\times4\times2}{2\times2\times2} = 3 \times 2 \times 1 = 6$. *Fast Method:* Divide each side by 2. *Distractor Note:* Dividing volume by 2 instead of 8.

Q10. If cylinder radius is doubled and height is halved, the volume:

(A) Halves (B) Remains same (C) Doubles (D) Quadruples

Answer: (C) $(2)^2 \times \frac{1}{2} = 2$. *Fast Method:* Scale factor $4 \times 0.5 = 2$. *Distractor Note:* Thinking it stays same.

Q11. Area of 4 walls of a room is also called:

(A) TSA (B) LSA (C) Volume (D) Perimeter

Answer: (B) 4 walls $= 2(l+b)h =$ LSA. *Fast Method:* Lateral Surface Area. *Distractor Note:* Picking TSA.

Q12. A cylinder has diameter 14 cm and volume 1540 cm^3. Its height is:

(A) 5 cm (B) 10 cm (C) 2.5 cm (D) 20 cm

Answer: (B) $r = 7 \implies \pi r^2 = 154 \implies h = 1540/154 = 10$ cm. *Fast Method:* Base area $= 154$. *Distractor Note:* Using $r = 14$.

H. Special Exam & MCQ Strategies

- Always write 'Given: $d = \cdots \implies r = d/2$' on the first line so you never plug diameter into r.
- Check unit consistency (cm vs m) before doing any multiplication.
- In word problems on tanks, boxes, or rooms, count how many faces are actually painted or covered.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Using diameter d directly in $\pi r^2 h$ or $2\pi r h$. $\longrightarrow$ **Correction:** Always halve diameter first: $r = d/2$.

[COMMON MISTAKE] **Error:** Converting $1\ \text{m}^2 = 100\ \text{cm}^2$. $\longrightarrow$ **Correction:** $1\ \text{m}^2 = 100\ \text{cm} \times 100\ \text{cm} = 10,000\ \text{cm}^2$, and $1\ \text{m}^3 = 1,000,000\ \text{cm}^3$.

[COMMON MISTAKE] **Error:** Using full TSA for open tanks or 4-wall room painting. $\longrightarrow$ **Correction:** Subtract the open face(s) or use LSA + base.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: Trapezium: $\frac{1}{2}(a+b)h$; Rhombus: $\frac{1}{2}d_1d_2$; Cuboid: $2(lb+bh+hl), lbh$; Cube: $6a^2, a^3$; Cylinder: $2\pi rh, 2\pi r(r+h), \pi r^2h$.

[QUICK REVISION] Key Theorems & Results: $1\text{ m}^3 = 1000\text{ L}$; $1\text{ L} = 1000\text{ cm}^3$; Road roller area $= N \times 2\pi rh$.

[QUICK REVISION] Shortcut Summary: $r = 7 \implies$ Base $= 154$, Circ $= 44$; Scaling by $k \implies$ Area k^2, Vol k^3.

[QUICK REVISION] High-Frequency Exam Patterns: Missing trapezium side, Room/cabinet painting, Road roller revolutions, Tank filling time, Paper-to-cylinder rolling.

Last-Minute Revision Checklist:

- ☐ Halve diameter to r
- ☐ Match all units
- ☐ Count faces (open vs closed)
- ☐ Convert $\text{m}^3 \to 1000$ L

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Area of trapezium with $a = 6, b = 10, h = 5$: (A) 40 (B) 80 (C) 30 (D) 50
2. Volume of cube with edge 3 cm: (A) 9 cm^3 (B) 27 cm^3 (C) 54 cm^3 (D) 18 cm^3
3. LSA of cube of side 5 cm: (A) 100 cm^2 (B) 150 cm^2 (C) 125 cm^2 (D) 25 cm^2
4. Volume of cuboid $5 \times 4 \times 2$: (A) 20 (B) 40 (C) 76 (D) 38

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Radius of cylinder with CSA 88 cm^2 and $h = 14$ cm: (A) 1 cm (B) 2 cm (C) 7 cm (D) 4 cm
2. Capacity in litres of a tank 2 m $\times$ 1.5 m $\times$ 1 m: (A) 3 L (B) 300 L (C) 3000 L (D) 30 L
3. Area of rhombus with diagonals 10 cm and 24 cm: (A) 120 cm^2 (B) 240 cm^2 (C) 60 cm^2 (D) 130 cm^2
4. Surface area of an open cuboidal box of $l = 4, b = 3, h = 2$: (A) 52 (B) 40 (C) 28 (D) 44

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If a cylinder's height is doubled and radius is halved, its volume: (A) Doubles (B) Halves (C) Stays same (D) Becomes $1/4$
2. Number of 3 cm cubes cut from a $9 \times 6 \times 3$ cm cuboid: (A) 6 (B) 9 (C) 18 (D) 27
3. TSA of a solid cylinder of $d = 14$ cm and $h = 3$ cm: (A) 440 cm^2 (B) 132 cm^2 (C) 286 cm^2 (D) 308 cm^2
4. Side of a cube whose volume numerically equals its TSA: (A) 4 (B) 6 (C) 8 (D) 12

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (A):** $\frac{1}{2}(16)(5) = 40$. **Q2 (B):** $3^3 = 27$. **Q3 (A):** $4(5)^2 = 100$. **Q4 (B):** $5 \times 4 \times 2 = 40$.
Exam Std	**Q1 (A):** $2 \times \frac{22}{7} \times r \times 14 = 88 \implies 88r = 88 \implies r = 1$ cm. **Q2 (C):** $3\ \text{m}^3 = 3000$ L. **Q3 (A):** $\frac{1}{2}(10)(24) = 120$. **Q4 (B):** $2(4+3)(2) + 4(3) = 28 + 12 = 40$.
Challenge	**Q1 (B):** $\pi(r/2)^2(2h) = \frac{1}{2}\pi r^2 h$. **Q2 (A):** $3 \times 2 \times 1 = 6$. **Q3 (A):** $2 \times \frac{22}{7} \times 7(7+3) = 440$. **Q4 (B):** $a^3 = 6a^2 \implies a = 6$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Trapezium, rhombus & polygon areas. Day 2: Cuboid & cube LSA/TSA/Volume. Day 3: Cylinder CSA/TSA/Volume & unit conversions.
- **How to Introduce the Concept:** Wrap a rectangular A4 sheet into a cylinder in front of the class to show why CSA = circumference $\times$ height $= 2\pi rh$!
- **Difficult Areas & Remediation:** Choosing between LSA and TSA in word problems, and $1\ \text{m}^3 = 1000$ L conversions.
- **Blackboard Teaching Examples:** Draw the 6 faces of a cuboid and cross out the lid for 'open box' problems.
- **Classroom Questions to Ask Students:** If you double the radius of a pizza, do you get twice as much pizza or four times as much?
- **Common Student Doubts & Answers:** Students confuse $1\ \text{m}^2 = 100\ \text{cm}^2$; draw a $100\ \text{cm} \times 100\ \text{cm}$ grid to prove $10,000\ \text{cm}^2$.
- **Homework & Revision Strategy:** Assign 3 trapezium/polygon problems, 3 cylinder problems, and 2 capacity/roller word problems.

[TEACHER'S TIP] Drilling $r = 7 \implies C = 44, A = 154$ makes 80% of school cylinder calculations effortless.

UNIT I: NUMBER SYSTEM TYBA COACHING CENTRE

Chapter 10: Exponents and Powers

Negative Exponents, Laws of Exponents & Scientific Standard Form (2026–27)

A. Chapter Introduction & Examination Relevance

Class 7 introduced positive exponents; Chapter 10 extends powers to negative integers and teaches how to read, compare, and compute extremely large astronomical numbers and ultra-tiny microscopic measurements using scientific standard form.

- **What You Will Learn:** Multiplicative inverse $a^{-m} = \frac{1}{a^m}$, all 6 laws of exponents for integer powers (positive, zero, and negative), expressing very large and very small numbers in standard form $k \times 10^n$ $(1 \leq k < 10)$, and solving exponential equations for m.
- **Why This Chapter Matters:** Powers and standard form are essential in Physics, Chemistry (Avogadro's number, atomic sizes), Class 9 Number System (rational exponents), and Class 11–12 Logarithms.
- **Real-Life Applications:** Measuring cell sizes (10^{-6} m), computer memory (2^{10} bytes), and distances between planets (1.496×10^{11} m).
- **Cross-Chapter Connections:** Builds on Class 7 Exponents and Chapter 1 Rational Numbers; leads into Class 9 Chapter 1 (Laws of Exponents for Real Numbers).
- **Exam & Foundation Relevance:** Unit I carries 20% weightage. Expect 1M–2M negative-exponent simplification and standard-form MCQs, plus 3M–4M 'find the value of m' and multi-law simplification questions.

B. Complete Concept Explanation & Derivations

1. Negative Exponents & Multiplicative Inverse

For any non-zero integer or rational number a and positive integer m, $a^{-m} = \frac{1}{a^m}$. Thus a^{-m} is the **multiplicative inverse** (reciprocal) of a^m, because $a^m \times a^{-m} = a^0 = 1$. For fractions, $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$!

[CONCEPT MASTER – 1. Negative Exponents & Multiplicative Inverse] Fraction Flip Rule: A negative sign in the exponent flips the fraction upside down—it does NOT make the answer negative!

2. Complete Laws of Exponents (All Integers)

All exponent laws hold for negative, zero, and positive integer exponents: (1) $a^m \times a^n = a^{m+n}$, (2) $a^m \div a^n = a^{m-n}$, (3) $(a^m)^n = a^{mn}$, (4) $a^m \times b^m = (ab)^m$, (5) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$, and (6) $a^0 = 1$ $(a \neq 0)$.

[CONCEPT MASTER – 2. Complete Laws of Exponents (All Integers)] Watch signs in division: $2^3 \div 2^{-5} = 2^{3-(-5)} = 2^{3+5} = 2^8$, NOT 2^{-2}!

3. Solving Exponential Equations ($a^x = a^y \implies x = y$)

To solve for an unknown exponent m in equations like $(-3)^{m+1} \times (-3)^5 = (-3)^7$, first reduce both sides to a single power of the SAME base, then equate the exponents ($m + 6 = 7 \implies m = 1$). Always prime-factorize bases like $4, 8, 9, 25, 125$ into $2, 3, 5$ first.

[CONCEPT MASTER – 3. Solving Exponential Equations ($a^x = a^y \implies x = y$)] If bases match $(a \neq 0, 1, -1)$, equate exponents: $a^x = a^y \implies x = y$!

4. Standard Form (Scientific Notation)

A number is in **standard form** if written as $k \times 10^n$, where $1 \leq k < 10$ (exactly ONE non-zero digit to the left of the decimal point) and n is an integer. Moving the decimal point LEFT gives a positive n; moving RIGHT (for tiny numbers < 1) gives a negative n.

[CONCEPT MASTER – 4. Standard Form (Scientific Notation)] LARS Rule: Decimal Left → Add to exponent; Decimal Right → Subtract from exponent!

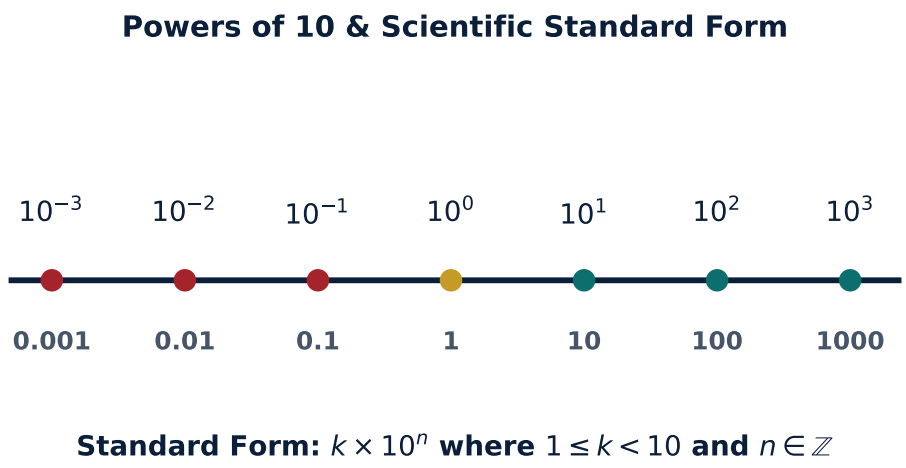


Figure 1: **Visual Model:** Powers of 10 scale from astronomical (10^{11}) down to microscopic (10^{-6}) standard form.

C. Complete Formula Bank

[FORMULA BANK – 1. Negative Exponent & Fraction Flip] **Expression:** $a^{-m} = \frac{1}{a^m}$; $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}$
Symbols: $a, b \neq 0$; $m \in \mathbb{Z}$.
Conditions & When to Use: Makes negative exponents positive.
Solved Mini-Example: $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^{3} = \frac{27}{8}$.
Common Mistake: Writing $-\left(\frac{2}{3}\right)^{3}$ instead of flipping.

[FORMULA BANK – 2. Product & Quotient Laws] **Expression:** $a^m \cdot a^n = a^{m+n}$; $\frac{a^m}{a^n} = a^{m-n}$
Symbols: Same base $a \neq 0$.
Conditions & When to Use: Bases must be identical.
Solved Mini-Example: $5^{-3} \times 5^{7} = 5^{4} = 625$.
Common Mistake: Multiplying exponents instead of adding.

[FORMULA BANK – 3. Power of a Power] **Expression:** $(a^m)^n = a^{mn}$
Symbols: $a \neq 0$; $m, n \in \mathbb{Z}$.
Conditions & When to Use: Nested exponents.
Solved Mini-Example: $\left((-2)^{-2}\right)^{3} = (-2)^{-6} = \frac{1}{64}$.
Common Mistake: Adding $m + n$ instead of multiplying mn.

[FORMULA BANK – 4. Same Exponent Laws] **Expression:** $a^m \cdot b^m = (ab)^m$; $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^{m}$
Symbols: $a, b \neq 0$; identical exponent m.
Conditions & When to Use: Combine different bases with same power.
Solved Mini-Example: $2^{-3} \times (-7)^{-3} = (-14)^{-3}$.
Common Mistake: Multiplying m by m.

[FORMULA BANK – 5. Zero Exponent Law] **Expression:** $a^0 = 1$ for all $a \neq 0$
Symbols: a: any non-zero real number.
Conditions & When to Use: 0^0 is undefined.
Solved Mini-Example: $(3^{-1} + 4^{-1} + 5^{-1})^0 = 1$.
Common Mistake: Writing $a^0 = 0$!

[FORMULA BANK – 6. Standard Form] **Expression:** $N = k \times 10^n$, where $1 \leq k < 10, n \in \mathbb{Z}$
Symbols: k: mantissa; n: order of magnitude.
Conditions & When to Use: Only 1 non-zero digit before decimal.
Solved Mini-Example: $0.000035 = 3.5 \times 10^{-5}$.
Common Mistake: Writing 35×10^{-6} (invalid since $35 \geq 10$).

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Sainik/Navodaya/Olympiad Foundation problem structures.

Pattern 1: Sum/Difference of Negative Powers

- **Identification Cue:** Evaluate $(3^{-1} + 4^{-1} + 5^{-1})^0$ or $(2^{-1} \times 4^{-1}) \div 2^{-2}$. | **Concept & Formula:** Convert $a^{-1} = 1/a$ inside $+$ or $-$; laws apply only to $\times, \div$.
- **Standard Step-by-Step:** Convert $(3^{-1} + 4^{-1})$ to $\frac{1}{3} + \frac{1}{4} = \frac{7}{12}$ BEFORE applying outer powers.
- **Fastest Valid Method:** Spot outer power 0: anything non-zero to power 0 is 1 instantly! | **Alternative:** None.
- **Common Trap:** Trying to apply $a^m + a^n = a^{m+n}$ (NO such law exists for $+$ or $-$!).
- **Practice Drill:** Ex. 3 & Ex. 5.

Pattern 2: Simplifying Prime-Power Fractions

- **Identification Cue:** Simplify $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$ or $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$. | **Concept & Formula:** Break composite bases ($10 = 2 \cdot 5, 6 = 2 \cdot 3, 25 = 5^2$) into primes.
- **Standard Step-by-Step:** Rewrite every number as powers of $2, 3, 5$; move negative powers across the fraction bar.
- **Fastest Valid Method:** Flip negative exponents between numerator and denominator to make all powers positive! | **Alternative:** Use a^{m-n} directly.
- **Common Trap:** Forgetting to distribute the exponent to BOTH factors of $10^{-5} = 2^{-5} \cdot 5^{-5}$.
- **Practice Drill:** Ex. 10 & Ex. 14.

Pattern 3: Solving for Unknown Exponent m

- **Identification Cue:** Find m so that $(-3)^{m+1} \times (-3)^5 = (-3)^7$ or $5^m \div 5^{-3} = 5^5$. | **Concept & Formula:** Combine left side, equate exponents.
- **Standard Step-by-Step:** $m + 1 + 5 = 7 \implies m = 1$; for division: $m - (-3) = 5 \implies m + 3 = 5 \implies m = 2$.
- **Fastest Valid Method:** Write the exponent equation directly once bases match. | **Alternative:** Check by plugging m back.
- **Common Trap:** Writing $m - 3 = 5 \implies m = 8$ instead of $m - (-3) = 5$!
- **Practice Drill:** Ex. 8 & Ex. 11.

Pattern 4: Standard Form & Usual Form Conversions

- **Identification Cue:** Express 0.00000000837 in standard form or 3.02×10^{-6} in usual form. | **Concept & Formula:** Count decimal shifts.
- **Standard Step-by-Step:** Shift decimal right 9 places $\rightarrow 8.37 \times 10^{-9}$.
- **Fastest Valid Method:** For $0.000...d$: count total zeros (including the one before decimal) to get the negative power! | **Alternative:** Divide by 10^k.
- **Common Trap:** Off-by-one error in counting zeros.

- **Practice Drill:** Ex. 6 & Ex. 12.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Elevator Rule for Negative Exponents] **Core Idea & Why It Works:** In a fraction, any factor with a negative exponent moves to the opposite floor (numerator ↔ denominator) and its exponent turns positive!
Valid Domain: All product/quotient expressions. | **Where It Fails:** Terms separated by $+$ or $-$ (must convert to fractions first).
Easy Example: $\frac{5^{-3}}{t^{-8}} = \frac{t^8}{5^3}$; $\frac{2^{-2}}{3^{-3}} = \frac{3^3}{2^2} = \frac{27}{4}$.
Exam-Level Example: Eliminates double-negative subtraction errors!
Advanced Example: Essential for NCERT Ex. 10.1 Q7.

[SMART SHORTCUT – 2. Zero-Count Trick for Small Standard Form] **Core Idea & Why It Works:** For any number $0.00\ldots d\ldots$, the negative exponent of 10 equals the TOTAL number of leading zeros (including the one before the decimal point)!
Valid Domain: All decimals $0 < x < 1$. | **Where It Fails:** Numbers ≥ 1.
Easy Example: 0.00085 has 4 zeros before $8 \implies 8.5 \times 10^{-4}$!
Exam-Level Example: Zero-error MCQ conversion in 1 second!
Advanced Example: Reverse: 3.02×10^{-6} has 6 zeros before $3 \to 0.00000302$.

[SMART SHORTCUT – 3. Instant Power-Zero Trap Detector] **Core Idea & Why It Works:** Whenever you see $(\text{long messy expression})^0$, check ONLY that the inside is non-zero—if so, the answer is immediately 1!
Valid Domain: Any non-zero base. | **Where It Fails:** If the inside simplifies to 0.
Easy Example: $(3^{-1} + 4^{-1} + 5^{-1})^0 = 1$ without adding fractions!
Exam-Level Example: Saves 45 seconds in exams.
Advanced Example: Also $(a^0 + b^0) = 1 + 1 = 2$, NOT 1!

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Evaluate: (i) 3^{-2}, (ii) $(-4)^{-2}$, (iii) $\left(\frac{1}{2}\right)^{-5}$.
Given & Required: Negative exponents. | **Concept & Formula:** $a^{-m} = \frac{1}{a^m}$.
Detailed Solution: (i) $\frac{1}{3^2} = \frac{1}{9}$. (ii) $\frac{1}{(-4)^2} = \frac{1}{16}$. (iii) $2^5 = 32$. **Final Answer: (i)** $\frac{1}{9}$**, (ii)** $\frac{1}{16}$**, (iii)** 32
Fastest Alternative / Shortcut: Flip the fraction, make power positive.

SOLVED EXAMPLE 2 [Beginner]
Question: Simplify and express in positive exponent: $(-4)^5 \div (-4)^8$.
Given & Required: Same base -4. | **Concept & Formula:** $a^m \div a^n = a^{m-n}$.
Detailed Solution: $(-4)^{5-8} = (-4)^{-3} = \frac{1}{(-4)^3}$. **Final Answer:** $\frac{1}{(-4)^3}$
Fastest Alternative / Shortcut: $5 - 8 = -3 \to$ denominator.

SOLVED EXAMPLE 3 [Beginner]
Question: Find the multiplicative inverse of: (i) 7^{-2}, (ii) 10^{-5}.
Given & Required: Negative powers. | **Concept & Formula:** Inverse of a^{-m} is a^m.
Detailed Solution: (i) $7^2 = 49$, (ii) 10^5. **Final Answer:** 7^2 **and** 10^5
Fastest Alternative / Shortcut: Negate the exponent!

SOLVED EXAMPLE 4 [Beginner]

Question: Express 3.61492×10^6 in usual form.

Given & Required: Power $+6$. | **Concept & Formula:** Shift decimal 6 places right.

Detailed Solution: $3.61492 \times 1000000 = 3614920$. **Final Answer:** 3614920

Fastest Alternative / Shortcut: 5 digits after decimal + 1 zero.

SOLVED EXAMPLE 5 [Intermediate]

Question: Find the value of $(3^0 + 4^{-1}) \times 2^2$.

Given & Required: $3^0 = 1, 4^{-1} = 1/4$. | **Concept & Formula:** Zero and negative powers.

Detailed Solution: $\left(1 + \frac{1}{4}\right) \times 4 = \frac{5}{4} \times 4 = 5$. **Final Answer:** 5

Fastest Alternative / Shortcut: Distribute 4: $4(1) + 4(1/4) = 4 + 1 = 5$!

SOLVED EXAMPLE 6 [Intermediate]

Question: Express in standard form: (i) 0.00000000837, (ii) 31860000000.

Given & Required: Very small and very large numbers. | **Concept & Formula:** $k \times 10^n, 1 \leq k < 10$.

Detailed Solution: (i) 9 leading zeros $\rightarrow 8.37 \times 10^{-9}$. (ii) Shift left 10 places $\rightarrow 3.186 \times 10^{10}$. **Final Answer: (i)** 8.37×10^{-9}, **(ii)** 3.186×10^{10}

Fastest Alternative / Shortcut: Zero-count rule for (i).

SOLVED EXAMPLE 7 [Intermediate]

Question: Evaluate $\left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}$.

Given & Required: Sum of negative powers. | **Concept & Formula:** Flip each fraction first.

Detailed Solution: $2^2 + 3^2 + 4^2 = 4 + 9 + 16 = 29$. **Final Answer:** 29

Fastest Alternative / Shortcut: $4 + 9 + 16 = 29$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Find the value of m for which $5^m \div 5^{-3} = 5^5$.

Given & Required: $5^m \div 5^{-3} = 5^5$. | **Concept & Formula:** $a^m \div a^n = a^{m-n}$.

Detailed Solution: $5^{m-(-3)} = 5^5 \implies m + 3 = 5 \implies m = 2$. **Final Answer:** $m = 2$

Fastest Alternative / Shortcut: $m + 3 = 5 \implies m = 2$.

SOLVED EXAMPLE 9 [Board Standard]

Question: Evaluate $\frac{8^{-1} \times 5^3}{2^{-4}}$.

Given & Required: $8 = 2^3$. | **Concept & Formula:** Convert 8 to base 2.

Detailed Solution: $\frac{(2^3)^{-1} \times 5^3}{2^{-4}} = \frac{2^{-3} \times 5^3}{2^{-4}} = 2^{-3-(-4)} \times 5^3 = 2^1 \times 125 = 250$. **Final Answer:** 250

Fastest Alternative / Shortcut: Elevator: $2^4/2^3 \times 125 = 250$.

SOLVED EXAMPLE 10 [Board Standard]

Question: Simplify $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}}$ $(t \neq 0)$.

Given & Required: $25 = 5^2, 10 = 2 \times 5$. | **Concept & Formula:** Prime bases + elevator rule.

Detailed Solution: $\frac{5^2 \times 5^3 \times t^8}{2 \times 5^1 \times t^4} = \frac{5^{5-1} \times t^{8-4}}{2} = \frac{625t^4}{2}$. **Final Answer:** $\frac{625}{2}t^4$

Fastest Alternative / Shortcut: Move negative powers across bar.

SOLVED EXAMPLE 11 [Board Standard]

Question: Evaluate $\left\{\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right\}^{-1}$.

Given & Required: Nested negative powers. | **Concept & Formula:** Inside-out.

Detailed Solution: $\{3 - 4\}^{-1} = (-1)^{-1} = -1$. **Final Answer:** -1

Fastest Alternative / Shortcut: $3 - 4 = -1$, reciprocal is -1.

SOLVED EXAMPLE 12 [Board Standard]

Question: In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness in standard form?

Given & Required: $5 \times 20 + 5 \times 0.016$. | **Concept & Formula:** Add, then convert.

Detailed Solution: Books $= 100$ mm; Sheets $= 0.080$ mm. Total $= 100.08$ mm $= 1.0008 \times 10^2$ mm. **Final Answer:** 1.0008×10^2 **mm**

Fastest Alternative / Shortcut: Keep all decimal digits!

SOLVED EXAMPLE 13 [Board Standard]

Question: Evaluate $\left(\frac{5}{8}\right)^{-7} \times \left(\frac{8}{5}\right)^{-4}$.

Given & Required: Reciprocal bases. | **Concept & Formula:** Flip one base to match.

Detailed Solution: $\left(\frac{8}{5}\right)^{7} \times \left(\frac{8}{5}\right)^{-4} = \left(\frac{8}{5}\right)^{7-4} = \left(\frac{8}{5}\right)^{3} = \frac{512}{125}$. **Final Answer:** $\frac{512}{125}$

Fastest Alternative / Shortcut: Match bases to $8/5$.

SOLVED EXAMPLE 14 [Challenge]

Question: Simplify: $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$.

Given & Required: $10 = 2 \cdot 5, 6 = 2 \cdot 3, 125 = 5^3$. | **Concept & Formula:** Prime factorize all bases.

Detailed Solution: $\frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}} = 3^0 \times 2^0 \times 5^{-5+3-(-7)} = 5^5 = 3125$. **Final Answer:** 3125

Fastest Alternative / Shortcut: 3^{-5} and 2^{-5} cancel completely!

SOLVED EXAMPLE 15 [Challenge]

Question: Find x if $\left(\frac{2}{9}\right)^{3} \times \left(\frac{2}{9}\right)^{-6} = \left(\frac{2}{9}\right)^{2x-1}$.

Given & Required: Same base $2/9$. | **Concept & Formula:** Equate exponents.

Detailed Solution: $3 + (-6) = 2x - 1 \implies -3 = 2x - 1 \implies 2x = -2 \implies x = -1$. **Final Answer:** $x = -1$

Fastest Alternative / Shortcut: $-3 = 2x - 1 \implies x = -1$.

SOLVED EXAMPLE 16 [Challenge]

Question: Compare the sizes of a red blood cell (0.000007 m) and a plant cell (0.00001275 m) by finding their approximate ratio.

Given & Required: 7×10^{-6} and 1.275×10^{-5}. | **Concept & Formula:** Convert to same power of 10 or divide.

Detailed Solution: Plant cell $= 1.275 \times 10^{-5} = 12.75 \times 10^{-6}$. Ratio $\frac{\text{RBC}}{\text{Plant}} = \frac{7}{12.75} \approx \frac{1}{2}$ (RBC is about half the plant cell). **Final Answer:** $\approx 1 : 2$

Fastest Alternative / Shortcut: Compare exponents first (10^{-6} vs 10^{-5}).

G. Competency MCQ Mastery Section

Q1. Value of 2^{-3} is:

(A) -6 (B) -8 (C) $\frac{1}{8}$ (D) $\frac{1}{6}$

Answer: (C) $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$. *Fast Method:* Reciprocal of 8. *Distractor Note:* Thinking negative power makes number negative.

Q2. Value of $(3^{-1} + 5^{-1} + 7^{-1})^0$ is:

(A) 0 (B) 1 (C) $\frac{71}{105}$ (D) 3

Answer: (B) Any non-zero number raised to 0 is 1. *Fast Method:* Power $0 \to 1$. *Distractor Note:* Writing 0.

Q3. Multiplicative inverse of $(-5)^{-3}$ is:

(A) 5^{-3} (B) $(-5)^3$ (C) 5^3 (D) $-\frac{1}{125}$

Answer: (B) Product $(-5)^{-3} \times (-5)^3 = 1$. *Fast Method:* Flip exponent sign only. *Distractor Note:* Dropping the minus sign on -5.

Q4. Standard form of 0.000045 is:

(A) 4.5×10^{-5} (B) 4.5×10^{-4} (C) 45×10^{-6} (D) 4.5×10^{5}

Answer: (A) 5 leading zeros $\rightarrow 4.5 \times 10^{-5}$. *Fast Method:* Zero-count rule. *Distractor Note:* Picking C ($45 \geq 10$).

Q5. If $3^{x} = \frac{1}{81}$, then x is:

(A) 4 (B) -4 (C) -3 (D) 3

Answer: (B) $\frac{1}{81} = 3^{-4} \implies x = -4$. *Fast Method:* $81 = 3^{4}$. *Distractor Note:* Picking $+4$.

Q6. Value of $(2^{-1} - 4^{-1}) \times 2^{2}$ is:

(A) 1 (B) 2 (C) -1 (D) 4

Answer: (A) $(\frac{1}{2} - \frac{1}{4}) \times 4 = \frac{1}{4} \times 4 = 1$. *Fast Method:* $2 - 1 = 1$. *Distractor Note:* Subtracting exponents.

Q7. Usual form of 2.05×10^{-4} is:

(A) 0.00205 (B) 0.000205 (C) 0.0000205 (D) 20500

Answer: (B) 4 leading zeros: 0.000205. *Fast Method:* Zero-count rule. *Distractor Note:* Miscounting zeros.

Q8. Value of $\left(\frac{1}{2}\right)^{-3}$ is:

(A) $\frac{1}{8}$ (B) -8 (C) 8 (D) 6

Answer: (C) $2^{3} = 8$. *Fast Method:* Flip to 2^{3}. *Distractor Note:* Picking -8.

Q9. $(-2)^{-3} \times (-2)^{-2}$ equals:

(A) $(-2)^{-5}$ (B) $(-2)^{6}$ (C) $(-2)^{-1}$ (D) 4^{5}

Answer: (A) $-3 + (-2) = -5$. *Fast Method:* Add exponents. *Distractor Note:* Multiplying $(-3)(-2) = 6$.

Q10. If $5^{2m-1} = 125$, then m is:

(A) 1 (B) 2 (C) 3 (D) 4

Answer: (B) $125 = 5^{3} \implies 2m - 1 = 3 \implies m = 2$. *Fast Method:* $2m = 4 \implies m = 2$. *Distractor Note:* Setting $2m - 1 = 125$.

Q11. $\left(\frac{3}{4}\right)^{0} + \left(\frac{4}{3}\right)^{0}$ equals:

(A) 0 (B) 1 (C) 2 (D) 7/12

Answer: (C) $1 + 1 = 2$. *Fast Method:* Two terms of 1. *Distractor Note:* Writing 1.

Q12. Which number is larger: 3.2×10^{-4} or 9.1×10^{-5}?

(A) 3.2×10^{-4} (B) 9.1×10^{-5} (C) Equal (D) Cannot say

Answer: (A) $-4 > -5$, so $10^{-4} > 10^{-5}$. *Fast Method:* Compare power of 10 first! *Distractor Note:* Comparing 3.2 vs 9.1.

H. Special Exam & MCQ Strategies

- Never apply exponent laws across $+$ or $-$ signs; convert negative powers to fractions first.
- In fraction simplifications, factorize $4, 6, 8, 9, 10, 12, 25, 125$ into primes $2, 3, 5$ immediately.
- Use the Zero-Count rule to check standard form conversions.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Writing $2^{-3} = -8$ or -6. $\longrightarrow$ **Correction:** A negative exponent means reciprocal ($\frac{1}{2^{3}} = \frac{1}{8}$), NOT a negative sign!

[COMMON MISTAKE] **Error:** Computing $5^{2} \div 5^{-3} = 5^{2-3} = 5^{-1}$. $\longrightarrow$ **Correction:** Subtracting a negative adds: $5^{2-(-3)} = 5^{5}$.

[COMMON MISTAKE] **Error:** Writing 24.5×10^5 as standard form. $\longrightarrow$ **Correction:** The multiplier k MUST satisfy $1 \leq k < 10$, so write 2.45×10^6.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $a^{-m} = \frac{1}{a^m}$; $a^m a^n = a^{m+n}$; $\frac{a^m}{a^n} = a^{m-n}$; $(a^m)^n = a^{mn}$; $a^m b^m = (ab)^m$; $a^0 = 1$.

[QUICK REVISION] Key Theorems & Results: $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$; Standard Form: $k \times 10^n$ $(1 \leq k < 10)$.

[QUICK REVISION] Shortcut Summary: Elevator rule (swap floor to flip exponent sign); Zero-count rule for $0.000\ldots$; LARS rule.

[QUICK REVISION] High-Frequency Exam Patterns: Negative power evaluation, Prime-base fraction simplification, Solve for m, Standard/Usual form.

Last-Minute Revision Checklist:

- ☐ Negative power $\neq$ negative number
- ☐ No law for $+$ or $-$
- ☐ Factorize composite bases
- ☐ $1 \leq k < 10$ in standard form

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Value of 5^{-2} is: (A) -25 (B) $\frac{1}{25}$ (C) -10 (D) 25
2. Value of $7^0 + 8^0$ is: (A) 0 (B) 1 (C) 2 (D) 15
3. Standard form of 450000 is: (A) 4.5×10^5 (B) 45×10^4 (C) 4.5×10^4 (D) 0.45×10^6
4. $(-3)^4 \div (-3)^2$ equals: (A) -9 (B) 9 (C) -6 (D) 81

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Value of $(2^{-1} + 3^{-1})^{-1}$ is: (A) 5 (B) 6/5 (C) 5/6 (D) 1/5
2. If $2^{m-1} = 32$, then m is: (A) 5 (B) 6 (C) 4 (D) 7
3. Standard form of 0.0000072 is: (A) 7.2×10^{-6} (B) 7.2×10^{-5} (C) 7.2×10^{-7} (D) 72×10^{-7}
4. Evaluate $4^{-1} \times 2^3$: (A) 2 (B) 1 (C) 4 (D) 1/2

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Simplify $\frac{2^{-3} \times 3^{-2}}{6^{-2}}$: (A) 1/2 (B) 2 (C) 1/6 (D) 6
2. If $10^x = 0.0001$, then x is: (A) -3 (B) -4 (C) -5 (D) 4
3. Value of $(1^{-1} + 2^{-1} + 3^{-1})^0 \times 5^{-1}$ is: (A) 1/5 (B) 0 (C) 5 (D) 1
4. If $2^x \times 4^x = 64$, then x is: (A) 1 (B) 2 (C) 3 (D) 4

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $1/5^2 = 1/25$. **Q2 (C):** $1+1=2$. **Q3 (A):** Shift 5 places left. **Q4 (B):** $(-3)^2 = 9$.
Exam Std	**Q1 (B):** $(5/6)^{-1} = 6/5$. **Q2 (B):** $32 = 2^5 \implies m-1=5 \implies m=6$. **Q3 (A):** 6 leading zeros. **Q4 (A):** $8/4 = 2$.
Challenge	**Q1 (A):** $\frac{6^2}{2^3\times 3^2} = \frac{36}{72} = \frac{1}{2}$. **Q2 (B):** $0.0001 = 10^{-4}$. **Q3 (A):** $1 \times 1/5 = 1/5$. **Q4 (B):** $2^{3x} = 2^6 \implies x = 2$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Negative exponents & multiplicative inverses. Day 2: All 6 laws & solving for m. Day 3: Scientific standard form & comparisons.
- **How to Introduce the Concept:** Write $2^3 = 8, 2^2 = 4, 2^1 = 2, 2^0 = ?, 2^{-1} = ?$ and ask students to continue dividing by 2—they discover $2^0 = 1$ and $2^{-1} = 1/2$ themselves!
- **Difficult Areas & Remediation:** Double-negative subtraction in $a^m \div a^{-n}$ and distributing powers over composite numbers like 10^{-5}.
- **Blackboard Teaching Examples:** Draw a two-story building for the Elevator Rule showing negative exponents moving floors.
- **Classroom Questions to Ask Students:** Why is 25×10^4 NOT in standard form?
- **Common Student Doubts & Answers:** Students ask why a^{-m} is positive when $a > 0$; remind them negative power means division, not subtraction.
- **Homework & Revision Strategy:** Assign 4 negative-exponent simplifications, 2 'find m' equations, and 4 standard-form conversions.

[TEACHER'S TIP] Emphasize prime-factoring $4, 6, 8, 9, 10, 25, 125$ as the golden key to every tough exponent problem.

UNIT III: RATIO & PROPORTION TYBA COACHING CENTRE

Chapter 11: Direct and Inverse Proportions

Direct Variation ($x/y = k$), Inverse Variation ($xy = k$), Speed–Time & Work–Men Problems (2026–27)

A. Chapter Introduction & Examination Relevance

Quantities in real life rarely change in isolation: more workers finish a wall faster, while more distance requires more fuel. Chapter 11 teaches how to identify Direct vs Inverse proportion instantly and solve ratio, speed–distance–time, and work-force problems with zero guesswork.

- **What You Will Learn:** Recognizing direct proportion ($\frac{x}{y} = k$) vs inverse proportion ($xy = k$), building proportion tables, solving multi-step unitary and cross-multiplication problems, and handling time–speed, gear–wheel, and food–provision problems.
- **Why This Chapter Matters:** Direct and inverse proportion is the foundation of Class 9–10 linear models, Physics (Ohm's Law, Boyle's Law, Speed–Time), and competitive exams (Navodaya, Sainik School, SSC, BSTC).
- **Real-Life Applications:** Map scale calculations, travel time estimation, recipe scaling, garrison food supply planning, and gear ratios.
- **Cross-Chapter Connections:** Connects Chapter 7 (Comparing Quantities) and Chapter 13 (Introduction to Graphs, where $y = kx$ is plotted).
- **Exam & Foundation Relevance:** Unit III carries 15% weightage. Expect 1M proportion-identification MCQs and 3M–5M tabular & word problems on maps, shadows, workers, and food provisions.

B. Complete Concept Explanation & Derivations

1. Direct Proportion ($x \propto y \iff \frac{x}{y} = k$)

Two quantities x and y are in **direct proportion** if they increase or decrease together in the same ratio, so $\frac{x}{y} = k$ (constant). For any two pairs (x_1, y_1) and (x_2, y_2), $\frac{x_1}{y_1} = \frac{x_2}{y_2}$. Examples: number of items vs total cost; distance travelled vs fuel consumed at constant speed; shadow length vs object height at the same time.

[CONCEPT MASTER – 1. Direct Proportion ($x \propto y \iff \frac{x}{y} = k$)] Test for Direct Proportion: Check if ALL ratios $\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{x_3}{y_3}$ are equal!

2. Inverse Proportion ($x \propto \frac{1}{y} \iff xy = k$)

Two quantities x and y are in **inverse proportion** if an increase in x causes a proportional decrease in y (and vice versa) such that their product remains constant: $x_1y_1 = x_2y_2 = k$. Examples: number of workers vs days to finish a job; speed vs travel time for a fixed distance; pipe count vs tank filling time.

[CONCEPT MASTER – 2. Inverse Proportion ($x \propto \frac{1}{y} \iff xy = k$)] Golden Check: Before setting up an equation, ask: 'If x DOUBLES, does y DOUBLE (Direct: $x_1/y_1 = x_2/y_2$) or HALVE (Inverse: $x_1y_1 = x_2y_2$)?'

3. Speed–Distance–Time & Map Scale Problems

At constant speed, Distance $\propto$ Time (Direct). For a fixed distance, Speed $\times$ Time = Distance (Inverse). On maps and scale models, Map Distance $\propto$ Actual Distance (Direct). Always convert units (minutes to hours, or cm to km) cleanly.

[CONCEPT MASTER – 3. Speed–Distance–Time & Map Scale Problems] Speed conversion: 1 km/h = $\frac{5}{18}$ m/s and 1 m/s = $\frac{18}{5}$ km/h.

4. Work–Men–Days & Provision Problems

For a fixed task or fixed food stock: (Workers) $\times$ (Days) = Constant, and (Persons) $\times$ (Days of food) = Constant. If M more persons join after d days, first subtract d from the initial days to get the remaining stock for the original group!

[CONCEPT MASTER – 4. Work–Men–Days & Provision Problems] Caution: Not every increasing relationship is direct proportion! E.g., age vs height, or $y = x + 5$ is NEITHER direct nor inverse.

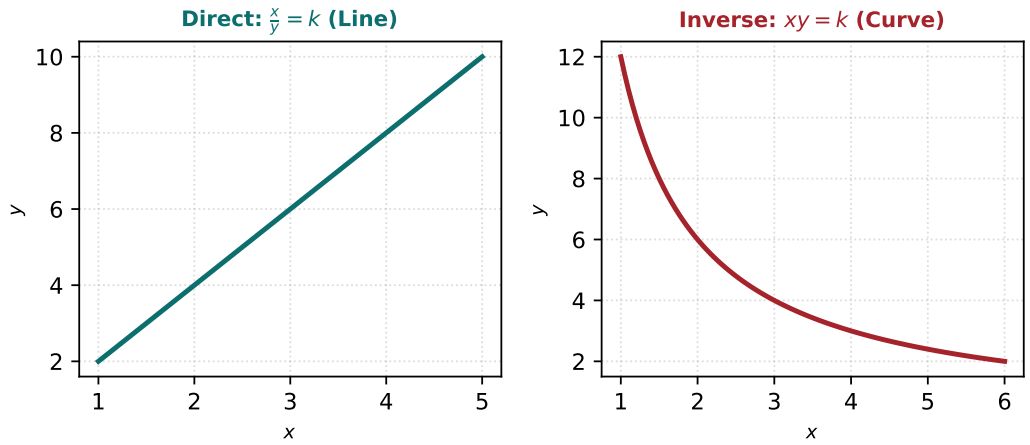


Figure 1: **Visual Model:** Side-by-side comparison of Direct Proportion ($y = kx$, straight line through origin) vs Inverse Proportion ($xy = k$, rectangular hyperbola).

C. Complete Formula Bank

[FORMULA BANK – 1. Direct Proportion Law] **Expression:** $\frac{x_1}{y_1} = \frac{x_2}{y_2} = k$ (constant)
Symbols: x_1, x_2: values of x; y_1, y_2: corresponding values of y.
Conditions & When to Use: Both quantities increase/decrease in same ratio.
Solved Mini-Example: Cost of 5 pens $= 40 \implies$ cost of 8 pens $= \frac{40}{5} \times 8 = 64$.
Common Mistake: Using $x_1 y_1 = x_2 y_2$ for direct variation.

[FORMULA BANK – 2. Inverse Proportion Law] **Expression:** $x_1 y_1 = x_2 y_2 = k$ (constant)
Symbols: x: e.g., workers or speed; y: e.g., days or time.
Conditions & When to Use: When x increases, y decreases proportionally.
Solved Mini-Example: 6 pipes fill a tank in 80 min $\implies$ 5 pipes take $\frac{6 \times 80}{5} = 96$ min.
Common Mistake: Cross-multiplying like direct proportion!

[FORMULA BANK – 3. Speed–Distance–Time Link] **Expression:** $D = S \times T$
Symbols: D: distance, S: speed, T: time.
Conditions & When to Use: Fixed $S \implies D \propto T$; Fixed $D \implies S \cdot T = k$ (inverse).
Solved Mini-Example: 60 km/h for 2 h covers 120 km; at 80 km/h, time $= \frac{120}{80} = 1.5$ h.
Common Mistake: Mixing minutes with km/h without dividing by 60.

[FORMULA BANK – 4. Man-Days Work Formula] **Expression:** $M_1 D_1 = M_2 D_2$
Symbols: M: workers; D: days (assuming equal work rate).
Conditions & When to Use: Same total work.
Solved Mini-Example: 15 workers build a wall in 48 h $\implies$ for 30 h, need $\frac{15 \times 48}{30} = 24$ workers.
Common Mistake: Saying 30 h needs fewer workers.

[FORMULA BANK – 5. Map Scale Formula] **Expression:** $\frac{\text{Map}_1}{\text{Actual}_1} = \frac{\text{Map}_2}{\text{Actual}_2}$
Symbols: Keep map units (cm) on top, actual units (km) below.
Conditions & When to Use: Direct proportion.
Solved Mini-Example: 1 cm : 18 km $\implies$ 4 cm $= 72$ km.
Common Mistake: Inverting one ratio.

[FORMULA BANK – 6. Work-Hours Per Day Extension] **Expression:** $M_1 D_1 H_1 = M_2 D_2 H_2$
Symbols: H: hours worked per day.
Conditions & When to Use: Same total work.
Solved Mini-Example: 10 men $\times 6$ days $\times 8$ h $= 480$ man-hours.
Common Mistake: Ignoring hours per day when given.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Sainik/Navodaya/Olympiad Foundation problem structures.

Pattern 1: Tabular Direct vs Inverse Check

- **Identification Cue:** Given a table of (x, y) values, find if x, y are in direct or inverse proportion and fill blanks. | **Concept & Formula:** Check x/y (direct) vs xy (inverse).
- **Standard Step-by-Step:** Compute x_1/y_1 and $x_1 y_1$ on complete columns to identify k, then solve missing cells.
- **Fastest Valid Method:** Multiply first column ($x_1 y_1$) and divide first column (x_1/y_1)—whichever stays constant in column 2 is the law! | **Alternative:** None.
- **Common Trap:** Assuming it's direct without checking column 2!
- **Practice Drill:** Ex. 1 & Ex. 5.

Pattern 2: Shadow, Map & Machine Production (Direct)

- **Identification Cue:** A 14 m pole casts a 10 m shadow; find height of tree casting 15 m shadow. | **Concept & Formula:** $\frac{x_1}{y_1} = \frac{x_2}{y_2}$.
- **Standard Step-by-Step:** $\frac{14}{10} = \frac{h}{15} \implies h = \frac{14 \times 15}{10} = 21$ m.
- **Fastest Valid Method:** Scale factor: $15 = 1.5 \times 10 \implies h = 1.5 \times 14 = 21$ m! | **Alternative:** Unitary method.
- **Common Trap:** Swapping numerator and denominator on one side.
- **Practice Drill:** Ex. 2 & Ex. 4.

Pattern 3: Workers, Pipes & Garrison Food (Inverse)

- **Identification Cue:** 6 pipes fill a tank in 1 h 20 min; how long will 5 pipes of the same type take? | **Concept & Formula:** $x_1 y_1 = x_2 y_2$.
- **Standard Step-by-Step:** Convert 1 h 20 min $= 80$ min. $6 \times 80 = 5 \times t \implies t = 96$ min $= 1$ h 36 min.
- **Fastest Valid Method:** $6 \times 80/5 = 6 \times 16 = 96$ min. | **Alternative:** None.
- **Common Trap:** Writing 1.20 hours instead of 80 minutes!
- **Practice Drill:** Ex. 7 & Ex. 9.

Pattern 4: Midway Change in Workers / Hostel Provisions

- **Identification Cue:** 100 students have food for 20 days; if 25 more join, how long will food last? | **Concept & Formula:** Inverse proportion on remaining/total man-days.
- **Standard Step-by-Step:** $100 \times 20 = 125 \times D \implies D = \frac{2000}{125} = 16$ days.
- **Fastest Valid Method:** Students increased by $\frac{5}{4}$ ($100 \to 125$), so days multiply by $\frac{4}{5}$ ($20 \to 16$)! | **Alternative:** None.
- **Common Trap:** Using 25 instead of $100 + 25 = 125$ total students!
- **Practice Drill:** Ex. 10 & Ex. 14.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. The 'Double-Test' Law Detector] **Core Idea & Why It Works:** Before writing any formula, ask: 'If the first quantity DOUBLES, does the second quantity DOUBLE (Direct: $\frac{x_1}{y_1} = \frac{x_2}{y_2}$) or HALVE (Inverse: $x_1 y_1 = x_2 y_2$)?'
Valid Domain: 100% of word problems. | **Where It Fails:** Non-proportional relations (e.g., $y = x + 3$).
Easy Example: More speed $\rightarrow$ LESS time to reach school $\implies$ Inverse ($S_1 T_1 = S_2 T_2$).
Exam-Level Example: Prevents setting up the wrong equation!
Advanced Example: Works for 3-variable problems too ($M_1 D_1 H_1 = M_2 D_2 H_2$).

[SMART SHORTCUT – 2. Ratio-Flip Shortcut for Inverse Proportion] **Core Idea & Why It Works:** If quantity A changes in the ratio $p : q$, an inversely proportional quantity B changes in the flipped ratio $q : p$!
Valid Domain: All inverse proportion problems. | **Where It Fails:** Direct proportion.
Easy Example: Speed increases from 60 to 80 (ratio $3 : 4$) $\implies$ Time scales by $\frac{3}{4}$: 40 min $\times \frac{3}{4} = 30$ min!
Exam-Level Example: Mental MCQ solver in 3 seconds!
Advanced Example: Workers $15 \rightarrow 20$ $(3 : 4)$ $\implies$ Days $48 \rightarrow 48 \times \frac{3}{4} = 36$.

[SMART SHORTCUT – 3. 1-Minute Time Unit Guard] **Core Idea & Why It Works:** ALWAYS convert 'hours + minutes' (like 1 hr 20 min) into pure minutes (80 min) BEFORE multiplying!
Valid Domain: All time/pipe/speed problems. | **Where It Fails:** None.
Easy Example: 1 h 20 min $= 80$ min (NOT 1.20 h!).
Exam-Level Example: Avoids the single biggest Class 8 decimal trap.
Advanced Example: Convert back at the end: 96 min $= 1$ h 36 min.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: If the cost of 5 metres of cloth is Rs. 210, what is the cost of 8 metres of cloth?
Given & Required: $x_1 = 5, y_1 = 210, x_2 = 8$. | **Concept & Formula:** Direct proportion.
Detailed Solution: $\frac{5}{210} = \frac{8}{y_2} \implies y_2 = \frac{210 \times 8}{5} = 42 \times 8 = 336$. **Final Answer: Rs.** 336
Fastest Alternative / Shortcut: 1 m $= 42 \implies 8$ m $= 336$.

SOLVED EXAMPLE 2 [Beginner]
Question: Check whether x and y are in direct or inverse proportion: $(x, y) = (2, 12), (4, 6), (6, 4)$.
Given & Required: 3 pairs. | **Concept & Formula:** Test xy vs x/y.
Detailed Solution: $2 \times 12 = 24$, $4 \times 6 = 24$, $6 \times 4 = 24$. Product xy is constant, so Inverse Proportion. **Final Answer: Inverse proportion ($xy = 24$)**
Fastest Alternative / Shortcut: x increases while y decreases.

SOLVED EXAMPLE 3 [Beginner]
Question: A machine fills 840 bottles in 6 hours. How many bottles will it fill in 5 hours?
Given & Required: 840 bottles in 6 h. | **Concept & Formula:** Direct proportion.
Detailed Solution: $\frac{840}{6} = \frac{x}{5} \implies x = 140 \times 5 = 700$ bottles. **Final Answer:** 700 **bottles**
Fastest Alternative / Shortcut: 140/hour $\times 5 = 700$.

SOLVED EXAMPLE 4 [Beginner]

Question: If 12 workers take 10 days to complete a job, how many days will 15 workers take?

Given & Required: $M_1 = 12, D_1 = 10, M_2 = 15$. | **Concept & Formula:** Inverse proportion.

Detailed Solution: $12 \times 10 = 15 \times D_2 \implies D_2 = \frac{120}{15} = 8$ days. **Final Answer:** 8 **days**

Fastest Alternative / Shortcut: $120/15 = 8$.

SOLVED EXAMPLE 5 [Intermediate]

Question: An electric pole, 14 metres high, casts a shadow of 10 metres. Find the height of a tree that casts a shadow of 15 metres under similar conditions.

Given & Required: 14 m pole $\rightarrow$ 10 m shadow. | **Concept & Formula:** Direct proportion.

Detailed Solution: $\frac{14}{10} = \frac{h}{15} \implies h = \frac{14 \times 15}{10} = 21$ m. **Final Answer:** 21 **m**

Fastest Alternative / Shortcut: $15 = 1.5 \times 10 \implies 1.5 \times 14 = 21$.

SOLVED EXAMPLE 6 [Intermediate]

Question: A photograph of a bacteria enlarged $50,000$ times attains a length of 5 cm. What is the actual length of the bacteria? If enlarged $20,000$ times, what would be its enlarged length?

Given & Required: $50000\times \rightarrow 5$ cm. | **Concept & Formula:** Direct proportion.

Detailed Solution: Actual $= \frac{5}{50000} = 10^{-4}$ cm. At $20000\times$: $\frac{5}{50000} = \frac{L}{20000} \implies L = 2$ cm. **Final Answer:** 10^{-4} **cm;** 2 **cm**

Fastest Alternative / Shortcut: $10000\times = 1$ cm $\implies 20000\times = 2$ cm.

SOLVED EXAMPLE 7 [Intermediate]

Question: A loaded truck travels 14 km in 25 minutes. If the speed remains the same, how far can it travel in 5 hours?

Given & Required: 14 km in 25 min; 5 h = 300 min. | **Concept & Formula:** Direct proportion (match units!).

Detailed Solution: $\frac{14}{25} = \frac{d}{300} \implies d = \frac{14 \times 300}{25} = 14 \times 12 = 168$ km. **Final Answer:** 168 **km**

Fastest Alternative / Shortcut: $300/25 = 12 \implies 14 \times 12 = 168$.

SOLVED EXAMPLE 8 [Intermediate]

Question: If a box of sweets is divided among 24 children, they will get 5 sweets each. How many would each get, if the number of the children is reduced by 4?

Given & Required: 24 children $\rightarrow$ 5 sweets; reduced by 4 $\implies$ 20 children. | **Concept & Formula:** Inverse proportion.

Detailed Solution: $24 \times 5 = 20 \times s \implies s = \frac{120}{20} = 6$ sweets each. **Final Answer:** 6 **sweets**

Fastest Alternative / Shortcut: Total sweets = 120; $120/20 = 6$.

SOLVED EXAMPLE 9 [Board Standard]

Question: Rashmi has a road map with a scale of 1 cm representing 18 km. She drives on a road for 72 km. What would be her distance covered in the map?

Given & Required: 1 cm : 18 km. | **Concept & Formula:** Direct proportion.

Detailed Solution: $\frac{1}{18} = \frac{x}{72} \implies x = \frac{72}{18} = 4$ cm. **Final Answer:** 4 **cm**

Fastest Alternative / Shortcut: $72/18 = 4$.

SOLVED EXAMPLE 10 [Board Standard]

Question: If 15 workers can build a wall in 48 hours, how many workers will be required to do the same work in 30 hours?

Given & Required: $M_1 = 15, T_1 = 48, T_2 = 30$. | **Concept & Formula:** Inverse proportion.

Detailed Solution: $15 \times 48 = M_2 \times 30 \implies M_2 = \frac{720}{30} = 24$ workers. **Final Answer:** 24 **workers**

Fastest Alternative / Shortcut: $15/30 = 1/2 \implies 48/2 = 24$.

SOLVED EXAMPLE 11 [Board Standard]

Question: 6 pipes are required to fill a tank in 1 hour 20 minutes. How long will it take if only 5 pipes of the same type are used?

Given & Required: 6 pipes $\rightarrow$ 80 min. | **Concept & Formula:** Inverse proportion.

Detailed Solution: $6 \times 80 = 5 \times t \implies t = \frac{480}{5} = 96$ min $= 1$ hour 36 minutes. **Final Answer:** 1 **hr** 36 **min**

Fastest Alternative / Shortcut: $480/5 = 96$ min.

SOLVED EXAMPLE 12 [Board Standard]

Question: There are 100 students in a hostel. Food provision for them is for 20 days. How long will these provisions last, if 25 more students join the group?

Given & Required: 100 students $\rightarrow$ 20 days; new count $= 125$. | **Concept & Formula:** Inverse proportion.

Detailed Solution: $100 \times 20 = 125 \times D \implies D = \frac{2000}{125} = 16$ days. **Final Answer:** 16 **days**

Fastest Alternative / Shortcut: $100 \rightarrow 125$ $(\times 5/4) \implies 20 \times 4/5 = 16$.

SOLVED EXAMPLE 13 [Board Standard]

Question: A car takes 2 hours to reach a destination by travelling at the speed of 60 km/h. How long will it take when the car travels at the speed of 80 km/h?

Given & Required: $S_1 = 60, T_1 = 2, S_2 = 80$. | **Concept & Formula:** Inverse proportion ($D = ST$).

Detailed Solution: $60 \times 2 = 80 \times T_2 \implies T_2 = \frac{120}{80} = 1.5$ hours (1 hr 30 min). **Final Answer:** 1 **hr** 30 **min**

Fastest Alternative / Shortcut: $120/80 = 1.5$ h.

SOLVED EXAMPLE 14 [Challenge]

Question: A batch of bottles were packed in 25 boxes with 12 bottles in each box. If the same batch is packed using 20 bottles in each box, how many boxes would be filled?

Given & Required: 25×12 total bottles. | **Concept & Formula:** Inverse proportion.

Detailed Solution: $25 \times 12 = B \times 20 \implies B = \frac{300}{20} = 15$ boxes. **Final Answer:** 15 **boxes**

Fastest Alternative / Shortcut: $300/20 = 15$.

SOLVED EXAMPLE 15 [Challenge]

Question: Two persons could fit new windows in a house in 3 days. (i) One of the persons fell ill before the work started. How long would the job take now? (ii) How many persons would be needed to fit the windows in one day?

Given & Required: 2 persons $\rightarrow$ 3 days. | **Concept & Formula:** Inverse proportion ($M \times D = 6$).

Detailed Solution: (i) $1 \times D = 6 \implies 6$ days. (ii) $M \times 1 = 6 \implies 6$ persons. **Final Answer:** **(i)** 6 **days, (ii)** 6 **persons**

Fastest Alternative / Shortcut: Total work $= 2 \times 3 = 6$ man-days.

SOLVED EXAMPLE 16 [Challenge]

Question: A fort had provisions for 300 men for 90 days. After 20 days, 50 men left the fort. How long would the remaining food last at the same rate?

Given & Required: Remaining after 20 days: 300 men for 70 days; new men $= 250$. | **Concept & Formula:** Inverse proportion on remaining days.

Detailed Solution: Remaining food $= 300 \times (90 - 20) = 300 \times 70$ man-days. With 250 men: $250 \times D = 21000 \implies D = 84$ days. **Final Answer:** 84 **days**

Fastest Alternative / Shortcut: Subtract 20 days FIRST: $300 \times 70/250 = 84$!

G. Competency MCQ Mastery Section

Q1. If x and y are in direct proportion, then:

(A) $xy =$ constant (B) $x/y =$ constant (C) $x + y =$ constant (D) $x - y =$ constant

Answer: (B) Ratio x/y remains constant. *Fast Method:* Direct $\rightarrow$ divide. *Distractor Note:* Confusing with $xy = k$ (inverse).

Q2. Which pair is in INVERSE proportion?

(A) Petrol and distance (B) Workers and days to finish a wall (C) Items and total cost (D) Radius and circumference

Answer: (B) More workers → fewer days. *Fast Method:* Opposite direction. *Distractor Note:* Picking A or C.

Q3. If $x \propto y$ and $x = 12$ when $y = 4$, find x when $y = 10$.

(A) 20 (B) 30 (C) 40 (D) 4.8

Answer: (B) $12/4 = x/10 \implies x = 30$. *Fast Method:* $x = 3y$. *Distractor Note:* Using $12 \times 4/10 = 4.8$.

Q4. If x and y vary inversely, and $x = 15$ when $y = 6$, find y when $x = 9$.

(A) 10 (B) 3.6 (C) 22.5 (D) 9

Answer: (A) $15 \times 6 = 9 \times y \implies y = 90/9 = 10$. *Fast Method:* $xy = 90$. *Distractor Note:* Using direct proportion.

Q5. A car travels 90 km in 2.5 hours. Distance in 4 hours at same speed is:

(A) 120 km (B) 144 km (C) 150 km (D) 160 km

Answer: (B) Speed $= 90/2.5 = 36$ km/h; $36 \times 4 = 144$ km. *Fast Method:* $36 \times 4 = 144$. *Distractor Note:* Arithmetic error.

Q6. 8 pipes fill a tank in 30 minutes. 12 pipes will fill it in:

(A) 45 min (B) 20 min (C) 15 min (D) 24 min

Answer: (B) $8 \times 30 = 12 \times t \implies t = 20$ min. *Fast Method:* $240/12 = 20$. *Distractor Note:* Using direct proportion (45 min).

Q7. If 18 notebooks cost Rs. 360, cost of 7 notebooks is:

(A) Rs. 140 (B) Rs. 120 (C) Rs. 160 (D) Rs. 180

Answer: (A) 1 notebook $= 20$; $7 \times 20 = 140$. *Fast Method:* Unitary. *Distractor Note:* None.

Q8. At 50 km/h, a train takes 6 hours. To cover the same distance in 5 hours, speed must be:

(A) 55 km/h (B) 60 km/h (C) 65 km/h (D) 41.6 km/h

Answer: (B) $50 \times 6 = S \times 5 \implies S = 60$ km/h. *Fast Method:* $300/5 = 60$. *Distractor Note:* Picking 41.6.

Q9. If $x = 5$ when $y = 15$ and $x = 10$ when $y = 30$, then x and y are in:

(A) Direct proportion (B) Inverse proportion (C) No proportion (D) None

Answer: (A) $x/y = 1/3$ in both. *Fast Method:* Both double together. *Distractor Note:* None.

Q10. Map scale is 1 cm = 25 km. Actual distance for 6.4 cm is:

(A) 150 km (B) 160 km (C) 175 km (D) 125 km

Answer: (B) $6.4 \times 25 = 160$ km. *Fast Method:* $25 \times 6.4 = 100 \times 1.6 = 160$. *Distractor Note:* Decimal shift error.

Q11. 40 cows graze a field in 16 days. How many cows graze it in 10 days?

(A) 25 (B) 64 (C) 50 (D) 60

Answer: (B) $40 \times 16 = C \times 10 \implies C = 64$. *Fast Method:* $640/10 = 64$. *Distractor Note:* Picking 25 (fewer cows for fewer days!).

Q12. If x and y are inversely proportional, which of the following is TRUE?

(A) $x_1/x_2 = y_1/y_2$ (B) $x_1/x_2 = y_2/y_1$ (C) $x_1 + y_1 = x_2 + y_2$ (D) $x_1 - x_2 = y_1 - y_2$

Answer: (B) $x_1 y_1 = x_2 y_2 \iff \frac{x_1}{x_2} = \frac{y_2}{y_1}$. *Fast Method:* Cross-multiply B to get $x_1 y_1 = x_2 y_2$. *Distractor Note:* Picking A.

H. Special Exam & MCQ Strategies

- Always run the 'Double-Test' first: if x doubles, does y double (Direct) or halve (Inverse)?
- Convert mixed hours-minutes into total minutes before multiplying in inverse proportion.
- In garrison/hostel food problems where people join or leave AFTER d days, subtract d from total days FIRST.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Using cross-multiplication ($x_1/y_1 = x_2/y_2$) for worker-days or speed-time problems. $\longrightarrow$ **Correction:** Worker-days and speed-time (fixed distance) are INVERSE: use $x_1 y_1 = x_2 y_2$.

[COMMON MISTAKE] **Error:** Writing 1 hour 20 minutes $= 1.20$ hours. $\longrightarrow$ **Correction:** 20 minutes $= \frac{20}{60} = \frac{1}{3}$ hour, or simply convert to 80 minutes!

[COMMON MISTAKE] **Error:** Using '25 more students joined' as $x_2 = 25$. $\longrightarrow$ **Correction:** $x_2 = 100 + 25 = 125$ total students!

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: Direct: $\frac{x_1}{y_1} = \frac{x_2}{y_2} = k$; Inverse: $x_1 y_1 = x_2 y_2 = k$; Work: $M_1 D_1 H_1 = M_2 D_2 H_2$.

[QUICK REVISION] Key Theorems & Results: Direct graph is a line through origin; Inverse graph is a hyperbola.

[QUICK REVISION] Shortcut Summary: Double-test; Ratio-flip ($p : q \to q : p$); Always convert hr+min to minutes.

[QUICK REVISION] High-Frequency Exam Patterns: Table completion, Pole-shadow/Map scale, Pipes/Workers/Speed-time, Hostel/Garrison provisions.

Last-Minute Revision Checklist:

- ☐ Run Double-Test
- ☐ Convert hr+min to min
- ☐ Add/subtract joining/leaving count
- ☐ Check reasonableness of answer

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. If 6 pens cost Rs. 72, cost of 10 pens is: (A) Rs. 100 (B) Rs. 120 (C) Rs. 144 (D) Rs. 90
2. If x and y vary directly and $x = 3, y = 15$, constant $k = x/y$ is: (A) 5 (B) 1/5 (C) 45 (D) 12
3. More speed for same distance means: (A) More time (B) Less time (C) Same time (D) Direct variation
4. If 10 men do a job in 6 days, 1 man takes: (A) 60 days (B) 16 days (C) 6 days (D) 10 days

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. 14 m pole casts 10 m shadow. Shadow of 21 m tree is: (A) 15 m (B) 14 m (C) 20 m (D) 12 m
2. 24 workers build a road in 15 days. Workers needed for 10 days: (A) 16 (B) 36 (C) 30 (D) 40
3. A tap fills a tank in 45 min. 3 identical taps fill it in: (A) 15 min (B) 135 min (C) 30 min (D) 20 min
4. Train at 75 km/h covers what distance in 20 minutes? (A) 15 km (B) 25 km (C) 30 km (D) 37.5 km

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. 50 students have food for 16 days. If 30 more join, food lasts: (A) 10 days (B) 12 days (C) 8 days (D) 25.6 days
2. If x increases by 25% and $xy = k$, then y decreases by: (A) 25% (B) 20% (C) 15% (D) 30%
3. A car at 60 km/h takes 4 hours. At 80 km/h it saves how much time? (A) 1 hr (B) 1.5 hr (C) 3 hr (D) 0.5 hr
4. Gears with 24 teeth and 16 teeth mesh. If the 24-tooth gear makes 10 turns, the 16-tooth gear makes: (A) 15 turns (B) 6.67 turns (C) 12 turns (D) 20 turns

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $12 \times 10 = 120$. **Q2 (B):** $3/15 = 1/5$. **Q3 (B):** Inverse proportion. **Q4 (A):** $10 \times 6 = 60$ days.
Exam Std	**Q1 (A):** $14/10 = 21/s \implies s = 15$ m. **Q2 (B):** $24 \times 15/10 = 36$. **Q3 (A):** $45/3 = 15$ min. **Q4 (B):** $75 \times \frac{20}{60} = 25$ km.
Challenge	**Q1 (A):** $50 \times 16/80 = 10$ days. **Q2 (B):** x becomes $5/4 \implies y$ becomes $4/5$ (down 20%). **Q3 (A):** New time $= 240/80 = 3$ h; saved $= 4 - 3 = 1$ hr. **Q4 (A):** $24 \times 10/16 = 15$ turns.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Direct proportion & tables. Day 2: Inverse proportion & speed/work. Day 3: Mixed word problems & provision shifts.
- **How to Introduce the Concept:** Ask: 'If 1 singer sings a song in 3 minutes, how long do 3 singers take?' Laugh together, then contrast with '1 worker paints a room in 3 hours'!
- **Difficult Areas & Remediation:** Distinguishing direct vs inverse in word problems and handling 'more men joined' additions.
- **Blackboard Teaching Examples:** Draw two side-by-side arrows: ↑↑ for Direct ($x/y = k$) and ↑↓ for Inverse ($xy = k$).
- **Classroom Questions to Ask Students:** If you double your walking speed, what happens to the time it takes to reach school?
- **Common Student Doubts & Answers:** Students think any two increasing columns are in direct proportion; test $y = x + 2$ to show ratios aren't equal.
- **Homework & Revision Strategy:** Assign 3 direct, 3 inverse, and 2 tricky time-unit problems.

[TEACHER'S TIP] Always ask students to predict whether the answer should be BIGGER or SMALLER before calculating.

UNIT II: ALGEBRA TYBA COACHING CENTRE

Chapter 12: Factorisation

Common Factors, Regrouping, Algebraic Identities, Middle-Term Splitting & Polynomial Division (2026–27)

A. Chapter Introduction & Examination Relevance

Factorisation is the exact reverse of multiplication—breaking a polynomial down into a product of its irreducible algebraic factors. Chapter 12 equips you with four systematic factorisation techniques plus polynomial division and error-spotting skills.

- **What You Will Learn:** Method of common factors, factorisation by regrouping terms, factorisation using identities $a^2 \pm 2ab + b^2$ and $a^2 - b^2$, splitting the middle term for $x^2 + bx + c$, division of polynomials (monomial by monomial, polynomial by monomial, and polynomial by polynomial via factors), and spotting common algebra errors.
- **Why This Chapter Matters:** Factorisation is one of the highest-utility skills in mathematics: used directly in Class 9 Polynomials, Class 10 Quadratic Equations, Class 11–12 Calculus, and JEE/Olympiad algebra.
- **Real-Life Applications:** Simplifying rational algebraic formulas, finding roots/zeros of equations, and determining unknown dimensions from polynomial areas.
- **Cross-Chapter Connections:** Exact inverse of Chapter 8 (Algebraic Expressions & Identities); directly leads into Class 9 Chapter 2 and Class 10 Chapter 4.
- **Exam & Foundation Relevance:** Unit II (Algebra) carries 25% weightage. Expect 2M common-factor/regrouping questions, 3M–4M identity ($a^2 - b^2$, $p^2 + 6p + 8$) factorisations, and 3M–4M polynomial division problems.

B. Complete Concept Explanation & Derivations

1. Method of Common Factors (HCF Extraction)

Always check for a **common factor FIRST**! Find the HCF of all numerical coefficients and the lowest power of every variable appearing in ALL terms. Factor it outside parentheses: $ab + ac = a(b + c)$. Never leave a common factor inside the brackets!

[CONCEPT MASTER – 1. Method of Common Factors (HCF Extraction)] First Commandment of Factorisation: ALWAYS pull out the Highest Common Factor (HCF) before trying any other method!

2. Factorisation by Regrouping Terms

When a 4-term expression has no single factor common to all 4 terms (e.g., $ax + ay + bx + by$), group terms in pairs that share a common factor: $a(x + y) + b(x + y) = (x + y)(a + b)$. Sometimes you must rearrange the terms first so the bracketed factors match identically.

[CONCEPT MASTER – 2. Factorisation by Regrouping Terms] If your two brackets have opposite signs like $(x - y)$ and $(y - x)$, factor out -1 from the second: $-(x - y)$!

3. Factorisation Using Identities & Middle-Term Splitting

For 2-term differences of squares, use $a^2 - b^2 = (a + b)(a - b)$. For 3-term perfect squares, check $a^2 \pm 2ab + b^2 = (a \pm b)^2$. For general quadratics $x^2 + (p + q)x + pq$, **split the middle term** by finding two numbers p, q whose sum is the middle coefficient and product is the constant term: $(x + p)(x + q)$.

[CONCEPT MASTER – 3. Factorisation Using Identities & Middle-Term Splitting] Quadratic Factor Check: For $x^2 + bx + c$, find p, q such that $p + q = b$ and $p \times q = c$!

4. Division of Algebraic Expressions & Error Spotting

To divide a polynomial by a monomial, divide **every term** of the numerator by the monomial: $\frac{A+B}{C} = \frac{A}{C} + \frac{B}{C}$. To divide a polynomial by a binomial/polynomial, **factorise the numerator completely first** and cancel common factor brackets! Never cancel individual terms across $+$ or $-$ signs.

[CONCEPT MASTER – 4. Division of Algebraic Expressions & Error Spotting] Deadly Trap: $\frac{3x+5}{3} \neq x+5$! You must divide BOTH terms: $x+\frac{5}{3}$.

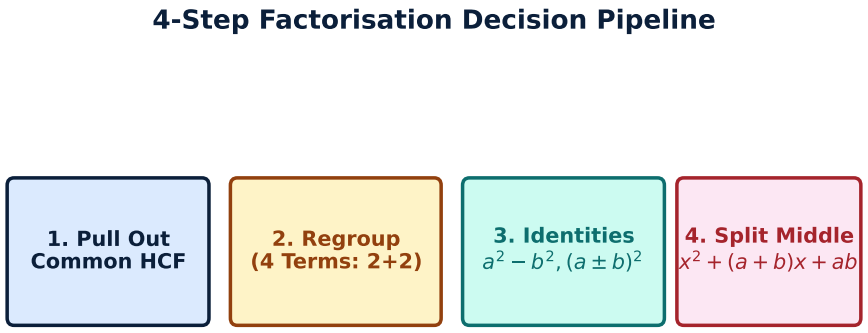


Figure 1: **Visual Model:** Four-step Factorisation Decision Tree: Common Factor → Regrouping → Identities → Middle-Term Split.

C. Complete Formula Bank

[FORMULA BANK – 1. Common Factor Extraction] **Expression:** $ab+ac=a(b+c)$
Symbols: a: HCF of all terms.
Conditions & When to Use: Always Step 1 of every factorisation.
Solved Mini-Example: $12a^2b+15ab^2=3ab(4a+5b)$.
Common Mistake: Leaving 1 out when an entire term is factored: $x^2+x=x(x+1)$, NOT $x(x)$.

[FORMULA BANK – 2. 4-Term Regrouping] **Expression:** $ac+ad+bc+bd=(a+b)(c+d)$
Symbols: Two matching binomials $(c+d)$.
Conditions & When to Use: 4-term polynomials.
Solved Mini-Example: $x^2+xy+8x+8y=x(x+y)+8(x+y)=(x+y)(x+8)$.
Common Mistake: Stopping at $a(c+d)+b(c+d)$ without pulling out $(c+d)$.

[FORMULA BANK – 3. Perfect Square Trinomials] **Expression:** $a^2\pm 2ab+b^2=(a\pm b)^2$
Symbols: a, b: square roots of first & last terms.
Conditions & When to Use: Verify middle term equals $\pm 2ab$.
Solved Mini-Example: $49y^2+84yz+36z^2=(7y+6z)^2$.
Common Mistake: Ignoring the middle-term check.

[FORMULA BANK – 4. Difference of Squares] **Expression:** $a^2-b^2=(a+b)(a-b)$
Symbols: a, b: expressions whose squares are subtracted.
Conditions & When to Use: Works for 2-term differences, or 4-term $(x^2+2xy+y^2)-z^2$.
Solved Mini-Example: $4p^2-9q^2=(2p+3q)(2p-3q)$.
Common Mistake: Trying to factor a^2+b^2 (sum of squares cannot be factored over reals).

[FORMULA BANK – 5. Middle-Term Splitting] **Expression:** $x^2 + (p+q)x + pq = (x+p)(x+q)$
Symbols: $p + q = b$, $pq = c$.
Conditions & When to Use: Non-square quadratics $x^2 + bx + c$.
Solved Mini-Example: $p^2 + 6p - 16 = (p+8)(p-2)$.
Common Mistake: Sign errors when $c < 0$.

[FORMULA BANK – 6. Term-by-Term Polynomial Division] **Expression:** $\frac{A+B+C}{D} = \frac{A}{D} + \frac{B}{D} + \frac{C}{D}$
Symbols: $D \neq 0$ (monomial divisor).
Conditions & When to Use: Every numerator term must be divided by D.
Solved Mini-Example: $\frac{10x-25}{5} = 2x - 5$.
Common Mistake: Dividing only the first term by D.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Sainik/Navodaya/Olympiad Foundation problem structures.

Pattern 1: Common Factor & Regrouping

- **Identification Cue:** Factorise $15xy - 6x + 5y - 2$ or $z - 7 + 7xy - xyz$. | **Concept & Formula:** Pair terms with matching inner brackets.
- **Standard Step-by-Step:** $3x(5y-2) + 1(5y-2) = (5y-2)(3x+1)$.
- **Fastest Valid Method:** If signs are reversed in pair 2, pull out a negative factor! | **Alternative:** Rearrange terms 2 and 3 if initial pairing fails.
- **Common Trap:** Dropping $+1$ when $(5y-2)$ appears by itself!
- **Practice Drill:** Ex. 3 & Ex. 5.

Pattern 2: Multi-Step $a^2 - b^2$ & Hidden Common Factor

- **Identification Cue:** Factorise $63a^2 - 112b^2$ or $p^4 - 81$ or $x^2 - 2xy + y^2 - z^2$. | **Concept & Formula:** Pull HCF first, or apply $(a+b)(a-b)$ repeatedly.
- **Standard Step-by-Step:** $63a^2 - 112b^2 = 7(9a^2 - 16b^2) = 7(3a+4b)(3a-4b)$.
- **Fastest Valid Method:** For $p^4 - 81$: $(p^2+9)(p^2-9) = (p^2+9)(p+3)(p-3)$! | **Alternative:** None.
- **Common Trap:** Stopping at $(p^2+9)(p^2-9)$ without factoring $p^2 - 9$!
- **Practice Drill:** Ex. 7 & Ex. 9.

Pattern 3: Quadratic Trinomial $x^2 + bx + c$

- **Identification Cue:** Factorise $q^2 - 10q + 21$ or $p^2 + 6p - 16$. | **Concept & Formula:** Find p, q with $p + q = b$ and $pq = c$.
- **Standard Step-by-Step:** For $p^2 + 6p - 16$: $+8$ and -2 give sum $+6$, product $-16 \implies (p+8)(p-2)$.
- **Fastest Valid Method:** Read signs from c: if $c > 0$, both p, q have the SAME sign as b; if $c < 0$, OPPOSITE signs (larger matches b)! | **Alternative:** Check by expanding.
- **Common Trap:** Picking -8 and $+2$ for $+6p$.
- **Practice Drill:** Ex. 10 & Ex. 11.

Pattern 4: Factor-Cancel Polynomial Division

- **Identification Cue:** Divide $44(x^4 - 5x^3 - 24x^2)$ by $11x(x-8)$. | **Concept & Formula:** Factorise numerator completely, then cancel.
- **Standard Step-by-Step:** $44x^2(x^2 - 5x - 24) = 44x^2(x-8)(x+3)$; divide by $11x(x-8) = 4x(x+3)$.
- **Fastest Valid Method:** Cancel constants ($44/11 = 4$) and powers ($x^2/x = x$) first, then factor the quadratic! | **Alternative:** Verify by multiplying quotient $\times$ divisor.
- **Common Trap:** Cancelling x^2 with x inside parentheses.
- **Practice Drill:** Ex. 13 & Ex. 14.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Instant Sign-Predictor for x^2+bx+c] **Core Idea & Why It Works:** Look at the sign of c first: (1) If $c>0$, BOTH factors have the sign of b; (2) If $c<0$, factors have OPPOSITE signs and the larger number takes the sign of b.
Valid Domain: All monic quadratics x^2+bx+c. | **Where It Fails:** Non-factorable quadratics.
Easy Example: x^2-5x+6 $(c>0, b<0)$ $\rightarrow$ both negative: $(x-2)(x-3)$. $x^2+6x-16$ $(c<0, b>0)$ $\rightarrow$ larger $+$: $(x+8)(x-2)$.
Exam-Level Example: Eliminates 90% of sign mistakes!
Advanced Example: Zero guesswork.

[SMART SHORTCUT – 2. Check Factorisation by $x=1$ Plug-In] **Core Idea & Why It Works:** In MCQs, plug $x=1$ (or $x=2$) into the original expression and into each option—only the true factorisation matches!
Valid Domain: All polynomial factorisations and divisions. | **Where It Fails:** If two options tie at $x=1$ (test $x=2$).
Easy Example: $x^2+6x-16$ at $x=2$: $4+12-16=0 \implies (x-2)$ MUST be a factor!
Exam-Level Example: Instant MCQ verification.
Advanced Example: Root test: if plugging $x=a$ gives 0, $(x-a)$ is a factor!

[SMART SHORTCUT – 3. Term-Count Factorisation Roadmap] **Core Idea & Why It Works:** After HCF: 2 terms $\rightarrow$ try a^2-b^2; 3 terms $\rightarrow$ try $(a\pm b)^2$ or middle-term split; 4 terms $\rightarrow$ try $2+2$ regrouping or $(a\pm b)^2-c^2$.
Valid Domain: Every Class 8–10 factorisation problem. | **Where It Fails:** None.
Easy Example: $x^2-2xy+y^2-z^2$ (4 terms, 3 form a square!) $\rightarrow (x-y)^2-z^2=(x-y+z)(x-y-z)$.
Exam-Level Example: Tells you which method to pick in 2 seconds!
Advanced Example: Covers NCERT Ex. 12.1–12.3 completely.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Factorise: $12a^2b+15ab^2$.
Given & Required: Two terms. | **Concept & Formula:** HCF $=3ab$.
Detailed Solution: $3ab(4a+5b)$. **Final Answer:** $3ab(4a+5b)$
Fastest Alternative / Shortcut: HCF of $12, 15$ is 3.

SOLVED EXAMPLE 2 [Beginner]
Question: Factorise: $10x^2-18x^3+14x^4$.
Given & Required: Three terms. | **Concept & Formula:** HCF $=2x^2$.
Detailed Solution: $2x^2(5-9x+7x^2)$. **Final Answer:** $2x^2(5-9x+7x^2)$
Fastest Alternative / Shortcut: Pull lowest power x^2.

SOLVED EXAMPLE 3 [Beginner]
Question: Factorise: $x^2+xy+8x+8y$.
Given & Required: 4 terms. | **Concept & Formula:** Regroup $2+2$.
Detailed Solution: $x(x+y)+8(x+y)=(x+y)(x+8)$. **Final Answer:** $(x+y)(x+8)$
Fastest Alternative / Shortcut: Group first two and last two.

SOLVED EXAMPLE 4 [Beginner]

Question: Divide: $(10x - 25) \div 5$.

Given & Required: Binomial ÷ monomial. | **Concept & Formula:** Factor numerator.

Detailed Solution: $\frac{5(2x-5)}{5} = 2x - 5$. **Final Answer:** $2x - 5$

Fastest Alternative / Shortcut: Divide both terms by 5.

SOLVED EXAMPLE 5 [Intermediate]

Question: Factorise: $15xy - 6x + 5y - 2$.

Given & Required: 4 terms. | **Concept & Formula:** Regrouping.

Detailed Solution: $3x(5y - 2) + 1(5y - 2) = (5y - 2)(3x + 1)$. **Final Answer:** $(5y - 2)(3x + 1)$

Fastest Alternative / Shortcut: Keep $+1$ outside second bracket!

SOLVED EXAMPLE 6 [Intermediate]

Question: Factorise: $49p^2 - 36$.

Given & Required: 2-term difference of squares. | **Concept & Formula:** $a^2 - b^2$.

Detailed Solution: $(7p)^2 - 6^2 = (7p + 6)(7p - 6)$. **Final Answer:** $(7p + 6)(7p - 6)$

Fastest Alternative / Shortcut: $a = 7p, b = 6$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Factorise: $4y^2 - 12y + 9$.

Given & Required: 3 terms. | **Concept & Formula:** $(a - b)^2$.

Detailed Solution: $(2y)^2 - 2(2y)(3) + 3^2 = (2y - 3)^2$. **Final Answer:** $(2y - 3)^2$

Fastest Alternative / Shortcut: Middle term $= 2(2y)(3) = 12y$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Divide: $(5p^2 - 25p + 20) \div (p - 1)$.

Given & Required: Quadratic ÷ linear. | **Concept & Formula:** Pull 5, factorise.

Detailed Solution: $\frac{5(p^2-5p+4)}{p-1} = \frac{5(p-1)(p-4)}{p-1} = 5(p - 4)$. **Final Answer:** $5(p - 4)$

Fastest Alternative / Shortcut: $-1 + (-4) = -5$.

SOLVED EXAMPLE 9 [Board Standard]

Question: Factorise completely: $63a^2 - 112b^2$.

Given & Required: 63 and 112 share 7. | **Concept & Formula:** HCF first, then $a^2 - b^2$.

Detailed Solution: $7(9a^2 - 16b^2) = 7(3a + 4b)(3a - 4b)$. **Final Answer:** $7(3a + 4b)(3a - 4b)$

Fastest Alternative / Shortcut: Divide both by 7 first!

SOLVED EXAMPLE 10 [Board Standard]

Question: Factorise completely: $p^4 - 81$.

Given & Required: $p^4 - 9^2$. | **Concept & Formula:** Apply $a^2 - b^2$ twice.

Detailed Solution: $(p^2)^2 - 9^2 = (p^2 + 9)(p^2 - 9) = (p^2 + 9)(p + 3)(p - 3)$. **Final Answer:** $(p^2 + 9)(p + 3)(p - 3)$

Fastest Alternative / Shortcut: Never leave $p^2 - 9$ unfactored!

SOLVED EXAMPLE 11 [Board Standard]

Question: Factorise: $p^2 + 6p - 16$.

Given & Required: $b = +6, c = -16$. | **Concept & Formula:** Middle-term split.

Detailed Solution: $p^2 + 8p - 2p - 16 = p(p + 8) - 2(p + 8) = (p + 8)(p - 2)$. **Final Answer:** $(p + 8)(p - 2)$

Fastest Alternative / Shortcut: $+8 - 2 = +6, 8(-2) = -16$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Factorise: $(l+m)^2 - 4lm$.

Given & Required: Expand and re-factor. | **Concept & Formula:** Identity $(a+b)^2 - 4ab = (a-b)^2$.

Detailed Solution: $l^2 + 2lm + m^2 - 4lm = l^2 - 2lm + m^2 = (l-m)^2$. **Final Answer:** $(l-m)^2$

Fastest Alternative / Shortcut: $(l+m)^2 - 4lm = (l-m)^2$.

SOLVED EXAMPLE 13 [Board Standard]

Question: Divide: $44(x^4 - 5x^3 - 24x^2)$ by $11x(x-8)$.

Given & Required: Polynomial division. | **Concept & Formula:** Pull x^2, then split middle term.

Detailed Solution: $\frac{44x^2(x^2-5x-24)}{11x(x-8)} = \frac{4x(x-8)(x+3)}{x-8} = 4x(x+3)$. **Final Answer:** $4x(x+3)$

Fastest Alternative / Shortcut: $-8+3=-5$.

SOLVED EXAMPLE 14 [Challenge]

Question: Factorise: $25a^2 - 4b^2 + 28bc - 49c^2$.

Given & Required: 4 terms. | **Concept & Formula:** Group last 3 terms as $-(2b-7c)^2$.

Detailed Solution: $25a^2 - (4b^2 - 28bc + 49c^2) = (5a)^2 - (2b-7c)^2 = (5a+2b-7c)(5a-2b+7c)$. **Final Answer:** $(5a+2b-7c)(5a-2b+7c)$

Fastest Alternative / Shortcut: Pull $-$ out of last 3 terms!

SOLVED EXAMPLE 15 [Challenge]

Question: Find and correct the error in the statement: $(2x)^2 + 5x = 4x + 5x = 9x$.

Given & Required: Erroneous student work. | **Concept & Formula:** $(2x)^2 = 4x^2$, not $4x$.

Detailed Solution: Error: $2^2x^2 = 4x^2$ was written as $4x$. Correct: $(2x)^2 + 5x = 4x^2 + 5x$. **Final Answer:** $4x^2 + 5x$

Fastest Alternative / Shortcut: Square BOTH 2 and x!

SOLVED EXAMPLE 16 [Challenge]

Question: Divide $z(5z^2 - 80)$ by $5z(z+4)$.

Given & Required: Factor 5 first. | **Concept & Formula:** HCF + difference of squares.

Detailed Solution: $\frac{5z(z^2-16)}{5z(z+4)} = \frac{(z+4)(z-4)}{z+4} = z-4$. **Final Answer:** $z-4$

Fastest Alternative / Shortcut: $z^2 - 16 = (z+4)(z-4)$.

G. Competency MCQ Mastery Section

Q1. The common factor of $12x$ and 36 is:

(A) 6 (B) 12 (C) $12x$ (D) $3x$

Answer: (B) 36 has no x, so HCF is 12. *Fast Method:* HCF(12, 36) = 12. *Distractor Note:* Including x.

Q2. Factorisation of $x^2 - 9$ is:

(A) $(x-3)^2$ (B) $(x+3)^2$ (C) $(x+3)(x-3)$ (D) $(x-9)(x+1)$

Answer: (C) $a^2 - b^2 = (a+b)(a-b)$. *Fast Method:* Difference of squares. *Distractor Note:* Picking $(x-3)^2$.

Q3. Factorisation of $x^2 + 5x + 6$ is:

(A) $(x+1)(x+6)$ (B) $(x+2)(x+3)$ (C) $(x-2)(x-3)$ (D) $(x+5)(x+1)$

Answer: (B) $2+3=5, 2\times3=6$. *Fast Method:* Sum 5, product 6. *Distractor Note:* Picking A $(1+6=7)$.

Q4. Value of $(4x^2 - 16) \div (2x+4)$ is:

(A) $2x+4$ (B) $2x-4$ (C) $2x-8$ (D) $x-4$

Answer: (B) $\frac{(2x+4)(2x-4)}{2x+4} = 2x-4$. *Fast Method:* $a^2 - b^2$. *Distractor Note:* Sign error.

Q5. Factors of $a^2 - 2ab + b^2$ are:

(A) $(a+b)(a+b)$ (B) $(a-b)(a-b)$ (C) $(a+b)(a-b)$ (D) $a^2 - b^2$

Answer: (B) $(a-b)^2$. *Fast Method:* Standard identity. *Distractor Note:* Picking C.

Q6. Divide $24x^2y^2z$ by $8xyz$:

(A) $3xy$ (B) $3xyz$ (C) $3x^2y$ (D) $16xy$

Answer: (A) $24/8 = 3$; z cancels. *Fast Method:* Subtract powers. *Distractor Note:* Keeping z.

Q7. Which is the correct factorisation of $ax - ay + bx - by$?

(A) $(a+b)(x+y)$ (B) $(a-b)(x-y)$ (C) $(a+b)(x-y)$ (D) $(a-b)(x+y)$

Answer: (C) $a(x-y) + b(x-y) = (a+b)(x-y)$. *Fast Method:* Regroup. *Distractor Note:* Sign of $(a+b)$.

Q8. Factorise $p^2 - 10p + 25$:

(A) $(p+5)^2$ (B) $(p-5)^2$ (C) $(p+5)(p-5)$ (D) $(p-25)(p+1)$

Answer: (B) $(p-5)^2$. *Fast Method:* $2(5) = 10, 5^2 = 25$. *Distractor Note:* Picking C.

Q9. Which of the following is equal to $\frac{4x+8}{4}$?

(A) $x+8$ (B) $4x+2$ (C) $x+2$ (D) $3x$

Answer: (C) $\frac{4(x+2)}{4} = x+2$. *Fast Method:* Divide BOTH terms by 4. *Distractor Note:* Picking A $(x+8)$!

Q10. Factors of $y^2 - 7y + 12$ are:

(A) $(y-3)(y-4)$ (B) $(y+3)(y+4)$ (C) $(y-6)(y-2)$ (D) $(y-12)(y+1)$

Answer: (A) $(-3) + (-4) = -7, (-3)(-4) = 12$. *Fast Method:* Both negative. *Distractor Note:* Picking C $(-6 - 2 = -8)$.

Q11. The irreducible factor form of $3x^2 - 12$ is:

(A) $3(x^2 - 4)$ (B) $3(x+2)(x-2)$ (C) $(3x+6)(x-2)$ (D) $3(x-2)^2$

Answer: (B) Factor $x^2 - 4$ completely! *Fast Method:* HCF + $a^2 - b^2$. *Distractor Note:* Stopping at A.

Q12. If $(x+a)$ is a factor of $x^2 + 3x - 10$ and $a > 0$, then a is:

(A) 2 (B) 5 (C) 10 (D) 3

Answer: (B) $x^2 + 3x - 10 = (x+5)(x-2) \implies a = 5$. *Fast Method:* $5 - 2 = 3$. *Distractor Note:* Picking 2 (it's $x-2$).

H. Special Exam & MCQ Strategies

- Follow the 4-Step Roadmap in order: (1) Common Factor → (2) Regrouping → (3) Identities → (4) Middle-Term Split.
- In polynomial division, NEVER cancel terms across + or − signs—factorise the numerator completely first!
- Always check whether a bracket like $(p^2 - 9)$ can be factored ONE MORE step.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Simplifying $\frac{3x+5}{3} = x + 5$ or $\frac{x+2}{2} = x$. ⟶ **Correction:** A monomial denominator divides EVERY term: $\frac{3x+5}{3} = x + \frac{5}{3}$.

[COMMON MISTAKE] **Error:** Writing $x^2 + 9 = (x+3)^2$ or $(x+3)(x-3)$. ⟶ **Correction:** The sum of two squares $x^2 + 9$ does NOT factorise over real numbers!

[COMMON MISTAKE] **Error:** Stopping at $p^4 - 16 = (p^2+4)(p^2-4)$. ⟶ **Correction:** Always factorise completely: $(p^2+4)(p+2)(p-2)$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $ab + ac = a(b + c)$; $a^2 - b^2 = (a + b)(a - b)$; $a^2 \pm 2ab + b^2 = (a \pm b)^2$; $x^2 + (p + q)x + pq = (x + p)(x + q)$.

[QUICK REVISION] Key Theorems & Results: Always extract HCF first; divide every term when dividing by a monomial.

[QUICK REVISION] Shortcut Summary: Sign predictor for $x^2 + bx + c$; plug $x = 1$ or $x = 2$ to verify factorisations.

[QUICK REVISION] High-Frequency Exam Patterns: HCF extraction, $2 + 2$ regrouping, $p^4 - 81$, Middle-term split, Factor-and-cancel division.

Last-Minute Revision Checklist:

- ☐ Extract HCF first
- ☐ Check for $(a^2 - b^2)$ twice
- ☐ Divide ALL terms in $\div$
- ☐ Verify by expanding

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Factorise $7x - 42$: (A) $7(x - 6)$ (B) $7(x - 7)$ (C) $6(x - 7)$ (D) $7x - 6$
2. Factorise $a^2 - 49$: (A) $(a - 7)^2$ (B) $(a + 7)(a - 7)$ (C) $(a + 7)^2$ (D) $(a - 49)(a + 1)$
3. Divide $15x^3$ by $3x$: (A) $5x^2$ (B) $5x^3$ (C) $12x^2$ (D) $5x$
4. Factorise $x^2 + 2x + 1$: (A) $(x + 1)^2$ (B) $(x - 1)^2$ (C) $(x + 1)(x - 1)$ (D) $x(x + 2)$

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Factorise $x^2 - 7x + 12$: (A) $(x - 3)(x - 4)$ (B) $(x + 3)(x + 4)$ (C) $(x - 6)(x - 2)$ (D) $(x - 12)(x + 1)$
2. Divide $(12x^2 - 27)$ by $(2x + 3)$: (A) $3(2x - 3)$ (B) $2x - 3$ (C) $6x - 9$ (D) Both A and C
3. Factorise $xy - x - y + 1$: (A) $(x - 1)(y - 1)$ (B) $(x + 1)(y - 1)$ (C) $(x - 1)(y + 1)$ (D) $(x + 1)(y + 1)$
4. Factorise $2x^2 - 18$ completely: (A) $2(x^2 - 9)$ (B) $2(x + 3)(x - 3)$ (C) $(2x + 6)(x - 3)$ (D) $2(x - 3)^2$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Factorise $x^4 - 16$ completely: (A) $(x^2 + 4)(x^2 - 4)$ (B) $(x^2 + 4)(x + 2)(x - 2)$ (C) $(x - 2)^4$ (D) $(x + 2)^2(x - 2)^2$
2. Factorise $x^2 + x - 12$: (A) $(x + 4)(x - 3)$ (B) $(x - 4)(x + 3)$ (C) $(x + 6)(x - 2)$ (D) $(x - 6)(x + 2)$
3. Divide $(x^2 - 10x + 25)$ by $(x - 5)$: (A) $x + 5$ (B) $x - 5$ (C) $x - 25$ (D) 5
4. Factorise $1 - (a - b)^2$: (A) $(1 + a - b)(1 - a + b)$ (B) $(1 + a - b)(1 - a - b)$ (C) $(1 - a + b)^2$ (D) $(a - b + 1)(a - b - 1)$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (A):** HCF $= 7 \implies 7(x-6)$. **Q2 (B):** $a^2 - 7^2 = (a+7)(a-7)$. **Q3 (A):** $15/3 = 5, x^{3-1} = x^2$. **Q4 (A):** $(x+1)^2$.
Exam Std	**Q1 (A):** $-3-4 = -7, (-3)(-4) = 12$. **Q2 (D):** $3(4x^2-9)/(2x+3) = 3(2x-3) = 6x-9$. **Q3 (A):** $x(y-1)-1(y-1) = (x-1)(y-1)$. **Q4 (B):** $2(x+3)(x-3)$.
Challenge	**Q1 (B):** $(x^2+4)(x+2)(x-2)$. **Q2 (A):** $4-3 = 1, 4(-3) = -12$. **Q3 (B):** $(x-5)^2/(x-5) = x-5$. **Q4 (A):** $1^2-(a-b)^2 = (1+a-b)(1-a+b)$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Common factors & regrouping. Day 2: Identities & middle-term splitting. Day 3: Polynomial division & error analysis.
- **How to Introduce the Concept:** Ask: 'What two numbers add to 7 and multiply to 12?' Everyone shouts 3 and 4—then reveal they just factored $x^2+7x+12$!
- **Difficult Areas & Remediation:** Pulling out -1 during regrouping and resisting the urge to cancel terms across $+$ signs in division.
- **Blackboard Teaching Examples:** Put the 4-Step Factorisation Flowchart on the left side of the board for the entire week.
- **Classroom Questions to Ask Students:** Why can we factor x^2-25 but NOT x^2+25?
- **Common Student Doubts & Answers:** Students write $\frac{x+5}{5} = x+1$ or x; show $\frac{10+5}{5} = 3$, not $10+1=11$!
- **Homework & Revision Strategy:** Assign 4 factorisations (one of each type) and 3 polynomial divisions.

[TEACHER'S TIP] Make students verify every quadratic factorisation by multiplying back in their heads.

UNIT VI: DATA HANDLING & GRAPHS TYBA COACHING CENTRE

Chapter 13: Introduction to Graphs

Line Graphs, Cartesian Coordinates (x, y), Independent vs Dependent Variables & Linear Graphs (2026–27)

A. Chapter Introduction & Examination Relevance

Numbers in tables come alive when plotted on a grid. Chapter 13 introduces line graphs for time-series data, teaches how to locate and plot points using (x, y) coordinates, and builds linear graphs for real-world relationships like cost, simple interest, and distance–time.

- **What You Will Learn:** Reading and interpreting line graphs, Cartesian plane basics (origin, x-axis, y-axis, coordinates (x, y)), plotting points and identifying axes intercepts, independent vs dependent variables, and drawing/reading linear graphs.
- **Why This Chapter Matters:** Coordinate geometry and linear graphs are the gateway to Class 9 Coordinate Geometry & Linear Equations in Two Variables, Class 10 Pair of Linear Equations, and Science motion/current–voltage graphs.
- **Real-Life Applications:** Tracking patient temperature, stock/weather charts, taxi fare vs distance, simple interest vs years, and speed calculation from distance–time graphs.
- **Cross-Chapter Connections:** Builds on Chapter 4 (Data Handling) and Chapter 11 (Direct Proportion $y = kx$); directly prepares for Class 9 Chapters 3 & 4.
- **Exam & Foundation Relevance:** Unit VI carries 12% weightage. Expect 1M coordinate/axis MCQs, 3M line-graph reading questions, and 4M–5M plotting linear graphs (Side–Perimeter, Interest–Deposit, Distance–Time) and reading intermediate values.

B. Complete Concept Explanation & Derivations

1. Line Graphs (Continuous Change Over Time)

A **line graph** displays data that changes continuously over periods of time (e.g., hourly temperature, yearly sales, or distance travelled). The horizontal axis (x-axis) usually shows time, and consecutive data points are joined by straight line segments so trends (steep rise, flat plateau, or drop) are immediately visible.

[CONCEPT MASTER – 1. Line Graphs (Continuous Change Over Time)] Steeper segment = faster rate of change! Horizontal flat segment = NO change during that interval.

2. Cartesian Coordinates (x, y) & Axes

Every point on a plane is uniquely located by an **ordered pair** (x, y): x (**abscissa**) is the horizontal distance from the y-axis, and y (**ordinate**) is the vertical distance from the x-axis. The two axes intersect at the **origin** $O(0, 0)$.

[CONCEPT MASTER – 2. Cartesian Coordinates (x, y) & Axes] Walk before you climb! Always move along x (horizontal) FIRST, then along y (vertical): $(x, y) \neq (y, x)$!

3. Points on the x-Axis and y-Axis

Any point lying ON the x-axis has y-coordinate $= 0$, so its form is $(x, 0)$ (e.g., $(4, 0)$). Any point lying ON the y-axis has x-coordinate $= 0$, so its form is $(0, y)$ (e.g., $(0, 5)$). If both $x = 0$ and $y = 0$, the point is the origin $(0, 0)$.

[CONCEPT MASTER – 3. Points on the x-Axis and y-Axis] Axis Zero Rule: On the x-axis, $y = 0$; on the y-axis, $x = 0$!

4. Independent/Dependent Variables & Linear Graphs

The **independent variable** (cause, e.g., Time, Side length, Deposit) is plotted on the x-axis; the **dependent variable** (effect, e.g., Distance, Area, Simple Interest) is plotted on the y-axis. When all plotted points lie on a single unbroken straight line, it is a **linear graph** (e.g., Perimeter $P = 4s$ is linear, whereas Area $A = s^2$ is curved, NOT linear!).

[CONCEPT MASTER – 4. Independent/Dependent Variables & Linear Graphs] Direct proportion $y = kx$ ALWAYS gives a straight line passing through the origin $(0, 0)$!

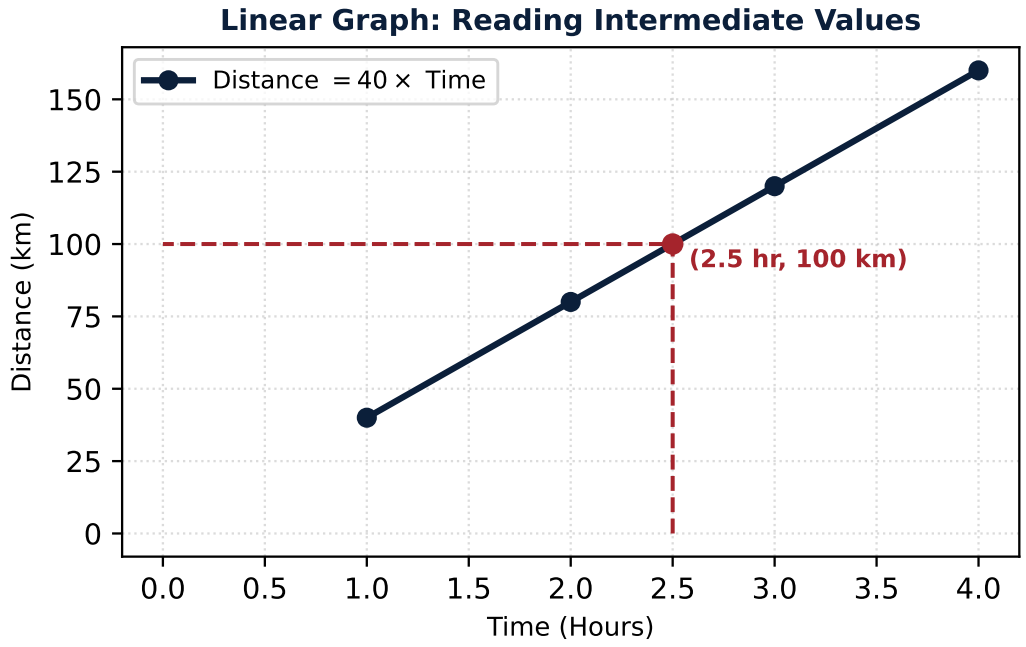


Figure 1: **Visual Model:** Cartesian Quadrant I linear graph of Distance vs Time ($d = 40t$) passing through $(0, 0), (1, 40), (2, 80), (3, 120)$.

C. Complete Formula Bank

[FORMULA BANK – 1. Ordered Pair Notation] **Expression:** $P(x, y)$: x = abscissa, y = ordinate
Symbols: x: distance from y-axis; y: distance from x-axis.
Conditions & When to Use: Order matters: $(2, 5) \neq (5, 2)$.
Solved Mini-Example: In $(3, 7)$, x-coordinate $= 3$, y-coordinate $= 7$.
Common Mistake: Swapping x and y while plotting.

[FORMULA BANK – 2. Axis & Origin Coordinates] **Expression:** Origin $= (0, 0)$; x-axis $= (a, 0)$; y-axis $= (0, b)$
Symbols: $a, b \in \mathbb{R}$.
Conditions & When to Use: Zero coordinate tells you which axis the point sits on.
Solved Mini-Example: $(0, 6)$ lies on the y-axis.
Common Mistake: Saying $(0, 6)$ lies on the x-axis!

[FORMULA BANK – 3. Square Perimeter vs Side (Linear)] **Expression:** $P = 4s$
Symbols: s: side (x-axis), P: perimeter (y-axis).
Conditions & When to Use: Linear graph passing through $(0, 0)$.
Solved Mini-Example: $(2, 8), (3, 12), (4, 16)$ lie on a straight line.
Common Mistake: Thinking $A = s^2$ is also linear.

[FORMULA BANK – 4. Square Area vs Side (Non-Linear)] **Expression:** $A = s^2$
Symbols: s: side, A: area.
Conditions & When to Use: NOT a linear graph (curves upward).
Solved Mini-Example: $(2, 4), (3, 9), (4, 16)$ do not have constant slope.
Common Mistake: Drawing a straight line through $(2, 4)$ and $(4, 16)$.

[FORMULA BANK – 5. Simple Interest vs Deposit (Linear)] **Expression:** $I = \frac{P \times R \times T}{100}$
Symbols: P: deposit (x-axis), I: interest (y-axis).
Conditions & When to Use: For fixed R, T, $I \propto P$ (linear through origin).
Solved Mini-Example: At 10% for 1 yr: $I = 0.1P$.
Common Mistake: Forgetting to label scale on both axes.

[FORMULA BANK – 6. Distance–Time & Speed] **Expression:** Speed $= \frac{\text{Change in Distance}}{\text{Change in Time}}$
Symbols: Slope of Distance–Time line.
Conditions & When to Use: Uniform speed gives a straight line.
Solved Mini-Example: From $(1, 40)$ to $(3, 120)$: Speed $= \frac{120-40}{3-1} = 40$ km/h.
Common Mistake: Dividing time by distance.

D. Exam & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and Sainik/Navodaya/Olympiad Foundation problem structures.

Pattern 1: Reading & Interpreting Line Graphs

- **Identification Cue:** Read a temperature, sales, or courier distance-time line graph to answer 4–5 sub-questions. | **Concept & Formula:** Locate x-value, move up to graph, read y-value (and vice versa).
- **Standard Step-by-Step:** Check axis scales carefully (e.g., 1 unit = 2 hours or 5°C) before reading.
- **Fastest Valid Method:** For 'maximum change', just spot the STEEPEST segment visually, then verify! | **Alternative:** None.
- **Common Trap:** Misreading half-grid ticks between labeled numbers.
- **Practice Drill:** Ex. 1 & Ex. 2.

Pattern 2: Plotting Points & Testing Collinearity

- **Identification Cue:** Plot $A(1, 1), B(2, 2), C(3, 3)$ or $K(2, 3), L(5, 3), M(5, 5)$ and state if they lie on a line. | **Concept & Formula:** Plot (x, y) and join with a ruler.
- **Standard Step-by-Step:** Plot each point carefully; check if a straightedge passes through all points.
- **Fastest Valid Method:** If y-coordinates are all equal (e.g., $(1, 3), (4, 3), (6, 3)$), they form a horizontal line! If x-coordinates match, vertical line! | **Alternative:** Check constant ratio y/x or constant difference.
- **Common Trap:** Plotting $(5, 3)$ as 3 right and 5 up!
- **Practice Drill:** Ex. 4 & Ex. 6.

Pattern 3: True/False Axis & Coordinate Rules

- **Identification Cue:** A point whose x-coordinate is 0 and y-coordinate is non-zero lies on the y-axis (T/F)? | **Concept & Formula:** $(0, y)$ has zero horizontal shift $\rightarrow$ y-axis.
- **Standard Step-by-Step:** Test with a concrete point like $(0, 4)$.
- **Fastest Valid Method:** Zero in x-slot $\implies$ y-axis; Zero in y-slot $\implies$ x-axis! | **Alternative:** Sketch tiny axes.
- **Common Trap:** Reversing which axis has which coordinate = 0.
- **Practice Drill:** Ex. 3 & Ex. 5.

Pattern 4: Drawing & Reading Linear Application Graphs

- **Identification Cue:** Draw a graph for Deposit vs Simple Interest (8% p.a.) or Distance vs Time (40 km/h) and read values. | **Concept & Formula:** Build 4-point table, choose clean scale, plot, join, interpolate.
- **Standard Step-by-Step:** Plot $(1000, 80), (2000, 160), (3000, 240)$, draw line through $(0, 0)$, read I at Rs. 2500 ($= 200$).
- **Fastest Valid Method:** Use $I = 0.08P$ to verify graph readings! | **Alternative:** None.
- **Common Trap:** Forgetting to state the scale on BOTH x- and y-axes.
- **Practice Drill:** Ex. 8 & Ex. 10.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Walk-Then-Climb Coordinate Rule] **Core Idea & Why It Works:** To plot (x, y), walk x steps RIGHT (along the floor) first, then climb y steps UP (like a ladder)!

Valid Domain: All Quadrant I points. | **Where It Fails:** None.

Easy Example: For $(4, 2)$: walk 4 right, climb 2 up.

Exam-Level Example: Never mix up $(4, 2)$ and $(2, 4)$ again.

Advanced Example: Also helps read points: drop down to floor for x, look left to wall for y.

[SMART SHORTCUT – 2. Zero-Slot Axis Identifier] **Core Idea & Why It Works:** Whichever slot is NON-ZERO names the axis! $(5, 0)$ has non-zero $x \to x$-axis; $(0, 5)$ has non-zero $y \to y$-axis!

Valid Domain: All points on axes. | **Where It Fails:** Origin $(0, 0)$ lies on BOTH axes.

Easy Example: $(0, 7)$ has non-zero y-coordinate $\implies$ lies on y-axis.

Exam-Level Example: 1-second MCQ solver.

Advanced Example: Works for negative coordinates too in Class 9!

[SMART SHORTCUT – 3. Linear vs Curved Instant Check] **Core Idea & Why It Works:** Compute $\Delta y/\Delta x$ (change in y for equal steps of x): if differences are CONSTANT, the graph is LINEAR; if they grow/shrink, it is CURVED!

Valid Domain: All tables in Ch. 13. | **Where It Fails:** None.

Easy Example: Square Side $1, 2, 3, 4$: Perimeter $4, 8, 12, 16$ ($+4, +4, +4 \to$ Linear!); Area $1, 4, 9, 16$ ($+3, +5, +7 \to$ NOT linear!).

Exam-Level Example: Answers NCERT Ex. 13.2 Q2 without plotting first!

Advanced Example: Foundation of slope in Class 10–11.

F. Fully Solved Examples (Beginner to Exam & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: Write the x-coordinate (abscissa) and y-coordinate (ordinate) of $P(4, 7)$.

Given & Required: $P(4, 7)$. | **Concept & Formula:** (x, y) format.

Detailed Solution: x-coordinate $= 4$, y-coordinate $= 7$. **Final Answer: Abscissa $= 4$, Ordinate $= 7$**

Fastest Alternative / Shortcut: First is x, second is y.

SOLVED EXAMPLE 2 [Beginner]

Question: State which axis the following points lie on: (i) $(5, 0)$, (ii) $(0, 8)$, (iii) $(0, 0)$.

Given & Required: Three points. | **Concept & Formula:** Zero-slot rule.

Detailed Solution: (i) x-axis ($y = 0$), (ii) y-axis ($x = 0$), (iii) Origin (both axes). **Final Answer: (i) x-axis, (ii) y-axis, (iii) Origin**

Fastest Alternative / Shortcut: Non-zero slot names the axis.

SOLVED EXAMPLE 3 [Beginner]

Question: Find the distance of point $A(3,5)$ from the x-axis and from the y-axis.

Given & Required: $A(3,5)$. | **Concept & Formula:** Distance from x-axis $= y$; from y-axis $= x$.

Detailed Solution: Distance from x-axis $= 5$ units; from y-axis $= 3$ units. **Final Answer:** 5 **units from** x**-axis;** 3 **units from** y**-axis**

Fastest Alternative / Shortcut: Opposite coordinate gives distance from axis!

SOLVED EXAMPLE 4 [Beginner]

Question: In a graph of 'Number of Apples vs Total Cost', which is the independent variable and which is the dependent variable?

Given & Required: Apples and Cost. | **Concept & Formula:** Cause (x) vs Effect (y).

Detailed Solution: Number of apples is independent (x-axis); Total cost is dependent (y-axis). **Final Answer: Independent: Apples; Dependent: Cost**

Fastest Alternative / Shortcut: Cost depends on apples.

SOLVED EXAMPLE 5 [Intermediate]

Question: Do the points $A(1,2), B(2,4), C(3,6), D(4,8)$ lie on a straight line? Does the line pass through the origin?

Given & Required: Four points. | **Concept & Formula:** Check y/x ratio.

Detailed Solution: For every point, $y = 2x$ (constant ratio 2). Yes, they lie on a straight line passing through $(0,0)$. **Final Answer: Yes; passes through** $(0,0)$

Fastest Alternative / Shortcut: $y = 2x$ is linear through origin.

SOLVED EXAMPLE 6 [Intermediate]

Question: Do the points $P(1,1), Q(2,4), R(3,9), S(4,16)$ form a linear graph?

Given & Required: $y = x^2$. | **Concept & Formula:** Check differences in y.

Detailed Solution: Differences are $4-1=3, 9-4=5, 16-9=7$ (not constant). No, it is curved, NOT linear. **Final Answer: No (non-linear)**

Fastest Alternative / Shortcut: Squares never form a straight line!

SOLVED EXAMPLE 7 [Intermediate]

Question: A car moves at a uniform speed of 40 km/h. Prepare a table of Distance vs Time for $t = 1, 2, 3, 4$ hours and find the distance at $t = 2.5$ hours.

Given & Required: $S = 40$ km/h. | **Concept & Formula:** $d = 40t$.

Detailed Solution: Points: $(1,40), (2,80), (3,120), (4,160)$. At $t = 2.5$ h, $d = 100$ km. **Final Answer:** 100 **km**

Fastest Alternative / Shortcut: Midpoint between 80 and 120 is 100.

SOLVED EXAMPLE 8 [Intermediate]

Question: Three vertices of a rectangle $PQRS$ are $P(1,1), Q(5,1), R(5,4)$. Find the coordinates of the fourth vertex S.

Given & Required: P, Q, R. | **Concept & Formula:** Opposite sides are horizontal/vertical.

Detailed Solution: S shares x-coordinate with P ($x = 1$) and y-coordinate with R ($y = 4$), so $S = (1,4)$. **Final Answer:** $S(1,4)$

Fastest Alternative / Shortcut: Match x of P and y of R!

SOLVED EXAMPLE 9 [Board Standard]

Question: A bank gives 10% Simple Interest per annum on deposits. Make a table of Deposit vs Annual Interest for Rs. $1000, 2000, 3000, 4000$, and find: (i) Interest on Rs. 2500, (ii) Deposit needed for Rs. 350 interest.

Given & Required: $R = 10\%, T = 1$. | **Concept & Formula:** $I = 0.1P$ (linear graph).

Detailed Solution: Points: $(1000, 100), (2000, 200), (3000, 300), (4000, 400)$. (i) At $P = 2500$, $I =$ Rs. 250. (ii) For $I = 350$, $P =$ Rs. 3500. **Final Answer: (i) Rs.** 250**, (ii) Rs.** 3500

Fastest Alternative / Shortcut: 10% of P.

SOLVED EXAMPLE 10 [Board Standard]

Question: A courier-person cycles from a town to a neighbouring suburban area: 8 a.m. (0 km), 9 a.m. (10 km), 10 a.m. (16 km), 10 : 30 a.m. (16 km), 11 : 30 a.m. (22 km). (i) When did he travel fastest? (ii) Did he stop anywhere?

Given & Required: Distance–time data. | **Concept & Formula:** Steepest = fastest; flat = rest.

Detailed Solution: (i) Between 8 a.m. and 9 a.m. (10 km in 1 hr). (ii) Yes, from 10 a.m. to 10 : 30 a.m. (distance stayed 16 km). **Final Answer: (i) 8–9 a.m.; (ii) Yes, 10–10 : 30 a.m.**

Fastest Alternative / Shortcut: Zero distance gain = rest!

SOLVED EXAMPLE 11 [Board Standard]

Question: Plot $A(2,3), B(5,3), C(5,5), D(2,5)$ and identify the geometric figure $ABCD$ and its area.

Given & Required: 4 points. | **Concept & Formula:** Horizontal & vertical differences.

Detailed Solution: Width $= 5-2 = 3$ units; Height $= 5-3 = 2$ units. $ABCD$ is a rectangle of area $3 \times 2 = 6$ sq. units. **Final Answer: Rectangle; Area $= 6$ sq. units**

Fastest Alternative / Shortcut: $\Delta x = 3, \Delta y = 2$.

SOLVED EXAMPLE 12 [Board Standard]

Question: For a square, plot Perimeter ($P = 4s$) for $s = 2, 3, 3.5, 5, 6$. Use the linear graph to read P when $s = 4$ cm and s when $P = 18$ cm.

Given & Required: $P = 4s$. | **Concept & Formula:** Linear graph.

Detailed Solution: Points: $(2,8), (3,12), (3.5,14), (5,20), (6,24)$. From graph: at $s = 4$, $P = 16$ cm; at $P = 18$, $s = 4.5$ cm. **Final Answer: $P = 16$ cm; $s = 4.5$ cm**

Fastest Alternative / Shortcut: $18/4 = 4.5$.

SOLVED EXAMPLE 13 [Board Standard]

Question: In a line graph of Temperature vs Time: 6 a.m. (37°C), 10 a.m. (40°C), 2 p.m. (38°C), 6 p.m. (35°C). During which 4-hour period did temperature fall fastest?

Given & Required: 4 readings. | **Concept & Formula:** Compare drops.

Detailed Solution: 10 a.m. to 2 p.m.: fell 2°C; 2 p.m. to 6 p.m.: fell 3°C. Fastest fall was 2 p.m. to 6 p.m. **Final Answer: 2 p.m. to 6 p.m.**

Fastest Alternative / Shortcut: $38 \to 35$ (-3°C).

SOLVED EXAMPLE 14 [Challenge]

Question: A straight line passes through $(2,5)$ and $(4,9)$. Find the y-coordinate of the point on this line whose x-coordinate is 6, and find where it crosses the y-axis.

Given & Required: $(2,5), (4,9)$. | **Concept & Formula:** Constant rate $\Delta y/\Delta x$.

Detailed Solution: As x increases by 2, y increases by 4 ($\implies +2$ per unit x). At $x = 6$, $y = 9+4 = 13$. Going back 2 units to $x = 0$: $y = 5-4 = 1 \implies (0,1)$. **Final Answer: $y = 13$; crosses y-axis at $(0,1)$**

Fastest Alternative / Shortcut: Pattern $y = 2x + 1$!

SOLVED EXAMPLE 15 [Challenge]

Question: If the points $(1,3), (2,5), (3,k)$ are collinear (lie on a straight line), find k.

Given & Required: $(1,3), (2,5), (3,k)$. | **Concept & Formula:** Equal step increase.

Detailed Solution: From $x = 1$ to $x = 2$, y rises by $5-3 = 2$. From $x = 2$ to $x = 3$, y rises by 2 again: $k = 5+2 = 7$. **Final Answer: $k = 7$**

Fastest Alternative / Shortcut: $3, 5, 7$.

SOLVED EXAMPLE 16 [Challenge]

Question: A triangle has vertices $O(0,0), A(6,0), B(0,4)$. Find its area.

Given & Required: Origin and two axis intercepts. | **Concept & Formula:** Right triangle on axes.

Detailed Solution: Base $OA = 6$ units along x-axis; Height $OB = 4$ units along y-axis. Area $= \frac{1}{2} \times 6 \times 4 = 12$ sq. units.

Final Answer: 12 **sq. units**

Fastest Alternative / Shortcut: $\frac{1}{2}(6)(4) = 12$.

G. Competency MCQ Mastery Section

Q1. The coordinates of the origin are:

(A) $(1,1)$ (B) $(0,1)$ (C) $(1,0)$ (D) $(0,0)$

Answer: (D) Axes meet at $(0,0)$. *Fast Method:* Zero both. *Distractor Note:* None.

Q2. Point $(0,5)$ lies on:

(A) x-axis (B) y-axis (C) Origin (D) Line $y = x$

Answer: (B) $x = 0 \implies$ on y-axis. *Fast Method:* Zero-slot rule. *Distractor Note:* Picking x-axis.

Q3. Abscissa of the point $(7,3)$ is:

(A) 3 (B) 7 (C) 10 (D) 4

Answer: (B) Abscissa is x-coordinate $= 7$. *Fast Method:* First coordinate. *Distractor Note:* Picking ordinate 3.

Q4. Distance of point $(4,9)$ from the x-axis is:

(A) 4 (B) 9 (C) 5 (D) 13

Answer: (B) Vertical distance from x-axis is $y = 9$. *Fast Method:* Height $= y$. *Distractor Note:* Picking $x = 4$.

Q5. A horizontal flat segment on a Distance–Time graph indicates:

(A) High speed (B) Rest (stopped) (C) Returning home (D) Acceleration

Answer: (B) Distance does not change with time. *Fast Method:* Flat = zero speed. *Distractor Note:* Thinking constant speed.

Q6. Which relation produces a LINEAR graph?

(A) $A = s^2$ (B) $V = a^3$ (C) $P = 4s$ (D) $xy = 12$

Answer: (C) Constant rate of change 4. *Fast Method:* Degree 1 in s. *Distractor Note:* Picking $A = s^2$.

Q7. If $A(2,1), B(6,1), C(6,5), D(2,5)$ are joined, $ABCD$ is a:

(A) Triangle (B) Square (C) Trapezium (D) Rhombus (non-square)

Answer: (B) Side $= 6 - 2 = 4$ and $5 - 1 = 4$ with 90° corners. *Fast Method:* 4×4 square. *Distractor Note:* Picking trapezium.

Q8. Which point lies on the line $y = 3x$?

(A) $(2,5)$ (B) $(3,9)$ (C) $(9,3)$ (D) $(1,4)$

Answer: (B) $3(3) = 9$. *Fast Method:* Plug $x = 3$. *Distractor Note:* Picking $(9,3)$.

Q9. A graph that displays data changing continuously over time is a:

(A) Pie chart (B) Histogram (C) Line graph (D) Pictograph

Answer: (C) Line graphs track continuous time-series data. *Fast Method:* Time $\rightarrow$ Line graph. *Distractor Note:* Picking histogram.

Q10. In a graph of Hours Studied vs Marks Scored, the independent variable is:

(A) Marks scored (B) Hours studied (C) Both (D) Neither

Answer: (B) Marks depend on hours studied. *Fast Method:* Cause on x-axis. *Distractor Note:* Reversing them.

Q11. Points $(1,4), (2,4), (5,4)$ lie on a line parallel to:

(A) x-axis (B) y-axis (C) Origin (D) Diagonal

Answer: (A) Constant $y = 4$ is a horizontal line parallel to x-axis. *Fast Method:* Same $y \rightarrow$ parallel to x-axis. *Distractor Note:* Picking y-axis.

Q12. Area of triangle with vertices $(0,0), (5,0), (0,8)$ is:

(A) 40 (B) 20 (C) 13 (D) 10

Answer: (B) $\frac{1}{2}(5)(8) = 20$. *Fast Method:* $\frac{1}{2}bh$. *Distractor Note:* Forgetting $\frac{1}{2}$.

H. Special Exam & MCQ Strategies

- Always label BOTH axes with variable names, units, and the exact Scale (e.g., 1 unit $= 10$ km) to earn full graph marks.
- Remember 'Walk before you Climb': move along x first, then up along y.
- Verify every linear graph by checking that at least 3 plotted points align along a single ruler edge.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Plotting $(3,5)$ as 5 units right and 3 units up. $\longrightarrow$ **Correction:** Always read (x,y) as (horizontal, vertical) — 3 right, 5 up!

[COMMON MISTAKE] **Error:** Stating that $(0,6)$ lies on the x-axis. $\longrightarrow$ **Correction:** Since $x = 0$, there is no horizontal shift, so $(0,6)$ lies on the y-axis.

[COMMON MISTAKE] **Error:** Connecting points of $A = s^2$ with a single straight ruler line. $\longrightarrow$ **Correction:** $A = s^2$ is non-linear; join points with a smooth freehand curve.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $P(x, y)$: x = abscissa, y = ordinate; Origin $= (0, 0)$; x-axis $= (a, 0)$; y-axis $= (0, b)$.

[QUICK REVISION] Key Theorems & Results: $y = kx$ (direct proportion) is a linear graph through $(0, 0)$; $A = s^2$ is curved.

[QUICK REVISION] Shortcut Summary: Walk-then-climb; Zero-slot axis rule; Constant $\Delta y/\Delta x$ means linear.

[QUICK REVISION] High-Frequency Exam Patterns: Line-graph reading, Axis identification, Rectangle/triangle vertices, Linear application graphs (I vs P, d vs t).

Last-Minute Revision Checklist:

- ☐ Label axes + units + scale
- ☐ Plot (x, y) horizontally first
- ☐ Flat segment = rest
- ☐ Check 3 points with ruler

K. Three-Tier Chapter Tests (Basic, Exam Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Ordinate of $(5, 9)$ is: (A) 5 (B) 9 (C) 14 (D) 4
2. Point $(8, 0)$ lies on: (A) x-axis (B) y-axis (C) Origin (D) Both axes
3. Coordinates of origin are: (A) $(0, 0)$ (B) $(1, 0)$ (C) $(0, 1)$ (D) $(1, 1)$
4. Independent variable is plotted on: (A) x-axis (B) y-axis (C) Either (D) Origin

PRACTICE ZONE: Exam Standard Practice Test (20 Min)

1. Distance of $(6, 2)$ from y-axis is: (A) 2 (B) 6 (C) 8 (D) 4
2. Fourth vertex of rectangle with $(2, 2), (7, 2), (7, 6)$ is: (A) $(2, 6)$ (B) $(6, 2)$ (C) $(2, 7)$ (D) $(7, 7)$
3. Which point lies on $y = 2x + 1$? (A) $(2, 4)$ (B) $(3, 7)$ (C) $(1, 2)$ (D) $(4, 8)$
4. Points $(3, 1), (3, 4), (3, 7)$ form a line parallel to: (A) x-axis (B) y-axis (C) Origin (D) Not a line

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Area of rectangle with vertices $(1, 2), (5, 2), (5, 5), (1, 5)$ is: (A) 12 (B) 15 (C) 20 (D) 14
2. A line passes through $(0, 0)$ and $(3, 15)$. At $x = 5$, y is: (A) 20 (B) 25 (C) 30 (D) 18
3. If $(2, 4), (4, 8), (6, k)$ are collinear, k is: (A) 10 (B) 12 (C) 14 (D) 16
4. Car goes from $(0, 0)$ to $(2\text{ h}, 110\text{ km})$. Its speed is: (A) 50 km/h (B) 55 km/h (C) 60 km/h (D) 110 km/h

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** Second coordinate is 9. **Q2 (A):** $y = 0 \implies x$-axis. **Q3 (A):** Intersection of axes. **Q4 (A):** Horizontal x-axis.
Exam Std	**Q1 (B):** Horizontal distance is $x = 6$. **Q2 (A):** $x = 2, y = 6 \implies (2, 6)$. **Q3 (B):** $2(3) + 1 = 7$. **Q4 (B):** Constant $x = 3 \implies$ vertical line parallel to y-axis.
Challenge	**Q1 (A):** $(5 - 1)(5 - 2) = 4 \times 3 = 12$. **Q2 (B):** $y = 5x \implies 5(5) = 25$. **Q3 (B):** $y = 2x \implies k = 12$. **Q4 (B):** $110/2 = 55$ km/h.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Reading line graphs. Day 2: Plotting (x, y) coordinates & axis rules. Day 3: Linear application graphs (Interest, Distance-Time).
- **How to Introduce the Concept:** Turn the classroom seating grid into a coordinate plane—call out $(2, 3)$ and have the student in row 2, column 3 stand up!
- **Difficult Areas & Remediation:** Confusing (x, y) with (y, x) and identifying which axis holds $(0, 5)$ vs $(5, 0)$.
- **Blackboard Teaching Examples:** Draw a big Quadrant I grid with labeled Scale box in the top-right corner.
- **Classroom Questions to Ask Students:** If a distance-time graph goes completely flat for 15 minutes, what is the cyclist doing?
- **Common Student Doubts & Answers:** Students ask why Area vs Side is not a straight line; plot $(1, 1), (2, 4), (3, 9)$ together to show the upward curve.
- **Homework & Revision Strategy:** Assign 2 line-graph interpretation sheets and 2 full linear graphs on graph paper.

[TEACHER'S TIP] Always deduct 0.5 marks in practice tests if students forget to write the Scale on their graph paper—they will never forget in the final exam!

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Complete Class 8 Formula Compendium, Shortcut Index & 2026–27 Annual Exam Model Paper with Answer Key

1. Master Formula Index (All 13 Chapters at a Glance)

Chapter	Essential Formulas, Identities & Rules
1. Rational Numbers	Closure $(+,-,\times)$, Commutative $(+,\times)$, Associative $(+,\times)$; Distributive: $a(b+c)=ab+ac$; Additive identity 0; Multiplicative identity 1; Additive inverse $-a$; Multiplicative inverse $1/a$ $(a\neq 0)$.
2. Linear Equations	Standard form $ax+b=0 \implies x=-b/a$; Transposition flips signs $(+\leftrightarrow -,\ \times\leftrightarrow \div)$; Cross-multiply $\frac{a}{b}=\frac{c}{d} \implies ad=bc$.
3. Quadrilaterals	Sum of interior angles of n-gon $=(n-2)\times 180^\circ$; Sum of all exterior angles $=360^\circ$; Regular polygon exterior angle $=360^\circ/n$; Parallelogram opposite sides/angles equal, diagonals bisect; Rhombus diagonals $\perp$; Rectangle diagonals equal; Square has all properties.
4. Data Handling	Central Angle $=\frac{\text{Value}}{\text{Total}}\times 360^\circ$; Probability $P(E)=\frac{\text{Favourable}}{\text{Total}}$, $0\le P(E)\le 1$, $P(E)+P(\text{not } E)=1$; Die $=6$, Coin $=2$, Cards $=52$.
5. Squares & Roots	Non-squares between $n^2,(n+1)^2=2n$; Sum of first n odds $=n^2$; Pythagorean triplet: $(2m,m^2-1,m^2+1)$; Never ends in $2,3,7,8$ or odd zeros; $(a5)^2=a(a+1)\vert 25$.
6. Cubes & Roots	Unit-digit swaps: $2\leftrightarrow 8, 3\leftrightarrow 7$; Odd-sum: n^3 is sum of n consecutive odds; Hardy–Ramanujan $1729=12^3+1^3=10^3+9^3$; Prime-factor triplets.
7. Comparing Quantities	% Change $=\frac{\text{Change}}{\text{Original}}\times 100$; Discount $=$ MP $-$ SP; GST on SP; CI: $A=P(1+\frac{R}{100})^n$, CI $=A-P$; Half-yearly: $R/2, 2n$.
8. Algebraic Expressions	$(a+b)^2=a^2+2ab+b^2$; $(a-b)^2=a^2-2ab+b^2$; $(a+b)(a-b)=a^2-b^2$; $(x+a)(x+b)=x^2+(a+b)x+ab$; $(a+b)^2-(a-b)^2=4ab$.
9. Mensuration	Trapezium $=\frac{1}{2}(a+b)h$; Rhombus $=\frac{1}{2}d_1d_2$; Cuboid: LSA $=2(l+b)h$, TSA $=2(lb+bh+hl)$, $V=lbh$; Cube: $4a^2, 6a^2, a^3$; Cylinder: CSA $=2\pi rh$, TSA $=2\pi r(r+h)$, $V=\pi r^2h$; $1\ \text{m}^3=1000$ L.
10. Exponents & Powers	$a^{-m}=1/a^m$; $(a/b)^{-m}=(b/a)^m$; $a^m a^n=a^{m+n}$; $a^m/a^n=a^{m-n}$; $(a^m)^n=a^{mn}$; $a^m b^m=(ab)^m$; $a^0=1$; Standard form $k\times 10^n$ $(1\le k<10)$.
11. Proportions	Direct: $x_1/y_1=x_2/y_2=k$; Inverse: $x_1y_1=x_2y_2=k$; $D=ST$; $M_1D_1H_1=M_2D_2H_2$.
12. Factorisation	$ab+ac=a(b+c)$; $ac+ad+bc+bd=(a+b)(c+d)$; $a^2-b^2=(a+b)(a-b)$; $a^2\pm 2ab+b^2=(a\pm b)^2$; $x^2+(p+q)x+pq=(x+p)(x+q)$.
13. Intro to Graphs	$P(x,y)$: x-axis $=(a,0)$, y-axis $=(0,b)$, Origin $=(0,0)$; Direct variation $y=kx$ is a linear graph through $(0,0)$; $A=s^2$ is curved.

2. Master Smart Shortcut & Mental Math Index

- **Ch. 1–2 (Fractions & Equations):** Clear denominators immediately by multiplying the entire equation by LCM of all denominators.
- **Ch. 3–4 (Geometry & Data):** Every convex polygon's exterior angles sum to 360°, so $n=360^\circ/\text{ext}$. For pie charts, multiply percentage by 3.6 to get degrees directly!
- **Ch. 5–6 (Squares & Cubes):** Square ending in 5: $a(a+1)\vert 25$. Long-division: Subtract $=R$; Add $=(Q+1)^2-N$. Cube root of up to 6-digit perfect cube: match last digit $(2\leftrightarrow 8, 3\leftrightarrow 7)$, drop last 3 digits, take floor cube root of remaining digits!
- **Ch. 7 (Comparing Quantities):** Successive %-change shortcut: $a+b+\frac{ab}{100}$! Use MP $=$ SP $\div(1-d/100)$ directly.
- **Ch. 8 & 12 (Algebra):** Plug $x=1$ (or $x=2$) into question and options to verify any expansion or factorisation in 3 seconds. For $x\pm 1/x=k$, $x^2+1/x^2=k^2\mp 2$.
- **Ch. 9–11 (Mensuration, Powers & Proportions):** Memorize $r=7 \implies C=44, A=154$. Elevator rule swaps negative exponents across fraction bar. Zero-count rule gives 10^{-n} for $0.00\ldots$ instantly. Double-test separates Direct vs Inverse proportion in 2 seconds.

3. Class 8 Mathematics Full-Length Annual Sample Paper (80 Marks)

Time Allowed: 3 Hours **Maximum Marks:** 80 **Session:** 2026–27 (NCERT/CBSE/RBSE)

General Instructions: Section A has 20 MCQs (1M each). Section B has 5 VSA (2M each). Section C has 6 SA (3M each). Section D has 4 LA (5M each). Section E has 3 Case-Based questions (4M each).

Section A: Multiple Choice Questions ($20 \times 1 = 20$ Marks)

1. Multiplicative inverse of $-1\frac{1}{8}$ is: (A) $\frac{9}{8}$ (B) $-\frac{8}{9}$ (C) $\frac{8}{9}$ (D) $1\frac{1}{8}$
2. If $5x - 3 = 3x + 7$, then $x =$ (A) 2 (B) 5 (C) 10 (D) 4
3. Each exterior angle of a regular polygon is $45°$. Number of sides is: (A) 6 (B) 8 (C) 9 (D) 10
4. A card is drawn from a well-shuffled deck of 52 cards. Probability of getting an ace is: (A) $\frac{1}{52}$ (B) $\frac{1}{13}$ (C) $\frac{4}{13}$ (D) $\frac{1}{26}$
5. How many natural numbers lie between 14^2 and 15^2? (A) 28 (B) 29 (C) 27 (D) 30
6. Value of $\sqrt[3]{1728}$ is: (A) 12 (B) 14 (C) 16 (D) 18
7. An item marked at Rs. 840 is sold for Rs. 714. Discount % is: (A) 12% (B) 15% (C) 18% (D) 20%
8. Value of $98^2 - 2^2$ is: (A) 9600 (B) 9604 (C) 9596 (D) 9200
9. Volume of a cube whose TSA is 216 cm^2 is: (A) 36 cm^3 (B) 216 cm^3 (C) 125 cm^3 (D) 64 cm^3
10. Standard form of 0.000035 is: (A) 3.5×10^{-5} (B) 3.5×10^{-4} (C) 35×10^{-6} (D) 3.5×10^{5}
11. If 12 workers build a wall in 15 hours, how many workers take 9 hours? (A) 18 (B) 20 (C) 24 (D) 16
12. Factorisation of $x^2 - 7x + 12$ is: (A) $(x-3)(x-4)$ (B) $(x+3)(x+4)$ (C) $(x-6)(x-2)$ (D) $(x-12)(x+1)$
13. Point $(0, 9)$ lies on: (A) x-axis (B) y-axis (C) Origin (D) Line $y = x$
14. Value of $(2^{-1} + 3^{-1})^0$ is: (A) 0 (B) 1 (C) $5/6$ (D) $6/5$
15. Sum of interior angles of a hexagon (6 sides) is: (A) $540°$ (B) $720°$ (C) $900°$ (D) $360°$
16. Smallest number to multiply 252 to make a perfect square is: (A) 2 (B) 3 (C) 7 (D) 14
17. If $x + \frac{1}{x} = 5$, then $x^2 + \frac{1}{x^2}$ is: (A) 25 (B) 23 (C) 27 (D) 21
18. 1 m^3 equals how many litres? (A) 100 (B) 1000 (C) 10000 (D) 10
19. **Assertion (A):** $(a+b)^2 = a^2 + 2ab + b^2$. **Reason (R):** An identity holds for all variable values. (A) Both A, R true & R explains A (B) Both true, R doesn't explain A (C) A true, R false (D) A false, R true
20. **Assertion (A):** Distance and time at constant speed are in inverse proportion. **Reason (R):** At constant speed, $D/T = S$. (A) Both true & R explains A (B) Both true, R doesn't explain A (C) A true, R false (D) A false, R true

Section B: Very Short Answer ($5 \times 2 = 10$ Marks)

21. Using distributive property, find: $\frac{7}{5} \times \left(-\frac{3}{12}\right) + \frac{7}{5} \times \frac{5}{12}$.
22. Solve: $\frac{x}{2} - \frac{1}{5} = \frac{x}{3} + \frac{1}{4}$.
23. Find the square root of 42.25 by long division.
24. Find the smallest number by which 675 must be multiplied to obtain a perfect cube.
25. Express 0.00000000837 in standard form and 3.02×10^{-6} in usual form.

Section C: Short Answer ($6 \times 3 = 18$ Marks)

26. In a parallelogram $RING$, if $\angle R = 70°$, find all the other angles.
27. A bag has 4 red balls and 2 yellow balls. A ball is drawn at random. Find the probability of getting (i) a red ball, (ii) a yellow ball.
28. Find the least number which must be added to 1750 so as to get a perfect square, and find its square root.
29. Using identities, evaluate: (i) 103^2, (ii) $51^2 - 49^2$, (iii) 103×97.
30. Factorise completely: (i) $63a^2 - 112b^2$, (ii) $p^2 + 6p - 16$.
31. 6 pipes are required to fill a tank in 1 hour 20 minutes. How long will it take if only 5 pipes of the same type are used?

Section D: Long Answer ($4 \times 5 = 20$ Marks)

32. Calculate the amount and compound interest on Rs. 10,800 for 3 years at $12\frac{1}{2}\%$ per annum compounded annually.
33. A road roller takes 750 complete revolutions to level a road. Find the area of the road if the diameter of the roller is 84 cm and its length is 1 m.
34. Simplify: $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$ and find m if $(-3)^{m+1} \times (-3)^5 = (-3)^7$.
35. Draw a linear graph for Deposit (P) vs Annual Simple Interest (I) at 10% p.a. for $P =$ Rs. 1000, 2000, 3000, 4000, and find: (i) Interest on Rs. 2500, (ii) Deposit for Rs. 350 interest.

Section E: Case-Based Questions ($3 \times 4 = 12$ Marks)

36. **School Hostel Survey (Pie Chart):** A hostel of 720 students speaks: Hindi (300), English (180), Marathi (120), Tamil (60), Bengali (60). Find: (a) Central angle for Hindi [1M], (b) Central angle for English [1M], (c) Combined percentage of Tamil & Bengali [2M].
37. **Water Reservoir Design (Mensuration):** A cuboidal water tank has dimensions 6 m $\times$ 5 m $\times$ 4.5 m. Find: (a) Volume in m^3 [1M], (b) Capacity in litres [1M], (c) Time in hours to fill if water pours at 750 L/min [2M].
38. **Garden Fencing (Algebra & Equations):** A rectangular garden has length $(2x + 5)$ m and breadth $(x + 2)$ m. Find: (a) Perimeter in terms of x [1M], (b) Area as a quadratic expression [2M], (c) Value of x if perimeter is 44 m [1M].

Complete Sample Paper Answer Key (Q1–Q38)

Section	Verified Answers & Key Steps
Sec A (1–20)	1:(B), 2:(B), 3:(B), 4:(B), 5:(A), 6:(A), 7:(B), 8:(A), 9:(B), 10:(A), 11:(B), 12:(A), 13:(B), 14:(B), 15:(B), 16:(C), 17:(B), 18:(B), 19:(A), 20:(D).
Sec B (21–25)	21: $\frac{7}{30}$; 22: $x = \frac{27}{10} = 2.7$; 23: 6.5; 24: Multiply by 5; 25: 8.37×10^{-9} and 0.00000302.
Sec C (26–31)	26: $\angle I = 110^\circ, \angle N = 70^\circ, \angle G = 110^\circ$; 27: (i) $2/3$, (ii) $1/3$; 28: Add 14, $\sqrt{1764} = 42$; 29: (i) 10609, (ii) 200, (iii) 9991; 30: (i) $7(3a + 4b)(3a - 4b)$, (ii) $(p + 8)(p - 2)$; 31: 1 hr 36 min.
Sec D (32–35)	32: $A =$ Rs. $15,377.34$, CI $=$ Rs. $4,577.34$; 33: 1980 m^2; 34: 3125 and $m = 1$; 35: (i) Rs. 250, (ii) Rs. 3500.
Sec E (36–38)	36: (a) 150°, (b) 90°, (c) $16\frac{2}{3}\%$; 37: (a) 135 m^3, (b) $135,000$ L, (c) 3 hours; 38: (a) $6x+14$, (b) $2x^2+9x+10$, (c) $x = 5$.