TYBA COACHING CENTRE, AJMER

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CLASS 11 MATHEMATICS

THE ULTIMATE COMPLETE MASTER TEXTBOOK

Dual-Board Edition: Rationalised NCERT / CBSE (Code 041) & RBSE (2026–27)

14 Complete Chapters • 12-Section Mastery Architecture • 100% English Edition

Concept Derivations • 14 Custom Vector Diagrams • Formula Banks • Smart Shortcuts • 224 Solved Examples • 168 Competency MCQs • 3-Tier Chapter Tests • Full 80-Mark Sample Paper

1. Edition Scope, Copyright & How to Use This Textbook

This master textbook is engineered for Class 11 students preparing simultaneously for **CBSE (Subject Code 041)**, **RBSE Class 11 Annual Examinations (2026–27)**, and foundational **JEE Main / CUET** mathematics. Every chapter follows a uniform **12-Section Mastery Architecture (Sections A–L)** so students build deep conceptual clarity alongside rapid exam-hall execution.

- **Sections A–C (Concept & Formula Core):** Exam relevance, intuitive derivations, custom vector figures, and complete formula banks with boundary conditions.
- **Sections D–F (Problem Classification & Worked Mastery):** High-frequency problem patterns, mathematically validated shortcuts, and 16 graded solved examples per chapter (Beginner to Challenge).
- **Sections G–J (MCQ Mastery, Strategy & Quick Revision):** 12 competency MCQs with distractor analysis, exam-hall tactics, common error fixes, and a dedicated 1-page revision sheet.
- **Sections K–L (3-Tier Assessment & Faculty Guide):** Basic, Board Standard, and Challenge tests with separate answer keys plus TYBA faculty classroom notes.

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2. Official CBSE & RBSE Class 11 Syllabus & Unit Weightage (2026–27)

CBSE Class 11 Mathematics (Code 041): Theory Examination = **80 Marks (3 Hours)** + Internal Assessment = **20 Marks** (Periodic Tests 10M + Mathematics Activities 10M) = **100 Marks**. **RBSE Class 11:** Follows the exact same 14 rationalised NCERT chapters (100 Marks, 3 Hours 15 Minutes).

Unit	Unit Name & Included Chapters	CBSE Marks	Periods
Unit I	**Sets and Functions** (Ch. 1 Sets, Ch. 2 Relations & Functions, Ch. 3 Trigonometric Functions)	**23**	60
Unit II	**Algebra** (Ch. 4 Complex Numbers, Ch. 5 Linear Inequalities, Ch. 6 Permutations & Combinations, Ch. 7 Binomial Theorem, Ch. 8 Sequences & Series)	**25**	50
Unit III	**Coordinate Geometry** (Ch. 9 Straight Lines, Ch. 10 Conic Sections, Ch. 11 Intro to 3D Geometry)	**12**	35
Unit IV	**Calculus** (Ch. 12 Limits and Derivatives)	**08**	40
Unit V	**Statistics and Probability** (Ch. 13 Statistics, Ch. 14 Probability)	**12**	25
Total	**All 5 Units / 14 Rationalised Chapters**	**80 (+20 IA)**	**210**

CBSE Question Paper Blueprint (80 Marks Theory)

- **Section A:** 18 Multiple Choice Questions (1 Mark each) + 2 Assertion-Reason Questions (1 Mark each) = **20 Marks**.
- **Section B:** 5 Very Short Answer (VSA) Questions (2 Marks each) = **10 Marks**.
- **Section C:** 6 Short Answer (SA) Questions (3 Marks each) = **18 Marks**.
- **Section D:** 4 Long Answer (LA) Questions (5 Marks each) = **20 Marks**.
- **Section E:** 3 Source/Case-Based Integrated Questions (4 Marks each) = **12 Marks**.

3. Complete 14-Chapter Table of Contents & Rationalised Scope

Ch.	Chapter Title	Core Focus & Rationalised Exclusions (2026–27)
01	Sets	Subsets, intervals, Venn diagrams, union/intersection. *(Excl: Power set exercises, complement word problems)*
02	Relations and Functions	Cartesian product, relations, domain/range, standard real graphs. *(Excl: $R \times R \times R$)*
03	Trigonometric Functions	Radians, ASTC signs, compound/multiple/C–D identities. *(Excl: General trig equations, sine/cosine rules)*
04	Complex Numbers	Algebra of i, $a+ib$, conjugate, modulus, Argand plane. *(Excl: Polar form, square root of z)*
05	Linear Inequalities	Algebraic inequalities in one variable on number line. *(Excl: 2-variable graphical shading)*
06	Permutations & Combinations	Counting principle, $n!$, ${}^nP_r$, ${}^nC_r$, repetitions. *(Full chapter retained)*
07	Binomial Theorem	Positive integral expansion $(a+b)^n$, Pascal's triangle. *(Excl: General/middle term exercises)*
08	Sequences and Series	G.P., n-th term, S_n, infinite G.P., G.M., $AM \geq GM$. *(Excl: A.P. exercises, $\sum n^2, \sum n^3$)*
09	Straight Lines	Slope, angle, line forms, point-line distance. *(Excl: Normal form, origin shift)*
10	Conic Sections	Circle, Parabola, Ellipse, Hyperbola standard forms. *(Excl: Degenerate cases)*
11	Intro to 3D Geometry	3D axes, octants, coordinates, 3D distance formula. *(Excl: 3D section formula)*
12	Limits and Derivatives	Standard limits, first principles, product/quotient rules. *(Excl: Complex chain rule)*
13	Statistics	Mean deviation (mean/median), variance, standard deviation. *(Excl: Equal-mean CV comparison)*
14	Probability	Axiomatic probability, $P(A \cup B)$, mutually exclusive/exhaustive events. *(Full axiomatic core retained)*

UNIT I: SETS AND FUNCTIONS TYBA COACHING CENTRE

Chapter 1: Sets

Roster & Set-Builder Forms, Subsets, Intervals, Venn Diagrams & Set Operations (2026–27)

A. Chapter Introduction & Examination Relevance

Set theory is the foundational language of modern mathematics. Introduced by Georg Cantor, sets allow us to group well-defined objects—numbers, functions, events, or geometric points—and reason about their relationships rigorously.

- **What You Will Learn:** Well-defined sets, Roster and Set-Builder forms, empty/finite/infinite/equal sets, subsets, intervals of $\mathbb{R}$, universal sets, Venn diagrams, union, intersection, difference, complement, and cardinality identities.
- **Why This Chapter Matters:** Every subsequent topic in Class 11 and Class 12—Relations, Functions, Domain/Range, Linear Inequalities, and Probability—is written in set notation. Mastering intervals and set operations prevents foundational errors across all units.
- **Real-Life Applications:** Database SQL queries (UNION, INTERSECT, EXCEPT), search-engine filtering, digital logic circuits, survey analytics, and sample-space modeling in probability.
- **Cross-Chapter Connections:** Directly feeds into Chapter 2 (Cartesian products $A \times B$), Chapter 5 (solution sets as intervals), Chapter 13 (data classes), and Chapter 14 (events as subsets of sample space S).
- **Board & Competitive Relevance:** Unit I carries 23/80 marks in CBSE. Chapter 1 contributes 4–6 marks in CBSE/RBSE via 1-mark MCQs on subsets/intervals and 2–3 mark proofs on De Morgan's laws and cardinality.

B. Complete Concept Explanation & Derivations

1. Representation of Sets & Types of Sets

A **set** is a well-defined collection of distinct objects. **Roster (Tabular) Form** lists elements separated by commas inside braces, e.g., $V = \{a, e, i, o, u\}$ (order and repetition do not matter). **Set-Builder Form** specifies a defining property $P(x)$: $A = \{x : x \in \mathbb{N},\ x < 6\}$. Key types include the **Empty Set** ($\emptyset$ or $\{\}$), **Singleton Set** (1 element), **Finite/Infinite Sets**, and **Equal Sets** ($A = B$ iff every element of A is in B and vice versa).

[CONCEPT MASTER – 1. Representation of Sets & Types of Sets] Note that $\{0\}$ and $\{\emptyset\}$ are singleton sets (each has 1 element), whereas $\emptyset = \{\}$ has 0 elements.

2. Subsets, Intervals of $\mathbb{R}$ & Power Set

A is a **subset** of B ($A \subseteq B$) if $x \in A \implies x \in B$. Every set is a subset of itself, and $\emptyset$ is a subset of every set. For a finite set with $n(A) = n$, the number of subsets is 2^n and the number of proper subsets is $2^n - 1$. Real subsets are written as intervals: open $(a, b) = \{x \in \mathbb{R} : a < x < b\}$, closed $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$, and half-open $[a, b)$ or $(a, b]$.

[CONCEPT MASTER – 2. Subsets, Intervals of $\mathbb{R}$ & Power Set] Always use parentheses (or) near ∞ and $-\infty$, never square brackets, because ∞ is not a real number.

3. Venn Diagrams & Algebraic Operations on Sets

Within a Universal Set U, the **Union** is $A \cup B = \{x : x \in A \text{ or } x \in B\}$, the **Intersection** is $A \cap B = \{x : x \in A \text{ and } x \in B\}$, the **Difference** is $A - B = \{x : x \in A \text{ and } x \notin B\}$, and the **Complement** is $A' = U - A$. Two sets are **disjoint** if $A \cap B = \emptyset$.

[CONCEPT MASTER – 3. Venn Diagrams & Algebraic Operations on Sets] De Morgan's Laws: $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$. Notice how complement flips $\cup$ to $\cap$ and $\cap$ to $\cup$.

4. Cardinality & Practical Union Identities

For finite sets A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, because elements in $A \cap B$ are counted twice in $n(A) + n(B)$. Also $n(A - B) = n(A) - n(A \cap B)$. For three sets, $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$.

[CONCEPT MASTER – 4. Cardinality & Practical Union Identities] Rationalised 2026–27 Scope: Complex power-set exercises and heavy complement word problems are excluded, while core union/intersection/De Morgan proofs remain essential.

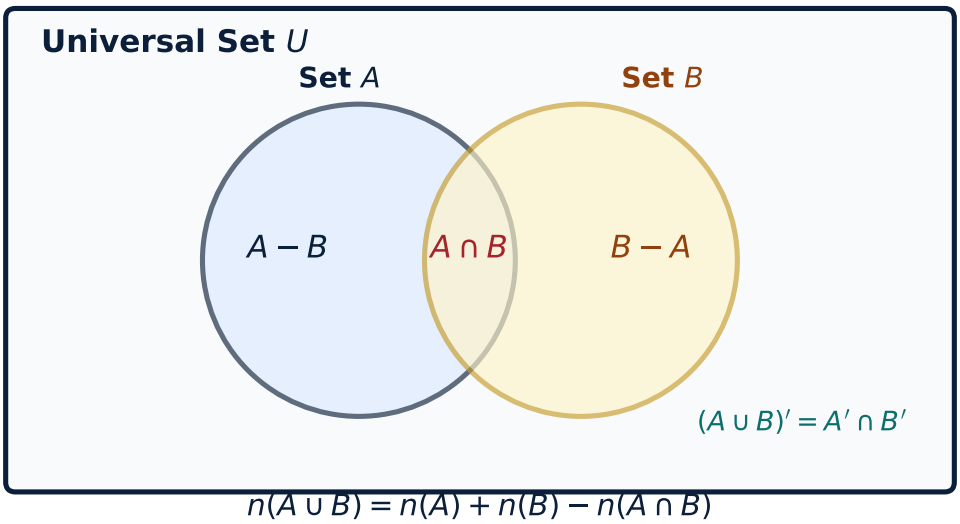


Figure 1: **Visual Model:** Venn diagram illustrating disjoint regions $A - B$, $A \cap B$, $B - A$, De Morgan's Law $(A \cup B)' = A' \cap B'$, and the inclusion-exclusion identity.

C. Complete Formula Bank

[FORMULA BANK – 1. Number of Subsets & Power Set Cardinality] **Expression:** $n(P(A)) = 2^n$, Proper Subsets $= 2^n - 1$
Symbols: $n = n(A)$ is the number of elements in finite set A.
Conditions & When to Use: Valid for any finite set A with $n \geq 0$.
Solved Mini-Example: If $A = \{1, 2, 3\}$, $n = 3 \implies 2^3 = 8$ subsets and 7 proper subsets.
Common Mistake: Confusing number of elements in A with number of elements in $P(A)$.

[FORMULA BANK – 2. Two-Set Union Cardinality] **Expression:** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
Symbols: $n(A)$: cardinality of A; $n(A \cap B)$: common elements.
Conditions & When to Use: Finite sets A, B. If disjoint, $n(A \cap B) = 0$.
Solved Mini-Example: $n(A) = 20, n(B) = 15, n(A \cap B) = 5 \implies n(A \cup B) = 20 + 15 - 5 = 30$.
Common Mistake: Adding $n(A) + n(B)$ without subtracting $n(A \cap B)$.

[FORMULA BANK – 3. Difference & Symmetric Difference] **Expression:** $n(A - B) = n(A) - n(A \cap B) = n(A \cup B) - n(B)$
Symbols: $A - B$: elements in A only.
Conditions & When to Use: Finite sets A, B. Note $A - B = A \cap B'$.
Solved Mini-Example: If $n(A) = 18, n(A \cap B) = 6$, then $n(A - B) = 12$.
Common Mistake: Assuming $A - B = B - A$; set difference is not commutative.

[FORMULA BANK – 4. De Morgan's Laws] **Expression:** $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$
Symbols: $A' = U - A$ is complement in universal set U.
Conditions & When to Use: Always true for any subsets $A, B \subseteq U$.
Solved Mini-Example: If $U = \{1,2,3,4\}, A = \{1,2\}, B = \{2,3\}$, $(A \cup B)' = \{4\} = A' \cap B'$.
Common Mistake: Writing $(A \cup B)' = A' \cup B'$ without flipping the operator.

[FORMULA BANK – 5. Distributive Laws of Sets] **Expression:** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Symbols: A, B, C: any three sets.
Conditions & When to Use: Universal set identities.
Solved Mini-Example: Used to simplify $(A \cup B) \cap (A \cup B') = A \cup (B \cap B') = A$.
Common Mistake: Dropping parentheses when mixing $\cup$ and $\cap$.

[FORMULA BANK – 6. Interval Length & Representation] **Expression:** $(a,b) = \{x : a < x < b\}$, $[a,b] = \{x : a \leq x \leq b\}$
Symbols: $a, b \in \mathbb{R}$ with $a < b$.
Conditions & When to Use: Used for continuous real-number subsets.
Solved Mini-Example: $\{x \in \mathbb{R} : -3 < x \leq 5\} = (-3, 5]$.
Common Mistake: Using $[a,b]$ for discrete integer sets like $\{1,2,3\}$.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and JEE/CUET Foundation problem structures.

Pattern 1: Roster $\leftrightarrow$ Set-Builder Conversion

- **Identification Cue:** Write in roster form or set-builder form. | **Concept & Formula:** Defining property $P(x)$ vs explicit listing.
- **Standard Step-by-Step:** Solve the defining equation/inequality over the given domain ($\mathbb{N}, \mathbb{Z}, \mathbb{R}$) and list unique elements.
- **Fastest Valid Method:** Test boundary integers directly. | **Alternative:** Factorize algebraic conditions first.
- **Common Trap:** Ignoring the domain restriction (e.g., $x \in \mathbb{N}$ vs $x \in \mathbb{Z}$).
- **Practice Drill:** Ex. 1 & Ex. 2.

Pattern 2: Subset & $\in$ vs $\subseteq$ Discrimination

- **Identification Cue:** Nested sets like $A = \{1, 2, \{3,4\}, 5\}$. | **Concept & Formula:** Element ($\in$) vs Subset ($\subseteq$).
- **Standard Step-by-Step:** Treat each comma-separated item at the top level as an element; enclose an element in $\{\}$ to make it a subset.
- **Fastest Valid Method:** Strip one pair of outer braces to check $\in$; keep them to check $\subseteq$. | **Alternative:** List all elements explicitly first.
- **Common Trap:** Writing $\{3,4\} \subseteq A$ instead of $\{3,4\} \in A$.
- **Practice Drill:** Ex. 5 & MCQ Q3.

Pattern 3: Interval Operations ($A \cup B, A \cap B, A - B$)

- **Identification Cue:** Find intersection/difference of intervals like $[-2, 5)$ and $(1, 7]$. | **Concept & Formula:** Number-line overlap.
- **Standard Step-by-Step:** Draw both intervals on a number line with open/closed dots and read the common or excluded segment.
- **Fastest Valid Method:** Intersection takes $(\max(a_1, a_2), \min(b_1, b_2))$ with stricter endpoint. | **Alternative:** Test a sample point at endpoints.
- **Common Trap:** Including an endpoint that is open in one of the sets during intersection.
- **Practice Drill:** Ex. 6 & Ex. 7.

Pattern 4: Two-Set & Three-Set Cardinality

- **Identification Cue:** Survey problems with 'likes A', 'likes B', 'both', 'neither'. | **Concept & Formula:** $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
- **Standard Step-by-Step:** Assign variables to Venn regions from innermost $A \cap B$ outward.
- **Fastest Valid Method:** Use $n(U) - n(\text{neither}) = n(A \cup B)$ directly. | **Alternative:** Algebraic inclusion-exclusion.
- **Common Trap:** Forgetting 'neither' when $n(A \cup B) < n(U)$.
- **Practice Drill:** Ex. 11 & Ex. 14.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Element vs Subset One-Brace Rule] **Core Idea & Why It Works:** An item x belongs to A ($x \in A$) iff $\{x\} \subseteq A$.
Valid Domain: All sets including nested sets. | **Where It Fails:** When x is inside a deeper nested brace.
Easy Example: In $A = \{1, \{2\}\}$, $2 \notin A$, $\{2\} \in A$, and $\{\{2\}\} \subseteq A$.
Board-Level Example: Check if $\{3, 4\} \subset \{1, 2, \{3, 4\}\}$: False, because $3 \notin A$.
Advanced Example: Power set membership: $B \in P(A) \iff B \subseteq A$.

[SMART SHORTCUT – 2. Venn-Region Innermost-First Fill] **Core Idea & Why It Works:** Always fill $n(A \cap B \cap C)$ or $n(A \cap B)$ first, then subtract to get 'only A' and 'only B'.
Valid Domain: All 2-set and 3-set survey problems. | **Where It Fails:** Infinite sets.
Easy Example: $n(A) = 10, n(B) = 8, n(A \cap B) = 3 \implies$ Only $A = 7$, Only $B = 5$.
Board-Level Example: In 60 students, 35 play cricket, 25 football, 10 both $\implies$ Total $= 25 + 10 + 15 = 50$, neither $= 10$.
Advanced Example: Find exact 'at least two' = sum of pairwise intersections minus $2n(A \cap B \cap C)$.

[SMART SHORTCUT – 3. Quick Subset Count $2^{n(A)}$] **Core Idea & Why It Works:** Each element has 2 choices (in or out), giving 2^n subsets.
Valid Domain: Finite sets. | **Where It Fails:** Infinite sets.
Easy Example: $n(A) = 4 \implies 16$ subsets.
Board-Level Example: If $2^m - 2^n = 56$, factor $2^n(2^{m-n} - 1) = 8 \times 7 \implies n = 3, m = 6$.
Advanced Example: Number of non-empty subsets of A is $2^n - 1$.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Write $A = \{x : x \text{ is an integer and } -3 \leq x < 7\}$ in roster form.
Given & Required: $x \in \mathbb{Z}, -3 \leq x < 7$. | **Concept & Formula:** Roster form.
Detailed Solution: Integers from -3 up to 6 are $-3, -2, -1, 0, 1, 2, 3, 4, 5, 6$. **Final Answer:** $\{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$
Fastest Alternative / Shortcut: Check endpoints: -3 included, 7 excluded.

SOLVED EXAMPLE 2 [Beginner]
Question: Write $B = \{2, 4, 8, 16, 32\}$ in set-builder form.
Given & Required: Powers of 2 from 2^1 to 2^5. | **Concept & Formula:** Set-builder representation.
Detailed Solution: Each element is 2^n where $n \in \mathbb{N}$ and $1 \leq n \leq 5$. **Final Answer:** $\{x : x = 2^n,\ n \in \mathbb{N},\ 1 \leq n \leq 5\}$
Fastest Alternative / Shortcut: Spot base 2 immediately.

SOLVED EXAMPLE 3 [Beginner]

Question: How many subsets does $A = \{a, b, c, d\}$ have?

Given & Required: $n(A) = 4$. | **Concept & Formula:** $n(P(A)) = 2^n$.

Detailed Solution: Total subsets $= 2^4 = 16$. **Final Answer:** 16

Fastest Alternative / Shortcut: $2^4 = 16$.

SOLVED EXAMPLE 4 [Beginner]

Question: Write $\{x : x \in \mathbb{R},\ -4 < x \leq 6\}$ as an interval and find its length.

Given & Required: $-4 < x \leq 6$. | **Concept & Formula:** Half-open interval.

Detailed Solution: Left open, right closed gives $(-4, 6]$. Length $= 6 - (-4) = 10$. **Final Answer:** $(-4, 6]$, **length** $= 10$

Fastest Alternative / Shortcut: Strict $<$ uses $($; $\leq$ uses $]$.

SOLVED EXAMPLE 5 [Intermediate]

Question: If $A = \{1, 2, \{3, 4\}, 5\}$, state whether $\{3, 4\} \subset A$ and $\{\{3, 4\}\} \subset A$ are true.

Given & Required: A has 4 elements: $1, 2, \{3, 4\}, 5$. | **Concept & Formula:** $\in$ vs $\subset$.

Detailed Solution: $3 \notin A$, so $\{3, 4\} \subset A$ is False (it is $\{3, 4\} \in A$). Enclosing it in braces gives $\{\{3, 4\}\} \subset A$, which is True.
Final Answer: **First False, Second True**

Fastest Alternative / Shortcut: One-brace rule.

SOLVED EXAMPLE 6 [Intermediate]

Question: If $A = [-2, 5)$ and $B = (1, 7]$, find $A \cap B$ and $A - B$.

Given & Required: Intervals A, B. | **Concept & Formula:** Interval operations.

Detailed Solution: Common part is $(1, 5)$. Elements in A not in B are $[-2, 1]$. **Final Answer:** $A \cap B = (1, 5)$, $A - B = [-2, 1]$

Fastest Alternative / Shortcut: Note $1 \notin B$, so $1 \in A - B$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3\}$, $B = \{3, 4, 5\}$. Verify $(A \cup B)' = A' \cap B'$.

Given & Required: U, A, B. | **Concept & Formula:** De Morgan's Law.

Detailed Solution: $A \cup B = \{2, 3, 4, 5\} \implies (A \cup B)' = \{1, 6\}$. Also $A' = \{1, 4, 5, 6\}, B' = \{1, 2, 6\} \implies A' \cap B' = \{1, 6\}$. Verified. **Final Answer:** **Both equal** $\{1, 6\}$

Fastest Alternative / Shortcut: Compute $A \cup B$ first.

SOLVED EXAMPLE 8 [Intermediate]

Question: In a group of 70 people, 37 like coffee, 52 like tea, and each person likes at least one. How many like both?

Given & Required: $n(C \cup T) = 70, n(C) = 37, n(T) = 52$. | **Concept & Formula:** Union cardinality.

Detailed Solution: $n(C \cap T) = 37 + 52 - 70 = 19$. **Final Answer:** 19

Fastest Alternative / Shortcut: $37 + 52 - 70 = 19$.

SOLVED EXAMPLE 9 [Board Standard]

Question: Two finite sets have m and n elements. The total number of subsets of the first set is 56 more than the second. Find m and n.

Given & Required: $2^m - 2^n = 56$. | **Concept & Formula:** Subset count.

Detailed Solution: $2^n(2^{m-n} - 1) = 8 \times 7 = 2^3(2^3 - 1)$. Equating even and odd factors gives $n = 3$ and $m - n = 3 \implies m = 6$.
Final Answer: $m = 6,\ n = 3$

Fastest Alternative / Shortcut: Factor $56 = 2^3 \times 7$.

SOLVED EXAMPLE 10 [Board Standard]

Question: Prove that for any sets A and B, $A \cup (B - A) = A \cup B$.

Given & Required: Arbitrary sets A, B. | **Concept & Formula:** Set algebra.

Detailed Solution: $A \cup (B - A) = A \cup (B \cap A') = (A \cup B) \cap (A \cup A') = (A \cup B) \cap U = A \cup B$. **Final Answer: Proved using distributive law**

Fastest Alternative / Shortcut: Replace $B - A$ with $B \cap A'$.

SOLVED EXAMPLE 11 [Board Standard]

Question: In a class of 35 students, 24 like cricket, 16 like football, and each student likes at least one. How many like both, and how many like only cricket?

Given & Required: $n(C \cup F) = 35, n(C) = 24, n(F) = 16$. | **Concept & Formula:** Venn regions.

Detailed Solution: $n(C \cap F) = 24 + 16 - 35 = 5$. Only cricket $= 24 - 5 = 19$. **Final Answer: Both $= 5$, Only cricket $= 19$**

Fastest Alternative / Shortcut: $24 + 16 - 35 = 5$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Show that $A \cap B = A \cap C$ need not imply $B = C$.

Given & Required: Counterexample needed. | **Concept & Formula:** Disproof by counterexample.

Detailed Solution: Take $A = \{1\}, B = \{1, 2\}, C = \{1, 3\}$. Then $A \cap B = \{1\} = A \cap C$, yet $B \neq C$. **Final Answer: Disproved via $A = \{1\}, B = \{1, 2\}, C = \{1, 3\}$**

Fastest Alternative / Shortcut: Make B, C differ outside A.

SOLVED EXAMPLE 13 [Board Standard]

Question: If $X = \{4^n - 3n - 1 : n \in \mathbb{N}\}$ and $Y = \{9(n - 1) : n \in \mathbb{N}\}$, prove $X \subseteq Y$.

Given & Required: Sets X, Y. | **Concept & Formula:** Binomial/factorization proof.

Detailed Solution: $4^n - 3n - 1 = (1 + 3)^n - 3n - 1 = 1 + 3n + 9k - 3n - 1 = 9k$ for integer $k \geq 0$. Thus every element of X is a non-negative multiple of 9, which lies in Y. **Final Answer: $X \subseteq Y$ proved**

Fastest Alternative / Shortcut: Test $n = 1, 2, 3$: $X = \{0, 9, 54, \dots\} \subset \{0, 9, 18, \dots\}$.

SOLVED EXAMPLE 14 [Challenge]

Question: If $n(U) = 100, n(A) = 50, n(B) = 70$, find the minimum and maximum possible values of $n(A \cap B)$.

Given & Required: $n(U) = 100, n(A) = 50, n(B) = 70$. | **Concept & Formula:** Cardinality bounds.

Detailed Solution: Max $n(A \cap B) = \min(50, 70) = 50$. Min occurs when $n(A \cup B)$ is max (100): $\min n(A \cap B) = 50 + 70 - 100 = 20$. **Final Answer: Min $= 20$, Max $= 50$**

Fastest Alternative / Shortcut: $\max(0, n(A) + n(B) - n(U)) \leq n(A \cap B) \leq \min(n(A), n(B))$.

SOLVED EXAMPLE 15 [Challenge]

Question: Simplify $(A \cap B) \cup (A - B)$.

Given & Required: Sets A, B. | **Concept & Formula:** Set identities.

Detailed Solution: $(A \cap B) \cup (A \cap B') = A \cap (B \cup B') = A \cap U = A$. **Final Answer:** A

Fastest Alternative / Shortcut: Distributive law in reverse.

SOLVED EXAMPLE 16 [Challenge]

Question: In a survey of 100 students, 40 study Maths, 45 Physics, 35 Chemistry, 15 M&P, 12 P&C, 10 M&C, and 5 all three. How many study none?

Given & Required: 3-set data, $n(U) = 100$. | **Concept & Formula:** 3-set inclusion-exclusion.

Detailed Solution: $n(M \cup P \cup C) = 40 + 45 + 35 - 15 - 12 - 10 + 5 = 88$. None $= 100 - 88 = 12$. **Final Answer:** 12

Fastest Alternative / Shortcut: $100 - (120 - 37 + 5) = 12$.

G. Competency MCQ Mastery Section

Q1. Which of the following is an empty set?

(A) $\{x \in \mathbb{R} : x^2 - 1 = 0\}$ (B) $\{x \in \mathbb{R} : x^2 + 1 = 0\}$ (C) $\{0\}$ (D) $\{\emptyset\}$

Answer: (B) $x^2 + 1 = 0$ has no real solution. *Fast Method:* $x^2 \geq 0$ for all $x \in \mathbb{R}$. *Distractor Note:* $\{0\}$ and $\{\emptyset\}$ have 1 element each.

Q2. If $A = \{x \in \mathbb{Z} : x^2 \leq 4\}$, then $n(A)$ is:

(A) 2 (B) 3 (C) 4 (D) 5

Answer: (D) $A = \{-2, -1, 0, 1, 2\}$, so $n(A) = 5$. *Fast Method:* Count negative integers too. *Distractor Note:* Forgetting $-2, -1, 0$ gives 2.

Q3. For $A = \{1, 2, \{3, 4\}\}$, which statement is correct?

(A) $3 \in A$ (B) $\{3, 4\} \subset A$ (C) $\{3, 4\} \in A$ (D) $\{1, 3\} \subset A$

Answer: (C) $\{3, 4\}$ is a single element of A. *Fast Method:* Check top-level commas. *Distractor Note:* Confusing $\in$ with $\subset$.

Q4. The number of proper subsets of $\{1, 2, 3, 4, 5\}$ is:

(A) 32 (B) 31 (C) 30 (D) 25

Answer: (B) $2^5 - 1 = 31$. *Fast Method:* Subtract A itself from 2^5. *Distractor Note:* 32 includes the improper subset A.

Q5. If $A = (-1, 4]$ and $B = [2, 6)$, then $A \cap B$ is:

(A) $[2, 4]$ (B) $(2, 4)$ (C) $[-1, 6)$ (D) $(2, 4]$

Answer: (A) Both 2 and 4 belong to both A and B. *Fast Method:* $\max(-1, 2) = 2$ closed, $\min(4, 6) = 4$ closed. *Distractor Note:* Using open brackets.

Q6. If $n(A) = 15, n(B) = 20, n(A \cup B) = 28$, then $n(A \cap B)$ is:

(A) 5 (B) 7 (C) 13 (D) 35

Answer: (B) $15 + 20 - 28 = 7$. *Fast Method:* Add and subtract union. *Distractor Note:* Adding $28 - 20$ gives 8.

Q7. $(A \cup B')'$ equals:

(A) $A' \cap B$ (B) $A' \cup B$ (C) $A \cap B'$ (D) $A \cup B$

Answer: (A) By De Morgan's Law, $(A \cup B')' = A' \cap (B')' = A' \cap B$. *Fast Method:* Flip $\cup$ to $\cap$ and double-complement B. *Distractor Note:* Forgetting $(B')' = B$.

Q8. If $A \subseteq B$, then $A \cap B$ equals:

(A) B (B) A (C) $\emptyset$ (D) U

Answer: (B) All elements of A are in B, so common part is A. *Fast Method:* Smaller set wins in $\cap$. *Distractor Note:* $A \cup B = B$, not $A \cap B$.

Q9. If $n(A) = 3, n(B) = 6$, the minimum value of $n(A \cup B)$ is:

(A) 3 (B) 6 (C) 9 (D) 0

Answer: (B) Minimum occurs when $A \subseteq B$, giving $n(A \cup B) = 6$. *Fast Method:* $\max(n(A), n(B)) = 6$. *Distractor Note:* 9 is the maximum, not minimum.

Q10. Which represents $\{x \in \mathbb{R} : 0 \leq x < 5\}$?

(A) $(0, 5)$ (B) $[0, 5]$ (C) $[0, 5)$ (D) $(0, 5]$

Answer: (C) 0 included, 5 excluded. *Fast Method:* Match $\leq$ with [and $<$ with). *Distractor Note:* Swapping brackets.

Q11. For disjoint sets A and B, $A - B$ equals:

(A) $\emptyset$ (B) A (C) B (D) $A \cup B$

Answer: (B) Since $A \cap B = \emptyset$, no element of A is removed. *Fast Method:* $n(A - B) = n(A) - 0$. *Distractor Note:* Thinking $A - B = \emptyset$.

Q12. If $n(U) = 50, n(A) = 25, n(B) = 20, n(A \cap B) = 5$, then $n((A \cup B)')$ is:

(A) 5 (B) 10 (C) 15 (D) 40

Answer: (B) $n(A \cup B) = 25 + 20 - 5 = 40$, so complement $= 50 - 40 = 10$. *Fast Method:* $50 - 40 = 10$. *Distractor Note:* Forgetting to subtract 5 first.

H. Special Exam & MCQ Strategies

- Always read the domain ($\mathbb{N}, \mathbb{W}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$) before listing elements in Roster form.
- For interval questions, sketch a quick 2-line number line with hollow circles for (and solid dots for [.
- In word problems, draw a 2-circle or 3-circle Venn diagram and fill the innermost intersection first.
- To verify set identities in MCQs, test with small concrete sets like $U = \{1, 2, 3, 4\}, A = \{1, 2\}, B = \{2, 3\}$.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Writing $\emptyset = \{0\}$ or $\{\emptyset\}$. $\longrightarrow$ **Correction:** $\emptyset = \{\}$ has 0 elements; $\{0\}$ and $\{\emptyset\}$ have 1 element.

[COMMON MISTAKE] **Error:** Writing $[2, \infty]$ for $x \geq 2$. $\longrightarrow$ **Correction:** Always use $[2, \infty)$; infinity is never enclosed in a square bracket.

[COMMON MISTAKE] **Error:** Confusing $A - B$ with $B - A$. $\longrightarrow$ **Correction:** $A - B$ keeps elements only in A; they are equal only when $A = B$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $n(P(A)) = 2^n$; $n(A \cup B) = n(A) + n(B) - n(A \cap B)$; $n(A - B) = n(A) - n(A \cap B)$.

[QUICK REVISION] Key Theorems & Results: De Morgan: $(A \cup B)' = A' \cap B'$, $(A \cap B)' = A' \cup B'$; $A - B = A \cap B'$.

[QUICK REVISION] Shortcut Summary: Element $\in$ vs Subset $\subseteq$ (add braces for subset); Min $n(A \cup B) = \max(n(A), n(B))$.

[QUICK REVISION] High-Frequency Exam Patterns: Roster/Set-builder, Nested-set membership, Interval intersection, 2-set/3-set survey.

Last-Minute Revision Checklist:

- ☐ Domain check ($\mathbb{N}$ vs $\mathbb{Z}$ vs $\mathbb{R}$)
- ☐ Open vs closed interval brackets
- ☐ De Morgan operator flip
- ☐ Venn diagram innermost-first fill

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Number of subsets of $\{x, y, z\}$ is: (A) 3 (B) 6 (C) 8 (D) 9
2. Roster form of $\{x \in \mathbb{N} : x^2 < 10\}$ is: (A) $\{1, 2, 3\}$ (B) $\{0, 1, 2, 3\}$ (C) $\{-3, \ldots, 3\}$ (D) $\{1, 2\}$
3. If $A = \{1, 2\}, B = \{2, 3\}$, then $A - B$ is: (A) $\{2\}$ (B) $\{1\}$ (C) $\{3\}$ (D) $\{1, 3\}$
4. Interval for $-2 < x < 5$ is: (A) $[-2, 5]$ (B) $(-2, 5]$ (C) $[-2, 5)$ (D) $(-2, 5)$

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. If $n(A) = 25, n(B) = 18, n(A \cup B) = 35$, find $n(A \cap B)$: (A) 7 (B) 8 (C) 10 (D) 17
2. If $A = [-3, 2)$ and $B = [0, 5]$, then $A \cup B$ is: (A) $[0, 2)$ (B) $[-3, 5]$ (C) $(-3, 5)$ (D) $[-3, 0)$
3. If $2^m - 2^n = 28$, then (m, n) is: (A) $(5, 2)$ (B) $(4, 2)$ (C) $(5, 3)$ (D) $(6, 2)$
4. Which equals $(A' \cup B')'$? (A) $A \cup B$ (B) $A \cap B$ (C) $A' \cap B'$ (D) $\emptyset$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If $n(U) = 80, n(A) = 45, n(B) = 55$, minimum $n(A \cap B)$ is: (A) 0 (B) 10 (C) 20 (D) 45
2. If $A = \{1, 2\}, B = \{1, 2, 3, 4\}$, number of sets C with $A \subseteq C \subseteq B$ is: (A) 2 (B) 4 (C) 6 (D) 8
3. $A - (A - B)$ equals: (A) $A \cup B$ (B) $A \cap B$ (C) B (D) A
4. If $n(A) = 4$ and $n(B) = 7$, maximum value of $n(A \cup B)$ is: (A) 7 (B) 11 (C) 4 (D) 28

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (C):** $2^3 = 8$. **Q2 (A):** $1^2 = 1, 2^2 = 4, 3^2 = 9 < 10$ in $\mathbb{N}$. **Q3 (B):** Remove 2 from A to get $\{1\}$. **Q4 (D):** Both strict inequalities use open parentheses.
Board Std	**Q1 (B):** $25 + 18 - 35 = 8$. **Q2 (B):** Union spans from -3 (closed) to 5 (closed). **Q3 (A):** $2^2(2^3 - 1) = 4 \times 7 = 28$, so $n = 2$ and $m = 5$. **Q4 (B):** By De Morgan's Law, $(A' \cup B')' = A \cap B$.
Challenge	**Q1 (C):** $45 + 55 - 80 = 20$. **Q2 (B):** Choices for $\{3, 4\}$ give $2^2 = 4$ sets. **Q3 (B):** Removing $A - B$ from A leaves $A \cap B$. **Q4 (B):** Max union when disjoint: $4 + 7 = 11$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Well-defined sets, Roster/Set-builder. Day 2: Types, subsets, intervals. Day 3: Venn diagrams, De Morgan's Laws. Day 4: Cardinality word problems.
- **How to Introduce the Concept:** Ask students to list 'good cricketers in India' vs 'players with $> 10,000$ ODI runs' to contrast subjective vs well-defined collections.
- **Difficult Areas & Remediation:** Distinguishing $\in$ from $\subseteq$ in nested sets; use physical boxes-inside-boxes analogy.
- **Blackboard Teaching Examples:** Draw $(A \cup B)'$ and $A' \cap B'$ side by side using shaded Venn diagrams.
- **Classroom Questions to Ask Students:** Why is $\emptyset$ a subset of every set? Is $0 \in \emptyset$?
- **Common Student Doubts & Answers:** Students ask why $\{1, 1, 2\} = \{1, 2\}$; explain that set membership is a yes/no property, not a count.
- **Homework & Revision Strategy:** Assign 10 interval conversions and 5 two-set/three-set survey problems.

[TEACHER'S TIP] Drilling open/closed intervals here eliminates 80% of sign errors in Chapter 2 and Chapter 5.

UNIT I: SETS AND FUNCTIONS TYBA COACHING CENTRE

Chapter 2: Relations and Functions

Cartesian Product, Relations, Domain, Range & Standard Real Function Graphs (2026–27)

A. Chapter Introduction & Examination Relevance

While Chapter 1 studies static collections, Chapter 2 connects two sets dynamically. Cartesian products pair elements, Relations filter those pairs, and Functions enforce a deterministic input-output rule.

- **What You Will Learn:** Ordered pairs, Cartesian product $A \times B$, number of relations, domain, codomain, range, function definition, and standard real functions (identity, constant, polynomial, rational, modulus, signum, greatest integer).
- **Why This Chapter Matters:** Functions are the backbone of Calculus (Chapter 12 and all of Class 12) and Trigonometry (Chapter 3). Understanding domain restrictions ($\frac{1}{g(x)}$, $\sqrt{g(x)}$) is vital for CBSE, RBSE, and JEE.
- **Real-Life Applications:** Computer programming functions, database foreign-key relations, tax brackets (step/GIF functions), and signal clipping (modulus/signum).
- **Cross-Chapter Connections:** Extends Chapter 1 (sets) directly into Chapter 3 (trigonometric functions) and Chapter 12 (limits and derivatives of real functions).
- **Board & Competitive Relevance:** Unit I carries 23/80 marks in CBSE; RBSE assigns up to 10 marks to Relations & Functions. Expect 2M/3M domain-range questions and graph/GIF MCQs.

B. Complete Concept Explanation & Derivations

1. Ordered Pairs & Cartesian Product $A \times B$

An **ordered pair** (a, b) has a as first component and b as second; $(a, b) = (c, d) \iff a = c$ and $b = d$. The **Cartesian Product** is $A \times B = \{(a, b) : a \in A,\ b \in B\}$. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$. If either A or B is empty, $A \times B = \emptyset$.

[CONCEPT MASTER – 1. Ordered Pairs & Cartesian Product $A \times B$] In general, $A \times B \neq B \times A$ unless $A = B$ or one of them is $\emptyset$.

2. Relations, Domain, Codomain & Range

A **relation** R from non-empty set A to B is any subset of $A \times B$. Total number of relations from A to B is 2^{pq}. The **Domain** of R is the set of all first elements; the **Range** is the set of all second elements; B is the **Codomain** (Range $\subseteq$ Codomain).

[CONCEPT MASTER – 2. Relations, Domain, Codomain & Range] Every relation is a subset of $A \times B$, so counting relations is simply counting subsets of $A \times B$: $2^{n(A)n(B)}$.

3. Functions & Vertical Line Test

A relation f from A to B is a **function** if every element $x \in A$ has *one and only one* image $y = f(x) \in B$. Graphically, any vertical line $x = c$ cuts the graph of a function at *at most one* point. Two distinct inputs may share the same output (many-one), but one input cannot map to two outputs.

[CONCEPT MASTER – 3. Functions & Vertical Line Test] Checklist for a relation to be a function: (1) No element of A is left unmapped, (2) No element of A has two different images.

4. Standard Real Functions & Domain Rules

Key real functions: **Modulus** $|x| = x$ $(x \geq 0)$, $-x$ $(x < 0)$, domain $\mathbb{R}$, range $[0, \infty)$; **Signum** $\text{sgn}(x) = 1$ $(x > 0)$, 0 $(x = 0)$, -1 $(x < 0)$, range $\{-1, 0, 1\}$; **Greatest Integer Function (GIF)** $\lfloor x \rfloor$ = greatest integer $\leq x$, domain $\mathbb{R}$, range $\mathbb{Z}$. For real domain: denominator $\neq 0$ and radicand inside $\sqrt{}$ must be ≥ 0.

[CONCEPT MASTER – 4. Standard Real Functions & Domain Rules] Watch out for negative numbers in GIF: $\lfloor -2.3 \rfloor = -3$, NOT -2, because $-3 \leq -2.3 < -2$.

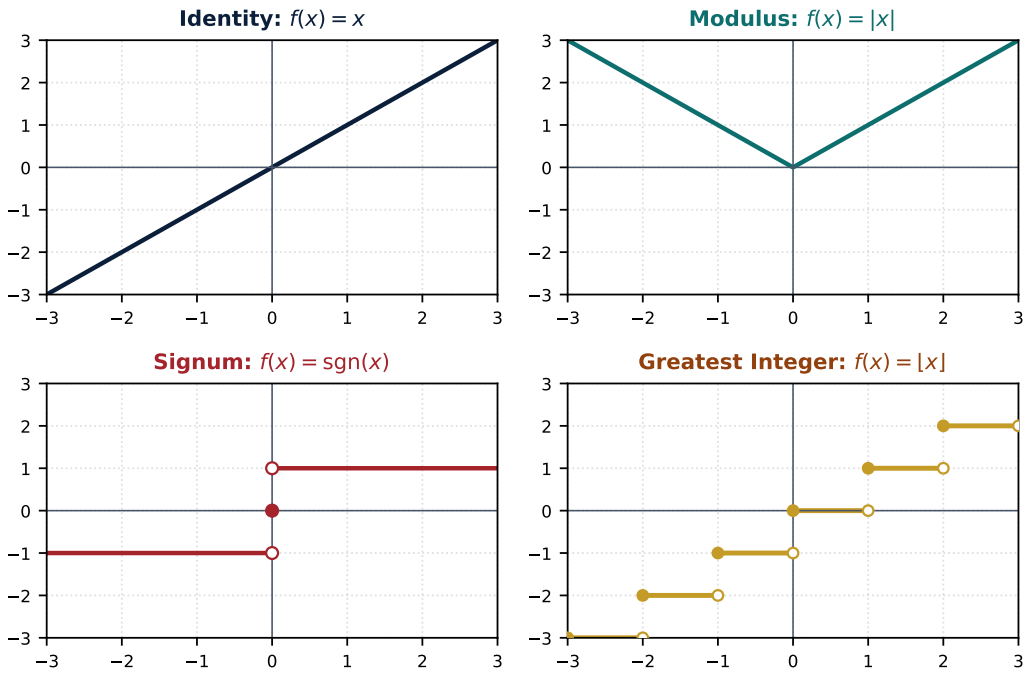


Figure 1: **Visual Model:** Graphs of four essential Class 11 real functions: Identity $f(x) = x$, Modulus $f(x) = |x|$, Signum $\text{sgn}(x)$, and Greatest Integer $\lfloor x \rfloor$.

C. Complete Formula Bank

[FORMULA BANK – 1. Equality of Ordered Pairs] **Expression:** $(a, b) = (c, d) \iff a = c$ and $b = d$

Symbols: a, b, c, d: any elements.

Conditions & When to Use: Order matters.

Solved Mini-Example: $(x + 1, y - 2) = (3, 1) \implies x = 2, y = 3$.

Common Mistake: Treating (a, b) like set $\{a, b\}$.

[FORMULA BANK – 2. Cardinality of $A \times B$ & Total Relations] **Expression:** $n(A \times B) = n(A) \cdot n(B)$; Total Relations $= 2^{n(A)n(B)}$

Symbols: A, B: finite sets.

Conditions & When to Use: If $A = \emptyset$ or $B = \emptyset$, $n(A \times B) = 0$.

Solved Mini-Example: $n(A) = 3, n(B) = 2 \implies n(A \times B) = 6$, relations $= 2^6 = 64$.

Common Mistake: Writing 2^{p+q} instead of 2^{pq}.

[FORMULA BANK – 3. Domain of Rational & Radical Functions] **Expression:** $\frac{1}{g(x)} \implies g(x) \neq 0$; $\sqrt{g(x)} \implies g(x) \geq 0$

Symbols: $g(x)$: real expression.

Conditions & When to Use: Real-valued functions.

Solved Mini-Example: Domain of $\sqrt{9 - x^2}$ is $[-3, 3]$.

Common Mistake: Writing $9 - x^2 > 0$ instead of ≥ 0 when radical is in numerator.

[FORMULA BANK – 4. Modulus Function] **Expression:** $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$
Symbols: $x \in \mathbb{R}$.
Conditions & When to Use: Range is $[0, \infty)$.
Solved Mini-Example: $|-5| = 5$; $|x-2| = 0 \implies x = 2$.
Common Mistake: Writing $\sqrt{x^2} = x$ instead of $|x|$.

[FORMULA BANK – 5. Signum Function] **Expression:** $\operatorname{sgn}(x) = \frac{|x|}{x}$ for $x \neq 0$, and 0 for $x = 0$
Symbols: $x \in \mathbb{R}$.
Conditions & When to Use: Range is $\{-1, 0, 1\}$.
Solved Mini-Example: $\operatorname{sgn}(-7) = -1$, $\operatorname{sgn}(0) = 0$.
Common Mistake: Saying range is $[-1, 1]$ instead of discrete $\{-1, 0, 1\}$.

[FORMULA BANK – 6. Greatest Integer Function (GIF)] **Expression:** $\lfloor x \rfloor \leq x < \lfloor x \rfloor + 1$
Symbols: $\lfloor x \rfloor \in \mathbb{Z}$.
Conditions & When to Use: Domain $\mathbb{R}$, Range $\mathbb{Z}$.
Solved Mini-Example: $\lfloor 3.9 \rfloor = 3$, $\lfloor -1.1 \rfloor = -2$.
Common Mistake: Rounding toward zero for negative decimals.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and JEE/CUET Foundation problem structures.

Pattern 1: Ordered Pair & Cartesian Product Identities

- **Identification Cue:** Find x, y or verify $A \times (B \cap C) = (A \times B) \cap (A \times C)$. | **Concept & Formula:** Component-wise equality.
- **Standard Step-by-Step:** Equate first components and second components separately.
- **Fastest Valid Method:** Check cardinality first for MCQ elimination. | **Alternative:** List pairs explicitly for small sets.
- **Common Trap:** Swapping coordinates in $A \times B$ vs $B \times A$.
- **Practice Drill:** Ex. 1 & Ex. 5.

Pattern 2: Relation Listing, Domain & Range

- **Identification Cue:** Relation $R = \{(x, y) : y = x + 5, x < 4\}$ on $\mathbb{N}$. | **Concept & Formula:** Subset of $A \times B$.
- **Standard Step-by-Step:** Test each allowed $x \in A$ and keep pairs where $y \in B$.
- **Fastest Valid Method:** Restrict x to values keeping y inside B. | **Alternative:** Arrow diagram.
- **Common Trap:** Including pairs whose y falls outside B.
- **Practice Drill:** Ex. 4 & Ex. 6.

Pattern 3: Domain of Algebraic Functions

- **Identification Cue:** Find domain of $\frac{x^2+2x+1}{x^2-8x+12}$ or $\sqrt{x-1}$. | **Concept & Formula:** Denominator $\neq 0$, radicand ≥ 0.
- **Standard Step-by-Step:** Factor denominator, exclude roots from $\mathbb{R}$, intersect with radical domain.
- **Fastest Valid Method:** Solve $g(x) = 0$ mentally and subtract from $\mathbb{R}$. | **Alternative:** Wavy-curve sign check.
- **Common Trap:** Canceling common factors before finding domain.
- **Practice Drill:** Ex. 7 & Ex. 8.

Pattern 4: Range of Quadratic, Rational & Modulus Functions

- **Identification Cue:** Find range of $2-3x$, x^2+2, $\frac{x^2}{1+x^2}$, or $-|x|$. | **Concept & Formula:** Bounding $y = f(x)$ or expressing x in terms of y.

- **Standard Step-by-Step:** Use $x^2 \geq 0$ or $|x| \geq 0$, or solve $y = f(x)$ for x and impose real-x conditions.
- **Fastest Valid Method:** Check extreme values at $x = 0$ and $x \to \pm\infty$. | **Alternative:** Graphical vertex/asymptote inspection.
- **Common Trap:** Missing $y = 1$ exclusion in $\frac{x^2}{1+x^2}$ (range is $[0, 1)$, not $[0, 1]$).
- **Practice Drill:** Ex. 10 & Ex. 13.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Vertical Line Test for Functions] **Core Idea & Why It Works:** A graph or set of ordered pairs is a function iff no two pairs have the same first element.

Valid Domain: All relations and graphs. | **Where It Fails:** Nothing—it is exact.

Easy Example: $\{(1, 2), (1, 3)\}$ is NOT a function.

Board-Level Example: $x^2 + y^2 = 1$ fails vertical line test $\implies$ not a function.

Advanced Example: Check if y is uniquely determined by x.

[SMART SHORTCUT – 2. Range via $x = g(y)$ Inversion] **Core Idea & Why It Works:** For rational $y = \frac{ax+b}{cx+d}$, solve for x in terms of y; excluded y-values are outside the range.

Valid Domain: Linear-over-linear and many algebraic functions. | **Where It Fails:** Check boundary exclusions if domain is restricted.

Easy Example: $y = \frac{x-1}{x+1} \implies x = \frac{1+y}{1-y} \implies y \neq 1$.

Board-Level Example: Range of $\frac{x^2}{1+x^2}$: $x^2 = \frac{y}{1-y} \geq 0 \implies y \in [0, 1)$.

Advanced Example: Combine with discriminant $\Delta \geq 0$ for quadratics.

[SMART SHORTCUT – 3. Count Relations in One Glance] **Core Idea & Why It Works:** Total relations from A (p elements) to B (q elements) is 2^{pq}.

Valid Domain: Finite sets. | **Where It Fails:** Infinite sets.

Easy Example: $p = 2, q = 3 \implies 2^6 = 64$.

Board-Level Example: Non-empty relations $= 2^{pq} - 1$.

Advanced Example: If $A \times A$ has 9 elements, $n(A) = 3$, relations on $A = 2^9 = 512$.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: If $(\frac{x}{3} + 1, y - \frac{2}{3}) = (\frac{5}{3}, \frac{1}{3})$, find x and y.

Given & Required: Equal ordered pairs. | **Concept & Formula:** Ordered pair equality.

Detailed Solution: $\frac{x}{3} + 1 = \frac{5}{3} \implies x = 2$; $y - \frac{2}{3} = \frac{1}{3} \implies y = 1$. **Final Answer:** $x = 2, y = 1$

Fastest Alternative / Shortcut: Equate coordinates directly.

SOLVED EXAMPLE 2 [Beginner]

Question: If $A = \{1, 2\}$ and $B = \{3, 4\}$, find $A \times B$ and total number of relations from A to B.

Given & Required: $n(A) = 2, n(B) = 2$. | **Concept & Formula:** Cartesian product.

Detailed Solution: $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$. Relations $= 2^4 = 16$. **Final Answer: 4 pairs; 16 relations**

Fastest Alternative / Shortcut: $2^{2\times 2} = 16$.

SOLVED EXAMPLE 3 [Beginner]

Question: Evaluate $\lfloor 4.7 \rfloor + \lfloor -4.7 \rfloor$.

Given & Required: GIF at positive and negative decimals. | **Concept & Formula:** Greatest integer function.

Detailed Solution: $\lfloor 4.7 \rfloor = 4$ and $\lfloor -4.7 \rfloor = -5$. Sum $= 4 + (-5) = -1$. **Final Answer:** -1

Fastest Alternative / Shortcut: For non-integer x, $\lfloor x \rfloor + \lfloor -x \rfloor = -1$.

SOLVED EXAMPLE 4 [Beginner]

Question: Is $R = \{(2,1),(5,1),(8,1),(11,1)\}$ a function? State its domain and range.

Given & Required: Set of 4 pairs. | **Concept & Formula:** Function test.

Detailed Solution: All first elements $2, 5, 8, 11$ are distinct, so it is a function. Domain $= \{2,5,8,11\}$, Range $= \{1\}$. **Final Answer: Yes; Domain** $\{2,5,8,11\}$**, Range** $\{1\}$

Fastest Alternative / Shortcut: Repeated y is allowed!

SOLVED EXAMPLE 5 [Intermediate]

Question: Let $A = \{1,2,3,4,5,6\}$. Define $R = \{(x,y) : y = x+1\}$ on A. Find domain, codomain, and range.

Given & Required: R on A. | **Concept & Formula:** Relation on a set.

Detailed Solution: $R = \{(1,2),(2,3),(3,4),(4,5),(5,6)\}$. Domain $= \{1,2,3,4,5\}$, Codomain $= A$, Range $= \{2,3,4,5,6\}$. **Final Answer: Dom** $\{1..5\}$**, Codom** A**, Range** $\{2..6\}$

Fastest Alternative / Shortcut: $(6,7)$ excluded since $7 \notin A$.

SOLVED EXAMPLE 6 [Intermediate]

Question: If $f(x) = x^2$, find $\frac{f(1.1)-f(1)}{1.1-1}$.

Given & Required: $f(x) = x^2$. | **Concept & Formula:** Function evaluation.

Detailed Solution: $\frac{1.21-1}{0.1} = \frac{0.21}{0.1} = 2.1$. **Final Answer:** 2.1

Fastest Alternative / Shortcut: $\frac{a^2-b^2}{a-b} = a+b = 1.1+1 = 2.1$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find the domain of $f(x) = \frac{x^2+3x+5}{x^2-5x+4}$.

Given & Required: Rational function. | **Concept & Formula:** Denominator $\neq 0$.

Detailed Solution: $x^2 - 5x + 4 = (x-1)(x-4) = 0 \implies x = 1, 4$. Domain $= \mathbb{R} - \{1,4\}$. **Final Answer:** $\mathbb{R} - \{1,4\}$

Fastest Alternative / Shortcut: Factor denominator.

SOLVED EXAMPLE 8 [Intermediate]

Question: Find the domain and range of $f(x) = \sqrt{x-1}$.

Given & Required: Square-root function. | **Concept & Formula:** Radicand ≥ 0.

Detailed Solution: $x - 1 \geq 0 \implies x \geq 1$, so Domain $= [1, \infty)$. Principal square root is ≥ 0, so Range $= [0, \infty)$. **Final Answer: Dom** $[1, \infty)$**, Range** $[0, \infty)$

Fastest Alternative / Shortcut: Radical output is non-negative.

SOLVED EXAMPLE 9 [Board Standard]

Question: Find the domain and range of $f(x) = \sqrt{9 - x^2}$.

Given & Required: Semicircle function. | **Concept & Formula:** $9 - x^2 \geq 0$.

Detailed Solution: $x^2 \leq 9 \implies x \in [-3,3]$. As x varies over $[-3,3]$, $9 - x^2 \in [0,9]$, so $f(x) \in [0,3]$. **Final Answer: Dom** $[-3,3]$**, Range** $[0,3]$

Fastest Alternative / Shortcut: Upper semicircle of radius 3.

SOLVED EXAMPLE 10 [Board Standard]

Question: Find the range of $f(x) = 2 - 3x$ for $x \in \mathbb{R}, x > 0$.

Given & Required: $x > 0$. | **Concept & Formula:** Inequality scaling.

Detailed Solution: $x > 0 \implies -3x < 0 \implies 2 - 3x < 2$. Range $= (-\infty, 2)$. **Final Answer:** $(-\infty, 2)$

Fastest Alternative / Shortcut: At $x = 0^+$, $f(x) = 2^-$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Find the domain and range of $f(x) = \frac{|x-1|}{x-1}$.

Given & Required: Shifted signum. | **Concept & Formula:** Modulus piecewise.

Detailed Solution: Denominator $\neq 0 \implies x \neq 1$, so Domain $= \mathbb{R} - \{1\}$. For $x > 1$, $f(x) = 1$; for $x < 1$, $f(x) = -1$. Range $= \{-1, 1\}$. **Final Answer: Dom** $\mathbb{R} - \{1\}$**, Range** $\{-1, 1\}$

Fastest Alternative / Shortcut: Shifted signum without 0.

SOLVED EXAMPLE 12 [Board Standard]

Question: Let $f = \{(1, 1), (2, 3), (0, -1), (-1, -3)\}$ be a linear function $f(x) = ax + b$. Find a and b.

Given & Required: 4 points on line. | **Concept & Formula:** Linear function.

Detailed Solution: $f(0) = -1 \implies b = -1$. $f(1) = a - 1 = 1 \implies a = 2$. Check $f(2) = 3, f(-1) = -3$. **Final Answer:** $a = 2, b = -1$

Fastest Alternative / Shortcut: Use $x = 0$ first for b.

SOLVED EXAMPLE 13 [Board Standard]

Question: Find the range of $f(x) = \frac{x^2}{1+x^2}$ for $x \in \mathbb{R}$.

Given & Required: $y = \frac{x^2}{1+x^2}$. | **Concept & Formula:** Inversion method.

Detailed Solution: $x^2 = \frac{y}{1-y} \geq 0$. Sign chart of $\frac{y}{1-y} \geq 0$ gives $0 \leq y < 1$. **Final Answer:** $[0, 1)$

Fastest Alternative / Shortcut: $0 \leq x^2 < 1 + x^2 \implies 0 \leq y < 1$.

SOLVED EXAMPLE 14 [Challenge]

Question: Find the domain of $f(x) = \frac{1}{\sqrt{|x|-x}}$.

Given & Required: Radical in denominator. | **Concept & Formula:** $|x| - x > 0$.

Detailed Solution: If $x \geq 0$, $|x| - x = 0$ (undefined). If $x < 0$, $|x| - x = -2x > 0$. Thus Domain $= (-\infty, 0)$. **Final Answer:** $(-\infty, 0)$

Fastest Alternative / Shortcut: $|x| > x$ holds iff $x < 0$.

SOLVED EXAMPLE 15 [Challenge]

Question: Find the domain of $f(x) = \frac{1}{\sqrt{x-\lfloor x \rfloor}}$.

Given & Required: Fractional part in denominator. | **Concept & Formula:** $\{x\} = x - \lfloor x \rfloor > 0$.

Detailed Solution: $x - \lfloor x \rfloor \geq 0$ always, and equals 0 iff $x \in \mathbb{Z}$. Thus Domain $= \mathbb{R} - \mathbb{Z}$. **Final Answer:** $\mathbb{R} - \mathbb{Z}$

Fastest Alternative / Shortcut: Fractional part is 0 at integers.

SOLVED EXAMPLE 16 [Challenge]

Question: If $f(x) = \frac{x-1}{x+1}$, express $f(2x)$ in terms of $f(x)$.

Given & Required: $y = f(x) = \frac{x-1}{x+1}$. | **Concept & Formula:** Functional algebra.

Detailed Solution: $x = \frac{1+y}{1-y}$. Then $f(2x) = \frac{2\frac{1+y}{1-y} - 1}{2\frac{1+y}{1-y} + 1} = \frac{2+2y-1+y}{2+2y+1-y} = \frac{3y+1}{y+3}$. **Final Answer:** $\frac{3f(x)+1}{f(x)+3}$

Fastest Alternative / Shortcut: Test $x = 1 \implies f(1) = 0, f(2) = 1/3$; $\frac{0+1}{0+3} = 1/3$.

G. Competency MCQ Mastery Section

Q1. If $n(A) = 3$ and $n(B) = 4$, total number of relations from A to B is:

(A) 12 (B) 2^7 (C) 2^{12} (D) 3^4

Answer: (C) $n(A \times B) = 12$, so 2^{12} subsets. *Fast Method:* 2^{pq}. *Distractor Note:* 2^{p+q} is wrong.

Q2. The domain of $f(x) = \sqrt{16 - x^2}$ is:

(A) $(-4, 4)$ (B) $[-4, 4]$ (C) $[0, 4]$ (D) $\mathbb{R}$

Answer: (B) $16 - x^2 \geq 0 \implies -4 \leq x \leq 4$. *Fast Method:* Closed interval. *Distractor Note:* $[0, 4]$ is the range, not domain.

Q3. The range of the signum function $f(x) = \text{sgn}(x)$ is:

(A) $[-1, 1]$ (B) $\{-1, 1\}$ (C) $\{-1, 0, 1\}$ (D) $\mathbb{R}$

Answer: (C) Outputs only $-1, 0, 1$. *Fast Method:* 3-element discrete set. *Distractor Note:* Forgetting 0.

Q4. The value of $\lfloor -3.2 \rfloor + \lfloor 3.2 \rfloor$ is:

(A) 0 (B) -1 (C) 1 (D) -7

Answer: (B) $-4 + 3 = -1$. *Fast Method:* Always -1 for non-integers. *Distractor Note:* $-3 + 3 = 0$ is wrong.

Q5. Which of the following relations is NOT a function?

(A) $\{(1, 2), (2, 2), (3, 2)\}$ (B) $\{(1, 2), (2, 3), (3, 4)\}$ (C) $\{(1, 2), (1, 3), (2, 4)\}$ (D) $\{(0, 0), (1, 1)\}$

Answer: (C) 1 maps to both 2 and 3. *Fast Method:* Duplicate x-coordinate. *Distractor Note:* Repeated y in (A) is valid.

Q6. Domain of $f(x) = \frac{1}{\sqrt{x-3}}$ is:

(A) $[3, \infty)$ (B) $(3, \infty)$ (C) $\mathbb{R} - \{3\}$ (D) $(-\infty, 3)$

Answer: (B) $x - 3 > 0 \implies x > 3$. *Fast Method:* Strict > 0 in denominator. *Distractor Note:* Including 3 divides by 0.

Q7. Range of $f(x) = -|x|$ for $x \in \mathbb{R}$ is:

(A) $[0, \infty)$ (B) $(-\infty, 0]$ (C) $\mathbb{R}$ (D) $(-\infty, 0)$

Answer: (B) $|x| \geq 0 \implies -|x| \leq 0$. *Fast Method:* Downward V-shape. *Distractor Note:* Omitting 0.

Q8. If $A = \{1, 2\}$ and $B = \{3, 4\}$, then $n(A \times B \times A)$ is:

(A) 6 (B) 8 (C) 12 (D) 16

Answer: (B) $2 \times 2 \times 2 = 8$. *Fast Method:* Multiply cardinalities. *Distractor Note:* Adding instead of multiplying.

Q9. Range of $f(x) = \frac{x-2}{2-x}$ $(x \neq 2)$ is:

(A) $\{1\}$ (B) $\{-1\}$ (C) $\{-1, 1\}$ (D) $\mathbb{R} - \{2\}$

Answer: (B) $\frac{x-2}{-(x-2)} = -1$ for all $x \neq 2$. *Fast Method:* Constant -1. *Distractor Note:* Confusing with $\frac{|x-2|}{x-2}$.

Q10. If $(x + y, x - y) = (6, 2)$, then (x, y) is:

(A) $(4, 2)$ (B) $(2, 4)$ (C) $(3, 3)$ (D) $(5, 1)$

Answer: (A) $x + y = 6, x - y = 2 \implies x = 4, y = 2$. *Fast Method:* Add equations. *Distractor Note:* Swapping x and y.

Q11. Domain of $f(x) = \frac{1}{|x|-x}$ is:

(A) $(0, \infty)$ (B) $(-\infty, 0)$ (C) $\mathbb{R} - \{0\}$ (D) $\mathbb{R}$

Answer: (B) $|x| - x \neq 0 \iff x < 0$. *Fast Method:* Test $x = 1$ (fails) and $x = -1$ (works). *Distractor Note:* Only excluding 0.

Q12. If $f(x) = x^2 - 3x + 2$, then $f(f(1))$ is:

(A) 0 (B) 1 (C) 2 (D) -2

Answer: (C) $f(1) = 0$, so $f(f(1)) = f(0) = 2$. *Fast Method:* Compute inner $f(1)$ first. *Distractor Note:* Stopping at $f(1) = 0$.

H. Special Exam & MCQ Strategies

- For domain questions, check all three filters: (1) denominator $\neq 0$, (2) even-root radicand ≥ 0, (3) strict > 0 if even root is in denominator.
- For range of rational functions, try $x = 0$, $x = \pm 1$, and large x first in MCQs, then confirm algebraically.
- Remember the difference between Codomain (given in definition $f : A \to B$) and Range (actual outputs).

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Evaluating $\lfloor -2.4 \rfloor = -2$. $\longrightarrow$ **Correction:** $\lfloor -2.4 \rfloor = -3$ because -3 is the greatest integer ≤ -2.4.

[COMMON MISTAKE] **Error:** Writing $\sqrt{x^2} = x$ for all real x. $\longrightarrow$ **Correction:** $\sqrt{x^2} = |x|$; for $x = -3$, $\sqrt{(-3)^2} = 3 = |-3|$.

[COMMON MISTAKE] **Error:** Canceling $(x - 2)$ in $\frac{x^2-4}{x-2}$ before finding domain. $\longrightarrow$ **Correction:** Always find domain BEFORE algebraic simplification ($x \neq 2$).

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $n(A \times B) = pq$; Relations $= 2^{pq}$; $\lfloor x \rfloor \leq x < \lfloor x \rfloor + 1$.

[QUICK REVISION] Key Theorems & Results: $|x| = \sqrt{x^2}$; $\text{sgn}(x) = |x|/x$ $(x \neq 0)$; Domain rules: denom $\neq 0$, $\sqrt{\cdot} \geq 0$.

[QUICK REVISION] Shortcut Summary: Vertical line test; Inversion $x = g(y)$ for range; $\lfloor x \rfloor + \lfloor -x \rfloor = -1$ $(x \notin \mathbb{Z})$.

[QUICK REVISION] High-Frequency Exam Patterns: Ordered pair equality, Relation listing, Domain of $\sqrt{a^2 - x^2}$, Range of rational functions.

Last-Minute Revision Checklist:

- ☐ $A \times B \neq B \times A$
- ☐ Function = unique output per input
- ☐ GIF on negative numbers
- ☐ Domain before simplification

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. If $n(A) = 2, n(B) = 3$, number of relations from A to B is: (A) 6 (B) 32 (C) 64 (D) 12
2. Value of $\lfloor -0.5 \rfloor$ is: (A) 0 (B) -1 (C) -0.5 (D) 1
3. Range of $f(x) = |x| + 2$ is: (A) $[0, \infty)$ (B) $[2, \infty)$ (C) $(2, \infty)$ (D) $\mathbb{R}$
4. If $(a - 1, b + 2) = (3, 5)$, then (a, b) is: (A) $(4, 3)$ (B) $(2, 7)$ (C) $(4, 7)$ (D) $(3, 4)$

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. Domain of $f(x) = \frac{1}{x^2 - 9}$ is: (A) $\mathbb{R} - \{3\}$ (B) $\mathbb{R} - \{-3, 3\}$ (C) $(-3, 3)$ (D) $[-3, 3]$
2. Range of $f(x) = \sqrt{25 - x^2}$ is: (A) $[-5, 5]$ (B) $[0, 5]$ (C) $(0, 5)$ (D) $[0, 25]$
3. If $A = \{1, 2\}, B = \{3, 4\}, C = \{4, 5\}$, then $n(A \times (B \cap C))$ is: (A) 2 (B) 4 (C) 6 (D) 8
4. Range of $f(x) = 5 - |x - 1|$ is: (A) $[5, \infty)$ (B) $(-\infty, 5]$ (C) $[1, \infty)$ (D) $\mathbb{R}$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Domain of $f(x) = \sqrt{x^2 - 5x + 6}$ is: (A) $[2, 3]$ (B) $(-\infty, 2] \cup [3, \infty)$ (C) $(2, 3)$ (D) $\mathbb{R}$
2. Range of $f(x) = \frac{1}{2 - \sin 3x}$ is: (A) $[1/3, 1]$ (B) $[1, 3]$ (C) $[-1, 1]$ (D) $[1/2, 1]$
3. If $f(x) = x + \frac{1}{x}$, then $f(x)^2 - f(x^2)$ equals: (A) 0 (B) 1 (C) 2 (D) 4
4. Number of functions from $A = \{1, 2, 3\}$ to $B = \{a, b\}$ is: (A) 6 (B) 8 (C) 9 (D) 64

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (C):** $2^{2 \times 3} = 2^6 = 64$. **Q2 (B):** Greatest integer ≤ -0.5 is -1. **Q3 (B):** $\|x\| \geq 0 \implies \|x\| + 2 \geq 2$. **Q4 (A):** $a = 4, b = 3$.
Board Std	**Q1 (B):** $x^2 - 9 \neq 0 \implies x \neq \pm 3$. **Q2 (B):** $0 \leq 25 - x^2 \leq 25 \implies 0 \leq f(x) \leq 5$. **Q3 (A):** $B \cap C = \{4\}$, so $2 \times 1 = 2$. **Q4 (B):** $\|x - 1\| \geq 0 \implies 5 - \|x - 1\| \leq 5$.
Challenge	**Q1 (B):** $(x - 2)(x - 3) \geq 0$ holds outside $[2, 3]$. **Q2 (A):** $-1 \leq \sin 3x \leq 1 \implies 1 \leq 2 - \sin 3x \leq 3 \implies [1/3, 1]$. **Q3 (C):** $(x + 1/x)^2 - (x^2 + 1/x^2) = 2$. **Q4 (B):** $n(B)^{n(A)} = 2^3 = 8$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Cartesian product. Day 2: Relations, domain, range. Day 3: Function test and standard graphs. Day 4: Domain/range problem solving.
- **How to Introduce the Concept:** Use a vending machine analogy: pressing one button (input) must give exactly one snack (output) — that's a function!
- **Difficult Areas & Remediation:** Negative inputs in GIF and finding range of rational functions.
- **Blackboard Teaching Examples:** Plot $y = |x|$, $y = \text{sgn}(x)$, and $y = \lfloor x \rfloor$ with hollow vs solid dots.
- **Classroom Questions to Ask Students:** Can a circle be the graph of a function? Why or why not?
- **Common Student Doubts & Answers:** Students confuse range with codomain; use arrow diagrams where one codomain element has no incoming arrow.
- **Homework & Revision Strategy:** Assign 8 domain problems and 6 range problems covering radicals and modulus.

[TEACHER'S TIP] Emphasize drawing the graph of $|x|$ and $\lfloor x \rfloor$ by hand — visual memory lasts till Class 12!

UNIT I: SETS AND FUNCTIONS TYBA COACHING CENTRE

Chapter 3: Trigonometric Functions

Radian Measure, ASTC Signs, Compound, Double, Triple & C–D Identities (2026–27)

A. Chapter Introduction & Examination Relevance

Trigonometry expands from right-angled triangles in Class 10 to arbitrary angles on the unit circle in Class 11. Measuring angles in radians turns trigonometric ratios into smooth periodic functions of real numbers.

- **What You Will Learn:** Degree-radian conversion, arc length $l = r\theta$, unit-circle definitions, ASTC quadrant sign rule, domain/range of all 6 functions, allied angles, compound angles $(A \pm B)$, C–D sum-to-product and product-to-sum formulas, and multiple/sub-multiple angle identities.
- **Why This Chapter Matters:** Trigonometric identities are used everywhere in Class 11 (Limits, Derivatives, Straight Lines, Conics) and Class 12 (Inverse Trig, Differentiation, Integration, Differential Equations).
- **Real-Life Applications:** AC alternating-current waveforms, sound/light wave interference, GPS triangulation, robotics joint angles, and astronomical orbital modeling.
- **Cross-Chapter Connections:** Builds on Chapter 2 (functions, domain, range) and directly powers Chapter 9 (slope $m = \tan\theta$) and Chapter 12 ($\lim_{x\to 0} \frac{\sin x}{x} = 1$).
- **Board & Competitive Relevance:** Unit I carries 23/80 marks in CBSE; RBSE assigns 10 marks to Chapter 3. Expect 3M/5M identity proofs and 1M/2M radian/allied-angle questions. (General trig equations and sine/cosine laws are excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Radian Measure & Arc Length

One **radian** is the angle subtended at the centre of a circle by an arc equal in length to the radius: $\theta = \frac{l}{r}$. Since a full circle has circumference $2\pi r$, 2π rad $= 360°$, giving π rad $= 180°$. Thus 1 rad $\approx 57°16'$.

[CONCEPT MASTER – 1. Radian Measure & Arc Length] Conversion: Radians $= \frac{\pi}{180} \times$ Degrees and Degrees $= \frac{180}{\pi} \times$ Radians.

2. Unit Circle, Signs (ASTC) & Allied Angles

On the unit circle, a point at angle x has coordinates $(\cos x, \sin x)$, so $\cos^2 x + \sin^2 x = 1$ and $|\sin x| \le 1, |\cos x| \le 1$. Quadrant signs follow **ASTC** (All, Sin/Cosec, Tan/Cot, Cos/Sec positive). For allied angles: $n\pi \pm \theta$ keeps the function unchanged; $(2n+1)\frac{\pi}{2} \pm \theta$ flips $\sin \leftrightarrow \cos$, $\tan \leftrightarrow \cot$, $\sec \leftrightarrow \csc$, with sign decided by the original quadrant.

[CONCEPT MASTER – 2. Unit Circle, Signs (ASTC) & Allied Angles] Even-Odd Parity: $\cos(-x) = \cos x$ and $\sec(-x) = \sec x$ (even); $\sin(-x), \tan(-x), \csc(-x), \cot(-x)$ pull out a minus sign (odd).

3. Compound, Multiple & Sub-Multiple Identities

From $\cos(x+y) = \cos x \cos y - \sin x \sin y$ and $\sin(x+y) = \sin x \cos y + \cos x \sin y$, we derive $\tan(x \pm y)$, double-angle formulas ($\sin 2x = 2\sin x \cos x$, $\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$), and triple-angle formulas ($\sin 3x = 3\sin x - 4\sin^3 x$, $\cos 3x = 4\cos^3 x - 3\cos x$).

[CONCEPT MASTER – 3. Compound, Multiple & Sub-Multiple Identities] Half-angle power reducers are golden for calculus too: $1 - \cos 2x = 2\sin^2 x$ and $1 + \cos 2x = 2\cos^2 x$.

4. Sum-to-Product (C–D) & Product-to-Sum Formulas

Adding and subtracting compound-angle formulas yields C–D identities: $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$, $\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$, $\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$, and $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2} = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$.

[CONCEPT MASTER – 4. Sum-to-Product (C–D) & Product-to-Sum Formulas] Rationalised 2026–27 Scope: General solutions of trigonometric equations ($\sin x = \sin y$) and sine/cosine triangle laws are excluded.

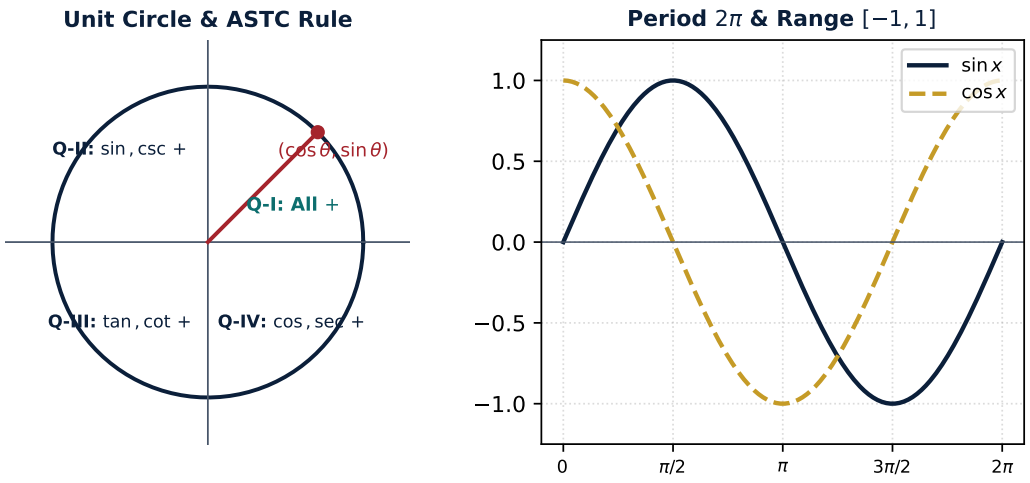


Figure 1: **Visual Model:** Unit circle coordinates $(\cos\theta, \sin\theta)$ with ASTC quadrant signs alongside $y = \sin x$ and $y = \cos x$ periodic waves.

C. Complete Formula Bank

[FORMULA BANK – 1. Radian-Degree & Arc Length] **Expression:** $\theta = \frac{l}{r}$; π rad $= 180^\circ$
Symbols: l: arc length, r: radius, θ: angle in radians.
Conditions & When to Use: θ MUST be in radians for $l = r\theta$.
Solved Mini-Example: Arc of $r = 10$ cm, $\theta = 60^\circ = \pi/3$ gives $l = 10\pi/3$ cm.
Common Mistake: Plugging degrees directly into $l = r\theta$.

[FORMULA BANK – 2. Compound Angle Identities] **Expression:** $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$; $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
Symbols: $A, B \in \mathbb{R}$.
Conditions & When to Use: All real angles.
Solved Mini-Example: $\sin 75^\circ = \sin(45^\circ + 30^\circ) = \frac{\sqrt{3}+1}{2\sqrt{2}}$.
Common Mistake: Sign error in $\cos(A + B)$ (it has a minus sign!).

[FORMULA BANK – 3. Tangent Compound Formula] **Expression:** $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
Symbols: $A, B, A \pm B \neq (2n+1)\frac{\pi}{2}$.
Conditions & When to Use: Denominator $\neq 0$.
Solved Mini-Example: $\tan(\frac{\pi}{4} + x) = \frac{1+\tan x}{1-\tan x}$.
Common Mistake: Using same sign in numerator and denominator.

[FORMULA BANK – 4. Double Angle Identities] **Expression:** $\sin 2x = 2\sin x \cos x = \frac{2\tan x}{1+\tan^2 x}$; $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$
Symbols: $x \in \mathbb{R}$.
Conditions & When to Use: Universal identities.
Solved Mini-Example: $\frac{1-\cos 2x}{\sin 2x} = \frac{2\sin^2 x}{2\sin x \cos x} = \tan x$.
Common Mistake: Writing $\sin 2x = 2\sin x$.

[FORMULA BANK – 5. Triple Angle Identities] **Expression:** $\sin 3x = 3\sin x - 4\sin^3 x$; $\cos 3x = 4\cos^3 x - 3\cos x$; $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$
Symbols: $x \in \mathbb{R}$.
Conditions & When to Use: For $\tan 3x$, $3x \neq (2n+1)\pi/2$.
Solved Mini-Example: $\sin 18^\circ$ is derived via $2x + 3x = 90^\circ \implies \sin 2x = \cos 3x$.
Common Mistake: Swapping coefficients 3 and 4 between $\sin 3x$ and $\cos 3x$.

[FORMULA BANK – 6. C–D Sum-to-Product Identities] **Expression:** $\sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$; $\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$
Symbols: $C, D \in \mathbb{R}$.
Conditions & When to Use: Always true.
Solved Mini-Example: $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \frac{2\sin 4x\cos x}{2\cos 4x\cos x} = \tan 4x$.
Common Mistake: Forgetting the minus sign in $\cos C - \cos D$.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and JEE/CUET Foundation problem structures.

Pattern 1: Degree ↔ Radian & Clock/Wheel Problems

- **Identification Cue:** Convert $40^\circ 20'$ or find angle turned by wheel/clock. | **Concept & Formula:** $1^\circ = 60'$, $1' = 60''$, π rad $= 180^\circ$.
- **Standard Step-by-Step:** Convert minutes/seconds to fractional degrees first, then multiply by $\frac{\pi}{180}$.
- **Fastest Valid Method:** Remember $30^\circ = \pi/6, 45^\circ = \pi/4, 60^\circ = \pi/3, 90^\circ = \pi/2$. | **Alternative:** Use $\pi \approx 22/7$ when converting radians to degrees-minutes-seconds.
- **Common Trap:** Writing 40.20° for $40^\circ 20'$ instead of $40\frac{1}{3}^\circ$.
- **Practice Drill:** Ex. 1 & Ex. 2.

Pattern 2: Allied Angles & Quadrant Value Evaluation

- **Identification Cue:** Find $\sin\frac{31\pi}{3}$ or given $\cos x = -3/5$ in Q-III, find $\tan x$. | **Concept & Formula:** ASTC + $2n\pi$ periodicity.
- **Standard Step-by-Step:** Split angle into $k\pi \pm \theta$ or $k\frac{\pi}{2} \pm \theta$, check function flip, then apply quadrant sign.
- **Fastest Valid Method:** Use a reference right triangle + ASTC sign. | **Alternative:** Pythagorean identity $\sin^2 x + \cos^2 x = 1$.
- **Common Trap:** Taking positive root when angle lies in Q-III or Q-IV.
- **Practice Drill:** Ex. 3 & Ex. 5.

Pattern 3: C–D Pairing in Identity Proofs

- **Identification Cue:** Prove $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$. | **Concept & Formula:** Pair angles with same average.
- **Standard Step-by-Step:** Group outer terms $(\sin 5x + \sin x)$ so $\frac{5x+x}{2} = 3x$ matches the middle term.
- **Fastest Valid Method:** Look for $(A+B)/2$ matching the remaining term. | **Alternative:** Convert everything to $\sin x, \cos x$.
- **Common Trap:** Pairing $\sin 5x$ and $\sin 3x$ which creates $4x$ instead of $3x$.
- **Practice Drill:** Ex. 9 & Ex. 11.

Pattern 4: Multiple/Sub-Multiple & Half-Angle Evaluation

- **Identification Cue:** Given $\tan x = -4/3$ in Q-II, find $\sin(x/2), \cos(x/2)$. | **Concept & Formula:** $1 \pm \cos x$ half-angle formulas.
- **Standard Step-by-Step:** Locate $x/2$ quadrant ($45^\circ < x/2 < 90^\circ$ is Q-I), find $\cos x$, then use $\sin^2(x/2) = \frac{1-\cos x}{2}$.
- **Fastest Valid Method:** Check quadrant of $x/2$ FIRST to lock signs. | **Alternative:** Use $\tan(x/2)$ quadratic.
- **Common Trap:** Using quadrant of x instead of $x/2$ for the sign.
- **Practice Drill:** Ex. 14 & Ex. 15.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Tangent Sum-Product Identity ($A+B+C$ or $3x-2x=x$)] **Core Idea & Why It Works:** Whenever $\alpha+\beta=\gamma$, $\tan\gamma-\tan\alpha-\tan\beta=\tan\gamma\tan\alpha\tan\beta$.
Valid Domain: Any $\alpha+\beta=\gamma$ with defined tangents. | **Where It Fails:** Odd multiples of $\pi/2$.
Easy Example: $3x=2x+x \implies \tan 3x-\tan 2x-\tan x=\tan 3x\tan 2x\tan x$.
Board-Level Example: Directly solves NCERT Misc. $\tan 3x\tan 2x\tan x$ proof in 2 lines!
Advanced Example: For $A+B=45^\circ$, $(1+\tan A)(1+\tan B)=2$.

[SMART SHORTCUT – 2. Max/Min of $a\sin x+b\cos x$] **Core Idea & Why It Works:** The expression $a\sin x+b\cos x$ always lies in $[-\sqrt{a^2+b^2},\sqrt{a^2+b^2}]$.
Valid Domain: All $x\in\mathbb{R}$. | **Where It Fails:** If arguments of sin and cos differ (e.g., $\sin x+\cos 2x$).
Easy Example: Max of $3\sin x+4\cos x$ is $\sqrt{9+16}=5$.
Board-Level Example: Range of $3\sin x+4\cos x+7$ is $[2,12]$.
Advanced Example: Used to find range of $\frac{1}{a\sin x+b\cos x+c}$.

[SMART SHORTCUT – 3. Product $\cos\theta\cos 2\theta\cos 4\theta\cdots$] **Core Idea & Why It Works:** $\cos\theta\cos 2\theta\cos 4\theta\cdots\cos 2^{n-1}\theta=\frac{\sin(2^n\theta)}{2^n\sin\theta}$.
Valid Domain: Angles doubling in G.P. | **Where It Fails:** When $\sin\theta=0$.
Easy Example: $\cos 20^\circ\cos 40^\circ\cos 80^\circ=\frac{\sin 160^\circ}{8\sin 20^\circ}=\frac{1}{8}$.
Board-Level Example: Instant check for $\cos\frac{\pi}{9}\cos\frac{2\pi}{9}\cos\frac{4\pi}{9}$.
Advanced Example: Multiply and divide by $2\sin\theta$ in subjective exams.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Convert $40^\circ 20'$ into radian measure.
Given & Required: $40^\circ 20'$. | **Concept & Formula:** $1^\circ=60'$, rad $=\frac{\pi}{180}$deg.
Detailed Solution: $40^\circ 20'=(40+\frac{20}{60})^\circ=\frac{121}{3}^\circ$. Radians $=\frac{121}{3}\times\frac{\pi}{180}=\frac{121\pi}{540}$. **Final Answer:** $\frac{121\pi}{540}$ **rad**
Fastest Alternative / Shortcut: Convert minutes to fraction first.

SOLVED EXAMPLE 2 [Beginner]
Question: Find the radius of a circle in which a central angle of 60° intercepts an arc of length 37.4 cm ($\pi=22/7$).
Given & Required: $\theta=60^\circ=\pi/3$, $l=37.4$ cm. | **Concept & Formula:** $l=r\theta$.
Detailed Solution: $r=\frac{l}{\theta}=\frac{37.4}{\pi/3}=\frac{37.4\times 3\times 7}{22}=1.7\times 21=35.7$ cm. **Final Answer:** 35.7 **cm**
Fastest Alternative / Shortcut: $37.4/22=1.7$.

SOLVED EXAMPLE 3 [Beginner]
Question: Find the value of $\sin\frac{31\pi}{3}$.
Given & Required: $\frac{31\pi}{3}$. | **Concept & Formula:** $2n\pi$ periodicity.
Detailed Solution: $\frac{31\pi}{3}=10\pi+\frac{\pi}{3}$. Thus $\sin(10\pi+\frac{\pi}{3})=\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$. **Final Answer:** $\frac{\sqrt{3}}{2}$
Fastest Alternative / Shortcut: Divide 31 by 6 (since $2\pi=6\pi/3$): remainder 1.

SOLVED EXAMPLE 4 [Beginner]

Question: Evaluate $\cos(-1710^\circ)$.

Given & Required: -1710°. | **Concept & Formula:** Even parity + 360° period.

Detailed Solution: $\cos(-1710^\circ) = \cos(1710^\circ) = \cos(5 \times 360^\circ - 90^\circ) = \cos 90^\circ = 0$. **Final Answer:** 0

Fastest Alternative / Shortcut: $1710 = 1800 - 90$.

SOLVED EXAMPLE 5 [Intermediate]

Question: If $\cos x = -\frac{3}{5}$ and x lies in the third quadrant, find $\sin x$ and $\tan x$.

Given & Required: $\cos x = -3/5$, Q-III. | **Concept & Formula:** ASTC + Pythagorean identity.

Detailed Solution: $\sin^2 x = 1 - \frac{9}{25} = \frac{16}{25}$. In Q-III, $\sin x < 0 \implies \sin x = -\frac{4}{5}$. Then $\tan x = \frac{-4/5}{-3/5} = \frac{4}{3}$. **Final Answer:** $\sin x = -4/5, \tan x = 4/3$

Fastest Alternative / Shortcut: 3-4-5 triangle with Q-III signs.

SOLVED EXAMPLE 6 [Intermediate]

Question: Find the value of $\tan \frac{13\pi}{12}$.

Given & Required: $13\pi/12 = \pi + \pi/12$. | **Concept & Formula:** Compound angle.

Detailed Solution: $\tan(\pi + \frac{\pi}{12}) = \tan 15^\circ = \tan(45^\circ - 30^\circ) = \frac{1-1/\sqrt{3}}{1+1/\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3}$. **Final Answer:** $2 - \sqrt{3}$

Fastest Alternative / Shortcut: $\tan 15^\circ = 2 - \sqrt{3}$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Prove that $\sin^2 6x - \sin^2 4x = \sin 2x \sin 10x$.

Given & Required: Difference of squares. | **Concept & Formula:** $\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$.

Detailed Solution: LHS $= \sin(6x + 4x)\sin(6x - 4x) = \sin 10x \sin 2x =$ RHS. **Final Answer:** **Proved**

Fastest Alternative / Shortcut: One-line identity.

SOLVED EXAMPLE 8 [Intermediate]

Question: Prove that $\frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x} = \tan 4x$.

Given & Required: Ratio of sums. | **Concept & Formula:** C–D formulas.

Detailed Solution: $\frac{2\sin 4x \cos x}{2\cos 4x \cos x} = \tan 4x$. **Final Answer:** **Proved**

Fastest Alternative / Shortcut: $\cos x$ cancels cleanly.

SOLVED EXAMPLE 9 [Board Standard]

Question: Prove that $\cos 4x = 1 - 8\sin^2 x \cos^2 x$.

Given & Required: $\cos 4x$. | **Concept & Formula:** Double-angle formula.

Detailed Solution: $\cos 4x = 1 - 2\sin^2 2x = 1 - 2(2\sin x \cos x)^2 = 1 - 8\sin^2 x \cos^2 x$. **Final Answer:** **Proved**

Fastest Alternative / Shortcut: Apply $\cos 2\theta = 1 - 2\sin^2\theta$ with $\theta = 2x$.

SOLVED EXAMPLE 10 [Board Standard]

Question: Prove that $\tan 4x = \frac{4\tan x(1-\tan^2 x)}{1 - 6\tan^2 x + \tan^4 x}$.

Given & Required: $\tan 4x$. | **Concept & Formula:** Repeated $\tan 2\theta$.

Detailed Solution: $\tan 4x = \frac{2\tan 2x}{1-\tan^2 2x}$. Substitute $\tan 2x = \frac{2t}{1-t^2}$ ($t = \tan x$): $\frac{4t(1-t^2)}{(1-t^2)^2 - 4t^2} = \frac{4t(1-t^2)}{1 - 6t^2 + t^4}$. **Final Answer:** **Proved**

Fastest Alternative / Shortcut: Let $t = \tan x$ to keep algebra clean.

SOLVED EXAMPLE 11 [Board Standard]

Question: Prove that $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2 \frac{x+y}{2}$.

Given & Required: Sum of squares. | **Concept & Formula:** C–D or expansion.

Detailed Solution: Expand: $(\cos^2 x+\sin^2 x)+(\cos^2 y+\sin^2 y)+2(\cos x\cos y-\sin x\sin y) = 2+2\cos(x+y) = 2(1+\cos(x+y)) = 4\cos^2\frac{x+y}{2}$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Expanding gives $\cos(x+y)$ directly.

SOLVED EXAMPLE 12 [Board Standard]

Question: Prove that $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \frac{1}{16}$.

Given & Required: Cosine product. | **Concept & Formula:** Product-to-sum.

Detailed Solution: $\cos 60^\circ = \frac{1}{2}$. Multiply remaining by $\frac{2\sin 20^\circ}{2\sin 20^\circ}$: $\frac{1}{2}\cdot\frac{\sin 160^\circ}{8\sin 20^\circ} = \frac{1}{16}$ since $\sin 160^\circ = \sin 20^\circ$. **Final Answer:** $\frac{1}{16}$

Fastest Alternative / Shortcut: $\frac{1}{2}\times\frac{1}{8} = \frac{1}{16}$.

SOLVED EXAMPLE 13 [Board Standard]

Question: Prove that $\cos^2 x + \cos^2(x+\frac{\pi}{3}) + \cos^2(x-\frac{\pi}{3}) = \frac{3}{2}$.

Given & Required: Sum of 3 squared cosines. | **Concept & Formula:** $\cos^2\theta = \frac{1+\cos 2\theta}{2}$.

Detailed Solution: LHS $= \frac{3}{2}+\frac{1}{2}[\cos 2x+\cos(2x+\frac{2\pi}{3})+\cos(2x-\frac{2\pi}{3})] = \frac{3}{2}+\frac{1}{2}[\cos 2x+2\cos 2x\cos\frac{2\pi}{3}] = \frac{3}{2}+\frac{1}{2}[\cos 2x-\cos 2x] = \frac{3}{2}$. **Final Answer:** $\frac{3}{2}$

Fastest Alternative / Shortcut: Test $x=0$: $1+1/4+1/4 = 3/2$.

SOLVED EXAMPLE 14 [Challenge]

Question: If $\tan x = -\frac{4}{3}$ and x is in Quadrant II, find $\sin\frac{x}{2}, \cos\frac{x}{2}, \tan\frac{x}{2}$.

Given & Required: $\tan x = -4/3$, $\frac{\pi}{2} < x < \pi$. | **Concept & Formula:** Half-angle formulas.

Detailed Solution: $\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$ (Q-I, all +). From Q-II, $\cos x = -\frac{3}{5}$. Thus $\sin\frac{x}{2} = \sqrt{\frac{1+3/5}{2}} = \frac{2}{\sqrt{5}}$, $\cos\frac{x}{2} = \sqrt{\frac{1-3/5}{2}} = \frac{1}{\sqrt{5}}$, $\tan\frac{x}{2} = 2$. **Final Answer:** $\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}, 2$

Fastest Alternative / Shortcut: Lock Q-I sign for $x/2$ first.

SOLVED EXAMPLE 15 [Challenge]

Question: Find the value of $\sin 18^\circ$.

Given & Required: $\theta = 18^\circ \implies 5\theta = 90^\circ$. | **Concept & Formula:** $2\theta = 90^\circ - 3\theta$.

Detailed Solution: $\sin 2\theta = \cos 3\theta \implies 2\sin\theta\cos\theta = 4\cos^3\theta - 3\cos\theta$. Divide by $\cos\theta \neq 0$: $2\sin\theta = 4(1-\sin^2\theta) - 3 \implies 4\sin^2\theta + 2\sin\theta - 1 = 0 \implies \sin 18^\circ = \frac{\sqrt{5}-1}{4}$ (>0). **Final Answer:** $\frac{\sqrt{5}-1}{4}$

Fastest Alternative / Shortcut: Remember $\sin 18^\circ = \frac{\sqrt{5}-1}{4} \approx 0.309$.

SOLVED EXAMPLE 16 [Challenge]

Question: Find the maximum and minimum values of $7\cos x + 24\sin x - 5$.

Given & Required: $a=24, b=7, c=-5$. | **Concept & Formula:** $[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$.

Detailed Solution: $\sqrt{7^2+24^2} = 25$. So $7\cos x + 24\sin x \in [-25, 25]$. Subtracting 5 gives $[-30, 20]$. **Final Answer: Min** $= -30$**, Max** $= 20$

Fastest Alternative / Shortcut: 7-24-25 Pythagorean triple!

G. Competency MCQ Mastery Section

Q1. The radian measure of 240° is:

(A) $\frac{2\pi}{3}$ (B) $\frac{4\pi}{3}$ (C) $\frac{3\pi}{4}$ (D) $\frac{5\pi}{3}$

Answer: (B) $240\times\frac{\pi}{180} = \frac{4\pi}{3}$. *Fast Method:* $4\times 60^\circ$. *Distractor Note:* Confusing with 120°.

Q2. Value of $\sin(765^\circ)$ is:

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) 1

Answer: (B) $765^\circ = 2(360^\circ) + 45^\circ$. *Fast Method:* $765 - 720 = 45^\circ$. *Distractor Note:* Sign error.

Q3. If $\sin x + \csc x = 2$, then $\sin^{20} x + \csc^{20} x$ equals:

(A) 2^{20} (B) 2 (C) 0 (D) 1

Answer: (B) $\sin x = 1 \implies 1^{20} + 1^{20} = 2$. *Fast Method:* $t + 1/t = 2 \implies t = 1$. *Distractor Note:* Guessing 2^{20}.

Q4. Value of $\tan 15^\circ$ is:

(A) $2 + \sqrt{3}$ (B) $2 - \sqrt{3}$ (C) $\sqrt{3} - 1$ (D) 1

Answer: (B) $\tan(45^\circ - 30^\circ) = 2 - \sqrt{3}$. *Fast Method:* $\tan 15^\circ < 1$, and $2 - \sqrt{3} \approx 0.268$. *Distractor Note:* $2 + \sqrt{3}$ is $\tan 75^\circ$.

Q5. Which of the following is positive in Quadrant III?

(A) $\sin x$ (B) $\cos x$ (C) $\tan x$ (D) $\sec x$

Answer: (C) By ASTC, $\tan$ and $\cot$ are positive in Q-III. *Fast Method:* ASTC: T = Tan. *Distractor Note:* Picking $\cos x$.

Q6. The maximum value of $5\sin x - 12\cos x$ is:

(A) 7 (B) 12 (C) 13 (D) 17

Answer: (C) $\sqrt{5^2 + (-12)^2} = 13$. *Fast Method:* 5-12-13 triple. *Distractor Note:* Adding $5 + 12 = 17$.

Q7. $\frac{1-\cos 2x}{1+\cos 2x}$ equals:

(A) $\cot^2 x$ (B) $\tan^2 x$ (C) $\sin^2 x$ (D) $\sec^2 x$

Answer: (B) $\frac{2\sin^2 x}{2\cos^2 x} = \tan^2 x$. *Fast Method:* Power-reduction identity. *Distractor Note:* Inverting to $\cot^2 x$.

Q8. If $\cos x = -\frac{1}{2}$ and $x \in (\pi, \frac{3\pi}{2})$, then $\sin x$ is:

(A) $\frac{\sqrt{3}}{2}$ (B) $-\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Answer: (B) Q-III has $\sin x < 0$. *Fast Method:* Reference angle 60°, negative sign. *Distractor Note:* Choosing $+\frac{\sqrt{3}}{2}$.

Q9. Value of $\cos 1^\circ \cos 2^\circ \cdots \cos 179^\circ$ is:

(A) 1 (B) -1 (C) 0 (D) $1/2$

Answer: (C) $\cos 90^\circ = 0$ is one of the factors. *Fast Method:* Look for $\cos 90^\circ = 0$. *Distractor Note:* Trying to pair cosines.

Q10. If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then $A + B$ (acute) is:

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

Answer: (B) $\tan(A+B) = \frac{1/2+1/3}{1-1/6} = 1 \implies A + B = \frac{\pi}{4}$. *Fast Method:* Classic $\frac{1}{2}, \frac{1}{3} \to 45^\circ$ pair. *Distractor Note:* Arithmetic slip in denominator.

Q11. Domain of $f(x) = \tan x$ is:

(A) $\mathbb{R}$ (B) $\mathbb{R} - \{n\pi\}$ (C) $\mathbb{R} - \{(2n+1)\frac{\pi}{2}\}$ (D) $[-1, 1]$

Answer: (C) $\cos x = 0$ at odd multiples of $\pi/2$. *Fast Method:* Denominator $\cos x \neq 0$. *Distractor Note:* Confusing with $\cot x$ domain $\mathbb{R} - \{n\pi\}$.

Q12. Value of $\sin 50^\circ - \sin 70^\circ + \sin 10^\circ$ is:

(A) 1 (B) 0 (C) $1/2$ (D) -1

Answer: (B) $\sin 50^\circ - \sin 70^\circ = -2\cos 60^\circ \sin 10^\circ = -\sin 10^\circ$. *Fast Method:* C–D on first two terms. *Distractor Note:* Sign error in $\sin 50^\circ - \sin 70^\circ$.

H. Special Exam & MCQ Strategies

- Write a 1-sheet formula grid of Compound, Double, Triple, and C–D identities and revise it for 5 minutes daily.
- In identity proofs, always start from the more complicated side and look for C–D pairing or $1 \pm \cos 2x$ substitutions.

- For MCQs, test $\theta = 0^\circ, 30^\circ, 45^\circ$ (avoiding $0/0$) to verify algebraic equivalence in 10 seconds.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Writing $\cos(A+B) = \cos A \cos B + \sin A \sin B$. $\longrightarrow$ **Correction:** $\cos(A+B) = \cos A \cos B - \sin A \sin B$ (opposite sign).

[COMMON MISTAKE] **Error:** Forgetting the minus sign in $\cos C - \cos D = -2 \sin \frac{C+D}{2} \sin \frac{C-D}{2}$. $\longrightarrow$ **Correction:** Use $2 \sin \frac{C+D}{2} \sin \frac{D-C}{2}$ if you prefer no leading minus.

[COMMON MISTAKE] **Error:** Using degrees in $l = r\theta$. $\longrightarrow$ **Correction:** Always convert degrees to radians before using $l = r\theta$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $l = r\theta$; π rad $= 180^\circ$; $\sin(A \pm B), \cos(A \pm B), \tan(A \pm B)$.

[QUICK REVISION] Key Theorems & Results: $1 - \cos 2x = 2\sin^2 x$; $1 + \cos 2x = 2\cos^2 x$; $\sin 3x = 3s - 4s^3$; $\cos 3x = 4c^3 - 3c$.

[QUICK REVISION] Shortcut Summary: $a\sin x + b\cos x \in [-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}]$; $\cos\theta\cos 2\theta \cdots \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n \sin\theta}$.

[QUICK REVISION] High-Frequency Exam Patterns: Radian conversion, Allied angles, C–D proofs, Half-angle signs.

Last-Minute Revision Checklist:

- ☐ ASTC quadrant signs
- ☐ Even/odd trig functions
- ☐ Half-angle quadrant of $x/2$
- ☐ C–D minus sign in $\cos C - \cos D$

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Convert 135° to radians: (A) $\frac{2\pi}{3}$ (B) $\frac{3\pi}{4}$ (C) $\frac{5\pi}{6}$ (D) $\frac{3\pi}{2}$
2. Value of $\cos(-60^\circ)$ is: (A) $-1/2$ (B) $1/2$ (C) $\sqrt{3}/2$ (D) $-\sqrt{3}/2$
3. Range of $f(x) = \sin x$ is: (A) $(-1, 1)$ (B) $[-1, 1]$ (C) $[0, 1]$ (D) $\mathbb{R}$
4. $1 + \cos 2x$ equals: (A) $2\sin^2 x$ (B) $2\cos^2 x$ (C) $\sin 2x$ (D) $\cos^2 x$

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. If $\sin x = 3/5$ and x is in Q-II, then $\cos x$ is: (A) $4/5$ (B) $-4/5$ (C) $3/4$ (D) $-3/4$
2. Value of $\cos 75^\circ$ is: (A) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (B) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$
3. $\frac{\sin 7x + \sin 5x}{\cos 7x + \cos 5x}$ equals: (A) $\tan 6x$ (B) $\tan x$ (C) $\cot 6x$ (D) $\tan 12x$
4. If an arc of length 20π cm subtends 144°, radius is: (A) 20 cm (B) 25 cm (C) 30 cm (D) 15 cm

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Value of $\sin 10^\circ \sin 50^\circ \sin 70^\circ$ is: (A) $1/4$ (B) $1/8$ (C) $1/16$ (D) $\sqrt{3}/8$
2. Minimum value of $3\cos x + 4\sin x + 8$ is: (A) 3 (B) 5 (C) 13 (D) 1
3. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then $\cos\theta - \sin\theta$ is: (A) $\sqrt{2}\sin\theta$ (B) $-\sqrt{2}\sin\theta$ (C) $\sqrt{2}\cos\theta$ (D) 0
4. Value of $(1 + \tan 20^\circ)(1 + \tan 25^\circ)$ is: (A) 1 (B) 2 (C) 4 (D) $\sqrt{2}$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $135 \times \frac{\pi}{180} = \frac{3\pi}{4}$. **Q2 (B):** $\cos(-x) = \cos x = 1/2$. **Q3 (B):** Closed interval $[-1, 1]$. **Q4 (B):** Standard double-angle identity.
Board Std	**Q1 (B):** Q-II has $\cos x < 0$, so $-4/5$. **Q2 (A):** $\cos(45^\circ + 30^\circ) = \frac{\sqrt{3}-1}{2\sqrt{2}}$. **Q3 (A):** $\frac{2\sin 6x \cos x}{2\cos 6x \cos x} = \tan 6x$. **Q4 (B):** $144^\circ = 4\pi/5$; $r = 20\pi/(4\pi/5) = 25$ cm.
Challenge	**Q1 (B):** $\cos 80^\circ \cos 40^\circ \cos 20^\circ = 1/8$. **Q2 (A):** $-5 + 8 = 3$. **Q3 (A):** Square both sides: $1 + \sin 2\theta = 2\cos^2\theta$, so $(\cos\theta - \sin\theta)^2 = 2 - 2\cos^2\theta = 2\sin^2\theta$. **Q4 (B):** $20^\circ + 25^\circ = 45^\circ \implies (1 + \tan A)(1 + \tan B) = 2$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Radians & arc length. Day 2: Unit circle, ASTC, allied angles. Day 3: Compound angles. Day 4: Double/Triple & C–D proofs.
- **How to Introduce the Concept:** Ask why a full circle has 360° (arbitrary) vs 2π radians (natural ratio l/r).
- **Difficult Areas & Remediation:** Choosing the right identity among the 3 forms of $\cos 2x$ and finding the quadrant of $x/2$.
- **Blackboard Teaching Examples:** Draw the ASTC circle and derive $\cos 2x$ forms side-by-side.
- **Classroom Questions to Ask Students:** Why is $\cos(-x) = \cos x$ while $\sin(-x) = -\sin x$ on the unit circle?
- **Common Student Doubts & Answers:** Students forget why π rad $= 180^\circ$ yet $\pi \approx 3.14$; clarify that π is a pure number of radians, not 180.
- **Homework & Revision Strategy:** Assign 12 identity proofs mixing C–D and double-angle formulas.

[TEACHER'S TIP] Remind students daily that 80% of Class 12 Integration relies on Class 11 Trig identities.

UNIT II: ALGEBRA TYBA COACHING CENTRE

Chapter 4: Complex Numbers and Quadratic Equations

Imaginary Unit i, Algebra of $a+ib$, Conjugate, Modulus & Argand Plane (2026–27)

A. Chapter Introduction & Examination Relevance

Real numbers cannot solve equations as simple as $x^2+1=0$. Extending $\mathbb{R}$ by introducing $i=\sqrt{-1}$ creates the Complex Number System $\mathbb{C}$, where every polynomial equation has a complete set of roots.

- **What You Will Learn:** Powers of i, standard form $z=a+ib$, equality and algebra of complex numbers, conjugate $\bar{z}$, modulus $|z|$, multiplicative inverse $z^{-1}=\bar{z}/|z|^2$, and geometric representation on the Argand plane.
- **Why This Chapter Matters:** Complex numbers unify algebra and 2D geometry. Modulus and conjugate identities are heavily tested in CBSE/RBSE Class 11 and form a core pillar of JEE Main.
- **Real-Life Applications:** AC impedance ($R+iX$) in electrical engineering, quantum wavefunctions, signal processing (Fourier transforms), and aerodynamic fluid flow.
- **Cross-Chapter Connections:** Extends real-number algebra and connects to Chapter 9 (2D coordinates (a,b)) and Chapter 10 (locus in the plane).
- **Board & Competitive Relevance:** Unit II (Algebra) carries 25/80 marks. Chapter 4 contributes 4–6 marks via $a+ib$ simplification, modulus/conjugate proofs, and Argand-plane MCQs. (Polar form and square root of z are excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Imaginary Unit i & Powers of i

We define $i=\sqrt{-1}$, so $i^2=-1$, $i^3=-i$, and $i^4=1$. For any integer k, $i^{4k+r}=i^r$, and the sum of any four consecutive powers of i is zero: $i^n+i^{n+1}+i^{n+2}+i^{n+3}=0$.

[CONCEPT MASTER – 1. Imaginary Unit i & Powers of i] Warning: $\sqrt{a}\sqrt{b}=\sqrt{ab}$ FAILS when both $a<0$ and $b<0$! Always convert $\sqrt{-4}=2i$ first: $\sqrt{-4}\sqrt{-9}=(2i)(3i)=-6$, NOT $+6$.

2. Standard Form $z=a+ib$ & Algebra

A complex number is $z=a+ib$ where $a=\mathrm{Re}(z)\in\mathbb{R}$ and $b=\mathrm{Im}(z)\in\mathbb{R}$. Two complex numbers are equal iff their real and imaginary parts are separately equal: $a+ib=c+id \iff a=c, b=d$. Complex numbers have **no order relation** ($2+3i>1+4i$ is meaningless).

[CONCEPT MASTER – 2. Standard Form $z=a+ib$ & Algebra] To simplify a fraction $\frac{z_1}{z_2}$, multiply numerator and denominator by the conjugate $\bar{z}_2$ to make the denominator real.

3. Conjugate $\bar{z}$ & Modulus $|z|$

The **conjugate** of $z=a+ib$ is $\bar{z}=a-ib$, and the **modulus** is $|z|=\sqrt{a^2+b^2}\geq 0$. Crucially, $z\bar{z}=a^2+b^2=|z|^2$. Thus the multiplicative inverse of $z\neq 0$ is $z^{-1}=\frac{\bar{z}}{|z|^2}$.

[CONCEPT MASTER – 3. Conjugate $\bar{z}$ & Modulus $|z|$] Modulus distributes over multiplication and division: $|z_1z_2|=|z_1||z_2|$ and $|z^n|=|z|^n$. Never expand $(a+ib)^n$ just to find its modulus!

4. The Argand Plane

Each complex number $z=x+iy$ corresponds uniquely to the point (x,y) on the **Argand (Complex) Plane**, where the x-axis is the Real axis and the y-axis is the Imaginary axis. Conjugate $\bar{z}$ is the reflection of z across the Real axis, and $|z|$ is the distance of (x,y) from the origin.

[CONCEPT MASTER – 4. The Argand Plane] Rationalised 2026–27 Scope: Polar representation ($r(\cos\theta+i\sin\theta)$) and complex-root quadratic equations are excluded from the 2026–27 NCERT syllabus.

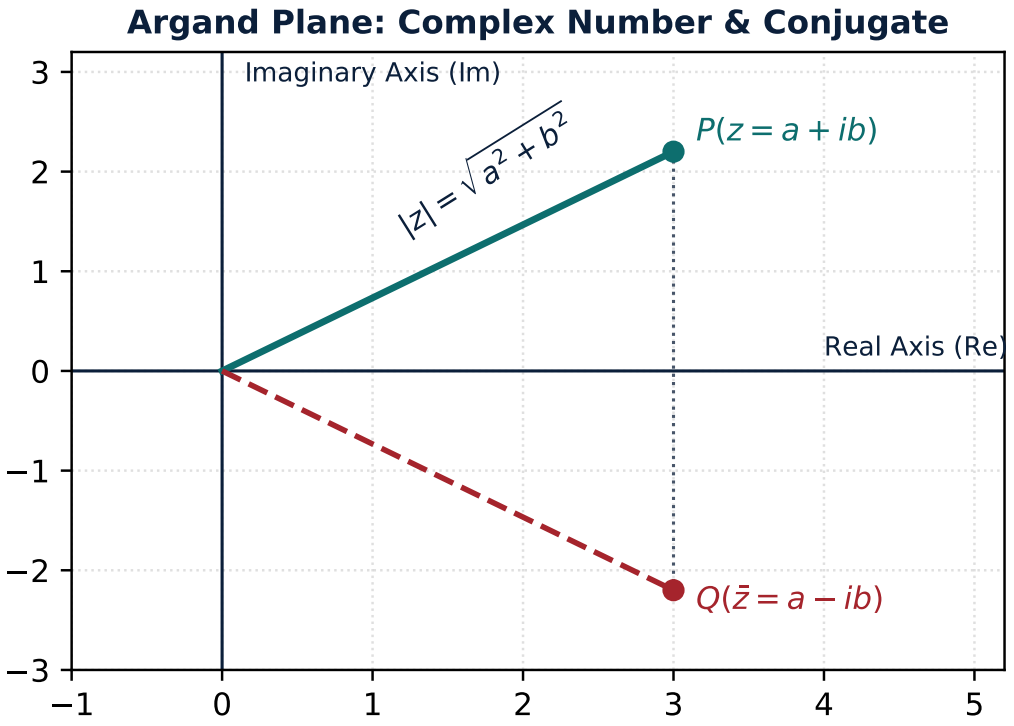


Figure 1: **Visual Model:** Argand plane representation of $z = a+ib$, its reflection conjugate $\bar{z} = a-ib$, and modulus $|z| = \sqrt{a^2+b^2}$.

C. Complete Formula Bank

[FORMULA BANK – 1. Cyclic Powers of i] **Expression:** $i^2 = -1,\ i^3 = -i,\ i^4 = 1;\ i^n + i^{n+1} + i^{n+2} + i^{n+3} = 0$
Symbols: $n \in \mathbb{Z}$.
Conditions & When to Use: Divide n by 4 and keep remainder.
Solved Mini-Example: $i^{39} = i^{4(9)+3} = i^3 = -i$.
Common Mistake: Writing $\sqrt{-a}\sqrt{-b} = \sqrt{ab}$.

[FORMULA BANK – 2. Modulus & Conjugate Identity] **Expression:** $|z| = \sqrt{a^2+b^2};\ \bar{z} = a - ib;\ z\bar{z} = |z|^2$
Symbols: $z = a + ib$ $(a, b \in \mathbb{R})$.
Conditions & When to Use: Always real and non-negative $|z|$.
Solved Mini-Example: $|3 - 4i| = \sqrt{9+16} = 5$.
Common Mistake: Including i inside $\sqrt{a^2 + (ib)^2}$.

[FORMULA BANK – 3. Multiplicative Inverse] **Expression:** $z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{a-ib}{a^2+b^2}$
Symbols: $z = a + ib \neq 0$.
Conditions & When to Use: $|z| \neq 0$.
Solved Mini-Example: $(4 - 3i)^{-1} = \frac{4+3i}{16+9} = \frac{4}{25} + \frac{3}{25}i$.
Common Mistake: Forgetting to change the sign of ib in numerator.

[FORMULA BANK – 4. Modulus Properties] **Expression:** $|z_1 z_2| = |z_1||z_2|;\ \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|};\ |z^n| = |z|^n$
Symbols: $z_1, z_2 \in \mathbb{C}, z_2 \neq 0$.
Conditions & When to Use: Always valid.
Solved Mini-Example: $\left|\frac{(1+i)^4}{(1-i)^2}\right| = \frac{(\sqrt{2})^4}{(\sqrt{2})^2} = 2$.
Common Mistake: Thinking $|z_1 + z_2| = |z_1| + |z_2|$ (only $\leq$ holds).

[FORMULA BANK – 5. Conjugate Properties] **Expression:** $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$; $\overline{z_1 z_2} = \bar{z}_1 \bar{z}_2$; $\overline{(z_1/z_2)} = \bar{z}_1/\bar{z}_2$
Symbols: $z_1, z_2 \in \mathbb{C}$.
Conditions & When to Use: Always valid.
Solved Mini-Example: z is purely real iff $z = \bar{z}$; purely imaginary iff $z = -\bar{z}$.
Common Mistake: Forgetting to conjugate constants? Real constants satisfy $\bar{c} = c$.

[FORMULA BANK – 6. Triangle Inequality] **Expression:** $||z_1| - |z_2|| \leq |z_1 + z_2| \leq |z_1| + |z_2|$
Symbols: $z_1, z_2 \in \mathbb{C}$.
Conditions & When to Use: All complex numbers.
Solved Mini-Example: If $|z| = 4$, then $|z + 3| \leq 4 + 3 = 7$.
Common Mistake: Assuming equality holds in general.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and JEE/CUET Foundation problem structures.

Pattern 1: Simplifying Powers of i & $a + ib$ Form

- **Identification Cue:** Express $\frac{(1+i)^2}{3-i}$ or i^{-35} in $a + ib$ form. | **Concept & Formula:** Remainder mod 4 and conjugate multiplication.
- **Standard Step-by-Step:** Reduce powers of i mod 4, expand numerator, multiply top and bottom by denominator's conjugate.
- **Fastest Valid Method:** Remember $(1 + i)^2 = 2i$, $(1 - i)^2 = -2i$, and $\frac{1+i}{1-i} = i$. | **Alternative:** Use $i^{-1} = -i$.
- **Common Trap:** Sign error when replacing i^2 with -1.
- **Practice Drill:** Ex. 1 & Ex. 6.

Pattern 2: Equality of Complex Numbers (x, y Solving)

- **Identification Cue:** Find real x, y if $(x + iy)(2 - 3i) = 4 + i$. | **Concept & Formula:** $a + ib = c + id \iff a = c, b = d$.
- **Standard Step-by-Step:** Expand LHS into $A + iB$, then equate $A = 4$ and $B = 1$.
- **Fastest Valid Method:** Divide by $(2 - 3i)$ via conjugate instead of solving 2x2 simultaneous equations. | **Alternative:** 2x2 elimination.
- **Common Trap:** Forgetting x, y must be real.
- **Practice Drill:** Ex. 7 & Ex. 10.

Pattern 3: Modulus Proof Without Expansion ($|z|^2 = z\bar{z}$)

- **Identification Cue:** If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, prove $(x^2 + y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$. | **Concept & Formula:** Taking conjugate of both sides and multiplying.
- **Standard Step-by-Step:** Write the conjugate equation $x - iy = \sqrt{\frac{a-ib}{c-id}}$ and multiply.
- **Fastest Valid Method:** Take modulus directly: $|x + iy|^2 = |\frac{a+ib}{c+id}|$. | **Alternative:** Square first, then take modulus.
- **Common Trap:** Trying to separate real and imaginary parts under a square root!
- **Practice Drill:** Ex. 11 & Ex. 12.

Pattern 4: Condition for Purely Real / Purely Imaginary

- **Identification Cue:** Find θ if $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real. | **Concept & Formula:** $\mathrm{Im}(z) = 0$ or $\mathrm{Re}(z) = 0$.
- **Standard Step-by-Step:** Multiply by conjugate of denominator and set $\mathrm{Im}(z) = 0$ (or $\mathrm{Re}(z) = 0$).
- **Fastest Valid Method:** Only compute the required part of the numerator. | **Alternative:** Use $z = \bar{z}$ (real) or $z = -\bar{z}$ (imaginary).
- **Common Trap:** Setting the entire expression to 0 instead of only its imaginary part.
- **Practice Drill:** Ex. 14.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Golden $(1 \pm i)$ Identities] **Core Idea & Why It Works:** $(1+i)^2 = 2i$, $(1-i)^2 = -2i$, $\frac{1+i}{1-i} = i$, and $\frac{1-i}{1+i} = -i$.
Valid Domain: All $1 \pm i$ powers and fractions. | **Where It Fails:** Nothing—exact algebraic identities.
Easy Example: $(\frac{1+i}{1-i})^{100} = i^{100} = 1$.
Board-Level Example: Find least positive m for $(\frac{1+i}{1-i})^m = 1 \implies i^m = 1 \implies m = 4$.
Advanced Example: $(1+i)^8 + (1-i)^8 = (2i)^4 + (-2i)^4 = 16 + 16 = 32$.

[SMART SHORTCUT – 2. Instant Modulus Without Rationalizing] **Core Idea & Why It Works:** Use $|z_1/z_2| = |z_1|/|z_2|$ directly.
Valid Domain: Whenever only $|z|$ is asked. | **Where It Fails:** If $a + ib$ form itself is required.
Easy Example: $|\frac{3+4i}{5-12i}| = \frac{5}{13}$.
Board-Level Example: Find $|\frac{1+i}{1-i} - \frac{1-i}{1+i}| = |i - (-i)| = |2i| = 2$.
Advanced Example: Prove $|z| = 1$ for $\frac{a+ib}{a-ib}$ in 1 second: $\frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2}} = 1$.

[SMART SHORTCUT – 3. Consecutive 4 Powers of i Vanish] **Core Idea & Why It Works:** $\sum_{k=n}^{n+3} i^k = 0$ for any integer n.
Valid Domain: Any 4 consecutive powers. | **Where It Fails:** Non-consecutive exponents.
Easy Example: $i^1 + i^2 + i^3 + i^4 = 0$.
Board-Level Example: $\sum_{n=1}^{13}(i^n + i^{n+1})$: group first 12 terms (vanish) + 13th term $= i^{13} + i^{14} = i - 1$.
Advanced Example: Works for any arithmetic progression of powers with odd common difference.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Evaluate i^{-39} in $a + ib$ form.
Given & Required: i^{-39}. | **Concept & Formula:** $i^4 = 1$.
Detailed Solution: $i^{-39} = \frac{1}{i^{39}} = \frac{1}{i^{36} \cdot i^3} = \frac{1}{-i} = \frac{i}{-i^2} = i$. **Final Answer:** $0 + i$
Fastest Alternative / Shortcut: Multiply by $i^{40} = 1$: $i^{-39} \cdot i^{40} = i$.

SOLVED EXAMPLE 2 [Beginner]
Question: Express $(5i)(-\frac{3}{5}i)$ in $a + ib$ form.
Given & Required: Product of two imaginary numbers. | **Concept & Formula:** $i^2 = -1$.
Detailed Solution: $(5)(-\frac{3}{5})i^2 = -3(-1) = 3$. **Final Answer:** $3 + 0i$
Fastest Alternative / Shortcut: $i^2 = -1$ cancels the minus sign.

SOLVED EXAMPLE 3 [Beginner]
Question: Find the multiplicative inverse of $2 - 3i$.
Given & Required: $z = 2 - 3i$. | **Concept & Formula:** $z^{-1} = \frac{\bar{z}}{|z|^2}$.
Detailed Solution: $\bar{z} = 2 + 3i$, $|z|^2 = 4 + 9 = 13$. So $z^{-1} = \frac{2}{13} + \frac{3}{13}i$. **Final Answer:** $\frac{2}{13} + \frac{3}{13}i$
Fastest Alternative / Shortcut: Conjugate over sum of squares.

SOLVED EXAMPLE 4 [Beginner]

Question: Evaluate $\sqrt{-16} \times \sqrt{-25}$.

Given & Required: Negative radicands. | **Concept & Formula:** $\sqrt{-a} = i\sqrt{a}$.

Detailed Solution: $(4i)(5i) = 20i^2 = -20$. **Final Answer:** -20

Fastest Alternative / Shortcut: Pull out i before multiplying!

SOLVED EXAMPLE 5 [Intermediate]

Question: Express $(1-i)^4$ in $a+ib$ form.

Given & Required: $(1-i)^4$. | **Concept & Formula:** $(1-i)^2 = -2i$.

Detailed Solution: $[(1-i)^2]^2 = (-2i)^2 = 4i^2 = -4$. **Final Answer:** $-4+0i$

Fastest Alternative / Shortcut: Square twice.

SOLVED EXAMPLE 6 [Intermediate]

Question: Find the conjugate of $\frac{(3-2i)(2+3i)}{(1+2i)(2-i)}$.

Given & Required: Rational complex expression. | **Concept & Formula:** Simplify then conjugate.

Detailed Solution: Num $= 6+9i-4i+6 = 12+5i$. Den $= 2-i+4i+2 = 4+3i$. Ratio $= \frac{(12+5i)(4-3i)}{16+9} = \frac{63-16i}{25}$. Conjugate $= \frac{63}{25} + \frac{16}{25}i$. **Final Answer:** $\frac{63}{25} + \frac{16}{25}i$

Fastest Alternative / Shortcut: Expand Num and Den separately first.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find real x and y if $(x-iy)(3+5i)$ is the conjugate of $-6-24i$.

Given & Required: Conjugate of $-6-24i$ is $-6+24i$. | **Concept & Formula:** Complex equality.

Detailed Solution: $x - iy = \frac{-6+24i}{3+5i} = \frac{(-6+24i)(3-5i)}{9+25} = \frac{-18+30i+72i+120}{34} = \frac{102+102i}{34} = 3+3i$. Thus $x=3, y=-3$. **Final Answer:** $x=3, y=-3$

Fastest Alternative / Shortcut: Divide instead of expanding.

SOLVED EXAMPLE 8 [Intermediate]

Question: Find the modulus of $\frac{1+i}{1-i} - \frac{1-i}{1+i}$.

Given & Required: Difference of two fractions. | **Concept & Formula:** Golden $(1 \pm i)$ identities.

Detailed Solution: $i-(-i) = 2i$. Modulus $= |2i| = 2$. **Final Answer:** 2

Fastest Alternative / Shortcut: Instant with $\frac{1+i}{1-i} = i$.

SOLVED EXAMPLE 9 [Board Standard]

Question: If $a+ib = \frac{(x+i)^2}{2x^2+1}$, prove that $a^2+b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$.

Given & Required: $a+ib = \frac{(x+i)^2}{2x^2+1}$. | **Concept & Formula:** $|z|^2 = a^2+b^2$.

Detailed Solution: Take modulus of both sides: $\sqrt{a^2+b^2} = \frac{|x+i|^2}{2x^2+1} = \frac{x^2+1}{2x^2+1}$. Squaring gives $a^2+b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Conjugate multiplication also works.

SOLVED EXAMPLE 10 [Board Standard]

Question: If $(x+iy)^3 = u+iv$, show that $\frac{u}{x} + \frac{v}{y} = 4(x^2-y^2)$.

Given & Required: $(x+iy)^3 = u+iv$. | **Concept & Formula:** Binomial expansion + equating parts.

Detailed Solution: $x^3-3xy^2+i(3x^2y-y^3) = u+iv$. So $u/x = x^2-3y^2$ and $v/y = 3x^2-y^2$. Adding gives $4x^2-4y^2 = 4(x^2-y^2)$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Expand $(x+iy)^3$ cleanly.

SOLVED EXAMPLE 11 [Board Standard]

Question: If α and β are different complex numbers with $|\beta| = 1$, find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$.

Given & Required: $|\beta| = 1 \implies \beta\bar{\beta} = 1$. | **Concept & Formula:** $|z|^2 = z\bar{z}$.

Detailed Solution: Replace 1 in denominator with $\beta\bar{\beta}$: $\left|\frac{\beta-\alpha}{\beta\bar{\beta}-\bar{\alpha}\beta}\right| = \frac{|\beta-\alpha|}{|\beta||\bar{\beta}-\bar{\alpha}|} = \frac{|\beta-\alpha|}{1\cdot|\beta-\alpha|} = 1$. **Final Answer:** 1

Fastest Alternative / Shortcut: Set $\beta = 1, \alpha = 0$ to verify: $|1/1| = 1$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Find the number of non-zero integral solutions of $|1-i|^x = 2^x$.

Given & Required: $|1-i|^x = 2^x$. | **Concept & Formula:** Modulus evaluation.

Detailed Solution: $|1-i| = \sqrt{2}$, so $(\sqrt{2})^x = 2^x \implies 2^{x/2} = 2^x \implies x/2 = x \implies x = 0$. Hence zero non-zero solutions. **Final Answer:** 0 **(no non-zero solution)**

Fastest Alternative / Shortcut: $(\sqrt{2})^x = 2^x$ only at $x = 0$.

SOLVED EXAMPLE 13 [Board Standard]

Question: If $z_1 = 2-i, z_2 = 1+i$, find $\left|\frac{z_1+z_2+1}{z_1-z_2+1}\right|$.

Given & Required: $z_1 = 2-i, z_2 = 1+i$. | **Concept & Formula:** Direct modulus ratio.

Detailed Solution: Num $= (2-i)+(1+i)+1 = 4$. Den $= (2-i)-(1+i)+1 = 2-2i$. Ratio $= \frac{|4|}{|2-2i|} = \frac{4}{\sqrt{4+4}} = \frac{4}{2\sqrt{2}} = \sqrt{2}$. **Final Answer:** $\sqrt{2}$

Fastest Alternative / Shortcut: Compute |Num|/|Den| directly.

SOLVED EXAMPLE 14 [Challenge]

Question: Find the least positive integer n such that $\left(\frac{1+i}{1-i}\right)^n = 1$.

Given & Required: $(\frac{1+i}{1-i})^n = 1$. | **Concept & Formula:** Golden $(1 \pm i)$ identity.

Detailed Solution: $\frac{1+i}{1-i} = \frac{(1+i)^2}{2} = i$. Thus $i^n = 1$, whose least positive integer solution is $n = 4$. **Final Answer:** 4

Fastest Alternative / Shortcut: $i^4 = 1$.

SOLVED EXAMPLE 15 [Challenge]

Question: Find real θ such that $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is purely real.

Given & Required: Purely real $\implies \mathrm{Im}(z) = 0$. | **Concept & Formula:** Rationalize denominator.

Detailed Solution: Num of product $= (3+2i\sin\theta)(1+2i\sin\theta) = (3-4\sin^2\theta) + i(8\sin\theta)$. Setting $\mathrm{Im} = 0$ gives $8\sin\theta = 0 \implies \sin\theta = 0 \implies \theta = n\pi, n \in \mathbb{Z}$. **Final Answer:** $\theta = n\pi,\ n \in \mathbb{Z}$

Fastest Alternative / Shortcut: Im part is $8\sin\theta/(1+4\sin^2\theta) = 0$.

SOLVED EXAMPLE 16 [Challenge]

Question: If $|z-3i| = |z+3i|$ for $z = x+iy$, find the locus of z in the Argand plane.

Given & Required: $|x+i(y-3)| = |x+i(y+3)|$. | **Concept & Formula:** Argand geometry.

Detailed Solution: $x^2+(y-3)^2 = x^2+(y+3)^2 \implies -6y = 6y \implies y = 0$ (the Real axis). **Final Answer:** **Real axis** $(y = 0)$

Fastest Alternative / Shortcut: Perpendicular bisector of $(0,3)$ and $(0,-3)$ is the x-axis.

G. Competency MCQ Mastery Section

Q1. Value of i^{101} is:

(A) 1 (B) -1 (C) i (D) $-i$

Answer: (C) $101 = 4(25)+1 \implies i^1 = i$. *Fast Method:* Remainder mod 4 is 1. *Distractor Note:* Dividing by 2 instead of 4.

Q2. The multiplicative inverse of $3+4i$ is:

(A) $\frac{3-4i}{5}$ (B) $\frac{3-4i}{25}$ (C) $\frac{3+4i}{25}$ (D) $3-4i$

Answer: (B) $\frac{\bar{z}}{|z|^2} = \frac{3-4i}{25}$. *Fast Method:* Denominator is $|z|^2 = 25$, not $|z| = 5$. *Distractor Note:* Forgetting to square $|z|$.

Q3. $\sqrt{-9} \times \sqrt{-16}$ equals:

(A) 12 (B) -12 (C) $12i$ (D) $-12i$

Answer: (B) $(3i)(4i) = 12i^2 = -12$. *Fast Method:* Two i's give -1. *Distractor Note:* Writing $+12$.

Q4. Value of $\left(\frac{1-i}{1+i}\right)^4$ is:

(A) 1 (B) -1 (C) i (D) $-i$

Answer: (A) $(-i)^4 = 1$. *Fast Method:* $\frac{1-i}{1+i} = -i$. *Distractor Note:* Sign error on $-i$.

Q5. If $z = \frac{1}{1-i}$, then $|z|$ is:

(A) $\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) 2 (D) 1

Answer: (B) $|z| = \frac{1}{|1-i|} = \frac{1}{\sqrt{2}}$. *Fast Method:* Direct modulus ratio. *Distractor Note:* Inverting to $\sqrt{2}$.

Q6. The conjugate of $(2+i)^2$ is:

(A) $3-4i$ (B) $3+4i$ (C) $4-4i$ (D) $5-4i$

Answer: (A) $(2+i)^2 = 3+4i$, conjugate $= 3-4i$. *Fast Method:* Expand then flip i. *Distractor Note:* Using $2^2+1^2=5$.

Q7. If $z = -\bar{z}$, then z is:

(A) Purely real (B) Purely imaginary (C) Zero only (D) Modulus 1

Answer: (B) $a+ib = -(a-ib) \implies a = 0$. *Fast Method:* Real part vanishes. *Distractor Note:* Confusing with $z = \bar{z}$.

Q8. Value of $1 + i^2 + i^4 + i^6 + \cdots + i^{20}$ is:

(A) 0 (B) 1 (C) -1 (D) 11

Answer: (B) $1-1+1-1+\cdots+1$ (11 terms) $= 1$. *Fast Method:* Odd count of alternating ± 1. *Distractor Note:* Assuming pairs cancel to 0.

Q9. If $|z_1| = 3$ and $|z_2| = 4$, the maximum value of $|z_1 + z_2|$ is:

(A) 1 (B) 5 (C) 7 (D) 12

Answer: (C) Triangle inequality: $|z_1 + z_2| \leq 3 + 4 = 7$. *Fast Method:* Add moduli. *Distractor Note:* 5 is for perpendicular vectors.

Q10. If $(1+i)(1+2i)(1+3i) = x+iy$, then x^2+y^2 is:

(A) 50 (B) 100 (C) 10 (D) 25

Answer: (B) $x^2+y^2 = |1+i|^2|1+2i|^2|1+3i|^2 = 2 \times 5 \times 10 = 100$. *Fast Method:* Multiply squared moduli! *Distractor Note:* Taking square root to get 10.

Q11. If $z = 3-5i$, its reflection in the Real axis on the Argand plane is:

(A) $(-3,5)$ (B) $(3,5)$ (C) $(-3,-5)$ (D) $(5,-3)$

Answer: (B) Reflection in Real axis is $\bar{z} = 3+5i \leftrightarrow (3,5)$. *Fast Method:* Flip sign of y. *Distractor Note:* Flipping x instead of y.

Q12. Sum $i + i^2 + i^3 + i^4$ equals:

(A) 1 (B) i (C) 0 (D) -1

Answer: (C) $i - 1 - i + 1 = 0$. *Fast Method:* 4 consecutive powers sum to 0. *Distractor Note:* Sign error on $i^3 = -i$.

H. Special Exam & MCQ Strategies

- Never expand large powers of fractions to find $|z|$; apply $|z_1/z_2| = |z_1|/|z_2|$ and $|z^n| = |z|^n$ directly.
- For proofs involving x^2+y^2 or a^2+b^2, take the conjugate of the entire equation and multiply the two equations.
- Always convert $\sqrt{-k}$ to $i\sqrt{k}$ before any multiplication or division.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Computing $\sqrt{-2}\sqrt{-3} = \sqrt{6}$. $\longrightarrow$ **Correction:** $(i\sqrt{2})(i\sqrt{3}) = i^2\sqrt{6} = -\sqrt{6}$.

[COMMON MISTAKE] **Error:** Writing $2 + 3i > 1 + 2i$. $\longrightarrow$ **Correction:** Inequalities $(<, >)$ do not exist in $\mathbb{C}$; only moduli $|z_1| > |z_2|$ can be compared.

[COMMON MISTAKE] **Error:** Forgetting $|z|^2$ in denominator of $z^{-1} = \frac{\bar{z}}{|z|^2}$. $\longrightarrow$ **Correction:** Divide $\bar{z}$ by $a^2 + b^2$, not $\sqrt{a^2 + b^2}$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $i^2 = -1, i^3 = -i, i^4 = 1$; $|z| = \sqrt{a^2+b^2}$; $z\bar{z} = |z|^2$; $z^{-1} = \bar{z}/|z|^2$.

[QUICK REVISION] Key Theorems & Results: $(1+i)^2 = 2i, (1-i)^2 = -2i$; $\frac{1+i}{1-i} = i$; $|z_1 z_2| = |z_1||z_2|$.

[QUICK REVISION] Shortcut Summary: Consecutive 4 powers of i sum to 0; use $|z|$ directly without rationalizing.

[QUICK REVISION] High-Frequency Exam Patterns: Powers of i, $a+ib$ simplification, a^2+b^2 proofs, Purely real/imaginary conditions.

Last-Minute Revision Checklist:

- ☐ Convert $\sqrt{-k} \to i\sqrt{k}$ first
- ☐ Modulus has NO i
- ☐ z^{-1} divides by a^2+b^2
- ☐ No inequalities in $\mathbb{C}$

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Value of $i^{24} + i^{25}$ is: (A) 0 (B) $1+i$ (C) $1-i$ (D) 2
2. Modulus of $6-8i$ is: (A) 2 (B) 10 (C) 14 (D) 100
3. Conjugate of $-4+7i$ is: (A) $4-7i$ (B) $-4-7i$ (C) $4+7i$ (D) $7-4i$
4. $(1+i)^2$ equals: (A) 2 (B) $2i$ (C) $-2i$ (D) 0

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. Multiplicative inverse of $1+i\sqrt{3}$ is: (A) $\frac{1-i\sqrt{3}}{4}$ (B) $\frac{1-i\sqrt{3}}{2}$ (C) $\frac{1+i\sqrt{3}}{4}$ (D) $1-i\sqrt{3}$
2. If $a+ib = \frac{3-4i}{4+3i}$, then a^2+b^2 is: (A) 1 (B) 5 (C) 25 (D) $1/5$
3. If $(1+i)z = 1-i$, then z is: (A) i (B) $-i$ (C) 1 (D) -1
4. If $x+iy = (1+i)(1+2i)$, then (x,y) is: (A) $(-1,3)$ (B) $(1,3)$ (C) $(3,-1)$ (D) $(-1,-3)$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Value of $(1+i)^6 + (1-i)^6$ is: (A) 0 (B) 16 (C) -16 (D) $16i$
2. If $|z| = 2$, minimum value of $|z-5|$ is: (A) 2 (B) 3 (C) 5 (D) 7
3. If $\frac{z-1}{z+1}$ is purely imaginary ($z \neq -1$), then $|z|$ is: (A) 0 (B) 1 (C) 2 (D) $1/2$
4. Sum $\sum_{k=1}^{100} i^k$ equals: (A) 0 (B) 1 (C) i (D) -1

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $i^{24}=1, i^{25}=i \implies 1+i$. **Q2 (B):** $\sqrt{36+64}=10$. **Q3 (B):** Flip sign of imaginary part only. **Q4 (B):** $1+2i+i^2 = 2i$.
Board Std	**Q1 (A):** $\bar{z}/|z|^2 = (1-i\sqrt{3})/(1+3)$. **Q2 (A):** $|3-4i|/|4+3i| = 5/5 = 1$. **Q3 (B):** $z = \frac{1-i}{1+i} = -i$. **Q4 (A):** $1+2i+i-2 = -1+3i$.
Challenge	**Q1 (A):** $(2i)^3 + (-2i)^3 = -8i + 8i = 0$. **Q2 (B):** $\|\|z|-5| = |2-5| = 3$. **Q3 (B):** Diameter endpoints $(-1,0), (1,0)$ give $|z| = 1$. **Q4 (A):** 100 is divisible by 4, so 25 complete zero-sum blocks.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: i, powers of i, $\sqrt{-a}\sqrt{-b}$ trap. Day 2: Algebra of $a+ib$, inverse. Day 3: Conjugate & modulus properties. Day 4: Argand plane.
- **How to Introduce the Concept:** Ask students to solve $x^2+1=0$, then show how i is a 90° rotation on a 2D plane.
- **Difficult Areas & Remediation:** The $\sqrt{-a}\sqrt{-b}$ trap and forgetting that $\mathbb{C}$ has no ordering.
- **Blackboard Teaching Examples:** Show $z, \bar{z}, -z$ on the Argand plane together.
- **Classroom Questions to Ask Students:** Why is $z\bar{z}$ always a non-negative real number?
- **Common Student Doubts & Answers:** Students ask where i is used in real life; mention AC circuits and 2D computer graphics rotations.
- **Homework & Revision Strategy:** Assign 10 $a+ib$ simplifications and 5 modulus/conjugate proofs.

[TEACHER'S TIP] Emphasize that Polar Form and Square Root of z are deleted in 2026–27 so students focus 100% on $a+ib$, modulus, and conjugate.

UNIT II: ALGEBRA TYBA COACHING CENTRE

Chapter 5: Linear Inequalities

Algebraic Inequalities in One Variable, Number-Line Intervals & Word Problems (2026–27)

A. Chapter Introduction & Examination Relevance

Real-world constraints—budgets, minimum marks, safe temperature ranges, and chemical concentrations—rarely demand exact equality; instead, they impose inequalities. Chapter 5 formalizes solving linear inequalities in one variable over $\mathbb{N}, \mathbb{Z}$, and $\mathbb{R}$.

- **What You Will Learn:** Inequality axioms, sign-reversal rule on multiplying/dividing by negatives, solving single and double linear inequalities in one variable, modulus inequalities $|x-a|<r$, number-line representation, and practical word problems.
- **Why This Chapter Matters:** Linear inequalities are indispensable for finding domains and ranges (Chapter 2), solving optimization problems in Class 12 Linear Programming, and analyzing derivatives ($f'(x)>0$).
- **Real-Life Applications:** Minimum-average grade calculations, manufacturing tolerance bands, safe acidity/pH ranges, and profit-threshold budgeting.
- **Cross-Chapter Connections:** Builds directly on Chapter 1 (interval notation) and feeds into Chapter 2 (domain inequalities) and Class 12 Chapter 12 (Linear Programming).
- **Board & Competitive Relevance:** Unit II carries 25/80 marks. Chapter 5 contributes 3–5 marks via number-line interval MCQs and 2M/3M word problems (acid dilution, average marks, triangle/length constraints). (2-variable graphical shading is excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Rules of Inequalities & Sign Reversal

Adding or subtracting any real number c on both sides preserves the inequality sign ($a<b \implies a+c<b+c$). Multiplying or dividing by a **positive** number preserves the sign, whereas multiplying or dividing by a **negative** number **reverses** the sign: $a<b$ and $k<0 \implies ka>kb$.

[CONCEPT MASTER – 1. Rules of Inequalities & Sign Reversal] Golden Rule: NEVER cross-multiply an inequality containing a variable x in the denominator unless you know its sign! Subtract and take LCM instead.

2. Domain-Dependent Solution Sets ($\mathbb{N}, \mathbb{Z}, \mathbb{R}$)

The exact same algebraic inequality yields different solution sets depending on the replacement set. For example, $x<3$ gives $\{1,2\}$ in $\mathbb{N}$, $\{\ldots,-1,0,1,2\}$ in $\mathbb{Z}$, and $(-\infty,3)$ in $\mathbb{R}$.

[CONCEPT MASTER – 2. Domain-Dependent Solution Sets ($\mathbb{N}, \mathbb{Z}, \mathbb{R}$)] On the number line, use a **hollow circle** ($\circ$) for $<$ or $>$, and a **filled dot** ($\bullet$) for $\leq$ or $\geq$.

3. System of Linear Inequalities & Double Inequalities

To solve a system $L_1(x)\geq 0$ AND $L_2(x)\leq 0$, solve each inequality independently and take the **intersection** of their solution intervals. For a compound chain $a<mx+c\leq b$, you may isolate x in the middle directly when x appears only in the middle term.

[CONCEPT MASTER – 3. System of Linear Inequalities & Double Inequalities] When x appears in the outer expressions too (e.g., $x+1<2x-3\leq x+5$), split into two separate inequalities and intersect!

4. Modulus Inequalities & Word Problems

For $r>0$: $|x-a|<r \iff a-r<x<a+r$, and $|x-a|>r \iff x<a-r$ or $x>a+r$. In word problems (marks average, acid solution dilution, consecutive odd/even integers), translate phrases like 'at least' ($\geq$), 'at most' ($\leq$), 'more than' ($>$), and 'less than' ($<$) carefully.

[CONCEPT MASTER – 4. Modulus Inequalities & Word Problems] Rationalised 2026–27 Scope: Graphical solution of linear inequalities in two variables (half-plane shading) is excluded from the 2026–27 NCERT syllabus.

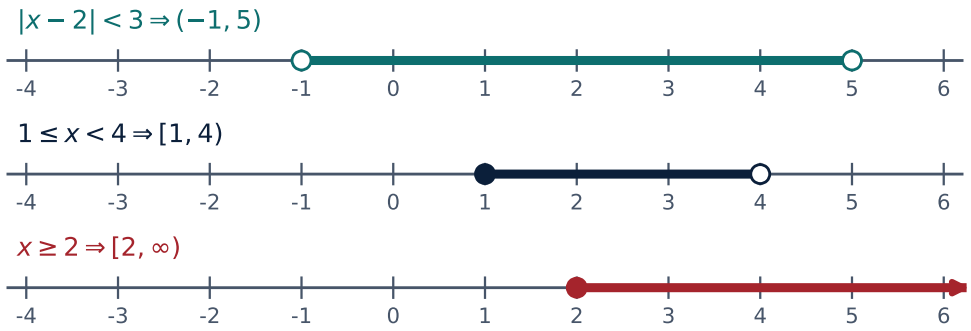


Figure 1: **Visual Model:** Number-line interval graphs showing open circles for strict inequalities ($<, >$) and closed dots for inclusive inequalities ($\leq, \geq$).

C. Complete Formula Bank

[FORMULA BANK – 1. Negative Scaling Reversal] **Expression:** $a < b$ and $c < 0 \implies ac > bc$ and $\frac{a}{c} > \frac{b}{c}$
Symbols: $a, b, c \in \mathbb{R}, c < 0$.
Conditions & When to Use: Always flip $<$ to $>$ and $\leq$ to $\geq$.
Solved Mini-Example: $-3x \geq 12 \implies x \leq -4$.
Common Mistake: Forgetting to flip the sign when dividing by -3.

[FORMULA BANK – 2. Reciprocal Inequality (Same Sign)] **Expression:** $0 < a < b \implies \frac{1}{a} > \frac{1}{b} > 0$
Symbols: a, b same sign, non-zero.
Conditions & When to Use: Fails if $a < 0 < b$ (opposite signs keep $\frac{1}{a} < \frac{1}{b}$).
Solved Mini-Example: $2 < x < 5 \implies \frac{1}{5} < \frac{1}{x} < \frac{1}{2}$.
Common Mistake: Inverting $-2 < x < 3$ blindly.

[FORMULA BANK – 3. Modulus Bounded Inequality] **Expression:** $|x - a| < r \iff a - r < x < a + r$
Symbols: $r > 0$.
Conditions & When to Use: If $r \leq 0$, $|x - a| < r$ has no solution.
Solved Mini-Example: $|x - 3| \leq 2 \implies 1 \leq x \leq 5$.
Common Mistake: Writing $x - 3 \leq 2$ only and forgetting $x - 3 \geq -2$.

[FORMULA BANK – 4. Modulus Unbounded Inequality] **Expression:** $|x - a| > r \iff x < a - r$ or $x > a + r$
Symbols: $r > 0$.
Conditions & When to Use: Union of two disjoint rays.
Solved Mini-Example: $|x + 1| > 4 \implies x \in (-\infty, -5) \cup (3, \infty)$.
Common Mistake: Writing $-4 < x + 1 < 4$ for $>$.

[FORMULA BANK – 5. System Intersection] **Expression:** $S = S_1 \cap S_2$
Symbols: S_1, S_2: individual solution intervals.
Conditions & When to Use: Both inequalities must hold simultaneously.
Solved Mini-Example: $x > 2$ and $x \leq 5 \implies (2, 5]$.
Common Mistake: Taking union instead of intersection.

[FORMULA BANK – 6. Average / Mixture Inequality] **Expression:** $L \leq \frac{\text{Sum}}{n} \leq U$; Mixture: $C_L \leq \frac{c_1V_1+c_2V_2}{V_1+V_2} \leq C_U$
Symbols: V_1, V_2: volumes; c_1, c_2: concentrations.
Conditions & When to Use: Positive volumes $V_1, V_2 > 0$.
Solved Mini-Example: Adding x litres of water (0%) to V litres of 30% acid gives $\frac{0.3V}{V+x}$.
Common Mistake: Putting V instead of $V + x$ in the denominator.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and JEE/CUET Foundation problem structures.

Pattern 1: Single Linear Inequality over $\mathbb{N}, \mathbb{Z}, \mathbb{R}$

- **Identification Cue:** Solve $30x < 200$ when $x \in \mathbb{N}$ or $x \in \mathbb{Z}$. | **Concept & Formula:** Divide and filter by domain.
- **Standard Step-by-Step:** Isolate x, then list integers or write real interval.
- **Fastest Valid Method:** Compute decimal bound $200/30 = 6.67$ first. | **Alternative:** Number-line plot.
- **Common Trap:** Including 0 in $\mathbb{N}$ or forgetting negative integers in $\mathbb{Z}$.
- **Practice Drill:** Ex. 1 & Ex. 2.

Pattern 2: Fractional Inequalities with Variable Numerator

- **Identification Cue:** Solve $\frac{2x-1}{3} \geq \frac{3x-2}{4} - \frac{2-x}{5}$. | **Concept & Formula:** Clear constant denominators via positive LCM.
- **Standard Step-by-Step:** Multiply entire inequality by positive LCM (60), collect x terms, divide.
- **Fastest Valid Method:** Never multiply by a negative LCM. | **Alternative:** Bring all terms to LHS.
- **Common Trap:** Sign error on $-(2 - x)$.
- **Practice Drill:** Ex. 5 & Ex. 6.

Pattern 3: System of Two Inequalities

- **Identification Cue:** Solve $3x - 7 > 2(x - 6)$ and $6 - x > 11 - 2x$. | **Concept & Formula:** Solve both and intersect.
- **Standard Step-by-Step:** Find S_1 and S_2 separately, plot on one number line, read overlap.
- **Fastest Valid Method:** Compare lower bounds (take $\max$) and upper bounds (take $\min$). | **Alternative:** Test boundary values.
- **Common Trap:** Writing $\emptyset$ when intersection is valid, or vice versa.
- **Practice Drill:** Ex. 9 & Ex. 10.

Pattern 4: Word Problems (Marks, Consecutive Integers, Acid Dilution)

- **Identification Cue:** How many litres of water/acid must be added so mixture is between 15% and 18%? | **Concept & Formula:** Double linear inequality modeling.
- **Standard Step-by-Step:** Set x = added volume, write total solute / total volume between lower and upper bounds.
- **Fastest Valid Method:** Multiply through by 100 first to eliminate decimals. | **Alternative:** Solve left and right bounds separately.
- **Common Trap:** Forgetting to add x to total volume in denominator.
- **Practice Drill:** Ex. 12 & Ex. 13.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Positive-Coefficient Isolation] **Core Idea & Why It Works:** Always move x-terms to the side that keeps the coefficient of x positive, avoiding negative division.
Valid Domain: All linear inequalities. | **Where It Fails:** Nothing.
Easy Example: $5x - 3 < 7x + 5 \implies -8 < 2x \implies x > -4$.
Board-Level Example: Saves sign-flip mistakes in multi-step fractional inequalities.
Advanced Example: In double inequalities, if middle x is negative, divide all 3 parts by $-k$ and reverse both $<$ signs.

[SMART SHORTCUT – 2. Modulus Distance Interpretation] **Core Idea & Why It Works:** Read $|x - a| < r$ as 'distance of x from a is less than r', so $x \in (a - r, a + r)$.
Valid Domain: All $|x - a|$ inequalities. | **Where It Fails:** When variable appears outside $|\cdot|$.
Easy Example: $|x - 5| \leq 3 \implies [5 - 3, 5 + 3] = [2, 8]$.
Board-Level Example: $|2x - 1| < 5 \implies |x - 0.5| < 2.5 \implies (-2, 3)$.
Advanced Example: For $a < |x| < b$, answer is $(-b, -a) \cup (a, b)$.

[SMART SHORTCUT – 3. Sign-Chart for Rational $\frac{x-a}{x-b} \geq 0$] **Core Idea & Why It Works:** Plot critical points on number line; sign alternates $+, -, +$ from right to left for distinct linear factors.
Valid Domain: All factored rational inequalities. | **Where It Fails:** Even powers $(x - a)^2$ do not flip sign.
Easy Example: $\frac{x-1}{x-3} > 0 \implies (-\infty, 1) \cup (3, \infty)$.
Board-Level Example: Handles $|x| - 1$ or domain problems instantly.
Advanced Example: Never include denominator roots even for ≥ 0.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 **[Beginner]**
Question: Solve $24x < 100$ when (i) $x \in \mathbb{N}$, (ii) $x \in \mathbb{Z}$.
Given & Required: $x < 100/24 = 4.166\ldots$ | **Concept & Formula:** Domain filtering.
Detailed Solution: (i) $\{1, 2, 3, 4\}$; (ii) $\{\ldots, -2, -1, 0, 1, 2, 3, 4\}$. **Final Answer:** $\{1, 2, 3, 4\}$; $\{\ldots, 2, 3, 4\}$
Fastest Alternative / Shortcut: $100/24 = 4\frac{1}{6}$.

SOLVED EXAMPLE 2 **[Beginner]**
Question: Solve $-12x > 30$ when $x \in \mathbb{Z}$ and $x \in \mathbb{R}$.
Given & Required: $-12x > 30$. | **Concept & Formula:** Negative division reversal.
Detailed Solution: Divide by -12: $x < -2.5$. For $x \in \mathbb{Z}$: $\{\ldots, -5, -4, -3\}$. For $x \in \mathbb{R}$: $(-\infty, -2.5)$. **Final Answer:** $\{\ldots, -4, -3\}$; $(-\infty, -5/2)$
Fastest Alternative / Shortcut: Flip $>$ to $<$ immediately.

SOLVED EXAMPLE 3 **[Beginner]**
Question: Solve $3(x - 1) \leq 2(x - 3)$ for $x \in \mathbb{R}$.
Given & Required: $3x - 3 \leq 2x - 6$. | **Concept & Formula:** Linear isolation.
Detailed Solution: $3x - 2x \leq -6 + 3 \implies x \leq -3$, i.e., $(-\infty, -3]$. **Final Answer:** $(-\infty, -3]$
Fastest Alternative / Shortcut: Move $2x$ left so coefficient stays $+1$.

SOLVED EXAMPLE 4 **[Beginner]**
Question: Solve $|x - 2| \leq 5$.
Given & Required: $|x - 2| \leq 5$. | **Concept & Formula:** Modulus interval.
Detailed Solution: $-5 \leq x - 2 \leq 5 \implies -3 \leq x \leq 7$. **Final Answer:** $[-3, 7]$
Fastest Alternative / Shortcut: $[2 - 5, 2 + 5] = [-3, 7]$.

SOLVED EXAMPLE 5 [Intermediate]

Question: Solve $\frac{x}{4} < \frac{5x-2}{3} - \frac{7x-3}{5}$ for $x \in \mathbb{R}$.

Given & Required: LCM of $4, 3, 5$ is 60. | **Concept & Formula:** Clearing denominators.

Detailed Solution: Multiply by 60: $15x < 20(5x-2) - 12(7x-3) = 100x - 40 - 84x + 36 = 16x - 4$. Thus $-x < -4 \implies x > 4$.

Final Answer: $(4, \infty)$

Fastest Alternative / Shortcut: Check $x = 5$: $1.25 < 23/3 - 32/5 = 1.267$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Solve $-8 \leq 5x - 3 < 7$.

Given & Required: Double inequality. | **Concept & Formula:** Isolate middle x.

Detailed Solution: Add 3: $-5 \leq 5x < 10$. Divide by 5: $-1 \leq x < 2$. **Final Answer:** $[-1, 2)$

Fastest Alternative / Shortcut: Operate on all 3 parts simultaneously.

SOLVED EXAMPLE 7 [Intermediate]

Question: Solve $-15 < \frac{3(x-2)}{5} \leq 0$.

Given & Required: Double inequality. | **Concept & Formula:** Scale by $+5/3$.

Detailed Solution: Multiply by $5/3$: $-25 < x - 2 \leq 0$. Add 2: $-23 < x \leq 2$. **Final Answer:** $(-23, 2]$

Fastest Alternative / Shortcut: Multiply by $5/3 > 0$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Solve $|2x - 3| > 7$.

Given & Required: Modulus > 7. | **Concept & Formula:** Two-ray union.

Detailed Solution: $2x - 3 < -7$ or $2x - 3 > 7 \implies 2x < -4$ or $2x > 10 \implies x < -2$ or $x > 5$. **Final Answer:** $(-\infty, -2) \cup (5, \infty)$

Fastest Alternative / Shortcut: Split into < -7 and > 7.

SOLVED EXAMPLE 9 [Board Standard]

Question: Solve the system: $3x - 7 > 2(x - 6)$ and $6 - x > 11 - 2x$.

Given & Required: Two inequalities. | **Concept & Formula:** System intersection.

Detailed Solution: (1) $3x - 7 > 2x - 12 \implies x > -5$. (2) $2x - x > 11 - 6 \implies x > 5$. Intersection of $(-5, \infty)$ and $(5, \infty)$ is $(5, \infty)$. **Final Answer:** $(5, \infty)$

Fastest Alternative / Shortcut: $\max(-5, 5) = 5$.

SOLVED EXAMPLE 10 [Board Standard]

Question: Solve the system: $5(2x - 7) - 3(2x + 3) \leq 0$ and $2x + 19 \leq 6x + 47$.

Given & Required: Two inequalities. | **Concept & Formula:** System intersection.

Detailed Solution: (1) $10x - 35 - 6x - 9 \leq 0 \implies 4x \leq 44 \implies x \leq 11$. (2) $-4x \leq 28 \implies x \geq -7$. Intersection: $[-7, 11]$.

Final Answer: $[-7, 11]$

Fastest Alternative / Shortcut: Combine lower and upper bounds.

SOLVED EXAMPLE 11 [Board Standard]

Question: Ravi obtained 70 and 75 marks in first two unit tests. Find minimum marks in the third test to get an average of at least 60 marks.

Given & Required: Marks: $70, 75, x$; avg ≥ 60. | **Concept & Formula:** Average inequality.

Detailed Solution: $\frac{70+75+x}{3} \geq 60 \implies 145 + x \geq 180 \implies x \geq 35$. **Final Answer:** 35 **marks**

Fastest Alternative / Shortcut: $3 \times 60 - 145 = 35$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11.

Given & Required: $x, x+2$ odd, $x+2<10$, $2x+2>11$. | **Concept & Formula:** Integer constraints.

Detailed Solution: $x<8$ and $2x>9 \implies 4.5<x<8$. Odd integers in $(4.5,8)$ are $5,7$. Pairs: $(5,7)$ and $(7,9)$. **Final Answer:** $(5,7)$ **and** $(7,9)$

Fastest Alternative / Shortcut: Check odd numbers <8.

SOLVED EXAMPLE 13 [Board Standard]

Question: How many litres of water will have to be added to 1125 litres of 45% acid solution so that the resulting mixture contains more than 25% but less than 30% acid?

Given & Required: 1125 L of 45% acid, x L water (0%). | **Concept & Formula:** Mixture double inequality.

Detailed Solution: $0.25(1125+x)<0.45(1125)<0.30(1125+x)$. Left: $1125+x<2025 \implies x<900$. Right: $1687.5<1125+x \implies x>562.5$. **Final Answer: Between** 562.5 **L and** 900 **L**

Fastest Alternative / Shortcut: Multiply by 100 first.

SOLVED EXAMPLE 14 [Challenge]

Question: Solve $\frac{x-3}{x+1} \leq 0$ for $x \in \mathbb{R}$.

Given & Required: Rational inequality. | **Concept & Formula:** Sign chart.

Detailed Solution: Critical points: $x=3$ (numerator, included) and $x=-1$ (denominator, excluded). Testing $x=0$ gives $-3 \leq 0$ (true). Thus $x \in (-1,3]$. **Final Answer:** $(-1,3]$

Fastest Alternative / Shortcut: Never include denominator root $x=-1$.

SOLVED EXAMPLE 15 [Challenge]

Question: Solve $1 \leq |x-2| \leq 3$.

Given & Required: Annulus on number line. | **Concept & Formula:** Distance from 2 is in $[1,3]$.

Detailed Solution: $x-2 \in [-3,-1] \cup [1,3] \implies x \in [-1,1] \cup [3,5]$. **Final Answer:** $[-1,1] \cup [3,5]$

Fastest Alternative / Shortcut: Shift $[-3,-1] \cup [1,3]$ by $+2$.

SOLVED EXAMPLE 16 [Challenge]

Question: IQ of a person is $IQ = \frac{MA}{CA} \times 100$. If $80 \leq IQ \leq 140$ for a group of 12-year-olds ($CA=12$), find the range of their mental age (MA).

Given & Required: $CA=12, 80 \leq \frac{MA}{12} \cdot 100 \leq 140$. | **Concept & Formula:** NCERT Misc. application.

Detailed Solution: $\frac{80\times12}{100} \leq MA \leq \frac{140\times12}{100} \implies 9.6 \leq MA \leq 16.8$ years. **Final Answer:** $9.6 \leq MA \leq 16.8$ **years**

Fastest Alternative / Shortcut: Multiply bounds by 0.12.

G. Competency MCQ Mastery Section

Q1. If $-3x+17<-13$, then:

(A) $x \in (10,\infty)$ (B) $x \in [10,\infty)$ (C) $x \in (-\infty,10)$ (D) $x \in (-10,10)$

Answer: (A) $-3x<-30 \implies x>10$. *Fast Method:* Flip $<$ on dividing by -3. *Distractor Note:* Forgetting sign flip gives (C).

Q2. Solution of $5x-3<3x+1$ when $x \in \mathbb{N}$ is:

(A) $\{1,2\}$ (B) $\{1\}$ (C) $\{0,1\}$ (D) $(-\infty,2)$

Answer: (B) $2x<4 \implies x<2$. Only $1 \in \mathbb{N}$. *Fast Method:* Strict <2 excludes 2. *Distractor Note:* Including 2 or 0.

Q3. If $|x+3| \geq 10$, then:

(A) $x \in (-13,7)$ (B) $x \in (-\infty,-13] \cup [7,\infty)$ (C) $x \in [-13,7]$ (D) $x \in (7,\infty)$

Answer: (B) $x+3 \leq -10$ or $x+3 \geq 10$. *Fast Method:* Outer rays for $\geq$. *Distractor Note:* Picking bounded interval.

Q4. If $a < b$ and $c < 0$, which is true?

(A) $ac < bc$ (B) $ac > bc$ (C) $a/c < b/c$ (D) $a - c > b - c$

Answer: (B) Negative multiplication reverses inequality. *Fast Method:* Test $a = 1, b = 2, c = -1$. *Distractor Note:* Thinking $a - c > b - c$.

Q5. Solution of $-12x > 30$ when $x \in \mathbb{Z}$ is:

(A) $\{-3, -4, \dots\}$ (B) $\{-2, -1, 0, \dots\}$ (C) $\{-2, -3, \dots\}$ (D) $\emptyset$

Answer: (A) $x < -2.5$, largest integer is -3. *Fast Method:* $-2 > -2.5$, so excluded. *Distractor Note:* Including -2.

Q6. Solution of $\frac{x-1}{x+2} < 0$ is:

(A) $(-2, 1)$ (B) $[-2, 1]$ (C) $(-\infty, -2) \cup (1, \infty)$ (D) $(-2, 1]$

Answer: (A) Opposite signs between -2 and 1. *Fast Method:* Test $x = 0 \implies -1/2 < 0$. *Distractor Note:* Including -2 or 1.

Q7. If $|x - 1| > 5$, then x lies in:

(A) $(-4, 6)$ (B) $(-\infty, -4) \cup (6, \infty)$ (C) $[-4, 6]$ (D) $(6, \infty)$

Answer: (B) $x - 1 < -5$ or $x - 1 > 5$. *Fast Method:* Distance > 5 from 1. *Distractor Note:* Missing left ray $(-\infty, -4)$.

Q8. System $x \geq -2$ and $x < 4$ represents:

(A) $(-2, 4)$ (B) $[-2, 4)$ (C) $[-2, 4]$ (D) $(-2, 4]$

Answer: (B) -2 closed, 4 open. *Fast Method:* Match $\geq$ with [and $<$ with). *Distractor Note:* Bracket mismatch.

Q9. If $0 < x < 1$, which is largest?

(A) x (B) x^2 (C) $1/x$ (D) $\sqrt{x}$

Answer: (C) For $x = 0.5$, $1/x = 2$ is largest. *Fast Method:* Test $x = 1/2$. *Distractor Note:* Assuming $x^2 > x$.

Q10. Solution of $2x - 1 \geq x + 3$ and $x < 2$ is:

(A) $[4, \infty)$ (B) $(-\infty, 2)$ (C) $\emptyset$ (D) $(2, 4]$

Answer: (C) $x \geq 4$ and $x < 2$ have no overlap. *Fast Method:* Disjoint rays. *Distractor Note:* Taking union instead of intersection.

Q11. If $3x + 5 \leq 20$ and x is a prime number, solution set is:

(A) $\{2, 3, 5\}$ (B) $\{1, 2, 3, 5\}$ (C) $\{2, 3\}$ (D) $\{1, 2, 3, 4, 5\}$

Answer: (A) $x \leq 5$; primes ≤ 5 are $2, 3, 5$. *Fast Method:* 1 is not prime. *Distractor Note:* Including 1.

Q12. Length of the solution interval of $-3 \leq 2x - 1 \leq 5$ is:

(A) 3 (B) 4 (C) 5 (D) 8

Answer: (B) $-1 \leq x \leq 3$, length $= 3 - (-1) = 4$. *Fast Method:* $(5 - (-3))/2 = 4$. *Distractor Note:* Forgetting to divide length by 2.

H. Special Exam & MCQ Strategies

- Always check the replacement set ($\mathbb{N}, \mathbb{Z}, \mathbb{R}$) at the very last step before writing your final answer.
- Keep the coefficient of x positive by moving x to the larger-coefficient side; this eliminates 90% of sign-reversal errors.
- For mixture word problems, write a quick 2-column table: Volume and Pure Acid Amount.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Dividing $-2x > 6$ by -2 to get $x > -3$. $\longrightarrow$ **Correction:** Dividing by a negative number MUST flip the sign: $x < -3$.

[COMMON MISTAKE] **Error:** Cross-multiplying $\frac{x-1}{x+2} > 0$ to get $x - 1 > 0$. $\longrightarrow$ **Correction:** Never multiply by a variable expression of unknown sign; use a sign chart.

[COMMON MISTAKE] **Error:** Writing $x \in [7, -13]$ for $|x+3| \geq 10$. $\longrightarrow$ **Correction:** Unbounded modulus gives a union of two rays: $(-\infty, -13] \cup [7, \infty)$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $ac > bc$ when $a < b, c < 0$; $|x-a| < r \iff (a-r, a+r)$; $|x-a| > r \iff (-\infty, a-r) \cup (a+r, \infty)$.

[QUICK REVISION] Key Theorems & Results: System solution $= S_1 \cap S_2$; Hollow circle $(<, >)$ vs Filled dot $(\leq, \geq)$.

[QUICK REVISION] Shortcut Summary: Move x to keep coefficient $+$; Sign-chart for $\frac{x-a}{x-b}$; Test $x = 0$ to verify.

[QUICK REVISION] High-Frequency Exam Patterns: Domain-filtered inequalities, Double inequalities, System intersections, Marks/Acid dilution.

Last-Minute Revision Checklist:

- ☐ Flip sign on negative multiply/divide
- ☐ Check $\mathbb{N}, \mathbb{Z}, \mathbb{R}$ domain
- ☐ Intersection for 'and'
- ☐ Denominator $\neq 0$

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Solve $-5x \leq 20$ for $x \in \mathbb{R}$: (A) $[-4, \infty)$ (B) $(-\infty, -4]$ (C) $[4, \infty)$ (D) $(-4, \infty)$
2. Solve $4x + 3 < 6x + 7$: (A) $(-2, \infty)$ (B) $(-\infty, -2)$ (C) $(2, \infty)$ (D) $[-2, \infty)$
3. Solution of $x < 3$ for $x \in \mathbb{N}$: (A) $\{1, 2, 3\}$ (B) $\{1, 2\}$ (C) $\{0, 1, 2\}$ (D) $(-\infty, 3)$
4. $|x| < 4$ means: (A) $x < 4$ (B) $-4 < x < 4$ (C) $x > -4$ (D) $x \leq 4$

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. Solve $-5 \leq \frac{5-3x}{2} \leq 8$: (A) $[-11/3, 5]$ (B) $[-5, 11/3]$ (C) $(-11/3, 5)$ (D) $[11/3, 5]$
2. System $2x - 1 > 3$ and $3x - 2 \leq 10$ gives: (A) $(2, 4]$ (B) $[2, 4]$ (C) $(2, 4)$ (D) $[2, 4)$
3. If $\frac{x+1}{x-2} \leq 0$, then x belongs to: (A) $[-1, 2]$ (B) $[-1, 2)$ (C) $(-1, 2)$ (D) $(-1, 2]$
4. Minimum marks in 3rd test to average ≥ 75 after $70, 72$: (A) 80 (B) 83 (C) 85 (D) 78

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Solve $|x-3| + |x-5| < 2$: (A) $(3, 5)$ (B) $[3, 5]$ (C) $\emptyset$ (D) $(2, 6)$
2. If $2 \leq |x-1| \leq 5$, then x lies in: (A) $[-4, -1] \cup [3, 6]$ (B) $[-4, 6]$ (C) $[-1, 3]$ (D) $(-4, -1) \cup (3, 6)$
3. Number of integer solutions of $\frac{x^2-4}{x-1} \leq 0$ with $x \geq -3$ is: (A) 3 (B) 4 (C) 5 (D) 6
4. If $a > b > 0$ and $c < d < 0$, which is always true? (A) $ac > bd$ (B) $a/c < b/d$ (C) $a - c > b - d$ (D) $ad < bc$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (A):** Divide by -5 and flip: $x \geq -4$. **Q2 (A):** $-4 < 2x \implies x > -2$. **Q3 (B):** Natural numbers < 3 are $1, 2$. **Q4 (B):** Bounded open interval $(-4, 4)$.
Board Std	**Q1 (A):** $-10 \leq 5 - 3x \leq 16 \implies -15 \leq -3x \leq 11 \implies -11/3 \leq x \leq 5$. **Q2 (A):** $x > 2$ and $x \leq 4 \implies (2, 4]$. **Q3 (B):** $x = -1$ included, $x = 2$ excluded. **Q4 (B):** $225 - 142 = 83$.
Challenge	**Q1 (C):** Min value of $\lvert x-3\rvert + \lvert x-5\rvert$ is $\lvert 5-3\rvert = 2$, never < 2. **Q2 (A):** $x - 1 \in [-5, -2] \cup [2, 5]$. Shift by $+1$. **Q3 (A):** Sign chart gives $(-\infty, -2] \cup (1, 2]$; integers ≥ -3 are $-3, -2, 2$ (3 integers). **Q4 (C):** $-c > -d > 0$ added to $a > b$ gives $a - c > b - d$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Inequality axioms & negative flip. Day 2: Double inequalities & systems. Day 3: Modulus inequalities. Day 4: Word problems.
- **How to Introduce the Concept:** Ask: 'If $5 > 3$, what happens when we multiply both sides by -1?' Let students verify $-5 < -3$ on a number line.
- **Difficult Areas & Remediation:** Double inequalities with negative x and mixture word problems.
- **Blackboard Teaching Examples:** Draw hollow vs filled circles on a giant number line for $(a, b], [a, b), (-\infty, a)$.
- **Classroom Questions to Ask Students:** Why can't we cross-multiply $\frac{x}{x-1} > 2$ by $(x-1)$?
- **Common Student Doubts & Answers:** Students ask why 2-variable graph shading is not in the book; clarify it was rationalised out of Class 11 (moved solely to Class 12 LPP).
- **Homework & Revision Strategy:** Assign 8 algebraic inequalities and 4 word problems.

[TEACHER'S TIP] Drilling sign-charts for $\frac{x-a}{x-b}$ here pays huge dividends in Class 12 Increasing/Decreasing Functions.

UNIT II: ALGEBRA TYBA COACHING CENTRE

Chapter 6: Permutations and Combinations

Fundamental Counting Principle, $n!$, ${}^nP_r$, ${}^nC_r$, Repetitions & Restrictions (2026–27)

A. Chapter Introduction & Examination Relevance

How many 4-digit PINs can be formed? In how many ways can a 5-member committee be chosen from 12 people? Chapter 6 replaces tedious listing with systematic counting principles.

- **What You Will Learn:** Fundamental Principle of Counting (Multiplication & Addition Rules), factorial notation $n!$, permutations ${}^nP_r$ (distinct and repeated objects), combinations ${}^nC_r$, and Pascal's identity ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$.
- **Why This Chapter Matters:** Full chapter is retained in the 2026–27 rationalised syllabus! It is one of the highest-scoring chapters for JEE/CUET and directly powers Binomial Theorem (Ch. 7) and Probability (Ch. 14).
- **Real-Life Applications:** Cryptography & password strength, DNA sequencing combinations, airline/tournament scheduling, and lottery/card probabilities.
- **Cross-Chapter Connections:** Directly supplies binomial coefficients ${}^nC_r$ to Chapter 7 and sample-space/event counts $n(E)/n(S)$ to Chapter 14.
- **Board & Competitive Relevance:** Unit II carries 25/80 marks. Expect 1M/2M MCQs on ${}^nC_r$ equations and 3M/5M word problems on word arrangements (MISSISSIPPI, DAUGHTER) and committee selection.

B. Complete Concept Explanation & Derivations

1. Fundamental Principle of Counting (FPC)

Multiplication Principle (AND): If an event occurs in m ways and a second independent event occurs in n ways, both together occur in $m \times n$ ways. **Addition Principle (OR):** If two mutually exclusive alternatives occur in m and n ways, either occurs in $m + n$ ways.

[CONCEPT MASTER – 1. Fundamental Principle of Counting (FPC)] Quick Cue: 'AND' means multiply ($\times$); mutually exclusive 'OR' means add ($+$).

2. Factorial Notation & Permutations (${}^nP_r$)

$n! = n(n-1)(n-2)\cdots 1$, with $0! = 1$. A **permutation** is an *ordered arrangement* of r objects from n distinct objects: ${}^nP_r = \frac{n!}{(n-r)!}$ $(0 \le r \le n)$. If n objects have p_1 alike of one kind, p_2 alike of another, ..., p_k alike, the number of distinct arrangements is $\frac{n!}{p_1!p_2!\cdots p_k!}$.

[CONCEPT MASTER – 2. Factorial Notation & Permutations (${}^nP_r$)] If repetition is allowed, each of r places has n choices, giving n^r arrangements.

3. Combinations (${}^nC_r$) & Key Identities

A **combination** is an *unordered selection* of r objects from n distinct objects: ${}^nC_r = \frac{n!}{r!(n-r)!}$. Key properties: ${}^nC_0 = {}^nC_n = 1$, ${}^nC_r = {}^nC_{n-r}$, ${}^nC_a = {}^nC_b \implies a = b$ or $a + b = n$, and Pascal's Rule: ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$.

[CONCEPT MASTER – 3. Combinations (${}^nC_r$) & Key Identities] Always remember: Select first (${}^nC_r$), then arrange ($r!$) if order matters: ${}^nP_r = {}^nC_r \times r!$.

4. Restricted Arrangements (String & Gap Methods)

Together (String Method): Tie the k objects that must stay together into 1 super-block, arrange $(n - k + 1)$ blocks, then multiply by $k!$ internal arrangements. **Never Together (Gap Method):** Arrange the other $(n - k)$ objects first, creating $(n - k + 1)$ gaps, and place the k restricted objects in ${}^{\text{gaps}}P_k$ ways.

[CONCEPT MASTER – 4. Restricted Arrangements (String & Gap Methods)] Warning: For 3 or more objects 'never together', Total − All-Together is WRONG because it misses '2 together'! Always use the Gap Method.

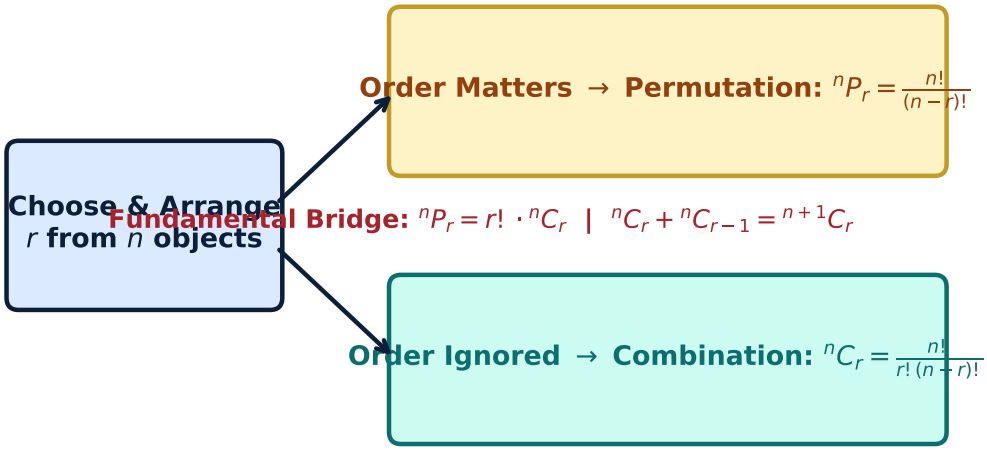


Figure 1: **Visual Model:** Decision tree distinguishing ordered Permutations (${}^nP_r$) from unordered Combinations (${}^nC_r$) and their bridge ${}^nP_r = r! \cdot {}^nC_r$.

C. Complete Formula Bank

[FORMULA BANK – 1. Factorial & Zero Factorial] **Expression:** $n! = n(n-1)!$; $0! = 1$, $1! = 1$
Symbols: $n \in \mathbb{W}$.
Conditions & When to Use: Factorials are undefined for negative integers and fractions.
Solved Mini-Example: $\frac{8!}{6!2!} = \frac{8\times7}{2} = 28$.
Common Mistake: Writing $0! = 0$.

[FORMULA BANK – 2. Permutations (Distinct Objects)] **Expression:** ${}^nP_r = \frac{n!}{(n-r)!} = n(n-1)\cdots(n-r+1)$
Symbols: $0 \le r \le n$.
Conditions & When to Use: No repetition, order matters.
Solved Mini-Example: ${}^5P_3 = 5 \times 4 \times 3 = 60$.
Common Mistake: Stopping at $n-r$ instead of $n-r+1$.

[FORMULA BANK – 3. Permutations with Identical Objects] **Expression:** $\frac{n!}{p_1!p_2!\cdots p_k!}$
Symbols: p_i: count of identical objects of type i.
Conditions & When to Use: All n objects used.
Solved Mini-Example: Letters of APPLE (5 letters, 2 P's): $\frac{5!}{2!} = 60$.
Common Mistake: Forgetting to divide by $p_i!$.

[FORMULA BANK – 4. Combinations (Selection)] **Expression:** ${}^nC_r = \frac{n!}{r!(n-r)!}$
Symbols: $0 \le r \le n$.
Conditions & When to Use: Order does NOT matter.
Solved Mini-Example: ${}^{10}C_3 = \frac{10\times9\times8}{3\times2\times1} = 120$.
Common Mistake: Using ${}^nP_r$ for committee/handshakes.

[FORMULA BANK – 5. Complementary & Equality Rule] **Expression:** ${}^nC_r = {}^nC_{n-r}$; ${}^nC_a = {}^nC_b \implies a = b$ or $a + b = n$
Symbols: $0 \le a, b \le n$.
Conditions & When to Use: Same upper index n.
Solved Mini-Example: ${}^nC_8 = {}^nC_2 \implies n = 8 + 2 = 10$.
Common Mistake: Assuming $a = b$ is the only possibility.

[FORMULA BANK – 6. Pascal's Identity] **Expression:** ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
Symbols: $1 \le r \le n$.
Conditions & When to Use: Same n, consecutive lower indices.
Solved Mini-Example: ${}^7C_3 + {}^7C_2 = {}^8C_3 = 56$.
Common Mistake: Incrementing both n and r improperly.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and JEE/CUET Foundation problem structures.

Pattern 1: Digit & Number Formation with Restrictions

- **Identification Cue:** How many 3-digit even numbers from $\{1, 2, 3, 4, 5, 6\}$ with/without repetition? | **Concept & Formula:** Box/slot method + FPC.
- **Standard Step-by-Step:** Fill the most restricted slot first (units digit for even/odd, hundreds digit $\neq 0$).
- **Fastest Valid Method:** Draw 3 boxes and write available choices inside each box. | **Alternative:** Use ${}^nP_r$ for remaining slots.
- **Common Trap:** Allowing 0 in the leading digit or forgetting repetition constraint.
- **Practice Drill:** Ex. 1 & Ex. 5.

Pattern 2: Word Arrangements (Vowels Together / Never Together)

- **Identification Cue:** Arrangements of DAUGHTER or MISSISSIPPI. | **Concept & Formula:** String method vs Gap method + identical letters.
- **Standard Step-by-Step:** Count total letters and repeats; bundle vowels for 'together' or use gaps for 'never together'.
- **Fastest Valid Method:** For 'no two vowels together', place consonants first, then vowels in $(C + 1)$ gaps. | **Alternative:** Total − Together (valid ONLY when exactly 2 items are restricted).
- **Common Trap:** Using Total − Together for 3+ vowels.
- **Practice Drill:** Ex. 7 & Ex. 9.

Pattern 3: Solving ${}^nP_r$ and ${}^nC_r$ Equations

- **Identification Cue:** Find n if ${}^nC_9 = {}^nC_8$ or ${}^{2n}C_3 : {}^nC_3 = 12 : 1$. | **Concept & Formula:** ${}^nC_a = {}^nC_b$ and factorial expansion.
- **Standard Step-by-Step:** Expand into factorials, cancel common factorial tails, solve polynomial.
- **Fastest Valid Method:** Use ${}^nC_a = {}^nC_b \implies n = a + b$ when $a \neq b$. | **Alternative:** Test small integer options in MCQs.
- **Common Trap:** Forgetting $n \ge r$ domain restriction.
- **Practice Drill:** Ex. 4 & Ex. 11.

Pattern 4: Committee Selection with 'At Least / At Most'

- **Identification Cue:** Committee of 5 from 6 men and 4 women with at least 2 women. | **Concept & Formula:** Case-by-case ${}^nC_r$ + Addition Principle.
- **Standard Step-by-Step:** List all valid (M, W) splits, compute ${}^6C_m \times {}^4C_w$ for each, and sum.
- **Fastest Valid Method:** Total − invalid cases when invalid cases are fewer (e.g., Total − 0W − 1W). | **Alternative:** Direct case enumeration.
- **Common Trap:** Adding instead of multiplying within a single case, or double-counting via ${}^4C_2 \times {}^8C_3$.
- **Practice Drill:** Ex. 12 & Ex. 13.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Most-Restricted-Slot-First] **Core Idea & Why It Works:** Always fill constrained positions (leading digit $\neq 0$, last digit even/5, fixed ends) BEFORE free positions.

Valid Domain: All digit, seating, and word problems. | **Where It Fails:** When two constraints overlap (then split into 2 cases, e.g., 0 at end vs non-zero at end).

Easy Example: 3-digit even from $\{1..5\}$ no repeat: Units $\{2,4\}$ (2), Hundreds (4), Tens (3) $\implies$ 24.

Board-Level Example: 4-digit numbers from $\{0,1,2,3,4,5\}$ divisible by 5: split Case 1 (ends in 0) and Case 2 (ends in 5).

Advanced Example: Handles divisibility by 4 (last 2 digits) cleanly.

[SMART SHORTCUT – 2. Geometry Counting Formulas] **Core Idea & Why It Works:** Lines from n non-collinear points $= {}^nC_2$; Triangles $= {}^nC_3$; Diagonals of n-gon $= {}^nC_2 - n = \frac{n(n-3)}{2}$.

Valid Domain: Polygon and point-grid problems. | **Where It Fails:** If k points are collinear, subtract ${}^kC_2 - 1$ for lines and ${}^kC_3$ for triangles.

Easy Example: Diagonals of octagon ($n=8$): $\frac{8\times5}{2} = 20$.

Board-Level Example: Handshakes among 12 people $= {}^{12}C_2 = 66$.

Advanced Example: Parallelograms from m parallel lines intersecting n parallel lines $= {}^mC_2 \times {}^nC_2$.

[SMART SHORTCUT – 3. Circular Permutations (Reference)] **Core Idea & Why It Works:** n distinct objects around a circle: $(n-1)!$ (and $\frac{(n-1)!}{2}$ for necklaces where clockwise = anticlockwise).

Valid Domain: Round-table seating. | **Where It Fails:** If seats are numbered (then it is linear $n!$).

Easy Example: 5 people at a round table: $4! = 24$.

Board-Level Example: 6 beads on a necklace: $5!/2 = 60$.

Advanced Example: Fix 1 person first, then arrange $(n-1)$ linearly.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: How many 3-digit numbers can be formed from digits $1,2,3,4,5$ if (i) repetition is allowed, (ii) repetition is not allowed?

Given & Required: 5 non-zero digits, 3 places. | **Concept & Formula:** FPC.

Detailed Solution: (i) $5\times5\times5 = 125$; (ii) $5\times4\times3 = 60$. **Final Answer: (i)** 125**, (ii)** 60

Fastest Alternative / Shortcut: Box method.

SOLVED EXAMPLE 2 [Beginner]

Question: Evaluate $\frac{8!}{6!\cdot2!}$.

Given & Required: $8!, 6!, 2!$. | **Concept & Formula:** Factorial cancellation.

Detailed Solution: $\frac{8\times7\times6!}{6!\times2} = 28$. **Final Answer:** 28

Fastest Alternative / Shortcut: Expand down to $6!$ only.

SOLVED EXAMPLE 3 [Beginner]

Question: In how many ways can 5 students sit on 3 chairs?

Given & Required: $n=5, r=3$. | **Concept & Formula:** ${}^nP_r$.

Detailed Solution: ${}^5P_3 = 5\times4\times3 = 60$. **Final Answer:** 60

Fastest Alternative / Shortcut: 5 choices for chair 1, 4 for chair 2, 3 for chair 3.

SOLVED EXAMPLE 4 [Beginner]

Question: Find ${}^{12}C_{10}$.

Given & Required: $n = 12, r = 10$. | **Concept & Formula:** ${}^nC_r = {}^nC_{n-r}$.

Detailed Solution: ${}^{12}C_{10} = {}^{12}C_2 = \frac{12\times11}{2} = 66$. **Final Answer:** 66

Fastest Alternative / Shortcut: Always flip $r > n/2$ to $n - r$.

SOLVED EXAMPLE 5 [Intermediate]

Question: Find n if ${}^nC_8 = {}^nC_2$, and hence find ${}^nC_2$.

Given & Required: ${}^nC_8 = {}^nC_2$. | **Concept & Formula:** ${}^nC_a = {}^nC_b \implies n = a + b$.

Detailed Solution: $n = 8 + 2 = 10$. Thus ${}^{10}C_2 = \frac{10\times9}{2} = 45$. **Final Answer:** $n = 10$, ${}^{10}C_2 = 45$

Fastest Alternative / Shortcut: Add subscripts $8 + 2 = 10$.

SOLVED EXAMPLE 6 [Intermediate]

Question: How many 4-digit numbers are there with no digit repeated?

Given & Required: Digits 0 to 9, 4 places. | **Concept & Formula:** Leading digit $\neq 0$.

Detailed Solution: Thousands place: 9 choices (1–9). Remaining 3 places from remaining 9 digits: ${}^9P_3 = 9 \times 8 \times 7 = 504$. Total $= 9 \times 504 = 4536$. **Final Answer:** 4536

Fastest Alternative / Shortcut: $9 \times 9 \times 8 \times 7 = 4536$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find the number of arrangements of the letters of the word INDEPENDENCE.

Given & Required: 12 letters: 3 N's, 4 E's, 2 D's. | **Concept & Formula:** Permutations with repeats.

Detailed Solution: $\frac{12!}{3!\cdot4!\cdot2!} = \frac{479001600}{6\times24\times2} = 1663200$. **Final Answer:** $1,663,200$

Fastest Alternative / Shortcut: Count every letter carefully and sum counts to 12.

SOLVED EXAMPLE 8 [Intermediate]

Question: How many chords can be drawn through 21 points on a circle?

Given & Required: 21 points, 2 per chord. | **Concept & Formula:** ${}^{21}C_2$.

Detailed Solution: ${}^{21}C_2 = \frac{21\times20}{2} = 210$. **Final Answer:** 210

Fastest Alternative / Shortcut: No 3 points on a circle are collinear.

SOLVED EXAMPLE 9 [Board Standard]

Question: Find r if $5 \cdot {}^4P_r = 6 \cdot {}^5P_{r-1}$.

Given & Required: $5 \cdot \frac{4!}{(4-r)!} = 6 \cdot \frac{5!}{(6-r)!}$. | **Concept & Formula:** Factorial equation.

Detailed Solution: $\frac{5!}{(4-r)!} = \frac{6\cdot5!}{(6-r)(5-r)(4-r)!} \implies (6-r)(5-r) = 6 \implies r^2 - 11r + 24 = 0 \implies r = 3$ (since $r \leq 4$, $r = 8$ rejected). **Final Answer:** $r = 3$

Fastest Alternative / Shortcut: Test $r = 1, 2, 3, 4$.

SOLVED EXAMPLE 10 [Board Standard]

Question: In how many ways can the letters of DAUGHTER be arranged so that all vowels always come together?

Given & Required: 8 distinct letters: 3 vowels (A,U,E), 5 consonants. | **Concept & Formula:** String method.

Detailed Solution: Treat (AUE) as 1 unit. Units $= 5 + 1 = 6$, arranged in $6! = 720$ ways. Vowels arrange internally in $3! = 6$ ways. Total $= 720 \times 6 = 4320$. **Final Answer:** 4320

Fastest Alternative / Shortcut: $6! \times 3! = 4320$.

SOLVED EXAMPLE 11 [Board Standard]

Question: How many words, with or without meaning, can be made from MONDAY (assuming no repetition) if 4 letters are used at a time, and all vowels are never together? Wait: find how many arrangements of EXAMINATION have no two vowels together? Let's solve: In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?

Given & Required: 5 girls, 3 boys. | **Concept & Formula:** Gap method.

Detailed Solution: Seat 5 girls in $5! = 120$ ways, creating 6 gaps. Seat 3 boys in ${}^{6}P_3 = 120$ ways. Total $= 120 \times 120 = 14400$. **Final Answer:** $14,400$

Fastest Alternative / Shortcut: Never use Total − Together for 3 boys!

SOLVED EXAMPLE 12 [Board Standard]

Question: A committee of 7 has to be formed from 9 boys and 4 girls. In how many ways can this be done when the committee consists of at least 3 girls?

Given & Required: 9B, 4G, size 7, $G \geq 3$. | **Concept & Formula:** Case enumeration.

Detailed Solution: Case 1 (3G, 4B): ${}^{4}C_3 \cdot {}^{9}C_4 = 4 \times 126 = 504$. Case 2 (4G, 3B): ${}^{4}C_4 \cdot {}^{9}C_3 = 1 \times 84 = 84$. Total $= 504 + 84 = 588$. **Final Answer:** 588

Fastest Alternative / Shortcut: Only 2 cases: 3G or 4G.

SOLVED EXAMPLE 13 [Board Standard]

Question: From a class of 25 students, 10 are to be chosen for an excursion. There are 3 students who decide that either all of them will join or none of them will join. In how many ways can the party be chosen?

Given & Required: 25 students, choose 10; group of 3 all-in or all-out. | **Concept & Formula:** Mutually exclusive cases.

Detailed Solution: Case 1 (all 3 join): choose 7 from remaining 22 $= {}^{22}C_7$. Case 2 (none join): choose 10 from 22 $= {}^{22}C_{10}$. Total $= {}^{22}C_7 + {}^{22}C_{10}$. **Final Answer:** ${}^{22}C_7 + {}^{22}C_{10}$

Fastest Alternative / Shortcut: Two clean cases.

SOLVED EXAMPLE 14 [Challenge]

Question: If the letters of the word MOTHER are written in all possible orders as in a dictionary, find the rank of MOTHER.

Given & Required: Alphabetical order: E, H, M, O, R, T. | **Concept & Formula:** Dictionary rank.

Detailed Solution: Words starting with E $(5! = 120)$, H $(5! = 120)$. M fixed; next E $(4! = 24)$, H $(4! = 24)$. MO fixed; E $(3! = 6)$, H $(3! = 6)$, R $(3! = 6)$. MOT fixed; E $(2! = 2)$, H $(2! = 2)$, MOTHER (1). Rank $= 240 + 48 + 18 + 2 + 1$? Wait: after MOT, remaining are E, H, R: MOTE (2!), MOTHE (1!) $\implies$ MOTHER is next! Wait: MOT-E.. = 2, MOT-H-E-R = 1st of MOTH! So $240 + 48 + 18 + 2 + 1 = 309$. **Final Answer:** 309

Fastest Alternative / Shortcut: $2(120) + 2(24) + 3(6) + 1(2) + 1 = 309$.

SOLVED EXAMPLE 15 [Challenge]

Question: Determine the number of 5-card combinations out of a deck of 52 cards if each selection has exactly one King.

Given & Required: 4 Kings, 48 non-Kings. | **Concept & Formula:** Select 1 King and 4 others.

Detailed Solution: ${}^{4}C_1 \times {}^{48}C_4 = 4 \times 194580 = 778320$. **Final Answer:** $778,320$

Fastest Alternative / Shortcut: $4 \times {}^{48}C_4$.

SOLVED EXAMPLE 16 [Challenge]

Question: How many diagonals does a convex decagon ($n = 10$) have?

Given & Required: $n = 10$ vertices. | **Concept & Formula:** ${}^{n}C_2 - n$.

Detailed Solution: ${}^{10}C_2 - 10 = 45 - 10 = 35$. **Final Answer:** 35

Fastest Alternative / Shortcut: $\frac{10 \times 7}{2} = 35$.

G. Competency MCQ Mastery Section

Q1. If ${}^{n}C_{12} = {}^{n}C_8$, then n equals:

(A) 20 (B) 12 (C) 8 (D) 4

Answer: (A) $n = 12 + 8 = 20$. *Fast Method:* Sum of lower indices. *Distractor Note:* Subtracting $12 - 8 = 4$.

Q2. Number of 3-digit numbers with all distinct digits is:

(A) 720 (B) 648 (C) 900 (D) 504

Answer: (B) $9 \times 9 \times 8 = 648$. *Fast Method:* First digit 1–9 (9 ways). *Distractor Note:* $10 \times 9 \times 8 = 720$ allows leading 0.

Q3. Value of ${}^{8}C_3 + {}^{8}C_2$ is:

(A) ${}^{8}C_5$ (B) ${}^{9}C_3$ (C) ${}^{9}C_2$ (D) ${}^{16}C_5$

Answer: (B) Pascal's identity: ${}^{8}C_3 + {}^{8}C_2 = {}^{9}C_3$. *Fast Method:* Increment n, keep larger r. *Distractor Note:* Adding $8 + 8$.

Q4. Number of arrangements of MISSISSIPPI is:

(A) $\frac{11!}{4!4!2!}$ (B) $\frac{11!}{4!4!}$ (C) $11!$ (D) $\frac{11!}{4!2!}$

Answer: (A) 11 letters: 4 I's, 4 S's, 2 P's. *Fast Method:* Divide by repeats. *Distractor Note:* Missing $2!$ for P's.

Q5. A polygon has 44 diagonals. Number of sides is:

(A) 10 (B) 11 (C) 12 (D) 9

Answer: (B) $\frac{n(n-3)}{2} = 44 \implies n(n-3) = 88 \implies n = 11$. *Fast Method:* $11 \times 8 = 88$. *Distractor Note:* Solving $n(n-1) = 88$.

Q6. If ${}^{n}P_4 = 20 \cdot {}^{n}P_2$, then n is:

(A) 5 (B) 6 (C) 7 (D) 8

Answer: (C) $(n-2)(n-3) = 20 \implies n - 2 = 5 \implies n = 7$. *Fast Method:* $5 \times 4 = 20$. *Distractor Note:* Arithmetic slip.

Q7. In how many ways can 4 prizes be given to 5 boys if a boy can get any number of prizes?

(A) 4^5 (B) 5^4 (C) ${}^{5}P_4$ (D) ${}^{5}C_4$

Answer: (B) Each of 4 prizes has 5 choices: $5^4 = 625$. *Fast Method:* Recipient$^{\text{item}}$. *Distractor Note:* Writing 4^5 instead of 5^4.

Q8. Number of triangles formed by 10 points of which 4 are collinear is:

(A) 120 (B) 116 (C) 110 (D) 96

Answer: (B) ${}^{10}C_3 - {}^{4}C_3 = 120 - 4 = 116$. *Fast Method:* Subtract collinear triples. *Distractor Note:* Forgetting to subtract ${}^{4}C_3$.

Q9. If ${}^{n}C_r$ is maximum for $n = 10$, then r is:

(A) 4 (B) 5 (C) 6 (D) 10

Answer: (B) Max at $r = n/2 = 5$. *Fast Method:* Middle binomial coefficient. *Distractor Note:* Picking $n = 10$.

Q10. Ways to answer 5 True/False questions is:

(A) 10 (B) 25 (C) 32 (D) 120

Answer: (C) $2^5 = 32$. *Fast Method:* 2 choices per question. *Distractor Note:* Multiplying $5 \times 2 = 10$.

Q11. How many ways can 6 persons sit around a circular table?

(A) 720 (B) 120 (C) 60 (D) 36

Answer: (B) $(6-1)! = 5! = 120$. *Fast Method:* $(n-1)!$. *Distractor Note:* $6! = 720$ is linear.

Q12. Value of $0! + 1! + 2! + 3!$ is:

(A) 8 (B) 9 (C) 10 (D) 6

Answer: (C) $1 + 1 + 2 + 6 = 10$. *Fast Method:* $0! = 1$. *Distractor Note:* Taking $0! = 0$ gives 9.

H. Special Exam & MCQ Strategies

- Decide FIRST whether order matters: arrangement/rank/PIN = Permutation (${}^{n}P_r$); committee/handshakes/subset = Combination (${}^{n}C_r$).
- In 'at least' committee problems, never use ${}^{n}C_k \times {}^{N-k}C_{r-k}$ because it overcounts! Always split into disjoint exact cases.

- In a^b distribution problems (prizes to boys, letters to postboxes), remember: (Number of Choices)$^{\text{Number of Items}}$.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Using Total − All-Together for 'no 3 vowels together'. $\longrightarrow$ **Correction:** That only excludes all 3 together, leaving 2-together cases! Use the Gap Method.

[COMMON MISTAKE] **Error:** Overcounting by selecting 1 girl first (${}^4C_1$) and 2 more from everyone (${}^{12}C_2$). $\longrightarrow$ **Correction:** Split into disjoint cases: (1G, 2B), (2G, 1B), (3G, 0B).

[COMMON MISTAKE] **Error:** Writing $0! = 0$ or 4^5 instead of 5^4. $\longrightarrow$ **Correction:** $0! = 1$, and each independent item chooses among available recipients.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $n! = n(n-1)!$, $0! = 1$; ${}^nP_r = \frac{n!}{(n-r)!}$; ${}^nC_r = \frac{n!}{r!(n-r)!}$; Repeats: $\frac{n!}{p_1!p_2!\cdots}$.

[QUICK REVISION] Key Theorems & Results: ${}^nC_r = {}^nC_{n-r}$; ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$; ${}^nP_r = r! \cdot {}^nC_r$.

[QUICK REVISION] Shortcut Summary: Most-restricted slot first; Gap method for 'never together'; Diagonals $= \frac{n(n-3)}{2}$.

[QUICK REVISION] High-Frequency Exam Patterns: Digit formation, Word arrangements, ${}^nC_r$ equations, 'At least' committees.

Last-Minute Revision Checklist:

- ☐ Order matters (P) vs selection (C)
- ☐ Leading digit $\neq 0$
- ☐ Gap method for 3+ never together
- ☐ Disjoint cases for 'at least'

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Value of ${}^7P_2$ is: (A) 21 (B) 42 (C) 14 (D) 49
2. Value of ${}^9C_7$ is: (A) 36 (B) 72 (C) 18 (D) 63
3. Number of 2-digit numbers from $\{1, 2, 3, 4\}$ with repetition: (A) 12 (B) 16 (C) 24 (D) 8
4. Ways to arrange 4 books on a shelf: (A) 16 (B) 24 (C) 12 (D) 4

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. If ${}^nC_6 = {}^nC_4$, then ${}^nC_2$ is: (A) 10 (B) 45 (C) 90 (D) 24
2. Arrangements of APPLE are: (A) 120 (B) 60 (C) 30 (D) 24
3. 3-digit even numbers from $\{1, 2, 3, 4, 5\}$ without repetition: (A) 24 (B) 36 (C) 48 (D) 60
4. Ways to select 3 boys and 2 girls from 5 boys and 4 girls: (A) 16 (B) 60 (C) 120 (D) 40

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Lines from 12 points where 5 are collinear: (A) 66 (B) 56 (C) 57 (D) 61
2. Ways 4 letters can be posted in 3 letter boxes: (A) 12 (B) 64 (C) 81 (D) 24
3. Sum of all 3-digit numbers formed by $1, 2, 3$ without repetition: (A) 1222 (B) 1332 (C) 666 (D) 1444
4. Rank of word SMALL in dictionary: (A) 58 (B) 59 (C) 60 (D) 57

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $7 \times 6 = 42$. **Q2 (A):** ${}^9C_2 = 9 \times 8/2 = 36$. **Q3 (B):** $4 \times 4 = 16$. **Q4 (B):** $4! = 24$.
Board Std	**Q1 (B):** $n = 10$, ${}^{10}C_2 = 45$. **Q2 (B):** $5!/2! = 60$. **Q3 (A):** Units 2 ways $(2, 4)$, remaining $4 \times 3 = 12$; $2 \times 12 = 24$. **Q4 (B):** ${}^5C_3 \times {}^4C_2 = 10 \times 6 = 60$.
Challenge	**Q1 (C):** ${}^{12}C_2 - {}^5C_2 + 1 = 66 - 10 + 1 = 57$. **Q2 (C):** $3^4 = 81$. **Q3 (B):** $2!(1 + 2 + 3) \times 111 = 12 \times 111 = 1332$. **Q4 (A):** Letters A,L,L,M,S. Before S: A($4!/2! = 12$), L($4! = 24$), M($4!/2! = 12$)=48. SA($3!/2! = 3$), SL($3! = 6$)=9. Next is SMALL (+1) = 58.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: FPC & $n!$. Day 2: ${}^nP_r$ & identical objects. Day 3: String/Gap methods. Day 4: ${}^nC_r$ & committee selections.
- **How to Introduce the Concept:** Use a 3-dial bike lock to demonstrate $10 \times 10 \times 10 = 1000$ codes in 10 seconds.
- **Difficult Areas & Remediation:** Overcounting in 'at least' committees and m^n vs n^m distribution.
- **Blackboard Teaching Examples:** Draw physical slots/boxes for digits and gap-slots $\square\, C \,\square\, C \,\square\, C \,\square$ for 'never together'.
- **Classroom Questions to Ask Students:** Why is $4!/2!$ correct for APPLE, and why does Total $-$ Together fail for 3 vowels?
- **Common Student Doubts & Answers:** Students confuse when to multiply vs add; use 'Step 1 AND Step 2' vs 'Case 1 OR Case 2'.
- **Homework & Revision Strategy:** Assign 5 digit problems, 4 word arrangements, and 4 committee problems.

[TEACHER'S TIP] Emphasize that every probability question in Chapter 14 is just two Chapter 6 counting problems divided!

UNIT II: ALGEBRA TYBA COACHING CENTRE

Chapter 7: Binomial Theorem

Positive Integral Expansion $(a+b)^n$, Pascal's Triangle & Applications (2026–27)

A. Chapter Introduction & Examination Relevance

Expanding $(a+b)^2$ or $(a+b)^3$ is easy, but what about $(x+2y)^6$ or $(1.01)^{10000}$? The Binomial Theorem gives a closed-form expansion for $(a+b)^n$ for any positive integer n using combinations ${}^nC_r$ from Chapter 6.

- **What You Will Learn:** Binomial Theorem for positive integral index n, Pascal's triangle, alternating signs in $(a-b)^n$, $(1+x)^n$ expansion, sum of coefficients 2^n, and divisibility / numerical comparison proofs.
- **Why This Chapter Matters:** Binomial expansions appear in Limits (Ch. 12), Probability (binomial distribution in Class 12), and numerical approximations across physics and finance.
- **Real-Life Applications:** Compound interest approximations, error propagation in physics experiments, algebraic divisibility proofs, and polynomial series.
- **Cross-Chapter Connections:** Uses ${}^nC_r$ directly from Chapter 6 and connects to Chapter 12 ($\lim_{x\to a}\frac{x^n-a^n}{x-a}$) and Set Theory ($2^n$ subsets in Ch. 1).
- **Board & Competitive Relevance:** Unit II carries 25/80 marks. Chapter 7 contributes 3–5 marks via direct expansions, $(a+b)^n+(a-b)^n$ cancellations, and divisibility/inequality proofs. (General term and middle term exercises are excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Binomial Theorem for Positive Integral Index

For any positive integer n and real/complex a, b: $(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r}b^r = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \cdots + {}^nC_nb^n$. Notice that the expansion has $n+1$ terms, and in every term the sum of exponents of a and b equals n.

[CONCEPT MASTER – 1. Binomial Theorem for Positive Integral Index] Total number of terms in $(a+b)^n$ is always $n+1$, and coefficients from both ends are equal since ${}^nC_r = {}^nC_{n-r}$.

2. Pascal's Triangle & Alternating Sign $(a-b)^n$

The coefficients ${}^nC_r$ form **Pascal's Triangle**, where each interior entry is the sum of the two entries directly above it (${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$). Replacing b with $-b$ gives $(a-b)^n = \sum_{r=0}^{n}(-1)^r {}^nC_r a^{n-r}b^r$, so signs alternate $+,-,+,-$.

[CONCEPT MASTER – 2. Pascal's Triangle & Alternating Sign $(a-b)^n$] Adding $(a+b)^n+(a-b)^n$ cancels all odd-r terms, leaving $2[{}^nC_0a^n + {}^nC_2a^{n-2}b^2 + \cdots]$.

3. Standard $(1+x)^n$ & Coefficient Sums

Setting $a=1, b=x$ gives $(1+x)^n = {}^nC_0+{}^nC_1x+{}^nC_2x^2+\cdots+{}^nC_nx^n$. Putting $x=1$ gives ${}^nC_0+{}^nC_1+\cdots+{}^nC_n = 2^n$. Putting $x=-1$ gives ${}^nC_0 - {}^nC_1 + {}^nC_2 - \cdots = 0$, so sum of even-indexed coefficients equals sum of odd-indexed coefficients $= 2^{n-1}$.

[CONCEPT MASTER – 3. Standard $(1+x)^n$ & Coefficient Sums] To find the sum of coefficients of ANY polynomial in x, y, simply set every variable equal to 1!

4. Numerical Approximations, Comparisons & Divisibility

To evaluate $(99)^5$, write $(100-1)^5$ and expand. To compare $(1.01)^{1000000}$ vs 10000, expand $(1+0.01)^{1000000} > 1+1000000(0.01) = 10001 > 10000$. To prove divisibility like $9^{n+1}-8n-9$ is divisible by 64, write $9^{n+1} = (1+8)^{n+1}$ and expand.

[CONCEPT MASTER – 4. Numerical Approximations, Comparisons & Divisibility] Rationalised 2026–27 Scope: Standalone General Term (T_{r+1}) and Middle Term exercises are excluded from the 2026–27 NCERT syllabus.

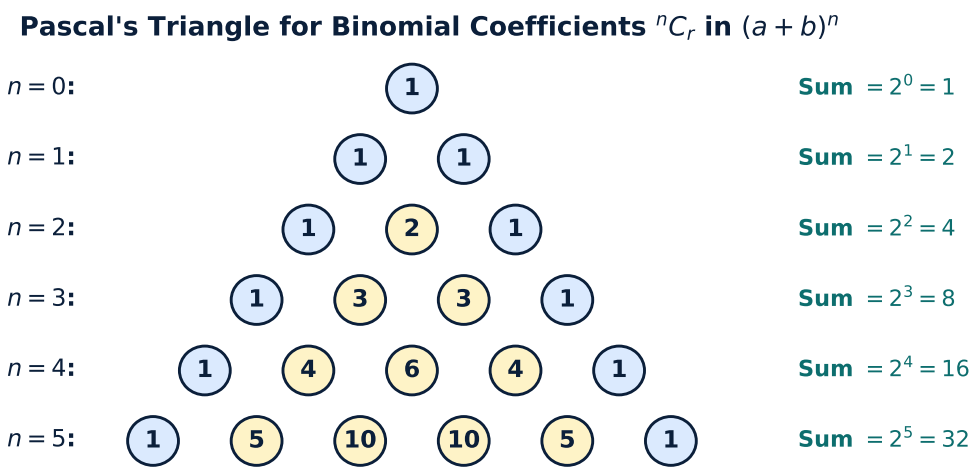


Figure 1: **Visual Model:** Pascal's Triangle for rows $n = 0$ to $n = 5$ showing symmetric binomial coefficients ${}^nC_r$ and row sums 2^n.

C. Complete Formula Bank

[FORMULA BANK – 1. Binomial Expansion] **Expression:** $(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r$
Symbols: $n \in \mathbb{N}$, $a, b \in \mathbb{R}$ or $\mathbb{C}$.
Conditions & When to Use: Positive integer n.
Solved Mini-Example: $(x+2)^3 = x^3 + 6x^2 + 12x + 8$.
Common Mistake: Forgetting to raise the coefficient 2 to power r.

[FORMULA BANK – 2. Alternating Binomial Expansion] **Expression:** $(a-b)^n = \sum_{r=0}^{n} (-1)^r {}^nC_r a^{n-r} b^r$
Symbols: $r = 0, 1, \ldots, n$.
Conditions & When to Use: Signs alternate $+, -, +, -$.
Solved Mini-Example: $(x-1)^4 = x^4 - 4x^3 + 6x^2 - 4x + 1$.
Common Mistake: Applying minus sign to even powers of b.

[FORMULA BANK – 3. Sum & Difference $(a+b)^n \pm (a-b)^n$] **Expression:** $(a+b)^n + (a-b)^n = 2\sum_{\text{even } r} {}^nC_r a^{n-r} b^r$; $(a+b)^n - (a-b)^n = 2\sum_{\text{odd } r} {}^nC_r a^{n-r} b^r$
Symbols: $n \in \mathbb{N}$.
Conditions & When to Use: Halves the number of terms.
Solved Mini-Example: $(\sqrt{2}+1)^4 + (\sqrt{2}-1)^4 = 2(4 + 6(2) + 1) = 34$.
Common Mistake: Forgetting the factor of 2 outside.

[FORMULA BANK – 4. Sum of Binomial Coefficients] **Expression:** ${}^nC_0 + {}^nC_1 + \cdots + {}^nC_n = 2^n$
Symbols: $n \in \mathbb{N}$.
Conditions & When to Use: Set $x = 1$ in $(1+x)^n$.
Solved Mini-Example: ${}^5C_0 + {}^5C_1 + \cdots + {}^5C_5 = 32$.
Common Mistake: Writing 2^{n-1} for full sum.

[FORMULA BANK – 5. Even & Odd Coefficient Sums] **Expression:** ${}^nC_0 + {}^nC_2 + \cdots = {}^nC_1 + {}^nC_3 + \cdots = 2^{n-1}$
Symbols: $n \geq 1$.
Conditions & When to Use: Set $x = -1$ in $(1+x)^n$.
Solved Mini-Example: ${}^6C_0 + {}^6C_2 + {}^6C_4 + {}^6C_6 = 2^5 = 32$.
Common Mistake: Dividing 2^n by n instead of 2.

[FORMULA BANK – 6. Binomial Inequality ($x > 0$)] **Expression:** $(1+x)^n > 1 + nx$ for $n \geq 2, x > 0$
Symbols: $x > 0, n \geq 2$.
Conditions & When to Use: All remaining terms ${}^nC_2 x^2 + \cdots > 0$.
Solved Mini-Example: $(1.01)^{1000} > 1 + 1000(0.01) = 11$.
Common Mistake: Using equality instead of $>$ when $n \geq 2$.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and JEE/CUET Foundation problem structures.

Pattern 1: Direct Expansion of $(ax \pm by)^n$ or $(\frac{x}{a} \pm \frac{a}{x})^n$

- **Identification Cue:** Expand $(2x-3)^6$ or $(\frac{x}{3} + \frac{1}{x})^5$. | **Concept & Formula:** Binomial formula with $n+1$ terms.
- **Standard Step-by-Step:** Write ${}^nC_0$ to ${}^nC_n$, apply powers to BOTH numbers and variables, and simplify.
- **Fastest Valid Method:** Use Pascal's row (e.g. $1, 5, 10, 10, 5, 1$ for $n = 5$) directly. | **Alternative:** Group signs separately.
- **Common Trap:** Writing $2x^3$ instead of $(2x)^3 = 8x^3$.
- **Practice Drill:** Ex. 1 & Ex. 5.

Pattern 2: Numerical Evaluation via $(10^k \pm d)^n$

- **Identification Cue:** Evaluate $(98)^5$ or $(102)^5$ using Binomial Theorem. | **Concept & Formula:** Split into round base 100 ± 2.
- **Standard Step-by-Step:** Expand $(100-2)^5$ and sum positive and negative terms separately.
- **Fastest Valid Method:** Group $+$ terms and $-$ terms cleanly before subtracting. | **Alternative:** Check last digit (8^5 ends in 8) to catch arithmetic slips.
- **Common Trap:** Place-value alignment errors when adding powers of 100.
- **Practice Drill:** Ex. 3 & Ex. 6.

Pattern 3: Simplifying $(a+b)^n \pm (a-b)^n$ with Surds

- **Identification Cue:** Evaluate $(\sqrt{3}+\sqrt{2})^4 - (\sqrt{3}-\sqrt{2})^4$. | **Concept & Formula:** Cancellation of half the terms.
- **Standard Step-by-Step:** Keep only odd-r terms (for minus) or even-r terms (for plus) and double the sum.
- **Fastest Valid Method:** Substitute $a = \sqrt{3}, b = \sqrt{2}$ symbolically first, plug numbers at the end. | **Alternative:** Expand both (slower).
- **Common Trap:** Forgetting the factor of 2 or keeping even-r terms for subtraction.
- **Practice Drill:** Ex. 8 & Ex. 9.

Pattern 4: Divisibility Proofs via $(1+d)^n$

- **Identification Cue:** Show that $9^{n+1} - 8n - 9$ is divisible by 64. | **Concept & Formula:** Expand $(1+8)^{n+1}$.
- **Standard Step-by-Step:** Write $9^{n+1} = (1+8)^{n+1} = 1 + 8(n+1) + 8^2[\cdots]$, cancel $1 + 8n + 8 = 8n + 9$, factor out 64.
- **Fastest Valid Method:** Always split base as $(1 + \sqrt{\text{divisor}})$ or $(1 + \text{factor})$. | **Alternative:** Mathematical induction.
- **Common Trap:** Forgetting that 9^{n+1} has $n+1$ in the exponent, not n.
- **Practice Drill:** Ex. 11 & Ex. 14.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Sum of Coefficients = Set Variables to 1] **Core Idea & Why It Works:** In any polynomial $(ax+by)^n$, setting $x = 1, y = 1$ immediately gives the sum of all numerical coefficients.
Valid Domain: All polynomials and binomial expansions. | **Where It Fails:** If negative powers like $1/x$ exist, it still gives sum of coefficients of all terms!
Easy Example: Sum of coefficients of $(2x-3y)^5$ is $(2-3)^5 = -1$.
Board-Level Example: Sum of coefficients of $(1+3x-2x^2)^{10}$ is $(1+3-2)^{10} = 2^{10} = 1024$.
Advanced Example: Sum of even-power coefficients $= \frac{f(1)+f(-1)}{2}$.

[SMART SHORTCUT – 2. Number of Terms in $(a+b+c)^n$] **Core Idea & Why It Works:** While $(a+b)^n$ has $n+1$ terms, trinomial $(a+b+c)^n$ has $\frac{(n+1)(n+2)}{2} = {}^{n+2}C_2$ terms.
Valid Domain: Distinct variable terms. | **Where It Fails:** If terms collapse (like $1+x+x^2$), count by degree 0 to $2n$ ($2n+1$ terms).
Easy Example: $(x+y+z)^4$ has ${}^6C_2 = 15$ terms.
Board-Level Example: $(1+2x+x^2)^5 = (1+x)^{10}$ has 11 terms!
Advanced Example: In general, $(x_1+\cdots+x_k)^n$ has ${}^{n+k-1}C_{k-1}$ terms.

[SMART SHORTCUT – 3. Instant $(1+x)^n$ First Two Terms for Comparisons] **Core Idea & Why It Works:** For $x>0, n\geq 2$, $(1+x)^n = 1+nx+$ positive terms $> 1+nx$.
Valid Domain: All $x>0, n\geq 2$. | **Where It Fails:** Negative x.
Easy Example: $(1.02)^{100} > 1+2 = 3$.
Board-Level Example: Compare $(1.01)^{1000000}$ vs 10000: $1+10000 = 10001 > 10000$.
Advanced Example: Use 3 terms $1+nx+\frac{n(n-1)}{2}x^2$ if 2 terms aren't enough.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Expand $(1-2x)^5$.
Given & Required: $a=1, b=-2x, n=5$. | **Concept & Formula:** Pascal row $1,5,10,10,5,1$.
Detailed Solution: $1-5(2x)+10(2x)^2-10(2x)^3+5(2x)^4-(2x)^5 = 1-10x+40x^2-80x^3+80x^4-32x^5$. **Final Answer:** $1-10x+40x^2-80x^3+80x^4-32x^5$
Fastest Alternative / Shortcut: Check sum of coefficients: $(1-2)^5 = -1$; $1-10+40-80+80-32 = -1$.

SOLVED EXAMPLE 2 [Beginner]
Question: How many terms are in the expansion of $(3x-5y)^{12}$?
Given & Required: $n=12$. | **Concept & Formula:** $n+1$ terms.
Detailed Solution: $12+1 = 13$ terms. **Final Answer:** 13
Fastest Alternative / Shortcut: Always $n+1$.

SOLVED EXAMPLE 3 [Beginner]
Question: Evaluate $(101)^4$ using the Binomial Theorem.
Given & Required: $(100+1)^4$. | **Concept & Formula:** Pascal row $1,4,6,4,1$.
Detailed Solution: $100^4+4(100)^3+6(100)^2+4(100)+1 = 100000000+4000000+60000+400+1 = 104060401$. **Final Answer:** $104,060,401$
Fastest Alternative / Shortcut: Blocks of 2 digits: $1|04|06|04|01$!

SOLVED EXAMPLE 4 [Beginner]
Question: Find the sum of all binomial coefficients for $n=8$.
Given & Required: $n=8$. | **Concept & Formula:** $\sum {}^nC_r = 2^n$.
Detailed Solution: $2^8 = 256$. **Final Answer:** 256
Fastest Alternative / Shortcut: $2^8 = 256$.

SOLVED EXAMPLE 5 [Intermediate]

Question: Expand $\left(\frac{x}{3}+\frac{1}{x}\right)^5$.

Given & Required: $a=x/3, b=1/x, n=5$. | **Concept & Formula:** Binomial expansion.

Detailed Solution: $\frac{x^5}{243}+5\frac{x^4}{81}\cdot\frac{1}{x}+10\frac{x^3}{27}\cdot\frac{1}{x^2}+10\frac{x^2}{9}\cdot\frac{1}{x^3}+5\frac{x}{3}\cdot\frac{1}{x^4}+\frac{1}{x^5}=\frac{x^5}{243}+\frac{5x^3}{81}+\frac{10x}{27}+\frac{10}{9x}+\frac{5}{3x^3}+\frac{1}{x^5}$. **Final Answer:** $\frac{x^5}{243}+\frac{5x^3}{81}+\frac{10x}{27}+\frac{10}{9x}+\frac{5}{3x^3}+\frac{1}{x^5}$

Fastest Alternative / Shortcut: Exponents drop by 2 each step.

SOLVED EXAMPLE 6 [Intermediate]

Question: Evaluate $(99)^5$ using Binomial Theorem.

Given & Required: $(100-1)^5$. | **Concept & Formula:** Alternating expansion.

Detailed Solution: $10^{10}-5(10^8)+10(10^6)-10(10^4)+5(10^2)-1=10010000500-500100001=9509900499$. **Final Answer:** $9,509,900,499$

Fastest Alternative / Shortcut: Group positive and negative terms.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find $(a+b)^4-(a-b)^4$. Hence evaluate $(\sqrt{3}+\sqrt{2})^4-(\sqrt{3}-\sqrt{2})^4$.

Given & Required: $n=4$. | **Concept & Formula:** Odd-index cancellation.

Detailed Solution: $(a+b)^4-(a-b)^4=2(4a^3b+4ab^3)=8ab(a^2+b^2)$. For $a=\sqrt{3}, b=\sqrt{2}$: $8\sqrt{6}(3+2)=40\sqrt{6}$. **Final Answer:** $8ab(a^2+b^2)$; $40\sqrt{6}$

Fastest Alternative / Shortcut: Factor $8ab(a^2+b^2)$ cleanly.

SOLVED EXAMPLE 8 [Intermediate]

Question: Find $(x+1)^6+(x-1)^6$. Hence evaluate $(\sqrt{2}+1)^6+(\sqrt{2}-1)^6$.

Given & Required: $n=6$. | **Concept & Formula:** Even-index doubling.

Detailed Solution: $2(x^6+15x^4+15x^2+1)$. At $x=\sqrt{2}$: $2(8+60+30+1)=198$. **Final Answer:** $2(x^6+15x^4+15x^2+1)$; 198

Fastest Alternative / Shortcut: $2\times 99=198$.

SOLVED EXAMPLE 9 [Board Standard]

Question: Using Binomial Theorem, indicate which number is larger: $(1.1)^{10000}$ or 1000.

Given & Required: $(1+0.1)^{10000}$. | **Concept & Formula:** First two terms lower bound.

Detailed Solution: $(1+0.1)^{10000}=1+10000(0.1)+\text{positive terms}=1+1000+\text{positive terms}>1000$. **Final Answer:** $(1.1)^{10000}$ **is larger**

Fastest Alternative / Shortcut: $10000\times 0.1=1000$, plus 1 and more.

SOLVED EXAMPLE 10 [Board Standard]

Question: Show that $9^{n+1}-8n-9$ is divisible by 64, whenever n is a positive integer.

Given & Required: $9^{n+1}=(1+8)^{n+1}$. | **Concept & Formula:** Binomial divisibility.

Detailed Solution: $(1+8)^{n+1}=1+(n+1)8+{}^{n+1}C_2 8^2+\cdots+8^{n+1}=9+8n+64k$. Subtracting $8n+9$ leaves $64k$, divisible by 64. **Final Answer: Proved**

Fastest Alternative / Shortcut: Test $n=1$: $81-8-9=64$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Prove that $\sum_{r=0}^{n}3^r\cdot{}^nC_r=4^n$.

Given & Required: $(1+x)^n$ at $x=3$. | **Concept & Formula:** Specializing $x=3$.

Detailed Solution: $(1+3)^n={}^nC_0+{}^nC_1(3)+\cdots+{}^nC_n(3^n)=\sum_{r=0}^{n}3^r{}^nC_r=4^n$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Recognize $a=1, b=3$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Find a if the coefficients of x^2 and x^3 in the expansion of $(3+ax)^9$ are equal.

Given & Required: $(3+ax)^9$. | **Concept & Formula:** Equating two terms.

Detailed Solution: Coeff of x^2 is ${}^9C_2 3^7 a^2$. Coeff of x^3 is ${}^9C_3 3^6 a^3$. Equating ($a \neq 0$): $36(3) = 84a \implies a = \frac{108}{84} = \frac{9}{7}$. **Final Answer:** $a = \frac{9}{7}$

Fastest Alternative / Shortcut: $\frac{{}^9C_2}{{}^9C_3} = \frac{3}{7}$, times 3 gives $9/7$.

SOLVED EXAMPLE 13 [Board Standard]

Question: Expand $(1+x+x^2)^3$ using Binomial Theorem.

Given & Required: Trinomial. | **Concept & Formula:** Group $(1+x)+x^2$.

Detailed Solution: $(1+x)^3 + 3(1+x)^2x^2 + 3(1+x)x^4 + x^6 = 1+3x+6x^2+7x^3+6x^4+3x^5+x^6$. **Final Answer:** $1+3x+6x^2+7x^3+6x^4+3x^5+x^6$

Fastest Alternative / Shortcut: Sum of coeffs: $3^3 = 27 = 1+3+6+7+6+3+1$.

SOLVED EXAMPLE 14 [Challenge]

Question: Find the remainder when 5^{99} is divided by 13.

Given & Required: $5^{99} = 5 \cdot (25)^{49}$. | **Concept & Formula:** $25 = 26 - 1$.

Detailed Solution: $5 \cdot (26-1)^{49} = 5(26k-1) = 13(10k) - 5 = 13(10k-1) + 8$. Remainder is 8. **Final Answer:** 8

Fastest Alternative / Shortcut: $25 \equiv -1 \pmod{13}$.

SOLVED EXAMPLE 15 [Challenge]

Question: Find the sum of coefficients in the expansion of $(1+2x-3x^2)^{20}$.

Given & Required: Polynomial power. | **Concept & Formula:** Set $x = 1$.

Detailed Solution: $(1+2-3)^{20} = 0^{20} = 0$. **Final Answer:** 0

Fastest Alternative / Shortcut: Plug $x = 1$ immediately.

SOLVED EXAMPLE 16 [Challenge]

Question: If $(1+x)^n = C_0 + C_1x + \cdots + C_nx^n$, evaluate $C_1 + 2C_2 + 3C_3 + \cdots + nC_n$.

Given & Required: Weighted binomial sum. | **Concept & Formula:** $r \cdot {}^nC_r = n \cdot {}^{n-1}C_{r-1}$.

Detailed Solution: $\sum_{r=1}^{n} n \cdot {}^{n-1}C_{r-1} = n \cdot 2^{n-1}$. **Final Answer:** $n \cdot 2^{n-1}$

Fastest Alternative / Shortcut: Test $n = 2$: $C_1 + 2C_2 = 2 + 2(1) = 4 = 2 \cdot 2^1$.

G. Competency MCQ Mastery Section

Q1. Number of terms in $(2x+3y)^9$ is:

(A) 8 (B) 9 (C) 10 (D) 11

Answer: (C) $n+1 = 9+1 = 10$. *Fast Method:* Always $n+1$. *Distractor Note:* Picking $n = 9$.

Q2. Sum of all coefficients in $(3x-2y)^7$ is:

(A) 1 (B) -1 (C) 5^7 (D) 2^7

Answer: (A) Set $x = 1, y = 1$: $(3-2)^7 = 1$. *Fast Method:* Plug $x = y = 1$. *Distractor Note:* Using 2^7 blindly.

Q3. Value of ${}^{10}C_0 + {}^{10}C_2 + {}^{10}C_4 + \cdots + {}^{10}C_{10}$ is:

(A) 2^{10} (B) 2^9 (C) 2^8 (D) 2^{11}

Answer: (B) Even-index sum $= 2^{n-1} = 2^9$. *Fast Method:* Half of 2^{10}. *Distractor Note:* Picking 2^{10}.

Q4. Number of terms in $(a+b)^5 + (a-b)^5$ is:

(A) 6 (B) 3 (C) 4 (D) 5

Answer: (B) Even $r = 0, 2, 4$ survive: 3 terms. *Fast Method:* $\frac{5+1}{2} = 3$. *Distractor Note:* Saying 6.

Q5. Coefficient of x^3 in $(1 - 2x)^4$ is:

(A) 32 (B) -32 (C) 16 (D) -16

Answer: (B) ${}^4C_3(-2)^3 = 4(-8) = -32$. *Fast Method:* Keep the minus sign. *Distractor Note:* Dropping minus sign.

Q6. $\sum_{r=0}^{n} 2^r {}^nC_r$ equals:

(A) 2^n (B) 3^n (C) 4^n (D) 2^{n+1}

Answer: (B) $(1 + 2)^n = 3^n$. *Fast Method:* Plug $x = 2$ in $(1 + x)^n$. *Distractor Note:* Picking 2^n.

Q7. Which is larger: $(1.01)^{10000}$ or 100?

(A) $(1.01)^{10000}$ (B) 100 (C) Equal (D) Cannot say

Answer: (A) $1 + 10000(0.01) + \cdots = 101 + \cdots > 100$. *Fast Method:* $1 + nx = 101 > 100$. *Distractor Note:* Thinking 1.01 stays small.

Q8. Number of distinct terms in $(x + y + z)^3$ is:

(A) 4 (B) 6 (C) 10 (D) 9

Answer: (C) ${}^{3+2}C_2 = {}^5C_2 = 10$. *Fast Method:* $(n + 1)(n + 2)/2$. *Distractor Note:* Saying $n + 1 = 4$.

Q9. Value of $(\sqrt{2} + 1)^4 + (\sqrt{2} - 1)^4$ is:

(A) 34 (B) 17 (C) 32 (D) 16

Answer: (A) $2(4 + 6(2) + 1) = 34$. *Fast Method:* $2(17) = 34$. *Distractor Note:* Forgetting factor 2.

Q10. For $n = 5$, the largest binomial coefficient ${}^5C_r$ is:

(A) 5 (B) 10 (C) 15 (D) 20

Answer: (B) ${}^5C_2 = {}^5C_3 = 10$. *Fast Method:* Row $1, 5, 10, 10, 5, 1$. *Distractor Note:* Picking 5.

Q11. Remainder when 7^{100} is divided by 6 is:

(A) 0 (B) 1 (C) 5 (D) 4

Answer: (B) $(1 + 6)^{100} = 1 + 6k$. *Fast Method:* $7 = 1 + 6$. *Distractor Note:* Overcomplicating.

Q12. In $(x^2 + 1)^5$, the highest power of x is:

(A) 5 (B) 7 (C) 10 (D) 25

Answer: (C) $(x^2)^5 = x^{10}$. *Fast Method:* Multiply exponents 2×5. *Distractor Note:* Picking 5.

H. Special Exam & MCQ Strategies

- For any expansion $(A + B)^n$, write the Pascal coefficients first, then decreasing powers of A and increasing powers of B, and simplify.
- Always verify an expansion by plugging $x = 1$ (or $x = y = 1$) into both the original expression and your expanded answer.
- In divisibility proofs, express the base as $(1 + k)$ or $(k - 1)$ where k is a factor of the divisor.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Expanding $(2x + 1)^3$ as $2x^3 + 3(2x^2) + 3(2x) + 1$. $\longrightarrow$ **Correction:** Raise the ENTIRE term $(2x)$ to the power: $(2x)^3 = 8x^3$, $(2x)^2 = 4x^2$.

[COMMON MISTAKE] **Error:** Using 2^n for the sum of coefficients of $(2x + 3)^n$. $\longrightarrow$ **Correction:** 2^n is only for $(1 + x)^n$. For $(2x + 3)^n$, set $x = 1$ to get 5^n.

[COMMON MISTAKE] **Error:** Forgetting the factor 2 in $(a+b)^n + (a-b)^n$. $\longrightarrow$ **Correction:** Surviving terms occur TWICE, so always multiply by 2.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r}b^r$; $\sum {}^nC_r = 2^n$; Even/Odd sum $= 2^{n-1}$.

[QUICK REVISION] Key Theorems & Results: $(1+x)^n > 1+nx$ $(x>0, n\geq 2)$; $(a+b)^n + (a-b)^n = 2\sum_{\text{even } r}$.

[QUICK REVISION] Shortcut Summary: Set variables $= 1$ for coefficient sum; $(1+x)^n$ at $x=k$ gives $\sum k^r {}^nC_r = (1+k)^n$.

[QUICK REVISION] High-Frequency Exam Patterns: Direct expansion, $(100 \pm d)^n$ evaluation, Surd cancellation, Divisibility proofs.

Last-Minute Revision Checklist:

- ☐ $n+1$ terms total
- ☐ Apply power to numerical coefficient too
- ☐ Verify with $x=1$
- ☐ Factor 2 in $(a+b)^n \pm (a-b)^n$

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Number of terms in $(x+2y)^7$ is: (A) 6 (B) 7 (C) 8 (D) 9
2. ${}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$ equals: (A) 8 (B) 15 (C) 16 (D) 32
3. Coefficient of x in $(1+3x)^4$ is: (A) 3 (B) 4 (C) 12 (D) 36
4. Third row of Pascal's triangle $(n=3)$ is: (A) $1,2,1$ (B) $1,3,3,1$ (C) $1,4,6,4,1$ (D) $1,3,1$

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. Sum of coefficients of $(1-4x)^5$ is: (A) -243 (B) 243 (C) 32 (D) -32
2. Value of $(102)^3$ using Binomial Theorem is: (A) 1061208 (B) 1061204 (C) 1040604 (D) 1060808
3. Number of terms in $(x+y)^6 - (x-y)^6$ is: (A) 3 (B) 4 (C) 6 (D) 7
4. ${}^8C_1 + {}^8C_3 + {}^8C_5 + {}^8C_7$ equals: (A) 256 (B) 128 (C) 64 (D) 127

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Remainder when 3^{20} is divided by 8 is: (A) 1 (B) 3 (C) 5 (D) 7
2. Number of terms in $(1+2x+x^2)^{10}$ is: (A) 11 (B) 20 (C) 21 (D) 66
3. Value of $\sum_{r=0}^{5}(-1)^r 3^{r\,5}C_r$ is: (A) 32 (B) -32 (C) 1024 (D) 0
4. If $C_0, \ldots, C_n$ are coefficients of $(1+x)^n$, then $\frac{C_1}{C_0} + 2\frac{C_2}{C_1} + \cdots + n\frac{C_n}{C_{n-1}}$ equals: (A) $n(n+1)/2$ (B) n^2 (C) 2^n (D) $n(n-1)/2$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (C):** $7+1=8$. **Q2 (C):** $2^4=16$. **Q3 (C):** ${}^4C_1(3)=12$. **Q4 (B):** ${}^3C_0..{}^3C_3 = 1,3,3,1$.
Board Std	**Q1 (A):** $(1-4)^5 = (-3)^5 = -243$. **Q2 (A):** $10^6 + 3(2)10^4 + 3(4)10^2 + 8 = 1061208$. **Q3 (A):** Odd $r = 1,3,5$ survive: 3 terms. **Q4 (B):** $2^{8-1} = 128$.
Challenge	**Q1 (A):** $3^{20} = 9^{10} = (1+8)^{10} = 1+8k$. **Q2 (C):** $(1+x)^{20}$ has 21 terms. **Q3 (B):** $(1-3)^5 = (-2)^5 = -32$. **Q4 (A):** $r\frac{{}^nC_r}{{}^nC_{r-1}} = n-r+1$, summing $n+(n-1)+\cdots+1 = \frac{n(n+1)}{2}$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Pascal's triangle & $(a+b)^n$. Day 2: $(100 \pm d)^n$ & surd cancellations. Day 3: Divisibility & inequalities.
- **How to Introduce the Concept:** Write $(a+b)^0, (a+b)^1, (a+b)^2, (a+b)^3$ and let students discover Pascal's Triangle addition pattern.
- **Difficult Areas & Remediation:** Applying exponents to numerical coefficients like $(2x)^3$ and setting up divisibility bases.
- **Blackboard Teaching Examples:** Show $(1+x)^n$ with $x=1$ and $x=-1$ side-by-side to derive 2^n and 2^{n-1}.
- **Classroom Questions to Ask Students:** Why does $(a+b)^n$ have $n+1$ terms instead of n?
- **Common Student Doubts & Answers:** Students ask about T_{r+1} general term; inform them it's excluded from 2026–27 boards but useful for JEE.
- **Homework & Revision Strategy:** Assign 4 expansions, 2 numerical evaluations, and 3 divisibility proofs.

[TEACHER'S TIP] Always tell students to check their expansion at $x=1$—it catches 95% of coefficient mistakes in 5 seconds.

UNIT II: ALGEBRA TYBA COACHING CENTRE

Chapter 8: Sequences and Series

Geometric Progressions (G.P.), Sum of n Terms, Infinite G.P., G.M. & $AM \geq GM$ (2026–27)

A. Chapter Introduction & Examination Relevance

From compound interest and radioactive decay to fractal geometry, Geometric Progressions (G.P.) model multiplicative growth and decay. Chapter 8 focuses on sequences, series, G.P. properties, and the fundamental $AM \geq GM$ inequality.

- **What You Will Learn:** Sequence vs Series, n-th term of a G.P. ($a_n = ar^{n-1}$), sum of n terms (S_n), infinite G.P. ($|r| < 1$), Geometric Mean ($G = \sqrt{ab}$), inserting n G.M.s, and the $AM \geq GM$ inequality.
- **Why This Chapter Matters:** G.P. sums and $AM \geq GM$ are among the most frequently tested algebra tools in CBSE, RBSE, JEE, and NDA/CUET exams.
- **Real-Life Applications:** Compound interest, population doubling, bouncing ball total distance, recurring decimals ($0.\overline{36}$), and optimization via $AM \geq GM$.
- **Cross-Chapter Connections:** Builds on Class 10 A.P. and connects to Chapter 7 (binomial sums) and Chapter 12 (limits of r^n as $n \to \infty$).
- **Board & Competitive Relevance:** Unit II carries 25/80 marks. Chapter 8 contributes 4–6 marks via S_n sums like $7 + 77 + 777 + \cdots$, AM-GM quadratic root problems, and G.P. word problems. (A.P. exercises and $\sum n^2, \sum n^3$ are excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Sequences, Series & Geometric Progression (G.P.)

A **sequence** is an ordered list $a_1, a_2, a_3, \ldots$ (function on $\mathbb{N}$), while a **series** is the sum $a_1 + a_2 + \cdots + a_n$. A sequence is a **G.P.** if the ratio $r = \frac{a_{k+1}}{a_k}$ is constant ($a \neq 0, r \neq 0$). Its general term is $a_n = ar^{n-1}$.

[CONCEPT MASTER – 1. Sequences, Series & Geometric Progression (G.P.)] Three consecutive terms in G.P. are best chosen as $\frac{a}{r}, a, ar$ when their PRODUCT is given!

2. Sum of n Terms & Infinite G.P.

For $r \neq 1$, $S_n = \frac{a(r^n-1)}{r-1} = \frac{a(1-r^n)}{1-r}$. If $r = 1$, $S_n = na$. When $|r| < 1$, $r^n \to 0$ as $n \to \infty$, giving the infinite G.P. sum $S_\infty = \frac{a}{1-r}$.

[CONCEPT MASTER – 2. Sum of n Terms & Infinite G.P.] If S_n is given, always recover the n-th term via $a_n = S_n - S_{n-1}$ (for $n \geq 2$) and $a_1 = S_1$.

3. Geometric Mean (G.M.) & Inserting n G.M.s

The **Geometric Mean** of two positive numbers a, b is $G = \sqrt{ab}$. Three positive numbers a, G, b are in G.P. iff $G^2 = ab$. To insert n G.M.s $G_1, G_2, \ldots, G_n$ between a and b, set $b = ar^{n+1}$, so $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ and $G_k = ar^k$.

[CONCEPT MASTER – 3. Geometric Mean (G.M.) & Inserting n G.M.s] Notice the exponent is $\frac{1}{n+1}$, NOT $\frac{1}{n}$, because there are $n + 2$ total terms from a to b.

4. Relationship Between A.M. and G.M. ($AM \geq GM$)

For any two positive real numbers a, b: $A = \frac{a+b}{2}$ and $G = \sqrt{ab}$. Since $A - G = \frac{(\sqrt{a}-\sqrt{b})^2}{2} \geq 0$, we always have $A \geq G$, with equality iff $a = b$. Moreover, the quadratic equation having a, b as roots is $x^2 - 2Ax + G^2 = 0$.

[CONCEPT MASTER – 4. Relationship Between A.M. and G.M. ($AM \geq GM$)] Rationalised 2026–27 Scope: Standalone A.P. exercises and special series $\sum n, \sum n^2, \sum n^3$ are excluded from the 2026–27 NCERT syllabus.

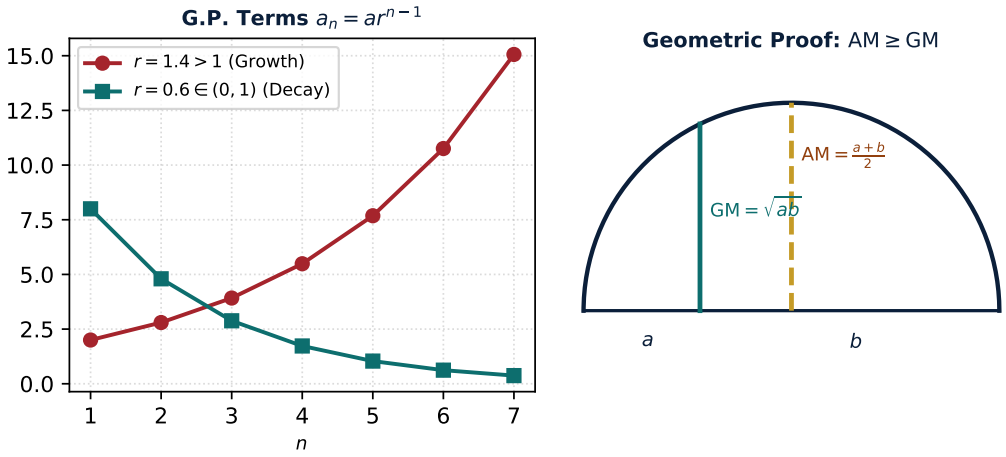


Figure 1: **Visual Model:** Exponential growth ($r > 1$) vs decay ($0 < r < 1$) of a G.P. alongside the geometric proof that AM $\geq$ GM.

C. Complete Formula Bank

[FORMULA BANK – 1. n-th Term of a G.P.] **Expression:** $a_n = ar^{n-1}$
Symbols: a: first term, r: common ratio, n: term position.
Conditions & When to Use: $a \neq 0, r \neq 0$.
Solved Mini-Example: In $3, 6, 12, \ldots$, $a_6 = 3 \cdot 2^5 = 96$.
Common Mistake: Using r^n instead of r^{n-1}.

[FORMULA BANK – 2. Sum of n Terms of a G.P.] **Expression:** $S_n = \frac{a(r^n-1)}{r-1}$ $(r \neq 1)$; $S_n = na$ $(r = 1)$
Symbols: a, r, n.
Conditions & When to Use: Use $\frac{a(1-r^n)}{1-r}$ when $|r| < 1$ for positive denominator.
Solved Mini-Example: $1 + 2 + 4 + \cdots + 2^7 = \frac{1(2^8-1)}{2-1} = 255$.
Common Mistake: Swapping $r^n - 1$ and $1 - r$ signs.

[FORMULA BANK – 3. Infinite G.P. Sum] **Expression:** $S_\infty = \frac{a}{1-r}$
Symbols: a: first term, r: common ratio.
Conditions & When to Use: Converges ONLY when $|r| < 1$ (i.e., $-1 < r < 1$).
Solved Mini-Example: $1 + \frac{1}{3} + \frac{1}{9} + \cdots = \frac{1}{1-1/3} = \frac{3}{2}$.
Common Mistake: Applying $\frac{a}{1-r}$ when $|r| \geq 1$.

[FORMULA BANK – 4. Geometric Mean & n G.M.s] **Expression:** $G = \sqrt{ab}$; $r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$
Symbols: $a, b > 0$.
Conditions & When to Use: Both a, b must have the same sign.
Solved Mini-Example: G.M. of 4 and 9 is $\sqrt{36} = 6$.
Common Mistake: Using $\frac{1}{n}$ instead of $\frac{1}{n+1}$ for r.

[FORMULA BANK – 5. $AM \geq GM$ Inequality] **Expression:** $\frac{a+b}{2} \geq \sqrt{ab}$
Symbols: $a, b > 0$.
Conditions & When to Use: Equality holds iff $a = b$.
Solved Mini-Example: Min value of $x + \frac{9}{x}$ $(x > 0)$ is $2\sqrt{9} = 6$.
Common Mistake: Applying $AM \geq GM$ to negative numbers.

[FORMULA BANK – 6. Quadratic from A.M. & G.M.] **Expression:** $x^2 - 2Ax + G^2 = 0$
Symbols: $A = \frac{a+b}{2}$, $G = \sqrt{ab}$.
Conditions & When to Use: Real roots exist iff $A \geq G$.
Solved Mini-Example: If $A = 5, G = 4$, $x^2 - 10x + 16 = 0 \implies 2, 8$.
Common Mistake: Writing $x^2 - Ax + G^2 = 0$ (missing factor 2).

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and JEE/CUET Foundation problem structures.

Pattern 1: Finding a, r, n from G.P. Term Relations

- **Identification Cue:** Sum of first 3 terms is $13/12$ and product is -1; or 4th term is x, 10th is y. | **Concept & Formula:** Symmetric G.P. choice $\frac{a}{r}, a, ar$ or $a_m = ar^{m-1}$.
- **Standard Step-by-Step:** Use $\frac{a}{r}, a, ar$ to get $a^3 =$ product immediately, then solve quadratic in r.
- **Fastest Valid Method:** Middle term $a = (\text{product})^{1/3}$. | **Alternative:** Use a, ar, ar^2.
- **Common Trap:** Forgetting the reciprocal root $1/r$ gives the reversed valid triple.
- **Practice Drill:** Ex. 5 & Ex. 9.

Pattern 2: Repdigit G.P. Sums $(7 + 77 + 777 + \cdots)$

- **Identification Cue:** Find sum of n terms of $8, 88, 888, \ldots$ or $0.6 + 0.66 + 0.666 + \cdots$. | **Concept & Formula:** Convert to $9, 99, 999 = 10^k - 1$.
- **Standard Step-by-Step:** Factor out the digit d, multiply/divide by 9, rewrite $9 = 10 - 1, 99 = 10^2 - 1$, sum G.P.
- **Fastest Valid Method:** Memorize template: $\frac{d}{9}[\frac{10(10^n-1)}{9} - n]$. | **Alternative:** For decimals $0.d + 0.dd + \cdots$, rewrite $0.9 = 1 - 10^{-1}$.
- **Common Trap:** Writing $1 + 1 + \cdots$ (n times) as 1 instead of n.
- **Practice Drill:** Ex. 10 & Ex. 11.

Pattern 3: Inserting n G.M.s & AM–GM Quadratic

- **Identification Cue:** If A.M. and G.M. of two numbers are 10 and 8, find the numbers. | **Concept & Formula:** $x^2 - 2Ax + G^2 = 0$.
- **Standard Step-by-Step:** Write $a + b = 2A$, $ab = G^2$, solve $(a - b)^2 = 4(A^2 - G^2)$.
- **Fastest Valid Method:** Directly solve $x^2 - 2Ax + G^2 = 0$. | **Alternative:** Test factor pairs of G^2.
- **Common Trap:** Taking $a + b = A$ instead of $2A$.
- **Practice Drill:** Ex. 8 & Ex. 12.

Pattern 4: Proofs with a, b, c, d in G.P.

- **Identification Cue:** If a, b, c, d are in G.P., prove $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) = (ab + bc + cd)^2$. | **Concept & Formula:** Substitute $b = ar, c = ar^2, d = ar^3$.
- **Standard Step-by-Step:** Express all terms via a and r, factor out powers of a and r, compare LHS and RHS.
- **Fastest Valid Method:** Set $a = 1, r = 2$ to check MCQs in 5 seconds. | **Alternative:** Use $b^2 = ac, c^2 = bd, bc = ad$.
- **Common Trap:** Missing common factor $(1 + r^2 + r^4)$.
- **Practice Drill:** Ex. 13.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Repdigit Sum Master Formula] **Core Idea & Why It Works:** $d + dd + ddd + \cdots$ (n terms) $= \frac{d}{81}[10^{n+1} - 9n - 10]$.
Valid Domain: Any single digit $d \in \{1..9\}$. | **Where It Fails:** Decimals (for $0.d + 0.dd + \cdots$, use $\frac{d}{9}[n - \frac{1}{9}(1 - 10^{-n})]$).
Easy Example: $n = 2, d = 7$: $\frac{7}{81}(1000 - 18 - 10) = \frac{7 \times 972}{81} = 84 = 7 + 77$.
Board-Level Example: Instant verification of 4-mark NCERT S_n questions.
Advanced Example: Also checks $5 + 55 + 555$ in MCQs.

[SMART SHORTCUT – 2. $x + \frac{k}{x} \geq 2\sqrt{k}$ $(AM \geq GM)$] **Core Idea & Why It Works:** For $x > 0$ and $k > 0$, $x + \frac{k}{x} \geq 2\sqrt{k}$, with minimum at $x = \sqrt{k}$.
Valid Domain: Positive x, k. | **Where It Fails:** Negative x (then max is $-2\sqrt{k}$).
Easy Example: Min of $x + \frac{16}{x}$ $(x > 0)$ is $2\sqrt{16} = 8$.
Board-Level Example: Min of $3^x + 3^{1-x}$ is $2\sqrt{3^x \cdot 3^{1-x}} = 2\sqrt{3}$.
Advanced Example: Extends to n terms: $\frac{x_1 + \cdots + x_n}{n} \geq (x_1 \cdots x_n)^{1/n}$.

[SMART SHORTCUT – 3. Recurring Decimal via Infinite G.P.] **Core Idea & Why It Works:** Every repeating decimal $0.\overline{ab} = \frac{ab}{99}$, $0.a\overline{b} = \frac{ab-a}{90}$.
Valid Domain: All repeating decimals. | **Where It Fails:** Non-repeating irrationals.
Easy Example: $0.\overline{27} = \frac{27}{99} = \frac{3}{11}$.
Board-Level Example: Write $0.27 + 0.0027 + \cdots = \frac{0.27}{1-0.01}$ for full steps.
Advanced Example: Exact fraction conversion in 2 seconds.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 **[Beginner]**
Question: Find the 10th and n-th terms of the G.P. $5, 25, 125, \ldots$
Given & Required: $a = 5, r = 5$. | **Concept & Formula:** $a_n = ar^{n-1}$.
Detailed Solution: $a_{10} = 5 \cdot 5^9 = 5^{10}$; $a_n = 5 \cdot 5^{n-1} = 5^n$. **Final Answer:** 5^{10} **and** 5^n
Fastest Alternative / Shortcut: Note $a = r = 5$.

SOLVED EXAMPLE 2 **[Beginner]**
Question: Which term of the G.P. $2, 2\sqrt{2}, 4, \ldots$ is 128?
Given & Required: $a = 2, r = \sqrt{2}, a_n = 128$. | **Concept & Formula:** $ar^{n-1} = 128$.
Detailed Solution: $2(\sqrt{2})^{n-1} = 128 \implies 2^{(n-1)/2} = 64 = 2^6 \implies n - 1 = 12 \implies n = 13$. **Final Answer:** 13**th term**
Fastest Alternative / Shortcut: Convert everything to base 2.

SOLVED EXAMPLE 3 **[Beginner]**
Question: Find the sum to n terms of $1, -a, a^2, -a^3, \ldots$ $(a \neq -1)$.
Given & Required: $a_1 = 1, r = -a$. | **Concept & Formula:** $S_n = \frac{a_1(1-r^n)}{1-r}$.
Detailed Solution: $S_n = \frac{1-(-a)^n}{1-(-a)} = \frac{1-(-a)^n}{1+a}$. **Final Answer:** $\frac{1-(-a)^n}{1+a}$
Fastest Alternative / Shortcut: Keep $(-a)^n$ in parentheses.

SOLVED EXAMPLE 4 **[Beginner]**
Question: Find the Geometric Mean of 3 and 27.
Given & Required: $a = 3, b = 27$. | **Concept & Formula:** $G = \sqrt{ab}$.
Detailed Solution: $G = \sqrt{3 \times 27} = \sqrt{81} = 9$. **Final Answer:** 9
Fastest Alternative / Shortcut: $3, 9, 27$ has $r = 3$.

SOLVED EXAMPLE 5 **[Intermediate]**
Question: In a G.P., the 3rd term is 24 and the 6th term is 192. Find the 10th term.
Given & Required: $ar^2 = 24, ar^5 = 192$. | **Concept & Formula:** Divide equations.
Detailed Solution: $r^3 = \frac{192}{24} = 8 \implies r = 2$. Then $a(4) = 24 \implies a = 6$. So $a_{10} = 6 \cdot 2^9 = 3072$. **Final Answer:** 3072
Fastest Alternative / Shortcut: $a_{10} = a_6 \cdot r^4 = 192 \times 16 = 3072$.

SOLVED EXAMPLE 6 [Intermediate]

Question: How many terms of the G.P. $3, 3^2, 3^3, \ldots$ are needed to give the sum 120?

Given & Required: $a = 3, r = 3, S_n = 120$. | **Concept & Formula:** $S_n = \frac{3(3^n-1)}{2}$.

Detailed Solution: $\frac{3(3^n-1)}{2} = 120 \implies 3^n - 1 = 80 \implies 3^n = 81 = 3^4 \implies n = 4$. **Final Answer:** 4 **terms**

Fastest Alternative / Shortcut: $3 + 9 + 27 + 81 = 120$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find the sum of the infinite G.P. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots$

Given & Required: $a = 1, r = 1/2$. | **Concept & Formula:** $S_\infty = \frac{a}{1-r}$.

Detailed Solution: $S_\infty = \frac{1}{1-1/2} = 2$. **Final Answer:** 2

Fastest Alternative / Shortcut: Classic Zeno sum = 2.

SOLVED EXAMPLE 8 [Intermediate]

Question: Insert 3 Geometric Means between 1 and 256.

Given & Required: $a = 1, b = 256, n = 3$. | **Concept & Formula:** $r = (b/a)^{1/(n+1)}$.

Detailed Solution: $r = (256)^{1/4} = \pm 4$. Taking $r = 4$: $4, 16, 64$ (or for $r = -4$: $-4, 16, -64$). **Final Answer:** $4, 16, 64$ **(or** $-4, 16, -64$**)**

Fastest Alternative / Shortcut: Exponent is $1/(3+1) = 1/4$.

SOLVED EXAMPLE 9 [Board Standard]

Question: The sum of first three terms of a G.P. is $\frac{13}{12}$ and their product is -1. Find the common ratio and terms.

Given & Required: $\frac{a}{r} + a + ar = \frac{13}{12}$, $(\frac{a}{r})(a)(ar) = -1$. | **Concept & Formula:** Symmetric G.P. terms.

Detailed Solution: $a^3 = -1 \implies a = -1$. Then $-1(\frac{1}{r} + 1 + r) = \frac{13}{12} \implies 12r^2 + 25r + 12 = 0 \implies (3r+4)(4r+3) = 0 \implies r = -\frac{4}{3}$ or $-\frac{3}{4}$. Terms: $\frac{3}{4}, -1, \frac{4}{3}$. **Final Answer:** $r = -4/3$ **or** $-3/4$**;** $\frac{3}{4}, -1, \frac{4}{3}$

Fastest Alternative / Shortcut: Middle term is $(-1)^{1/3} = -1$.

SOLVED EXAMPLE 10 [Board Standard]

Question: Find the sum of n terms of the sequence $8, 88, 888, 8888, \ldots$

Given & Required: Repdigit series with $d = 8$. | **Concept & Formula:** Rewrite via $10^k - 1$.

Detailed Solution: $S_n = \frac{8}{9}(9 + 99 + 999 + \cdots) = \frac{8}{9}[(10-1) + (10^2-1) + \cdots + (10^n - 1)] = \frac{8}{9}[\frac{10(10^n-1)}{9} - n]$. **Final Answer:** $\frac{80}{81}(10^n - 1) - \frac{8n}{9}$

Fastest Alternative / Shortcut: Verify at $n = 1$: $\frac{80}{81}(9) - \frac{8}{9} = 8$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Find the sum of n terms of $0.6 + 0.66 + 0.666 + \cdots$

Given & Required: Decimal repdigit series. | **Concept & Formula:** Factor $6/9$.

Detailed Solution: $S_n = \frac{6}{9}(0.9 + 0.99 + \cdots) = \frac{2}{3}[(1-0.1) + (1-0.01) + \cdots] = \frac{2}{3}[n - \frac{0.1(1-0.1^n)}{0.9}] = \frac{2}{3}[n - \frac{1}{9}(1 - 10^{-n})]$.
Final Answer: $\frac{2}{3}n - \frac{2}{27}(1 - 10^{-n})$

Fastest Alternative / Shortcut: Check $n = 1$: $2/3 - 2/27(0.9) = 0.6$.

SOLVED EXAMPLE 12 [Board Standard]

Question: If A.M. and G.M. of roots of a quadratic equation are 8 and 5 respectively, obtain the quadratic equation.

Given & Required: $A = 8, G = 5$. | **Concept & Formula:** $x^2 - 2Ax + G^2 = 0$.

Detailed Solution: Sum of roots $= 2A = 16$, product $= G^2 = 25$. Quadratic: $x^2 - 16x + 25 = 0$. **Final Answer:** $x^2 - 16x + 25 = 0$

Fastest Alternative / Shortcut: Direct substitution into $x^2 - 2Ax + G^2 = 0$.

SOLVED EXAMPLE 13 [Board Standard]

Question: If a, b, c, d are in G.P., prove that $(a^2+b^2+c^2)(b^2+c^2+d^2) = (ab+bc+cd)^2$.

Given & Required: $b = ar, c = ar^2, d = ar^3$. | **Concept & Formula:** Parametric G.P. substitution.

Detailed Solution: LHS $= a^2(1+r^2+r^4)\cdot a^2r^2(1+r^2+r^4) = a^4r^2(1+r^2+r^4)^2$. RHS $= [a^2r(1+r^2+r^4)]^2 = a^4r^2(1+r^2+r^4)^2$. Equal! **Final Answer: Proved**

Fastest Alternative / Shortcut: Factor a^2 and a^2r^2 cleanly.

SOLVED EXAMPLE 14 [Challenge]

Question: If A and G be A.M. and G.M. between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.

Given & Required: $x^2 - 2Ax + G^2 = 0$. | **Concept & Formula:** Quadratic formula.

Detailed Solution: $x = \frac{2A \pm \sqrt{4A^2-4G^2}}{2} = A \pm \sqrt{A^2-G^2} = A \pm \sqrt{(A+G)(A-G)}$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Quadratic formula + $A^2 - G^2$ factoring.

SOLVED EXAMPLE 15 [Challenge]

Question: Find the minimum value of $4^x + 4^{1-x}$ for $x \in \mathbb{R}$.

Given & Required: $4^x > 0, 4^{1-x} > 0$. | **Concept & Formula:** $AM \geq GM$.

Detailed Solution: $\frac{4^x+4^{1-x}}{2} \geq \sqrt{4^x \cdot 4^{1-x}} = \sqrt{4} = 2 \implies 4^x + 4^{1-x} \geq 4$. **Final Answer:** 4 **(at $x = 1/2$)**

Fastest Alternative / Shortcut: Exponents cancel in product!

SOLVED EXAMPLE 16 [Challenge]

Question: A ball is dropped from 120 m and rebounds $\frac{2}{3}$ of its previous height on each bounce. Find the total vertical distance traveled before coming to rest.

Given & Required: $h = 120, r = 2/3$. | **Concept & Formula:** Infinite G.P. (up + down).

Detailed Solution: Down: $120 + 80 + \cdots = \frac{120}{1-2/3} = 360$. Up: $80 + \cdots = \frac{80}{1-2/3} = 240$. Total $= 360 + 240 = 600$ m. **Final Answer:** 600 **m**

Fastest Alternative / Shortcut: $h\left(\frac{1+r}{1-r}\right) = 120 \cdot \frac{5/3}{1/3} = 600$ m.

G. Competency MCQ Mastery Section

Q1. The 7th term of the G.P. $3, 6, 12, \ldots$ is:

(A) 96 (B) 192 (C) 384 (D) 64

Answer: (B) $3 \cdot 2^6 = 192$. *Fast Method:* $3 \times 64 = 192$. *Distractor Note:* Using 2^7 instead of 2^6.

Q2. If $x, 2x+2, 3x+3$ are in G.P., then the 4th term is:

(A) -27 (B) 27 (C) -13.5 (D) 13.5

Answer: (C) $(2x+2)^2 = x(3x+3) \implies x^2 + 5x + 4 = 0 \implies x = -4$ (since $x = -1$ gives 0). Terms $-4, -6, -9$ ($r = 3/2$), 4th term $= -13.5$. *Fast Method:* Reject $x = -1$ since r undefined. *Distractor Note:* Accepting $x = -1$.

Q3. Sum of the infinite G.P. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots$ is:

(A) 2 (B) $2/3$ (C) $3/2$ (D) $1/2$

Answer: (B) $\frac{1}{1-(-1/2)} = \frac{2}{3}$. *Fast Method:* $1 - r = 1 + 1/2 = 3/2$. *Distractor Note:* Writing $1 - 1/2$ in denominator.

Q4. If A.M. of two numbers is 13 and G.M. is 12, the numbers are:

(A) 18, 8 (B) 16, 9 (C) 20, 6 (D) 24, 2

Answer: (A) $a + b = 26, ab = 144 \implies 18, 8$. *Fast Method:* Check $18 + 8 = 26$ and $18 \times 8 = 144$. *Distractor Note:* Using $a + b = 13$.

Q5. Minimum value of $x + \frac{1}{x}$ for $x > 0$ is:

(A) 1 (B) 2 (C) 0 (D) 4

Answer: (B) By $AM \geq GM$, $x + 1/x \geq 2\sqrt{1} = 2$. *Fast Method:* Equality at $x = 1$. *Distractor Note:* Picking 1.

Q6. If a, b, c are in G.P., then $\log a, \log b, \log c$ are in:

(A) G.P. (B) A.P. (C) H.P. (D) None

Answer: (B) $b^2 = ac \implies 2\log b = \log a + \log c$. *Fast Method:* Log turns $\times$ into $+$. *Distractor Note:* Saying G.P.

Q7. Sum of $1 + 2 + 4 + \cdots + 512$ is:

(A) 1023 (B) 1024 (C) 511 (D) 2047

Answer: (A) $2^{10} - 1 = 1023$. *Fast Method:* Next power of 2 minus 1. *Distractor Note:* Forgetting -1.

Q8. The G.M. between $\frac{1}{4}$ and 16 is:

(A) 4 (B) 2 (C) 8 (D) 1

Answer: (B) $\sqrt{16/4} = 2$. *Fast Method:* $\sqrt{4} = 2$. *Distractor Note:* Dividing instead of $\sqrt{\cdot}$.

Q9. If n-th term of a sequence is $a_n = \frac{n(n-2)}{n+3}$, then a_{20} is:

(A) $360/23$ (B) $380/23$ (C) $340/23$ (D) $400/23$

Answer: (A) $\frac{20\times 18}{23} = \frac{360}{23}$. *Fast Method:* Direct plug-in. *Distractor Note:* Arithmetic slip.

Q10. If 3rd term of a G.P. is 4, product of first 5 terms is:

(A) 4^3 (B) 4^4 (C) 4^5 (D) 2^5

Answer: (C) $\frac{a}{r^2} \cdot \frac{a}{r} \cdot a \cdot ar \cdot ar^2 = a^5 = 4^5$. *Fast Method:* (Middle term)^5. *Distractor Note:* Picking 4^3.

Q11. Sum of infinite G.P. exists only when:

(A) $r > 1$ (B) $|r| < 1$ (C) $|r| > 1$ (D) $r = 1$

Answer: (B) $r^n \to 0$ iff $|r| < 1$. *Fast Method:* $-1 < r < 1$. *Distractor Note:* Including $r = 1$.

Q12. Value of $0.\overline{45}$ in lowest fraction is:

(A) $9/20$ (B) $5/11$ (C) $45/100$ (D) $4/9$

Answer: (B) $45/99 = 5/11$. *Fast Method:* Divide by 99. *Distractor Note:* Dividing by 100.

H. Special Exam & MCQ Strategies

- Whenever the product of 3 terms in G.P. is given, ALWAYS choose $\frac{a}{r}, a, ar$ so r cancels instantly.
- Remember the Repdigit Sum pipeline: factor d, multiply/divide by 9, convert $99\cdots9$ to $10^k - 1$, then sum the G.P.
- In $AM \geq GM$, always verify all terms are strictly positive before applying $\frac{a+b}{2} \geq \sqrt{ab}$.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Using $r = (b/a)^{1/n}$ when inserting n G.M.s between a and b. $\longrightarrow$ **Correction:** There are $n + 2$ terms total, so $b = ar^{n+1} \implies r = (b/a)^{1/(n+1)}$.

[COMMON MISTAKE] **Error:** Writing $x^2 - Ax + G^2 = 0$ for the quadratic with A.M. A and G.M. G. $\longrightarrow$ **Correction:** Sum of roots is $2A$, so the equation is $x^2 - 2Ax + G^2 = 0$.

[COMMON MISTAKE] **Error:** Applying $S_\infty = \frac{a}{1-r}$ when $r = 2$ or $r = -1$. $\longrightarrow$ **Correction:** $S_\infty = \frac{a}{1-r}$ is valid ONLY when $-1 < r < 1$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $a_n = ar^{n-1}$; $S_n = \frac{a(r^n-1)}{r-1}$; $S_\infty = \frac{a}{1-r}$ ($|r| < 1$); $G = \sqrt{ab}$.

[QUICK REVISION] Key Theorems & Results: $AM \geq GM$: $\frac{a+b}{2} \geq \sqrt{ab}$; Quadratic: $x^2 - 2Ax + G^2 = 0$.

[QUICK REVISION] Shortcut Summary: Product of k symmetric G.P. terms $=$ (middle)^k; $0.\overline{ab} = \frac{ab}{99}$.

[QUICK REVISION] High-Frequency Exam Patterns: Symmetric G.P. product, Repdigit $7 + 77 + \cdots$, n G.M.s, AM–GM optimization.

Last-Minute Revision Checklist:

- ☐ r^{n-1} not r^n
- ☐ $|r| < 1$ for S_∞
- ☐ $1/(n+1)$ exponent for n GMs
- ☐ Factor $2A$ in quadratic

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. 5th term of G.P. $2, 6, 18, \ldots$ is: (A) 54 (B) 162 (C) 486 (D) 108
2. G.M. of 2 and 32 is: (A) 17 (B) 8 (C) 16 (D) 64
3. Infinite sum of $3 + 1 + 1/3 + \cdots$ is: (A) $9/2$ (B) 4 (C) $3/2$ (D) 6
4. Sum of first 4 terms of $1, 3, 9, \ldots$ is: (A) 27 (B) 40 (C) 81 (D) 120

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. If $a_4 = 54$ and $a_7 = 1458$ in a G.P., common ratio r is: (A) 2 (B) 3 (C) 4 (D) 9
2. Insert 2 G.M.s between 3 and 81: (A) $9, 27$ (B) $6, 18$ (C) $9, 36$ (D) $12, 36$
3. If A.M. $= 15$ and G.M. $= 9$, quadratic equation is: (A) $x^2 - 30x + 81 = 0$ (B) $x^2 - 15x + 81 = 0$ (C) $x^2 - 30x + 9 = 0$ (D) $x^2 + 30x + 81 = 0$
4. Value of $5 + 55 + 555$ is: (A) 615 (B) 605 (C) 625 (D) 610

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Minimum value of $9\tan^2\theta + 4\cot^2\theta$ is: (A) 6 (B) 12 (C) 13 (D) 36
2. If $S_\infty = 15$ and sum of squares of terms is 45, first term a is: (A) 5 (B) 10 (C) 3 (D) 6
3. If a, b, c are in G.P. and $a^{1/x} = b^{1/y} = c^{1/z}$, then x, y, z are in: (A) A.P. (B) G.P. (C) H.P. (D) None
4. Sum of $1 + \frac{2}{3} + \frac{4}{9} + \cdots$ to ∞ is: (A) 2 (B) 3 (C) 4 (D) $3/2$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $2 \cdot 3^4 = 162$. **Q2 (B):** $\sqrt{64} = 8$. **Q3 (A):** $\frac{3}{1-1/3} = 9/2$. **Q4 (B):** $1+3+9+27 = 40$.
Board Std	**Q1 (B):** $r^3 = 1458/54 = 27 \implies r = 3$. **Q2 (A):** $r = (81/3)^{1/3} = 3 \implies 9, 27$. **Q3 (A):** $x^2 - 2(15)x + 9^2 = 0$. **Q4 (A):** $5 + 55 + 555 = 615$.
Challenge	**Q1 (B):** $2\sqrt{9 \times 4} = 12$. **Q2 (A):** $\frac{a}{1-r} = 15$, $\frac{a^2}{1-r^2} = 45 \implies \frac{a}{1+r} = 3 \implies 1 + r = 5(1-r) \implies r = 2/3, a = 5$. **Q3 (A):** $k^{2y} = k^{x+z} \implies 2y = x + z$. **Q4 (B):** $\frac{1}{1-2/3} = 3$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: G.P. n-th term & symmetric selection. Day 2: S_n, repdigit sums, S_∞. Day 3: G.M. & $AM \geq GM$.
- **How to Introduce the Concept:** Ask: 'Would you take Rs. 10 lakh today, or 1 paisa doubled every day for 30 days?' (2^{29} paise $>$ Rs. 53 lakh!).
- **Difficult Areas & Remediation:** Repdigit series $0.6 + 0.66 + \cdots$ and distinguishing A vs $2A$ in the quadratic.
- **Blackboard Teaching Examples:** Draw the semicircle geometric proof of $AM \geq GM$ (radius vs altitude).
- **Classroom Questions to Ask Students:** Why does $1 + 2 + 4 + 8 + \cdots$ diverge while $1 + 1/2 + 1/4 + \cdots = 2$?
- **Common Student Doubts & Answers:** Students ask about $\sum n^2$ and $\sum n^3$; clarify they are excluded from 2026–27 NCERT Class 11.
- **Homework & Revision Strategy:** Assign 4 G.P. problems, 2 repdigit sums, and 3 $AM \geq GM$ problems.

[TEACHER'S TIP] Remind students that $AM \geq GM$ is one of the fastest optimization tools in competitive math.

UNIT III: COORDINATE GEOMETRY TYBA COACHING CENTRE

Chapter 9: Straight Lines

Slope, Angle Between Lines, All Standard Line Forms & Distance Formulas (2026–27)

A. Chapter Introduction & Examination Relevance

Coordinate geometry bridges algebra and Euclidean geometry. Chapter 9 turns every straight line into a linear equation $Ax + By + C = 0$ and gives exact algebraic tools for slope, angles, intercepts, and distances.

- **What You Will Learn:** Slope of a line ($m = \tan\theta = \frac{y_2-y_1}{x_2-x_1}$), parallelism ($m_1 = m_2$) and perpendicularity ($m_1 m_2 = -1$), angle between two lines, horizontal/vertical, point-slope, two-point, slope-intercept, intercept forms, and point-to-line / parallel-line distances.
- **Why This Chapter Matters:** Straight lines are the foundation for Conic Sections (Ch. 10), 3D lines (Class 12), and tangents/normals in Calculus.
- **Real-Life Applications:** Linear regression best-fit lines in data science, computer graphics ray-tracing, road/ramp gradients, and economics supply-demand curves.
- **Cross-Chapter Connections:** Uses $\tan\theta$ from Chapter 3 and directly supports Chapter 10 (axes, directrices, tangents) and Chapter 12 (slope of tangent = $f'(x)$).
- **Board & Competitive Relevance:** Unit III carries 12/80 marks. Chapter 9 contributes 4–6 marks via angle-between-lines problems, intercept-form lines, and perpendicular distance. (Normal form and origin shifting are excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Slope (Gradient), Collinearity & Angle Between Lines

If a line makes angle θ ($\theta \neq 90°$) with the positive x-axis, its **slope** is $m = \tan\theta = \frac{y_2-y_1}{x_2-x_1}$. Vertical lines have undefined slope; horizontal lines have $m = 0$. Two non-vertical lines are **parallel** iff $m_1 = m_2$, and **perpendicular** iff $m_1 m_2 = -1$. The acute angle θ between them satisfies $\tan\theta = \left|\frac{m_2-m_1}{1+m_1 m_2}\right|$.

[CONCEPT MASTER – 1. Slope (Gradient), Collinearity & Angle Between Lines] Three points A, B, C are collinear iff slope(AB) = slope(BC)—much faster than the area-of-triangle formula!

2. Standard Equations of a Line

Point-Slope: $y - y_1 = m(x - x_1)$. **Two-Point:** $y - y_1 = \frac{y_2-y_1}{x_2-x_1}(x - x_1)$. **Slope-Intercept:** $y = mx + c$. **Intercept Form:** $\frac{x}{a} + \frac{y}{b} = 1$ (where a, b are x- and y-intercepts). **General Form:** $Ax + By + C = 0$ has slope $m = -\frac{A}{B}$, x-intercept $-\frac{C}{A}$, y-intercept $-\frac{C}{B}$.

[CONCEPT MASTER – 2. Standard Equations of a Line] A line parallel to $Ax + By + C = 0$ is $Ax + By + k = 0$; a line perpendicular to it is $Bx - Ay + k = 0$.

3. Distance of a Point from a Line & Parallel Lines

The perpendicular distance from $P(x_1, y_1)$ to $Ax+By+C = 0$ is $d = \frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$. For two parallel lines $Ax+By+C_1 = 0$ and $Ax + By + C_2 = 0$, the distance between them is $d = \frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$.

[CONCEPT MASTER – 3. Distance of a Point from a Line & Parallel Lines] Before using $\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$, make sure the coefficients A and B of x and y are IDENTICAL in both equations!

4. Foot of Perpendicular & Image (Board Extension)

To find the foot of perpendicular (h, k) from (x_1, y_1) to $Ax + By + C = 0$, use the ratio $\frac{h-x_1}{A} = \frac{k-y_1}{B} = -\frac{Ax_1+By_1+C}{A^2+B^2}$. For the mirror image (h, k), multiply the RHS by 2: $-2\frac{Ax_1+By_1+C}{A^2+B^2}$.

[CONCEPT MASTER – 4. Foot of Perpendicular & Image (Board Extension)] Rationalised 2026–27 Scope: Normal form ($x\cos\omega + y\sin\omega = p$), general equation reduction, and shifting of origin are excluded from the 2026–27 NCERT syllabus.

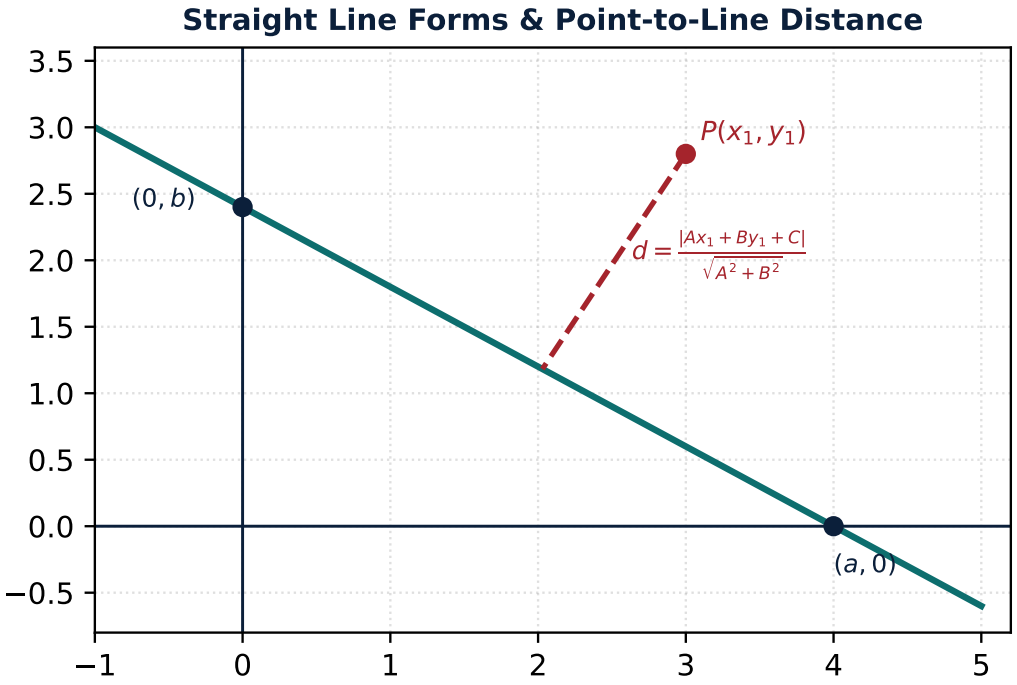


Figure 1: **Visual Model:** Straight line $Ax + By + C = 0$ with intercepts $(a, 0)$ and $(0, b)$ and perpendicular distance d from external point $P(x_1, y_1)$.

C. Complete Formula Bank

[FORMULA BANK – 1. Slope & Two-Point Slope] **Expression:** $m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$
Symbols: $(x_1, y_1), (x_2, y_2)$ with $x_1 \neq x_2$.
Conditions & When to Use: Undefined if $x_1 = x_2$ (vertical line).
Solved Mini-Example: Slope through $(1, 2)$ and $(4, 8)$ is $\frac{6}{3} = 2$.
Common Mistake: Writing $\frac{x_2 - x_1}{y_2 - y_1}$ (run over rise).

[FORMULA BANK – 2. Angle Between Two Lines] **Expression:** $\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$
Symbols: m_1, m_2: slopes; θ: acute angle.
Conditions & When to Use: $1 + m_1 m_2 \neq 0$ (if 0, lines are $\perp$).
Solved Mini-Example: Slopes 1 and 0 give $\tan\theta = 1 \implies \theta = 45°$.
Common Mistake: Forgetting absolute value $|\cdot|$ for acute angle.

[FORMULA BANK – 3. Point-Slope & Slope-Intercept] **Expression:** $y - y_1 = m(x - x_1)$; $y = mx + c$
Symbols: m: slope, c: y-intercept.
Conditions & When to Use: Non-vertical lines.
Solved Mini-Example: Slope 3 through $(1, -2)$: $y + 2 = 3(x - 1) \implies 3x - y - 5 = 0$.
Common Mistake: Sign errors with negative x_1, y_1.

[FORMULA BANK – 4. Intercept Form] **Expression:** $\frac{x}{a}+\frac{y}{b}=1$
Symbols: a: x-intercept, b: y-intercept.
Conditions & When to Use: $a\neq 0, b\neq 0$ (does not pass through origin).
Solved Mini-Example: Intercepts 3 and -4: $\frac{x}{3}-\frac{y}{4}=1 \implies 4x-3y=12$.
Common Mistake: Using intercept form for lines through $(0,0)$.

[FORMULA BANK – 5. Point-to-Line Distance] **Expression:** $d=\frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$
Symbols: Point (x_1,y_1), line $Ax+By+C=0$.
Conditions & When to Use: Equation must be in $=0$ form.
Solved Mini-Example: From $(0,0)$ to $3x+4y-10=0$: $d=\frac{10}{5}=2$.
Common Mistake: Forgetting $|\cdot|$ or $\sqrt{A^2+B^2}$.

[FORMULA BANK – 6. Distance Between Parallel Lines] **Expression:** $d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$
Symbols: $Ax+By+C_1=0$ and $Ax+By+C_2=0$.
Conditions & When to Use: Equal A, B in both lines.
Solved Mini-Example: $3x-4y+5=0$ and $3x-4y-15=0$: $d=\frac{20}{5}=4$.
Common Mistake: Not equalizing A, B first (e.g., $3x+4y=5$ vs $6x+8y=15$).

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and JEE/CUET Foundation problem structures.

Pattern 1: Slope, Collinearity & Angle Between Lines

- **Identification Cue:** Find slope/angle or prove 3 points collinear, or find line making 45° with given line. | **Concept & Formula:** $\tan\theta=\left|\frac{m_2-m_1}{1+m_1m_2}\right|$.
- **Standard Step-by-Step:** Plug known m_1 and $\tan\theta$, drop $|\cdot|$ to get $\pm$, solve for TWO possible slopes m_2.
- **Fastest Valid Method:** Always expect TWO lines at angle θ unless $\theta=90^\circ$. | **Alternative:** Vector/rotation.
- **Common Trap:** Dropping the $\pm$ sign and finding only one line.
- **Practice Drill:** Ex. 5 & Ex. 9.

Pattern 2: Intercept-Form Word Problems

- **Identification Cue:** Line passes through $(2,3)$ and cuts intercepts whose sum is 10 or is bisected at $(2,3)$. | **Concept & Formula:** $\frac{x}{a}+\frac{y}{b}=1$.
- **Standard Step-by-Step:** Use midpoint $(\frac{a}{2},\frac{b}{2})=(x_1,y_1)$ or substitute (x_1,y_1) into $\frac{x}{a}+\frac{y}{b}=1$ with $a+b=k$.
- **Fastest Valid Method:** Bisected at $(p,q) \implies a=2p, b=2q \implies \frac{x}{2p}+\frac{y}{2q}=1$. | **Alternative:** Point-slope form.
- **Common Trap:** Assuming a, b are positive lengths instead of signed intercepts.
- **Practice Drill:** Ex. 7 & Ex. 10.

Pattern 3: Parallel & Perpendicular Line Equations

- **Identification Cue:** Find line perpendicular to $x-7y+5=0$ having x-intercept 3. | **Concept & Formula:** $Bx-Ay+k=0$.
- **Standard Step-by-Step:** Swap A, B and flip middle sign to $7x+y+k=0$, plug $(3,0)$ to find k.
- **Fastest Valid Method:** Direct $Bx-Ay=Bx_1-Ay_1$. | **Alternative:** Find m_1, take $-1/m_1$, use point-slope.
- **Common Trap:** Sign slip when taking negative reciprocal.
- **Practice Drill:** Ex. 6 & Ex. 8.

Pattern 4: Perpendicular Distance & Foot/Image

- **Identification Cue:** Find distance between $15x+8y-34=0$ and $15x+8y+31=0$ or foot of $\perp$. | **Concept & Formula:** $d=\frac{|C_1-C_2|}{\sqrt{A^2+B^2}}$.
- **Standard Step-by-Step:** Normalize A, B if needed, apply distance formula or foot ratio.
- **Fastest Valid Method:** Foot/image 1-line ratio formula. | **Alternative:** Solve intersection of line and perpendicular line.
- **Common Trap:** Using $C_1 - C_2$ when one constant is on the RHS.
- **Practice Drill:** Ex. 11 & Ex. 14.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Instant Parallel & Perpendicular Line Templates] **Core Idea & Why It Works:** Through (x_1, y_1): Parallel to $Ax+By+C=0$ is $A(x-x_1)+B(y-y_1)=0$; Perpendicular is $B(x-x_1)-A(y-y_1)=0$.
Valid Domain: All lines including vertical/horizontal. | **Where It Fails:** Nothing.
Easy Example: $\perp$ to $2x+3y=5$ through $(1,4)$: $3(x-1)-2(y-4)=0 \implies 3x-2y+5=0$.
Board-Level Example: Saves 3 steps of finding m_1 and $-1/m_1$.
Advanced Example: Works even when $B=0$ (vertical line)!

[SMART SHORTCUT – 2. Foot & Image in One Ratio Line] **Core Idea & Why It Works:** For $P(x_1,y_1)$ and $Ax+By+C=0$: $\frac{h-x_1}{A}=\frac{k-y_1}{B}=-\lambda\frac{Ax_1+By_1+C}{A^2+B^2}$, where $\lambda=1$ for Foot and $\lambda=2$ for Image.
Valid Domain: All points and lines. | **Where It Fails:** Nothing.
Easy Example: Image of $(1,2)$ in $x+y-5=0$: $\frac{h-1}{1}=\frac{k-2}{1}=-2(\frac{1+2-5}{2})=2 \implies (3,4)$.
Board-Level Example: Fastest check for NCERT foot-of-perpendicular 5M problems.
Advanced Example: Also reflects across $x=y$ or $ax+by+c=0$.

[SMART SHORTCUT – 3. Midpoint-Bisected Intercept Segment] **Core Idea & Why It Works:** If a segment between the axes is bisected at (p,q), its equation is $\frac{x}{2p}+\frac{y}{2q}=1$.
Valid Domain: $p\neq 0, q\neq 0$. | **Where It Fails:** If $p=0$ or $q=0$.
Easy Example: Bisected at $(3,-4) \implies \frac{x}{6}-\frac{y}{8}=1$.
Board-Level Example: Solves NCERT Ex. 9.2 Q16 in 5 seconds.
Advanced Example: Area of triangle formed with axes $=2|pq|$.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Find the slope of the line passing through $(3,-2)$ and $(-1,4)$.
Given & Required: Two points. | **Concept & Formula:** $m=\frac{y_2-y_1}{x_2-x_1}$.
Detailed Solution: $m=\frac{4-(-2)}{-1-3}=\frac{6}{-4}=-\frac{3}{2}$. **Final Answer:** $-\frac{3}{2}$
Fastest Alternative / Shortcut: Keep order consistent.

SOLVED EXAMPLE 2 [Beginner]
Question: Find the equation of the line passing through $(-4,3)$ with slope $\frac{1}{2}$.
Given & Required: $(-4,3), m=1/2$. | **Concept & Formula:** Point-slope form.
Detailed Solution: $y-3=\frac{1}{2}(x+4) \implies 2y-6=x+4 \implies x-2y+10=0$. **Final Answer:** $x-2y+10=0$
Fastest Alternative / Shortcut: Multiply by 2 immediately.

SOLVED EXAMPLE 3 [Beginner]

Question: Find the equation of the line cutting off x-intercept -3 and y-intercept 2.

Given & Required: $a = -3, b = 2$. | **Concept & Formula:** Intercept form.

Detailed Solution: $\frac{x}{-3} + \frac{y}{2} = 1 \implies -2x + 3y = 6 \implies 2x - 3y + 6 = 0$. **Final Answer:** $2x - 3y + 6 = 0$

Fastest Alternative / Shortcut: Multiply by -6.

SOLVED EXAMPLE 4 [Beginner]

Question: Find the distance of the point $(3, -5)$ from the line $3x - 4y - 26 = 0$.

Given & Required: $(3, -5)$, $3x - 4y - 26 = 0$. | **Concept & Formula:** Point-to-line distance.

Detailed Solution: $d = \frac{|3(3)-4(-5)-26|}{\sqrt{3^2+(-4)^2}} = \frac{|9+20-26|}{5} = \frac{3}{5}$. **Final Answer:** $\frac{3}{5}$

Fastest Alternative / Shortcut: 3-4-5 denominator.

SOLVED EXAMPLE 5 [Intermediate]

Question: Without using Pythagoras theorem, show that $(4, 4), (3, 5), (-1, -1)$ are vertices of a right-angled triangle.

Given & Required: $A(4, 4), B(3, 5), C(-1, -1)$. | **Concept & Formula:** $m_1 m_2 = -1$.

Detailed Solution: $m_{AB} = \frac{5-4}{3-4} = -1$, $m_{AC} = \frac{-1-4}{-1-4} = 1$. Since $m_{AB} \cdot m_{AC} = -1$, $\angle A = 90°$. **Final Answer: Right-angled at** $A(4, 4)$

Fastest Alternative / Shortcut: Compute all 3 slopes.

SOLVED EXAMPLE 6 [Intermediate]

Question: Find the angle between the lines $y - \sqrt{3}x - 5 = 0$ and $\sqrt{3}y - x + 6 = 0$.

Given & Required: $m_1 = \sqrt{3}, m_2 = 1/\sqrt{3}$. | **Concept & Formula:** $\tan\theta = |\frac{m_2 - m_1}{1 + m_1 m_2}|$.

Detailed Solution: $\tan\theta = \left|\frac{1/\sqrt{3} - \sqrt{3}}{1+1}\right| = \frac{2/\sqrt{3}}{2} = \frac{1}{\sqrt{3}} \implies \theta = 30°$ (acute) or $150°$ (obtuse). **Final Answer:** $30°$ **or** $150°$

Fastest Alternative / Shortcut: Slopes are $60°$ and $30°$, difference $= 30°$!

SOLVED EXAMPLE 7 [Intermediate]

Question: Find the equation of the line perpendicular to $x - 7y + 5 = 0$ and having x-intercept 3.

Given & Required: $x - 7y + 5 = 0$, $(3, 0)$. | **Concept & Formula:** Perpendicular template.

Detailed Solution: Perpendicular line: $7x + y + k = 0$. Passes through $(3, 0) \implies 21 + 0 + k = 0 \implies k = -21$. Equation: $7x + y - 21 = 0$. **Final Answer:** $7x + y - 21 = 0$

Fastest Alternative / Shortcut: $7(x - 3) + 1(y - 0) = 0$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Find the distance between parallel lines $3x - 4y + 7 = 0$ and $3x - 4y + 5 = 0$.

Given & Required: $C_1 = 7, C_2 = 5$. | **Concept & Formula:** $\frac{|C_1 - C_2|}{\sqrt{A^2+B^2}}$.

Detailed Solution: $d = \frac{|7-5|}{\sqrt{9+16}} = \frac{2}{5}$. **Final Answer:** $\frac{2}{5}$

Fastest Alternative / Shortcut: A, B already match.

SOLVED EXAMPLE 9 [Board Standard]

Question: If the angle between two lines is $\frac{\pi}{4}$ and slope of one line is $\frac{1}{2}$, find the slope of the other line.

Given & Required: $\theta = 45°, m_1 = 1/2$. | **Concept & Formula:** $|\frac{m - 1/2}{1 + m/2}| = 1$.

Detailed Solution: $\frac{m-1/2}{1+m/2} = \pm 1$. Case $+1$: $m - 1/2 = 1 + m/2 \implies m = 3$. Case -1: $m - 1/2 = -1 - m/2 \implies m = -\frac{1}{3}$.

Final Answer: 3 **or** $-\frac{1}{3}$

Fastest Alternative / Shortcut: Note $3 \times (-1/3) = -1$ (the two lines are $\perp$!).

SOLVED EXAMPLE 10 [Board Standard]

Question: Find the equation of a line passing through $(2,2)$ and cutting off intercepts on the axes whose sum is 9.

Given & Required: $a+b=9$, passes through $(2,2)$. | **Concept & Formula:** Intercept form.

Detailed Solution: $\frac{2}{a}+\frac{2}{9-a}=1 \implies 18=a(9-a) \implies a^2-9a+18=0 \implies a=3$ or 6. Lines: $\frac{x}{3}+\frac{y}{6}=1$ $(2x+y=6)$ and $\frac{x}{6}+\frac{y}{3}=1$ $(x+2y=6)$. **Final Answer:** $2x+y=6$ **and** $x+2y=6$

Fastest Alternative / Shortcut: Symmetric in a,b, so $(3,6)$ and $(6,3)$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Find the coordinates of the foot of perpendicular from $(-1,3)$ to the line $3x-4y-16=0$.

Given & Required: $P(-1,3)$, line $3x-4y-16=0$. | **Concept & Formula:** Foot ratio formula.

Detailed Solution: $\frac{h+1}{3}=\frac{k-3}{-4}=-\frac{3(-1)-4(3)-16}{9+16}=\frac{31}{25}$. Thus $h=\frac{93}{25}-1=\frac{68}{25}$, $k=3-\frac{124}{25}=-\frac{49}{25}$. **Final Answer:** $(\frac{68}{25},-\frac{49}{25})$

Fastest Alternative / Shortcut: Check: $3(68/25)-4(-49/25)=400/25=16$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Find the equation of the right bisector of the line segment joining $(3,4)$ and $(-1,2)$.

Given & Required: $A(3,4), B(-1,2)$. | **Concept & Formula:** Midpoint + perpendicular slope.

Detailed Solution: Midpoint $=(1,3)$. Slope of $AB=\frac{2-4}{-1-3}=\frac{1}{2} \implies m_\perp=-2$. Line: $y-3=-2(x-1) \implies 2x+y-5=0$. **Final Answer:** $2x+y-5=0$

Fastest Alternative / Shortcut: Midpoint $(1,3)$ satisfies $2(1)+3-5=0$.

SOLVED EXAMPLE 13 [Board Standard]

Question: If p is the length of perpendicular from the origin to $\frac{x}{a}+\frac{y}{b}=1$, prove $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$.

Given & Required: $bx+ay-ab=0$, point $(0,0)$. | **Concept & Formula:** Point-to-line distance.

Detailed Solution: $p=\frac{|-ab|}{\sqrt{b^2+a^2}} \implies p^2=\frac{a^2b^2}{a^2+b^2} \implies \frac{1}{p^2}=\frac{a^2+b^2}{a^2b^2}=\frac{1}{a^2}+\frac{1}{b^2}$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Invert p^2 directly.

SOLVED EXAMPLE 14 [Challenge]

Question: Find the equations of lines through $(3,2)$ which make an angle of 45° with the line $x-2y=3$.

Given & Required: $P(3,2), m_1=1/2, \theta=45^\circ$. | **Concept & Formula:** Angle formula + point-slope.

Detailed Solution: From Ex. 9, slopes are $m=3$ and $m=-1/3$. Lines: $y-2=3(x-3) \implies 3x-y-7=0$ and $y-2=-\frac{1}{3}(x-3) \implies x+3y-9=0$. **Final Answer:** $3x-y-7=0$ **and** $x+3y-9=0$

Fastest Alternative / Shortcut: Reuse $m=3,-1/3$.

SOLVED EXAMPLE 15 [Challenge]

Question: Find the distance between the parallel lines $3x+4y=9$ and $6x+8y=15$.

Given & Required: Different scales of A, B. | **Concept & Formula:** Equalize A, B first.

Detailed Solution: Write first line as $6x+8y-18=0$ and second as $6x+8y-15=0$. Then $d=\frac{|-18-(-15)|}{\sqrt{36+64}}=\frac{3}{10}$. **Final Answer:** $\frac{3}{10}$

Fastest Alternative / Shortcut: Never subtract $9-15$ without scaling!

SOLVED EXAMPLE 16 [Challenge]

Question: Find the value of k so that the lines $3x-y-2=0$, $5x+ky-3=0$, and $2x+y-3=0$ are concurrent.

Given & Required: Three lines. | **Concept & Formula:** Intersection of two lies on third.

Detailed Solution: Add 1st and 3rd: $5x-5=0 \implies x=1, y=1$. Plug $(1,1)$ into 2nd: $5+k-3=0 \implies k=-2$. **Final Answer:** $k=-2$

Fastest Alternative / Shortcut: Solve the two known lines first.

G. Competency MCQ Mastery Section

Q1. Slope of a line parallel to the y-axis is:

(A) 0 (B) 1 (C) -1 (D) Undefined

Answer: (D) $\tan 90^\circ$ is undefined. *Fast Method:* Vertical line. *Distractor Note:* Picking 0 (that's x-axis).

Q2. If $(k, 2), (2, 4), (3, 2)$ are collinear, then k is:

(A) 1 (B) 3 (C) 2 (D) 0

Answer: (B) Wait: $y = 2$ at both $(k, 2)$ and $(3, 2)$! For collinearity with $(2, 4)$? If $k = 3$, two points coincide $(3, 2)$ and $(3, 2)$! So $k = 3$. *Fast Method:* If $k = 3$, only 2 distinct points. *Distractor Note:* Dividing by 0.

Q3. Slope of the line $3x + 4y - 7 = 0$ is:

(A) $3/4$ (B) $-3/4$ (C) $4/3$ (D) $-4/3$

Answer: (B) $m = -A/B = -3/4$. *Fast Method:* $-A/B$. *Distractor Note:* Forgetting minus sign.

Q4. Lines $2x + 3y = 5$ and $6x + ky = 10$ are perpendicular if k is:

(A) 9 (B) -4 (C) 4 (D) -9

Answer: (B) $(-2/3)(-6/k) = -1 \implies k = -4$. *Fast Method:* $A_1A_2 + B_1B_2 = 0 \implies 12 + 3k = 0$. *Distractor Note:* Using $k = 9$ (parallel).

Q5. Distance of $(0, 0)$ from $5x - 12y + 26 = 0$ is:

(A) 2 (B) $26/13$ (C) 13 (D) Both A and B

Answer: (A) $26/\sqrt{25 + 144} = 26/13 = 2$. *Fast Method:* 5-12-13 triple. *Distractor Note:* Forgetting square root.

Q6. Intercepts of $4x - 3y = 12$ on the axes are:

(A) $3, 4$ (B) $3, -4$ (C) $-3, 4$ (D) $4, -3$

Answer: (B) $\frac{x}{3} + \frac{y}{-4} = 1$. *Fast Method:* Divide by 12. *Distractor Note:* Dropping minus on y-intercept.

Q7. Acute angle between $y = x$ and $y = \sqrt{3}x$ is:

(A) 15° (B) 30° (C) 45° (D) 60°

Answer: (A) $60^\circ - 45^\circ = 15^\circ$. *Fast Method:* Subtract inclinations. *Distractor Note:* Picking 30°.

Q8. Distance between $x + y = 2$ and $x + y = 6$ is:

(A) 4 (B) $2\sqrt{2}$ (C) $\sqrt{2}$ (D) 2

Answer: (B) $\frac{|6-2|}{\sqrt{1+1}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$. *Fast Method:* Divide 4 by $\sqrt{2}$. *Distractor Note:* Answering 4 directly.

Q9. Area of triangle formed by $\frac{x}{4} + \frac{y}{3} = 1$ with axes is:

(A) 12 (B) 6 (C) 7 (D) 24

Answer: (B) $\frac{1}{2}|ab| = \frac{1}{2}(4)(3) = 6$. *Fast Method:* $\frac{1}{2}ab$. *Distractor Note:* Forgetting $\frac{1}{2}$.

Q10. Equation of line through $(2, -3)$ parallel to x-axis is:

(A) $x = 2$ (B) $y = -3$ (C) $y = 2$ (D) $x = -3$

Answer: (B) Horizontal line has constant $y = -3$. *Fast Method:* Parallel to x-axis $\implies y = k$. *Distractor Note:* Writing $x = 2$.

Q11. If $3x + 4y = 12$, its x-intercept is:

(A) 3 (B) 4 (C) 12 (D) -4

Answer: (B) Set $y = 0 \implies x = 4$. *Fast Method:* $12/3 = 4$. *Distractor Note:* Reading coefficient of x directly.

Q12. Image of $(2, 3)$ across the x-axis is:

(A) $(-2, 3)$ (B) $(2, -3)$ (C) $(-2, -3)$ (D) $(3, 2)$

Answer: (B) Flip y-coordinate. *Fast Method:* x-axis reflection negates y. *Distractor Note:* Negating x.

H. Special Exam & MCQ Strategies

- Always draw a quick rough coordinate sketch showing intercepts and quadrants—it prevents sign errors immediately.
- Remember $A_1A_2 + B_1B_2 = 0$ for perpendicular lines and $A_1/A_2 = B_1/B_2$ for parallel lines.
- To find distance between parallel lines, ALWAYS equalize the x, y coefficients before subtracting constants.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Computing distance between $3x + 4y = 5$ and $6x + 8y = 15$ as $\frac{|15-5|}{10} = 1$. $\longrightarrow$ **Correction:** Equalize A, B first: $6x + 8y = 10$ and $6x + 8y = 15 \implies d = \frac{5}{10} = 0.5$.

[COMMON MISTAKE] **Error:** Finding only ONE slope when solving $\left|\frac{m_2-m_1}{1+m_1m_2}\right| = \tan\theta$. $\longrightarrow$ **Correction:** Absolute value gives $\pm\tan\theta$, producing two lines.

[COMMON MISTAKE] **Error:** Writing slope of $Ax + By + C = 0$ as A/B. $\longrightarrow$ **Correction:** Slope is $-A/B$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $m = \frac{y_2-y_1}{x_2-x_1}$; $y - y_1 = m(x - x_1)$; $\frac{x}{a} + \frac{y}{b} = 1$; $d = \frac{|Ax_1+By_1+C|}{\sqrt{A^2+B^2}}$.

[QUICK REVISION] Key Theorems & Results: Parallel: $m_1 = m_2$; Perpendicular: $m_1 m_2 = -1$; $\tan\theta = |\frac{m_2-m_1}{1+m_1m_2}|$.

[QUICK REVISION] Shortcut Summary: Perp to $Ax + By + C = 0$ is $Bx - Ay + k = 0$; Foot/Image ratio formula; Bisected at $(p, q) \implies \frac{x}{2p} + \frac{y}{2q} = 1$.

[QUICK REVISION] High-Frequency Exam Patterns: Angle between lines, Intercept word problems, Foot of perpendicular, Parallel-line distance.

Last-Minute Revision Checklist:

- ☐ Equalize A, B before parallel distance
- ☐ $\pm$ in angle formula
- ☐ Slope is $-A/B$
- ☐ Collinearity via equal slopes

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Slope of line through $(0, 0)$ and $(2, 6)$: (A) $1/3$ (B) 3 (C) 2 (D) 6
2. Line through $(1, 2)$ parallel to y-axis: (A) $y = 2$ (B) $x = 1$ (C) $x = 2$ (D) $y = 1$
3. y-intercept of $2x - y + 5 = 0$: (A) 5 (B) -5 (C) $5/2$ (D) $-5/2$
4. Slope of line perpendicular to $y = 4x + 1$: (A) 4 (B) -4 (C) $-1/4$ (D) $1/4$

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. Distance of $(1, -2)$ from $3x + 4y - 5 = 0$: (A) 2 (B) $10/5$ (C) 2 (D) Both A and C
2. Line with equal intercepts passing through $(2, 3)$: (A) $x + y = 5$ (B) $x - y = -1$ (C) $2x + 3y = 13$ (D) $x + y = 6$
3. Distance between $5x - 12y + 10 = 0$ and $5x - 12y - 16 = 0$: (A) 2 (B) 1 (C) $6/13$ (D) 26
4. Acute angle between $x + y = 3$ and $x - y = 1$: (A) $45°$ (B) $60°$ (C) $90°$ (D) $30°$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Foot of perpendicular from $(2, 3)$ to $x + y - 1 = 0$: (A) $(0, 1)$ (B) $(1, 0)$ (C) $(-1, 2)$ (D) $(2, -1)$
2. Number of lines through $(1, 2)$ at distance 3 from $(4, 6)$: (A) 0 (B) 1 (C) 2 (D) Infinite
3. If p_1, p_2 are distances from origin to $x\cos\theta - y\sin\theta = k\cos 2\theta$ and $x\sec\theta + y\csc\theta = k$, then $p_1^2 + 4p_2^2$ equals: (A) k^2 (B) $4k^2$ (C) $2k^2$ (D) $k^2/2$
4. Area of parallelogram bounded by $y = x, y = x + 2, y = -x, y = -x + 2$: (A) 1 (B) 2 (C) 4 (D) $2\sqrt{2}$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $6/2 = 3$. **Q2 (B):** Vertical line $x = 1$. **Q3 (A):** $y = 2x + 5 \implies c = 5$. **Q4 (C):** $-1/m = -1/4$.
Board Std	**Q1 (A):** $\frac{\lvert 3-8-5\rvert}{5} = \frac{10}{5} = 2$. **Q2 (A):** $x/a + y/a = 1 \implies a = 2 + 3 = 5$. **Q3 (A):** $\lvert 10 - (-16)\rvert/13 = 26/13 = 2$. **Q4 (C):** $m_1 = -1, m_2 = 1 \implies m_1 m_2 = -1 \implies 90^\circ$.
Challenge	**Q1 (A):** $h - 2 = k - 3 = -(2 + 3 - 1)/2 = -2 \implies (0, 1)$. **Q2 (C):** Distance between points is $5 > 3$, so 2 tangents exist. **Q3 (A):** $p_1 = \lvert k \cos 2\theta\rvert$, $p_2 = \frac{\lvert k\rvert}{\sqrt{\sec^2\theta + \operatorname{csc}^2\theta}} = \frac{\lvert k\rvert \sin 2\theta}{2}$, so $p_1^2 + 4p_2^2 = k^2$. **Q4 (B):** Square of side $2/\sqrt{2} = \sqrt{2}$, area $= 2$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Slope, collinearity, angle between lines. Day 2: All 4 line forms. Day 3: Point-to-line & parallel distances.
- **How to Introduce the Concept:** Connect $m = \tan\theta$ to road-sign incline percentages and wheelchair ramps.
- **Difficult Areas & Remediation:** Two-slope $\pm$ cases in $\tan\theta$ and equalizing A, B in parallel-line distance.
- **Blackboard Teaching Examples:** Show all 4 line forms converting into $Ax + By + C = 0$ in one flowchart.
- **Classroom Questions to Ask Students:** Why does $Ax + By + C = 0$ fail to use $y = mx + c$ when $B = 0$?
- **Common Student Doubts & Answers:** Students ask about Normal Form; confirm it is excluded in 2026–27.
- **Homework & Revision Strategy:** Assign 6 line-equation problems and 4 distance/foot problems.

[TEACHER'S TIP] Emphasize the $Bx - Ay + k = 0$ perpendicular template—it saves time in both Ch. 9 and Ch. 10.

UNIT III: COORDINATE GEOMETRY TYBA COACHING CENTRE

Chapter 10: Conic Sections

Circle, Parabola, Ellipse & Hyperbola: Standard Equations, Foci, Eccentricity & Latus Rectum (2026–27)

A. Chapter Introduction & Examination Relevance

Slicing a double-napped cone with a plane produces four famous curves: Circle, Parabola, Ellipse, and Hyperbola. Chapter 10 studies their standard equations, foci, vertices, eccentricity, and latus rectum.

- **What You Will Learn:** Circle $(x-h)^2+(y-k)^2=r^2$ and general form $x^2+y^2+2gx+2fy+c=0$; Parabola ($y^2=\pm 4ax, x^2=\pm 4ay$); Ellipse ($\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$); Hyperbola ($\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$).
- **Why This Chapter Matters:** Conics are essential for Class 12 Area Under Curves (Application of Integrals) and remain one of the highest-weightage coordinate-geometry chapters in JEE.
- **Real-Life Applications:** Planetary orbits (ellipses), satellite dish & car headlight reflectors (parabolas), telescope mirrors & LORAN navigation (hyperbolas), and gear/wheel design (circles).
- **Cross-Chapter Connections:** Builds directly on Chapter 9 (distance formula, axes) and feeds into Class 12 Chapter 8 (Areas of circles, parabolas, ellipses).
- **Board & Competitive Relevance:** Unit III carries 12/80 marks. Expect 1M/2M MCQs on circle centre/radius or eccentricity, and 3M/5M problems finding focus/vertices/LR of parabola/ellipse/hyperbola or equation of a circle through 3 points.

B. Complete Concept Explanation & Derivations

1. Circle: Standard & General Equation

A **circle** with centre (h,k) and radius r has equation $(x-h)^2+(y-k)^2=r^2$. Expanding gives the general form $x^2+y^2+2gx+2fy+c=0$, whose centre is $(-g,-f)$ and radius is $r=\sqrt{g^2+f^2-c}$ (real circle iff $g^2+f^2-c>0$).

[CONCEPT MASTER – 1. Circle: Standard & General Equation] To get centre $(-g,-f)$ from $x^2+y^2+Dx+Ey+F=0$, simply take HALF the coefficients of x and y and FLIP their signs!

2. Parabola ($e=1$)

A **parabola** is the locus of points equidistant from a fixed point (focus) and a fixed line (directrix). Standard forms with vertex $(0,0)$: $y^2=4ax$ (opens right, focus $(a,0)$, directrix $x=-a$), $y^2=-4ax$ (left), $x^2=4ay$ (upward, focus $(0,a)$, directrix $y=-a$), and $x^2=-4ay$ (downward). Length of **Latus Rectum** $=4a$ in all four.

[CONCEPT MASTER – 2. Parabola ($e=1$)] The linear variable tells you the axis of symmetry: $y^2=4ax$ is symmetric about the x-axis; $x^2=4ay$ is symmetric about the y-axis.

3. Ellipse ($0<e<1$)

An **ellipse** is the locus of points whose sum of distances from two foci is constant $(2a)$. Standard form: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ($a>b$, major axis along x-axis). Here $c=\sqrt{a^2-b^2}$, foci $(\pm c,0)$, vertices $(\pm a,0)$, eccentricity $e=\frac{c}{a}<1$, and Latus Rectum $=\frac{2b^2}{a}$. If the larger denominator is under y^2, the major axis is vertical.

[CONCEPT MASTER – 3. Ellipse ($0<e<1$)] In an Ellipse, a is ALWAYS the larger semi-axis ($a>b$) and $c^2=a^2-b^2$ (MINUS sign).

4. Hyperbola ($e>1$)

A **hyperbola** is the locus of points whose difference of distances from two foci is constant $(2a)$. Standard form: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ (transverse axis along x-axis). Here $c=\sqrt{a^2+b^2}$, foci $(\pm c,0)$, vertices $(\pm a,0)$, eccentricity $e=\frac{c}{a}>1$,

and Latus Rectum $= \frac{2b^2}{a}$. Note that in $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, the transverse axis is the y-axis regardless of whether $a > b$ or $a < b$.

[CONCEPT MASTER – 4. Hyperbola ($e > 1$)] In a Hyperbola, a^2 is ALWAYS under the POSITIVE term, and $c^2 = a^2 + b^2$ (PLUS sign)!

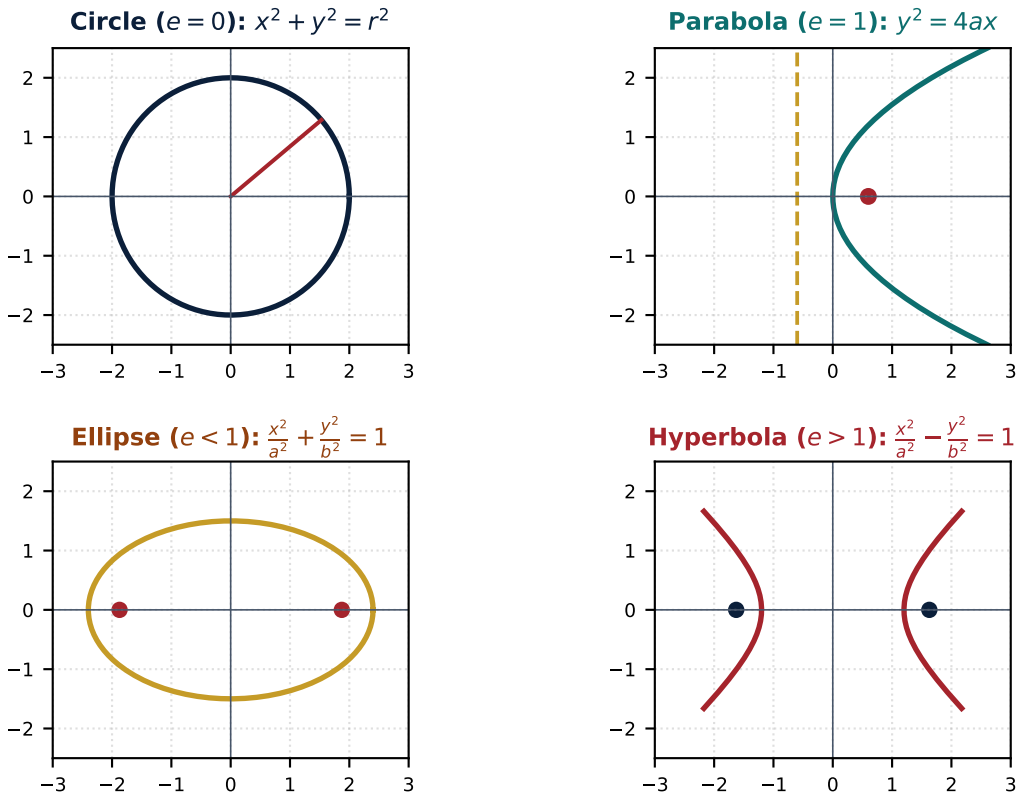


Figure 1: **Visual Model:** Side-by-side comparison of Circle ($e = 0$), Parabola ($e = 1$), Ellipse ($e < 1$), and Hyperbola ($e > 1$) with their foci and directrix.

C. Complete Formula Bank

[FORMULA BANK – 1. Circle General Form] **Expression:** $x^2 + y^2 + 2gx + 2fy + c = 0$; Centre $(-g, -f)$, $r = \sqrt{g^2 + f^2 - c}$
Symbols: $g, f, c \in \mathbb{R}$.
Conditions & When to Use: Coefficients of x^2 and y^2 must be 1 (divide first if $Ax^2 + Ay^2$).
Solved Mini-Example: $x^2 + y^2 - 4x + 6y - 12 = 0 \implies$ Centre $(2, -3)$, $r = \sqrt{4 + 9 + 12} = 5$.
Common Mistake: Forgetting to divide by A when $2x^2 + 2y^2 + \cdots$ is given.

[FORMULA BANK – 2. Parabola Standard Forms] **Expression:** $y^2 = \pm 4ax$ or $x^2 = \pm 4ay$; LR $= 4a$
Symbols: $a > 0$: focal distance from vertex.
Conditions & When to Use: Vertex at $(0, 0)$.
Solved Mini-Example: $y^2 = -12x \implies 4a = 12 \implies a = 3$, focus $(-3, 0)$, directrix $x = 3$.
Common Mistake: Writing $a = -3$ and getting negative LR length.

[FORMULA BANK – 3. Ellipse Standard Form ($a > b$)] **Expression:** $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$; $c^2 = a^2 - b^2$; $e = \frac{c}{a}$; $\text{LR} = \frac{2b^2}{a}$
Symbols: $2a$: major axis, $2b$: minor axis, $2c$: focal distance.
Conditions & When to Use: $0 < e < 1$.
Solved Mini-Example: $\frac{x^2}{25} + \frac{y^2}{9} = 1 \implies a = 5, b = 3, c = 4, e = 4/5, \text{LR} = 18/5$.
Common Mistake: Using $c^2 = a^2 + b^2$ (that's hyperbola!).

[FORMULA BANK – 4. Hyperbola Standard Form] **Expression:** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; $c^2 = a^2 + b^2$; $e = \frac{c}{a}$; $\text{LR} = \frac{2b^2}{a}$
Symbols: $2a$: transverse axis, $2b$: conjugate axis.
Conditions & When to Use: $e > 1$; a^2 is under the $+$ term.
Solved Mini-Example: $\frac{x^2}{9} - \frac{y^2}{16} = 1 \implies a = 3, b = 4, c = 5, e = 5/3, \text{LR} = 32/3$.
Common Mistake: Picking larger denominator as a^2 in a hyperbola.

[FORMULA BANK – 5. Diameter Form of Circle] **Expression:** $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$
Symbols: $(x_1, y_1), (x_2, y_2)$: diameter endpoints.
Conditions & When to Use: Any circle.
Solved Mini-Example: Endpoints $(1, 2)$ and $(3, 4)$: $(x - 1)(x - 3) + (y - 2)(y - 4) = 0$.
Common Mistake: Putting a minus sign between the two products.

[FORMULA BANK – 6. Conic Recognition ($Ax^2 + By^2 + \cdots = 0$)] **Expression:** $A = B \neq 0$: Circle; $AB = 0$: Parabola; $AB > 0, A \neq B$: Ellipse; $AB < 0$: Hyperbola
Symbols: A, B: coefficients of x^2, y^2.
Conditions & When to Use: No xy term.
Solved Mini-Example: $9x^2 - 4y^2 = 36$ has $A = 9, B = -4 \implies AB < 0 \implies$ Hyperbola.
Common Mistake: Confusing $9x^2 + 4y^2 = 36$ (Ellipse) with Circle.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and JEE/CUET Foundation problem structures.

Pattern 1: Circle Centre/Radius & Circle Through 3 Points

- **Identification Cue:** Find centre/radius of $2x^2 + 2y^2 - x = 0$, or circle through $(4, 1), (6, 5)$ with centre on $4x + y = 16$. | **Concept & Formula:** $(x - h)^2 + (y - k)^2 = r^2$.
- **Standard Step-by-Step:** For centre on a line, equate distances from (h, k) to both points and plug (h, k) into the line equation.
- **Fastest Valid Method:** Divide by leading coeff first for general form. | **Alternative:** Perpendicular bisector of chord passes through centre.
- **Common Trap:** Forgetting to divide $2x^2 + 2y^2$ by 2 before reading $-g, -f$.
- **Practice Drill:** Ex. 5 & Ex. 9.

Pattern 2: Parabola Parameters & Equation from Focus/Directrix

- **Identification Cue:** Find focus, directrix, LR of $x^2 = -16y$ or parabola with vertex $(0, 0)$ passing through $(5, 2)$ symmetric about y-axis. | **Concept & Formula:** Match $y^2 = \pm 4ax$ or $x^2 = \pm 4ay$.
- **Standard Step-by-Step:** Identify axis from symmetry/focus/directrix, write standard form, plug point to get $4a$.
- **Fastest Valid Method:** Always keep $a > 0$; handle direction via the $\pm$ sign in the equation. | **Alternative:** Distance to focus = distance to directrix.
- **Common Trap:** Writing $y^2 = 4ax$ when focus is on y-axis.
- **Practice Drill:** Ex. 2 & Ex. 6.

Pattern 3: Ellipse Analysis & Construction

- **Identification Cue:** Find foci, vertices, e, LR of $36x^2+4y^2=144$ or ellipse with vertices $(0,\pm13)$, foci $(0,\pm5)$. | **Concept & Formula:** $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a>b)$ or $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$.
- **Standard Step-by-Step:** Divide RHS to 1, check which denominator is larger to pick horizontal vs vertical major axis, compute $c=\sqrt{a^2-b^2}$.
- **Fastest Valid Method:** Look at $a>b$ first; $b^2=a^2-c^2$. | **Alternative:** Use $c=ae$.
- **Common Trap:** Putting a^2 under x^2 when vertices are on the y-axis.
- **Practice Drill:** Ex. 7 & Ex. 11.

Pattern 4: Hyperbola Analysis & Construction

- **Identification Cue:** Find foci, vertices, e, LR of $16x^2-9y^2=576$ or hyperbola with foci $(\pm3\sqrt{5},0)$, LR $=8$. | **Concept & Formula:** $c^2=a^2+b^2$ and LR $=\frac{2b^2}{a}$.
- **Standard Step-by-Step:** Divide RHS to 1, take a^2 from the positive term, compute $c=\sqrt{a^2+b^2}$, then $e=c/a$.
- **Fastest Valid Method:** Substitute $b^2=c^2-a^2$ into LR $=\frac{2b^2}{a}$ to solve a quadratic in a. | **Alternative:** Use $b^2=a^2(e^2-1)$.
- **Common Trap:** Using $c^2=a^2-b^2$ instead of a^2+b^2.
- **Practice Drill:** Ex. 8 & Ex. 13.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Circle Centre-Radius in 5 Seconds] **Core Idea & Why It Works:** For $Ax^2+Ay^2+Dx+Ey+F=0$, Centre $=\left(-\frac{D}{2A},-\frac{E}{2A}\right)$ and $r=\frac{\sqrt{D^2+E^2-4AF}}{2|A|}$.
Valid Domain: All circles. | **Where It Fails:** If $D^2+E^2-4AF\le0$ (point or imaginary).
Easy Example: $2x^2+2y^2-x=0 \implies$ Centre $(\frac{1}{4},0)$, $r=\frac{1}{4}$.
Board-Level Example: No need to complete the square in MCQs!
Advanced Example: Length of x-intercept $=2\sqrt{g^2-c}$.

[SMART SHORTCUT – 2. Ellipse vs Hyperbola c^2 Memory Hook] **Core Idea & Why It Works:** Ellipse ($+$ in equation) has $-$ in $c^2=a^2-b^2$; Hyperbola ($-$ in equation) has $+$ in $c^2=a^2+b^2$.
Valid Domain: All standard ellipses and hyperbolas. | **Where It Fails:** Nothing—opposite signs every time!
Easy Example: Ellipse $25,16 \implies c=\sqrt{25-16}=3$; Hyperbola $25,16 \implies c=\sqrt{25+16}=\sqrt{41}$.
Board-Level Example: Prevents the #1 formula swap error in Conics.
Advanced Example: Also $e_{\text{ellipse}}<1<e_{\text{hyperbola}}$.

[SMART SHORTCUT – 3. Equilateral / Rectangular Hyperbola] **Core Idea & Why It Works:** When $a=b$ in $\frac{x^2}{a^2}-\frac{y^2}{a^2}=1$, eccentricity is ALWAYS $e=\sqrt{2}$.
Valid Domain: All rectangular hyperbolas ($x^2-y^2=a^2$ or $xy=c^2$). | **Where It Fails:** If $a\neq b$.
Easy Example: $x^2-y^2=16 \implies e=\sqrt{2}$.
Board-Level Example: Instant 1-mark MCQ answer.
Advanced Example: LR $=2a$.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Find the centre and radius of the circle $(x+5)^2+(y-3)^2=36$.
Given & Required: Standard circle. | **Concept & Formula:** $(x-h)^2+(y-k)^2=r^2$.
Detailed Solution: Centre $(-5,3)$, radius $r=\sqrt{36}=6$. **Final Answer: Centre** $(-5,3)$**,** $r=6$
Fastest Alternative / Shortcut: Flip signs inside brackets.

SOLVED EXAMPLE 2 [Beginner]

Question: Find the focus, directrix, and latus rectum of $y^2 = 12x$.

Given & Required: $4a = 12$. | **Concept & Formula:** Right-opening parabola.

Detailed Solution: $a = 3$. Focus $(3, 0)$, directrix $x = -3$, LR $= 4a = 12$. **Final Answer: Focus** $(3, 0)$**,** $x = -3$**, LR** $= 12$

Fastest Alternative / Shortcut: $a = 12/4 = 3$.

SOLVED EXAMPLE 3 [Beginner]

Question: Find the centre and radius of $x^2 + y^2 - 8x + 10y - 12 = 0$.

Given & Required: $2g = -8, 2f = 10, c = -12$. | **Concept & Formula:** General circle.

Detailed Solution: Centre $(4, -5)$, $r = \sqrt{4^2 + (-5)^2 - (-12)} = \sqrt{16 + 25 + 12} = \sqrt{53}$. **Final Answer: Centre** $(4, -5)$**,** $r = \sqrt{53}$

Fastest Alternative / Shortcut: Half and flip signs.

SOLVED EXAMPLE 4 [Beginner]

Question: Find the eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

Given & Required: $a = 4, b = 3$. | **Concept & Formula:** $c = \sqrt{a^2 - b^2}, e = c/a$.

Detailed Solution: $c = \sqrt{16 - 9} = \sqrt{7}$, so $e = \frac{\sqrt{7}}{4}$. **Final Answer:** $\frac{\sqrt{7}}{4}$

Fastest Alternative / Shortcut: $e = \sqrt{1 - b^2/a^2}$.

SOLVED EXAMPLE 5 [Intermediate]

Question: Find the centre and radius of $2x^2 + 2y^2 - x = 0$.

Given & Required: Leading coeff 2. | **Concept & Formula:** Divide by 2 first.

Detailed Solution: $x^2 + y^2 - \frac{1}{2}x = 0$. Here $g = -\frac{1}{4}, f = 0, c = 0$. Centre $(\frac{1}{4}, 0)$, $r = \frac{1}{4}$. **Final Answer: Centre** $(\frac{1}{4}, 0)$**,** $r = \frac{1}{4}$

Fastest Alternative / Shortcut: Divide by 2 before reading g, f.

SOLVED EXAMPLE 6 [Intermediate]

Question: Find the equation of the parabola with focus $(0, -3)$ and directrix $y = 3$.

Given & Required: Focus on $-y$-axis. | **Concept & Formula:** $x^2 = -4ay$.

Detailed Solution: Vertex $(0, 0)$, $a = 3$, opens downward: $x^2 = -12y$. **Final Answer:** $x^2 = -12y$

Fastest Alternative / Shortcut: Focus is at $(0, -a)$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find the foci, vertices, and latus rectum of $9x^2 + 4y^2 = 36$.

Given & Required: $\frac{x^2}{4} + \frac{y^2}{9} = 1$. | **Concept & Formula:** Vertical ellipse $(9 > 4)$.

Detailed Solution: $a = 3, b = 2, c = \sqrt{9 - 4} = \sqrt{5}$. Foci $(0, \pm\sqrt{5})$, vertices $(0, \pm 3)$, LR $= \frac{2b^2}{a} = \frac{8}{3}$. **Final Answer: Foci** $(0, \pm\sqrt{5})$**, Vertices** $(0, \pm 3)$**, LR** $= 8/3$

Fastest Alternative / Shortcut: Larger denominator under $y^2 \implies$ vertical.

SOLVED EXAMPLE 8 [Intermediate]

Question: Find the foci, eccentricity, and latus rectum of $\frac{y^2}{9} - \frac{x^2}{27} = 1$.

Given & Required: Vertical hyperbola. | **Concept & Formula:** $a^2 = 9, b^2 = 27$.

Detailed Solution: $a = 3, b = 3\sqrt{3}, c = \sqrt{9 + 27} = 6$. Foci $(0, \pm 6)$, $e = \frac{6}{3} = 2$, LR $= \frac{2(27)}{3} = 18$. **Final Answer: Foci** $(0, \pm 6)$**,** $e = 2$**, LR** $= 18$

Fastest Alternative / Shortcut: a^2 is under $+y^2$, NOT 27!

SOLVED EXAMPLE 9 [Board Standard]

Question: Find the equation of the circle passing through $(4,1)$ and $(6,5)$ and whose centre lies on $4x+y=16$.

Given & Required: Two points on circle, centre (h,k) on line. | **Concept & Formula:** Equidistant centre.

Detailed Solution: $(h-4)^2+(k-1)^2=(h-6)^2+(k-5)^2 \implies 4h+8k=44 \implies h+2k=11$. Solving with $4h+k=16$ gives $h=3, k=4$. $r^2=(3-4)^2+(4-1)^2=10$. Circle: $(x-3)^2+(y-4)^2=10$. **Final Answer:** $x^2+y^2-6x-8y+15=0$

Fastest Alternative / Shortcut: Solve $h+2k=11$ and $4h+k=16$.

SOLVED EXAMPLE 10 [Board Standard]

Question: Find the equation of the ellipse with vertices $(\pm 13,0)$ and foci $(\pm 5,0)$.

Given & Required: $a=13, c=5$ on x-axis. | **Concept & Formula:** $b^2=a^2-c^2$.

Detailed Solution: $b^2=169-25=144$. Equation: $\frac{x^2}{169}+\frac{y^2}{144}=1$. **Final Answer:** $\frac{x^2}{169}+\frac{y^2}{144}=1$

Fastest Alternative / Shortcut: 5-12-13 triple gives $b=12$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Find the equation of the hyperbola with vertices $(0,\pm 3)$ and foci $(0,\pm 5)$.

Given & Required: Vertical hyperbola: $a=3, c=5$. | **Concept & Formula:** $b^2=c^2-a^2$.

Detailed Solution: $b^2=25-9=16$. Equation: $\frac{y^2}{9}-\frac{x^2}{16}=1$. **Final Answer:** $\frac{y^2}{9}-\frac{x^2}{16}=1$

Fastest Alternative / Shortcut: 3-4-5 triple gives $b=4$.

SOLVED EXAMPLE 12 [Board Standard]

Question: An arch is in the form of a parabola with its axis vertical, 10 m high and 5 m wide at the base. How wide is it 2 m from the vertex?

Given & Required: Downward parabola through $(2.5,-10)$. | **Concept & Formula:** $x^2=-4ay$.

Detailed Solution: $(2.5)^2=-4a(-10) \implies 4a=\frac{6.25}{10}=0.625$. At $y=-2$: $x^2=0.625(2)=1.25 \implies x=\frac{\sqrt{5}}{2}$. Width $=2x=\sqrt{5}\approx 2.24$ m. **Final Answer:** $\sqrt{5}$ **m (≈ 2.24 m)**

Fastest Alternative / Shortcut: $x^2 \propto y \implies (w/5)^2=2/10 \implies w=\sqrt{5}$.

SOLVED EXAMPLE 13 [Board Standard]

Question: A rod AB of length 15 cm rests between coordinate axes with A on x-axis and B on y-axis. Find the locus of point $P(x,y)$ on AB such that $AP=6$ cm.

Given & Required: $AP=6, PB=9$. | **Concept & Formula:** Parametric ellipse.

Detailed Solution: If rod makes angle θ with x-axis, $x=PB\cos\theta=9\cos\theta$ and $y=AP\sin\theta=6\sin\theta$. Squaring and adding: $\frac{x^2}{81}+\frac{y^2}{36}=1$. **Final Answer:** $\frac{x^2}{81}+\frac{y^2}{36}=1$

Fastest Alternative / Shortcut: Wait: $x=9\cos\theta, y=6\sin\theta$ immediately.

SOLVED EXAMPLE 14 [Challenge]

Question: Find the equation of the hyperbola whose foci are $(\pm 3\sqrt{5},0)$ and length of latus rectum is 8.

Given & Required: $c=3\sqrt{5}, \frac{2b^2}{a}=8$. | **Concept & Formula:** $b^2=4a$, $a^2+b^2=45$.

Detailed Solution: $a^2+4a-45=0 \implies (a+9)(a-5)=0 \implies a=5$ $(a>0)$. Then $b^2=20$. Equation: $\frac{x^2}{25}-\frac{y^2}{20}=1$. **Final Answer:** $\frac{x^2}{25}-\frac{y^2}{20}=1$

Fastest Alternative / Shortcut: Solve $a^2+4a=45$.

SOLVED EXAMPLE 15 [Challenge]

Question: Find the area of an equilateral triangle inscribed in the parabola $y^2=4ax$ with one vertex at the origin.

Given & Required: Equilateral $\triangle OAB$, $O(0,0)$. | **Concept & Formula:** Symmetry at 30°.

Detailed Solution: Side OA makes 30° with x-axis: $A=(L\cos 30^\circ, L\sin 30^\circ)$ on $y^2=4ax \implies \frac{L^2}{4}=4a\frac{\sqrt{3}L}{2} \implies L=8\sqrt{3}a$. Area $=\frac{\sqrt{3}}{4}L^2=48\sqrt{3}a^2$. **Final Answer:** $48\sqrt{3}a^2$

Fastest Alternative / Shortcut: Side $L=8\sqrt{3}a$.

SOLVED EXAMPLE 16 [Challenge]

Question: If e_1 and e_2 are eccentricities of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and its conjugate $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, prove $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$.

Given & Required: $e_1^2 = \frac{a^2+b^2}{a^2}$, $e_2^2 = \frac{a^2+b^2}{b^2}$. | **Concept & Formula:** Invert and add.

Detailed Solution: $\frac{1}{e_1^2} + \frac{1}{e_2^2} = \frac{a^2}{a^2+b^2} + \frac{b^2}{a^2+b^2} = 1$. **Final Answer: Proved**

Fastest Alternative / Shortcut: 2-line reciprocal sum.

G. Competency MCQ Mastery Section

Q1. Centre of the circle $x^2 + y^2 + 6x - 8y - 11 = 0$ is:

(A) $(3, -4)$ (B) $(-3, 4)$ (C) $(6, -8)$ (D) $(-6, 8)$

Answer: (B) Half and flip signs: $(-3, 4)$. *Fast Method:* $(-g, -f)$. *Distractor Note:* Forgetting sign flip.

Q2. Focus of the parabola $x^2 = -16y$ is:

(A) $(-4, 0)$ (B) $(4, 0)$ (C) $(0, -4)$ (D) $(0, 4)$

Answer: (C) Downward parabola, $a = 4$, focus $(0, -4)$. *Fast Method:* x^2 means y-axis. *Distractor Note:* Putting -4 in x-coordinate.

Q3. Eccentricity of the ellipse $9x^2 + 25y^2 = 225$ is:

(A) $3/5$ (B) $4/5$ (C) $5/4$ (D) $4/3$

Answer: (B) $a = 5, b = 3, c = 4 \implies e = 4/5$. *Fast Method:* 3-4-5 triangle. *Distractor Note:* Picking $e > 1$ for ellipse.

Q4. Length of latus rectum of $y^2 = 8x$ is:

(A) 2 (B) 4 (C) 8 (D) 16

Answer: (C) LR $= 4a = 8$. *Fast Method:* Read coefficient of x directly. *Distractor Note:* Dividing by 2 or 4.

Q5. For the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$, the foci are:

(A) $(\pm 5, 0)$ (B) $(\pm\sqrt{7}, 0)$ (C) $(0, \pm 5)$ (D) $(\pm 4, 0)$

Answer: (A) $c = \sqrt{16 + 9} = 5$. *Fast Method:* Add denominators for hyperbola. *Distractor Note:* Subtracting to get $\sqrt{7}$.

Q6. Eccentricity of the rectangular hyperbola $x^2 - y^2 = 25$ is:

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) $1/\sqrt{2}$

Answer: (B) $a = b = 5 \implies c = 5\sqrt{2} \implies e = \sqrt{2}$. *Fast Method:* Always $\sqrt{2}$. *Distractor Note:* Picking 1 (parabola).

Q7. Which conic has eccentricity $e = 1$?

(A) Circle (B) Ellipse (C) Parabola (D) Hyperbola

Answer: (C) Circle 0, Ellipse $(0, 1)$, Parabola 1, Hyperbola > 1. *Fast Method:* $e = 1$ defines parabola. *Distractor Note:* Confusing with circle $e = 0$.

Q8. Radius of the circle $3x^2 + 3y^2 - 6x + 12y - 12 = 0$ is:

(A) 3 (B) 4 (C) 9 (D) $\sqrt{17}$

Answer: (A) Divide by 3: $x^2 + y^2 - 2x + 4y - 4 = 0 \implies r = \sqrt{1 + 4 + 4} = 3$. *Fast Method:* Divide by 3 first. *Distractor Note:* Not dividing by 3.

Q9. Directrix of $y^2 = -8x$ is:

(A) $x = -2$ (B) $x = 2$ (C) $y = 2$ (D) $y = -2$

Answer: (B) Opens left ($a = 2$), directrix behind vertex at $x = 2$. *Fast Method:* Opposite side of focus $(-2, 0)$. *Distractor Note:* Writing $x = -2$ (that's focus line).

Q10. Latus rectum of ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$ is:

(A) $16/3$ (B) $8/3$ (C) $32/3$ (D) 6

Answer: (A) $\frac{2b^2}{a} = \frac{2(16)}{6} = \frac{16}{3}$. *Fast Method:* $2b^2/a$. *Distractor Note:* Forgetting factor 2.

Q11. Equation of circle with diameter endpoints $(0,0)$ and $(4,6)$ is:

(A) $x^2+y^2-4x-6y=0$ (B) $x^2+y^2+4x+6y=0$ (C) $x^2+y^2=52$ (D) $x^2+y^2-2x-3y=0$

Answer: (A) $x(x-4)+y(y-6)=0$. *Fast Method:* Centre $(2,3)$ passes through $(0,0)$. *Distractor Note:* Halving coefficients.

Q12. If major axis of an ellipse is 10 and minor axis is 6, distance between foci is:

(A) 4 (B) 8 (C) 6 (D) 10

Answer: (B) $a=5, b=3 \implies c=4$, distance $2c=8$. *Fast Method:* $2c=8$. *Distractor Note:* Giving $c=4$ instead of $2c$.

H. Special Exam & MCQ Strategies

- Always write a 3-line summary before solving any Ellipse/Hyperbola problem: Axis (x or y), $a=\ldots, b=\ldots$, and $c=\ldots$.
- In Circle equations, check the coefficients of x^2 and y^2 first—divide by A if $A \neq 1$.
- For Parabolas, check which variable is squared (y^2 = horizontal axis, x^2 = vertical axis) and the sign ($+$ right/up, $-$ left/down).

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Using $c^2=a^2+b^2$ for Ellipse or $c^2=a^2-b^2$ for Hyperbola. $\longrightarrow$ **Correction:** Ellipse: $c^2=a^2-b^2$; Hyperbola: $c^2=a^2+b^2$.

[COMMON MISTAKE] **Error:** Picking the larger denominator as a^2 in a Hyperbola. $\longrightarrow$ **Correction:** In a Hyperbola, a^2 is ALWAYS under the positive term, even if it is smaller than b^2!

[COMMON MISTAKE] **Error:** Computing centre $(-g,-f)$ before dividing $Ax^2+Ay^2+Dx+Ey+F=0$ by A. $\longrightarrow$ **Correction:** Always normalize x^2+y^2 to coefficient 1 first.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: Circle: $(x-h)^2+(y-k)^2=r^2$; Parabola: $y^2=\pm 4ax, x^2=\pm 4ay$; Ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$; Hyperbola: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

[QUICK REVISION] Key Theorems & Results: Ellipse: $c^2=a^2-b^2, e<1$; Hyperbola: $c^2=a^2+b^2, e>1$; LR $=\frac{2b^2}{a}$ (both).

[QUICK REVISION] Shortcut Summary: Rectangular hyperbola $e=\sqrt{2}$; Circle centre $(-D/2A, -E/2A)$; $2c$ = focal distance.

[QUICK REVISION] High-Frequency Exam Patterns: Circle through 2 points + line, Parabola focus/directrix, Ellipse/Hyperbola parameters, Arch word problems.

Last-Minute Revision Checklist:

- ☐ Normalize x^2+y^2 coeff to 1
- ☐ Ellipse $c^2=a^2-b^2$, Hyperbola $c^2=a^2+b^2$
- ☐ Hyperbola a^2 under $+$ term
- ☐ LR of parabola is $4a$

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Radius of $x^2+y^2-2x+4y-4=0$ is: (A) 2 (B) 3 (C) 4 (D) 9
2. Focus of $y^2=20x$ is: (A) $(5,0)$ (B) $(0,5)$ (C) $(-5,0)$ (D) $(10,0)$
3. Eccentricity of circle is: (A) 1 (B) 0 (C) ∞ (D) $1/2$
4. Latus rectum of $x^2=16y$ is: (A) 4 (B) 8 (C) 16 (D) 32

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. Foci of $\frac{x^2}{169}+\frac{y^2}{144}=1$ are: (A) $(\pm 5,0)$ (B) $(0,\pm 5)$ (C) $(\pm 12,0)$ (D) $(\pm 13,0)$
2. Eccentricity of $9x^2-16y^2=144$ is: (A) $5/4$ (B) $5/3$ (C) $4/5$ (D) $3/5$
3. Parabola with vertex $(0,0)$ passing through $(2,3)$ symmetric to x-axis: (A) $2y^2=9x$ (B) $9y^2=2x$ (C) $x^2=4y/3$ (D) $y^2=9x$
4. Circle concentric with $x^2+y^2-4x-6y=0$ passing through $(0,0)$: (A) $x^2+y^2-4x-6y=0$ (B) $x^2+y^2+4x+6y=0$ (C) $(x-2)^2+(y-3)^2=1$ (D) $x^2+y^2=13$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If LR of an ellipse is half of its minor axis, its eccentricity is: (A) $\sqrt{3}/2$ (B) $1/2$ (C) $1/\sqrt{2}$ (D) $\sqrt{3}/4$
2. Sum of focal distances of any point on $\frac{x^2}{25}+\frac{y^2}{16}=1$ is: (A) 8 (B) 10 (C) 6 (D) 20
3. Circle touching both axes in Q-I with radius 4: (A) $x^2+y^2-8x-8y+16=0$ (B) $x^2+y^2-4x-4y+16=0$ (C) $x^2+y^2-8x-8y=0$ (D) $(x-4)^2+(y-4)^2=4$
4. Difference of focal distances of any point on $\frac{x^2}{9}-\frac{y^2}{16}=1$ is: (A) 6 (B) 8 (C) 10 (D) 3

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $\sqrt{1+4+4}=3$. **Q2 (A):** $4a=20 \implies a=5$. **Q3 (B):** $c=0 \implies e=0$. **Q4 (C):** $4a=16$.
Board Std	**Q1 (A):** $c=\sqrt{169-144}=5$ on x-axis. **Q2 (A):** $a=4, b=3, c=5 \implies e=5/4$. **Q3 (A):** $y^2=4ax \implies 9=8a \implies 2y^2=9x$. **Q4 (A):** $(0,0)$ already lies on it!
Challenge	**Q1 (A):** $\frac{2b^2}{a}=b \implies 2b=a \implies e=\sqrt{1-1/4}=\sqrt{3}/2$. **Q2 (B):** $2a=10$. **Q3 (A):** Centre $(4,4), r=4 \implies x^2+y^2-8x-8y+16=0$. **Q4 (A):** $2a=6$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Circle. Day 2: Parabola. Day 3: Ellipse. Day 4: Hyperbola & comparison table.
- **How to Introduce the Concept:** Use a flashlight beam against a wall at different tilt angles to show circle, ellipse, parabola, and hyperbola live!
- **Difficult Areas & Remediation:** Swapping $c^2=a^2 \pm b^2$ between ellipse and hyperbola, and identifying major/transverse axis.
- **Blackboard Teaching Examples:** Keep a 4-column comparison table (Circle, Parabola, Ellipse, Hyperbola) on the side of the board all week.
- **Classroom Questions to Ask Students:** Why can't an ellipse have $e>1$?
- **Common Student Doubts & Answers:** Students ask why a^2 in hyperbola can be smaller than b^2; emphasize that sign ($+$ vs $-$) decides a^2, not size.
- **Homework & Revision Strategy:** Assign 3 problems from each conic section.

[TEACHER'S TIP] Have students build a 1-page Conics Cheat Sheet summarizing all 4 curves.

UNIT III: COORDINATE GEOMETRY TYBA COACHING CENTRE

Chapter 11: Introduction to Three Dimensional Geometry

3D Coordinate Axes, Coordinate Planes, Octants & 3D Distance Formula (2026–27)

A. Chapter Introduction & Examination Relevance

Real-world objects—buildings, satellites, drones, and molecules—exist in three dimensions, not flat 2D planes. Chapter 11 extends Cartesian coordinates from (x, y) to (x, y, z) and establishes the 3D distance formula.

- **What You Will Learn:** Mutually perpendicular coordinate axes (X, Y, Z), coordinate planes (XY, YZ, ZX), sign scheme of the 8 octants, coordinates of points on axes and planes, and the 3D distance formula.
- **Why This Chapter Matters:** Chapter 11 is the direct prerequisite for Class 12 Vectors and Three Dimensional Geometry (which carries 14/80 marks in Class 12 CBSE!).
- **Real-Life Applications:** 3D CAD modeling, GPS altitude/latitude/longitude, video game engines, robotics kinematics, and aviation tracking.
- **Cross-Chapter Connections:** Extends Chapter 9 (2D distance) to 3D and leads directly into Class 12 Chapter 10 (Vectors) and Chapter 11 (3D Lines & Planes).
- **Board & Competitive Relevance:** Unit III carries 12/80 marks. Chapter 11 contributes 2–4 marks via 1M MCQs on octants/axis projections and 2M/3M distance-based triangle/collinearity/equidistant-locus problems. (3D Section formula is excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Coordinate Axes, Coordinate Planes & Octants

Three mutually perpendicular lines $X'OX, Y'OY, Z'OZ$ intersecting at origin $O(0, 0, 0)$ form a rectangular 3D coordinate system. The three coordinate planes (XY, YZ, ZX) divide space into **8 octants**. A point on the x-axis has form $(x, 0, 0)$, on the y-axis $(0, y, 0)$, on the z-axis $(0, 0, z)$. A point in the XY-plane has $z = 0$; in the YZ-plane, $x = 0$; in the ZX-plane, $y = 0$.

[CONCEPT MASTER – 1. Coordinate Axes, Coordinate Planes & Octants] Octant Numbering Rule: Octants I–IV have $z > 0$ (matching 2D Quadrants I–IV for (x, y)); Octants V–VIII have $z < 0$ (same (x, y) signs as I–IV!).

2. Projections & Reflections in 3D

The foot of perpendicular from $P(x, y, z)$ onto the XY-plane is $(x, y, 0)$, onto the YZ-plane is $(0, y, z)$, and onto the x-axis is $(x, 0, 0)$. Its perpendicular distance from the XY-plane is $|z|$, while its distance from the x-axis is $\sqrt{y^2 + z^2}$.

[CONCEPT MASTER – 2. Projections & Reflections in 3D] Notice the contrast: distance from XY-**plane** is $|z|$, whereas distance from Z-**axis** is $\sqrt{x^2 + y^2}$!

3. Distance Formula in 3D Space

The distance between $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ is $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$. Distance of $P(x, y, z)$ from origin O is $\sqrt{x^2 + y^2 + z^2}$.

[CONCEPT MASTER – 3. Distance Formula in 3D Space] Three points A, B, C are collinear in 3D iff the sum of two smaller distances equals the largest: $AB + BC = AC$.

4. Rationalised Scope & Equidistant Loci

Using $PA = PB$ or $PA^2 + PB^2 = AB^2$, we prove triangles are isosceles/right-angled, check parallelogram vertices (opposite sides equal, or midpoints of diagonals coincide), and find equations of planes/spheres equidistant from given points.

[CONCEPT MASTER – 4. Rationalised Scope & Equidistant Loci] Rationalised 2026–27 Scope: The 3D Section Formula ($\frac{mx_2+nx_1}{m+n}$) is excluded from the 2026–27 Class 11 NCERT syllabus.

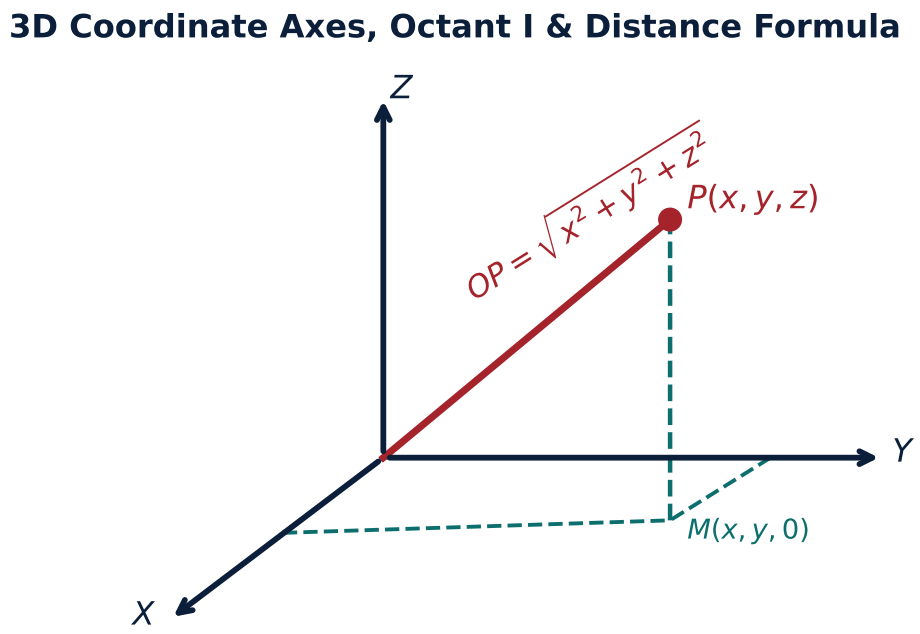


Figure 1: **Visual Model:** Right-handed 3D Cartesian axes (X, Y, Z), Octant I point $P(x, y, z)$, XY-plane projection $M(x, y, 0)$, and 3D distance $OP = \sqrt{x^2 + y^2 + z^2}$.

C. Complete Formula Bank

[FORMULA BANK – 1. 3D Distance Formula] **Expression:** $PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
Symbols: $P(x_1, y_1, z_1), Q(x_2, y_2, z_2)$.
Conditions & When to Use: Always non-negative.
Solved Mini-Example: Distance between $(1, -3, 4)$ and $(-4, 1, 2)$ is $\sqrt{25 + 16 + 4} = \sqrt{45} = 3\sqrt{5}$.
Common Mistake: Forgetting parentheses on negative differences.

[FORMULA BANK – 2. Distance from Axes & Planes] **Expression:** From XY-plane: $|z|$; From X-axis: $\sqrt{y^2 + z^2}$
Symbols: Point $P(x, y, z)$.
Conditions & When to Use: Always non-negative.
Solved Mini-Example: Distance of $(3, 4, 12)$ from z-axis is $\sqrt{3^2 + 4^2} = 5$.
Common Mistake: Writing $|x|$ as distance from x-axis.

[FORMULA BANK – 3. Equation of Coordinate Planes & Axes] **Expression:** XY-plane: $z = 0$; YZ-plane: $x = 0$; ZX-plane: $y = 0$
Symbols: $x, y, z \in \mathbb{R}$.
Conditions & When to Use: x-axis is $y = 0, z = 0$.
Solved Mini-Example: A point on y-axis has form $(0, y, 0)$.
Common Mistake: Writing $x = 0$ for x-axis.

[FORMULA BANK – 4. Collinearity in 3D] **Expression:** $AB + BC = AC$ (for ordered A, B, C)
Symbols: AB, BC, AC: pairwise distances.
Conditions & When to Use: Always compute all 3 distances.
Solved Mini-Example: If $AB = \sqrt{14}, BC = 2\sqrt{14}, AC = 3\sqrt{14}$, points are collinear.
Common Mistake: Trying to use 2D slope in 3D.

[FORMULA BANK – 5. Parallelogram Vertex Test] **Expression:** $\frac{A+C}{2} = \frac{B+D}{2} \iff A + C = B + D$
Symbols: A, B, C, D: vertices in cyclic order.
Conditions & When to Use: Midpoints of diagonals AC and BD coincide.
Solved Mini-Example: Given A, B, C, 4th vertex $D = A + C - B$.
Common Mistake: Ignoring cyclic vertex order $A - B - C - D$.

[FORMULA BANK – 6. Centroid of Triangle in 3D] **Expression:** $G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$
Symbols: Vertices A, B, C.
Conditions & When to Use: Any triangle in 3D.
Solved Mini-Example: Centroid of $(1,0,0), (0,2,0), (0,0,3)$ is $(\frac{1}{3}, \frac{2}{3}, 1)$.
Common Mistake: Dividing by 2 instead of 3.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and JEE/CUET Foundation problem structures.

Pattern 1: Octant Identification & Axis/Plane Coordinates

- **Identification Cue:** Name the octant in which $(-3, 1, -2)$ lies, or distance from y-axis. | **Concept & Formula:** Quadrant + z-sign rule.
- **Standard Step-by-Step:** Check (x, y) quadrant $q \in \{1, 2, 3, 4\}$; if $z > 0$ octant is q, if $z < 0$ octant is $q + 4$.
- **Fastest Valid Method:** Q-I..IV for $z > 0$; add 4 when $z < 0$. | **Alternative:** Memorize 8-row sign table.
- **Common Trap:** Swapping Octant II and Octant IV.
- **Practice Drill:** Ex. 1 & Ex. 2.

Pattern 2: 3D Collinearity & Triangle Type

- **Identification Cue:** Verify if $(0, 7, -10), (1, 6, -6), (4, 9, -6)$ form an isosceles or right triangle, or are collinear. | **Concept & Formula:** Compute AB, BC, CA.
- **Standard Step-by-Step:** Calculate AB^2, BC^2, CA^2 first (avoids radicals for right-triangle check), then take roots for collinearity.
- **Fastest Valid Method:** Leave in squared form $AB^2 + BC^2 = AC^2$ for right-angle test. | **Alternative:** Vector dot product (in Class 12).
- **Common Trap:** Arithmetic slip with double negatives.
- **Practice Drill:** Ex. 5 & Ex. 7.

Pattern 3: Parallelogram 4th Vertex / Verification

- **Identification Cue:** Show that A, B, C, D are vertices of a parallelogram, or find D. | **Concept & Formula:** Equal opposite sides & unequal diagonals (or $A + C = B + D$).
- **Standard Step-by-Step:** Check $AB = CD$ and $BC = DA$ (if $AC = BD$ too, it's a rectangle).
- **Fastest Valid Method:** For finding D, use $D = A + C - B$ directly. | **Alternative:** Midpoint equality.
- **Common Trap:** Forgetting to check diagonals are unequal when distinguishing from rectangle.
- **Practice Drill:** Ex. 8 & Ex. 11.

Pattern 4: Equidistant Point Locus

- **Identification Cue:** Find the point on y-axis equidistant from $(3,1,2)$ and $(5,5,2)$, or locus $PA = PB$. | **Concept & Formula:** Set $PA^2 = PB^2$.
- **Standard Step-by-Step:** Assume point $(0,y,0)$ on y-axis, equate $PA^2 = PB^2$, expand and cancel y^2.
- **Fastest Valid Method:** Square both sides immediately to avoid $\sqrt{}$. | **Alternative:** Perpendicular bisector plane.
- **Common Trap:** Writing $(y,0,0)$ for y-axis.
- **Practice Drill:** Ex. 9 & Ex. 12.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Octant Number = Quadrant + (4 if $z<0$)] **Core Idea & Why It Works:** Find the 2D quadrant $Q \in \{1,2,3,4\}$ of (x,y). If $z>0$, Octant $= Q$. If $z<0$, Octant $= Q+4$.

Valid Domain: All 8 octants. | **Where It Fails:** Points on coordinate planes ($x=0, y=0$, or $z=0$).

Easy Example: $(-3,1,-2)$: $(-3,1)$ is Q-II, $z<0 \implies 2+4=6$ (Octant VI).

Board-Level Example: Zero memorization of an 8-row table!

Advanced Example: $(2,-3,-5)$: Q-IV + 4 = Octant VIII.

[SMART SHORTCUT – 2. 4th Vertex of Parallelogram $D = A + C - B$] **Core Idea & Why It Works:** In cyclic parallelogram $ABCD$, $D = A + C - B$ coordinate-wise.

Valid Domain: All parallelograms, rectangles, squares, rhombuses. | **Where It Fails:** Non-cyclic vertex ordering.

Easy Example: $A(3,-1,2), B(1,2,-4), C(-1,1,2) \implies D = (3-1-1, -1+1-2, 2+2+4) = (1,-2,8)$.

Board-Level Example: Solves NCERT Misc. Q1 in 10 seconds!

Advanced Example: Check with $AB^2 = CD^2$.

[SMART SHORTCUT – 3. Drop Missing Coordinate for Plane/Axis Projection] **Core Idea & Why It Works:** Projection onto XY-plane drops z to 0; projection onto X-axis drops y, z to 0.

Valid Domain: All points. | **Where It Fails:** Nothing.

Easy Example: Projection of $(3,-4,5)$ on ZX-plane is $(3,0,5)$.

Board-Level Example: Distance to projection gives distance to that plane/axis.

Advanced Example: Reflection across XY-plane flips $z \to -z$.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: Name the octant in which $(-3,-1,6)$ and $(2,-4,-7)$ lie.

Given & Required: Two 3D points. | **Concept & Formula:** Quadrant $+4 (z<0)$ rule.

Detailed Solution: $(-3,-1)$ is Q-III, $z>0 \implies$ Octant III. $(2,-4)$ is Q-IV, $z<0 \implies 4+4=$ Octant VIII. **Final Answer:** **Octant III and Octant VIII**

Fastest Alternative / Shortcut: Q-III and Q-IV+4.

SOLVED EXAMPLE 2 [Beginner]

Question: Find the distance between $P(1,-3,4)$ and $Q(-4,1,2)$.

Given & Required: Two points. | **Concept & Formula:** 3D distance.

Detailed Solution: $PQ = \sqrt{(-4-1)^2+(1+3)^2+(2-4)^2} = \sqrt{25+16+4} = \sqrt{45} = 3\sqrt{5}$. **Final Answer:** $3\sqrt{5}$

Fastest Alternative / Shortcut: $\sqrt{25+16+4}$.

SOLVED EXAMPLE 3 [Beginner]

Question: A point lies on the x-axis. What are its y- and z-coordinates?

Given & Required: Point on x-axis. | **Concept & Formula:** Axis equations.

Detailed Solution: Both $y = 0$ and $z = 0$; form $(x, 0, 0)$. **Final Answer:** $y = 0, z = 0$

Fastest Alternative / Shortcut: Missing axes are zero.

SOLVED EXAMPLE 4 [Beginner]

Question: Find the distance of $(3, -4, 12)$ from the origin and from the z-axis.

Given & Required: $P(3, -4, 12)$. | **Concept & Formula:** Origin vs axis distance.

Detailed Solution: From origin: $\sqrt{9 + 16 + 144} = 13$. From z-axis: $\sqrt{3^2 + (-4)^2} = 5$. **Final Answer:** 13 **and** 5

Fastest Alternative / Shortcut: 3-4-5 and 5-12-13 triples!

SOLVED EXAMPLE 5 [Intermediate]

Question: Show that the points $P(-2, 3, 5), Q(1, 2, 3), R(7, 0, -1)$ are collinear.

Given & Required: P, Q, R. | **Concept & Formula:** $PQ + QR = PR$.

Detailed Solution: $PQ = \sqrt{9 + 1 + 4} = \sqrt{14}$; $QR = \sqrt{36 + 4 + 16} = \sqrt{56} = 2\sqrt{14}$; $PR = \sqrt{81 + 9 + 36} = \sqrt{126} = 3\sqrt{14}$. Since $PQ + QR = PR$, collinear. **Final Answer: Collinear**

Fastest Alternative / Shortcut: All are multiples of $\sqrt{14}$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Verify that $(0, 7, -10), (1, 6, -6), (4, 9, -6)$ are vertices of an isosceles triangle.

Given & Required: A, B, C. | **Concept & Formula:** Two equal sides.

Detailed Solution: $AB = \sqrt{1 + 1 + 16} = \sqrt{18}$; $BC = \sqrt{9 + 9 + 0} = \sqrt{18}$; $AC = \sqrt{16 + 4 + 16} = 6$. Since $AB = BC$, isosceles. **Final Answer: Isosceles ($AB = BC = 3\sqrt{2}$)**

Fastest Alternative / Shortcut: Compare squared distances.

SOLVED EXAMPLE 7 [Intermediate]

Question: Verify that $(0, 7, 10), (-1, 6, 6), (-4, 9, 6)$ are vertices of a right-angled triangle.

Given & Required: A, B, C. | **Concept & Formula:** Pythagoras theorem.

Detailed Solution: $AB^2 = 1 + 1 + 16 = 18$; $BC^2 = 9 + 9 + 0 = 18$; $AC^2 = 16 + 4 + 16 = 36$. Since $AB^2 + BC^2 = AC^2$, right-angled at B. **Final Answer: Right-angled at B**

Fastest Alternative / Shortcut: $18 + 18 = 36$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Find the equation of the set of points equidistant from $(1, 2, 3)$ and $(3, 2, -1)$.

Given & Required: $PA = PB$. | **Concept & Formula:** $PA^2 = PB^2$.

Detailed Solution: $(x-1)^2+(y-2)^2+(z-3)^2 = (x-3)^2+(y-2)^2+(z+1)^2 \implies -2x+1-6z+9 = -6x+9+2z+1 \implies 4x - 8z = 0 \implies x - 2z = 0$. **Final Answer:** $x - 2z = 0$

Fastest Alternative / Shortcut: $(y - 2)^2$ cancels immediately.

SOLVED EXAMPLE 9 [Board Standard]

Question: Find the point on the y-axis which is at a distance of $5\sqrt{2}$ from $(3, -2, 5)$.

Given & Required: Point $(0, y, 0)$, distance $5\sqrt{2}$. | **Concept & Formula:** 3D distance.

Detailed Solution: $(0-3)^2+(y+2)^2+(0-5)^2 = 50 \implies 9+(y+2)^2+25 = 50 \implies (y+2)^2 = 16 \implies y+2 = \pm 4 \implies y = 2$ or -6. **Final Answer:** $(0, 2, 0)$ **and** $(0, -6, 0)$

Fastest Alternative / Shortcut: Do not forget the -4 root!

SOLVED EXAMPLE 10 [Board Standard]

Question: Find the equation of the set of points P such that $PA^2 + PB^2 = 2k^2$, where $A(3,4,5)$ and $B(-1,3,-7)$.

Given & Required: A, B, $PA^2 + PB^2 = 2k^2$. | **Concept & Formula:** 3D locus.

Detailed Solution: $2(x^2+y^2+z^2) - 4x - 14y + 4z + 50 + 59 = 2k^2 \implies x^2+y^2+z^2-2x-7y+2z+\frac{109}{2} - k^2 = 0$. **Final Answer:** $2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 - 2k^2 = 0$

Fastest Alternative / Shortcut: Sum squares cleanly.

SOLVED EXAMPLE 11 [Board Standard]

Question: Show that $(-1,2,1), (1,-2,5), (4,-7,8), (2,-3,4)$ are vertices of a parallelogram.

Given & Required: A, B, C, D. | **Concept & Formula:** Midpoints of diagonals coincide.

Detailed Solution: Midpoint of $AC = (\frac{3}{2}, -\frac{5}{2}, \frac{9}{2})$; Midpoint of $BD = (\frac{3}{2}, -\frac{5}{2}, \frac{9}{2})$. Also $AB = \sqrt{4+16+16} = 6$, $BC = \sqrt{9+25+9} = \sqrt{43}$, $AC = \sqrt{25+81+49} = \sqrt{155} \neq BD$. Hence parallelogram. **Final Answer: Proved**

Fastest Alternative / Shortcut: Midpoint test is fastest.

SOLVED EXAMPLE 12 [Board Standard]

Question: Three vertices of a parallelogram $ABCD$ are $A(3,-1,2), B(1,2,-4), C(-1,1,2)$. Find the fourth vertex D.

Given & Required: A, B, C. | **Concept & Formula:** $D = A + C - B$.

Detailed Solution: $D = (3-1-1,\ -1+1-2,\ 2+2-(-4)) = (1,-2,8)$. **Final Answer:** $(1,-2,8)$

Fastest Alternative / Shortcut: Equate midpoints of AC and BD.

SOLVED EXAMPLE 13 [Board Standard]

Question: Find the equation of the set of points P whose sum of distances from $A(4,0,0)$ and $B(-4,0,0)$ is 10.

Given & Required: $PA + PB = 10$. | **Concept & Formula:** Square twice to remove radicals.

Detailed Solution: $\sqrt{(x-4)^2+y^2+z^2} = 10 - \sqrt{(x+4)^2+y^2+z^2}$. Squaring and simplifying gives $5\sqrt{(x+4)^2+y^2+z^2} = 25 + 4x$. Squaring again: $9x^2 + 25y^2 + 25z^2 = 225$. **Final Answer:** $9x^2 + 25y^2 + 25z^2 = 225$

Fastest Alternative / Shortcut: 3D ellipsoid with $a = 5, c = 4 \implies b^2 = 9$!

SOLVED EXAMPLE 14 [Challenge]

Question: If a triangle has vertices $(a,1,3), (-2,b,-5), (4,7,c)$ and its centroid is at the origin $(0,0,0)$, find a, b, c.

Given & Required: Centroid $(0,0,0)$. | **Concept & Formula:** Centroid average.

Detailed Solution: $\frac{a-2+4}{3} = 0 \implies a = -2$; $\frac{1+b+7}{3} = 0 \implies b = -8$; $\frac{3-5+c}{3} = 0 \implies c = 2$. **Final Answer:** $a = -2, b = -8, c = 2$

Fastest Alternative / Shortcut: Sum of coordinates $= 0$.

SOLVED EXAMPLE 15 [Challenge]

Question: Find the reflection of the point $P(3,-4,5)$ in the XZ-plane and in the origin.

Given & Required: $P(3,-4,5)$. | **Concept & Formula:** 3D symmetry.

Detailed Solution: Across XZ-plane, flip y: $(3,4,5)$. Across origin, flip all three: $(-3,4,-5)$. **Final Answer:** $(3,4,5)$ **and** $(-3,4,-5)$

Fastest Alternative / Shortcut: Flip missing coordinate for plane reflection.

SOLVED EXAMPLE 16 [Challenge]

Question: Find the lengths of the medians of the triangle with vertices $A(0,0,6), B(0,4,0), C(6,0,0)$.

Given & Required: A, B, C. | **Concept & Formula:** Midpoints + distance.

Detailed Solution: Midpoint D of BC is $(3,2,0) \implies AD = \sqrt{9+4+36} = 7$. Similarly $BE = \sqrt{9+16+9} = \sqrt{34}$ and $CF = \sqrt{36+4+9} = 7$. **Final Answer:** $7, \sqrt{34}, 7$

Fastest Alternative / Shortcut: Symmetric A, C give equal medians 7.

G. Competency MCQ Mastery Section

Q1. The point $(-4, 2, -5)$ lies in which octant?

(A) II (B) IV (C) VI (D) VIII

Answer: (C) Q-II $(-,+)$ with $z<0 \implies 2+4=6$ (Octant VI). *Fast Method:* Q-II + 4. *Distractor Note:* Picking Octant II.

Q2. Distance of $(3, 4, 5)$ from the XY-plane is:

(A) 3 (B) 4 (C) 5 (D) $\sqrt{50}$

Answer: (C) Distance from XY-plane is $|z|=5$. *Fast Method:* $|z|$. *Distractor Note:* Computing $\sqrt{3^2+4^2}$ (that's from z-axis!).

Q3. Distance of $(3, 4, 5)$ from the z-axis is:

(A) 5 (B) $\sqrt{34}$ (C) $\sqrt{41}$ (D) 3

Answer: (A) $\sqrt{3^2+4^2}=5$. *Fast Method:* 3-4-5 triple. *Distractor Note:* Using z-coordinate.

Q4. Any point in the YZ-plane has the form:

(A) $(x, y, 0)$ (B) $(0, y, z)$ (C) $(x, 0, z)$ (D) $(x, 0, 0)$

Answer: (B) $x=0$ in the YZ-plane. *Fast Method:* Missing coordinate is 0. *Distractor Note:* Setting $y=0$.

Q5. Distance between $(2, -1, 3)$ and $(-2, 1, 3)$ is:

(A) $2\sqrt{5}$ (B) $\sqrt{20}$ (C) Both A and B (D) $\sqrt{10}$

Answer: (A) $\sqrt{16+4+0}=2\sqrt{5}$. *Fast Method:* Standard simplified surd $2\sqrt{5}$. *Distractor Note:* Sign error on $2-(-2)$.

Q6. Centroid of $(1, 2, 3), (3, 4, 5), (5, 6, 7)$ is:

(A) $(3, 4, 5)$ (B) $(9, 12, 15)$ (C) $(3, 3, 3)$ (D) $(4, 5, 6)$

Answer: (A) Average of A.P. is the middle term $(3, 4, 5)$. *Fast Method:* Middle term. *Distractor Note:* Forgetting to divide by 3.

Q7. Number of octants into which coordinate planes divide space is:

(A) 4 (B) 6 (C) 8 (D) 12

Answer: (C) $2^3=8$ sign combinations. *Fast Method:* 8 octants. *Distractor Note:* Saying 4 (quadrants) or 6 (planes/faces).

Q8. Image of $(-2, 3, 4)$ in the XY-plane is:

(A) $(2, 3, 4)$ (B) $(-2, -3, 4)$ (C) $(-2, 3, -4)$ (D) $(2, -3, -4)$

Answer: (C) Negate z-coordinate. *Fast Method:* Flip z. *Distractor Note:* Negating x or y.

Q9. Foot of perpendicular from $(5, -2, 7)$ on the x-axis is:

(A) $(5, 0, 0)$ (B) $(0, -2, 0)$ (C) $(0, 0, 7)$ (D) $(5, -2, 0)$

Answer: (A) Keep x, zero y, z. *Fast Method:* Drop y, z. *Distractor Note:* Keeping x, y.

Q10. If distance between $(a, 0, 1)$ and $(0, 1, 2)$ is $\sqrt{27}$, then a is:

(A) ± 5 (B) 5 (C) ± 25 (D) $\sqrt{25}$

Answer: (A) $a^2+1+1=27 \implies a^2=25 \implies a=\pm 5$. *Fast Method:* Both $+5$ and -5! *Distractor Note:* Omitting -5.

Q11. Equation of the ZX-plane is:

(A) $x=0$ (B) $y=0$ (C) $z=0$ (D) $x+z=0$

Answer: (B) Missing coordinate $y=0$. *Fast Method:* $y=0$. *Distractor Note:* Writing $x=0, z=0$.

Q12. Distance of $(1, 1, 1)$ from the origin is:

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 3

Answer: (C) $\sqrt{1+1+1}=\sqrt{3}$. *Fast Method:* Cube diagonal. *Distractor Note:* Answering 3.

H. Special Exam & MCQ Strategies

- Use the Quadrant + $4(z < 0)$ rule to identify octants in 2 seconds with zero memorization.
- Keep squared distances AB^2, BC^2, CA^2 when testing Pythagoras theorem; take square roots only for collinearity.
- Distinguish carefully between distance from a PLANE (1 coordinate) and distance from an AXIS (2 coordinates under $\sqrt{\ }$).

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Writing distance of (x, y, z) from the x-axis as $|x|$. $\longrightarrow$ **Correction:** Distance from x-axis is $\sqrt{y^2 + z^2}$; $|x|$ is the distance from the YZ-plane.

[COMMON MISTAKE] **Error:** Forgetting the negative root when solving $(y + 2)^2 = 16$ for a point on an axis. $\longrightarrow$ **Correction:** Always take $\pm$ when taking a square root in distance problems.

[COMMON MISTAKE] **Error:** Using $3D$ section formula when it is excluded in 2026–27. $\longrightarrow$ **Correction:** Use midpoint/distance methods for parallelogram and triangle problems.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $PQ = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$; $G = \frac{A+B+C}{3}$; $D = A + C - B$ (parallelogram).

[QUICK REVISION] Key Theorems & Results: From XY-plane: $|z|$; From X-axis: $\sqrt{y^2 + z^2}$; XY-plane: $z = 0$.

[QUICK REVISION] Shortcut Summary: Octant $= Q + (4 \text{ if } z < 0)$; Projection drops missing coordinates to 0.

[QUICK REVISION] High-Frequency Exam Patterns: Octant naming, 3D collinearity, Right/Isosceles triangle, Equidistant point on axis.

Last-Minute Revision Checklist:

- ☐ Octant rule ($z < 0$ adds 4)
- ☐ Plane distance vs Axis distance
- ☐ $\pm$ roots in distance eqns
- ☐ Cyclic order in parallelogram

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Octant of $(1, -2, 3)$ is: (A) I (B) II (C) IV (D) VIII
2. Distance of $(2, 3, 6)$ from origin is: (A) 7 (B) 11 (C) 49 (D) $\sqrt{11}$
3. Point on z-axis has form: (A) $(z, 0, 0)$ (B) $(0, z, 0)$ (C) $(0, 0, z)$ (D) $(0, z, z)$
4. Equation of XY-plane is: (A) $x = 0$ (B) $y = 0$ (C) $z = 0$ (D) $x + y = 0$

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. Distance of $(-3, 4, 7)$ from the y-axis is: (A) 4 (B) $\sqrt{58}$ (C) 5 (D) $\sqrt{65}$
2. Point on x-axis equidistant from $(7, 6, 0)$ and $(-3, 4, 0)$: (A) $(3, 0, 0)$ (B) $(2, 0, 0)$ (C) $(1, 0, 0)$ (D) $(4, 0, 0)$
3. 4th vertex D of parallelogram $ABCD$ with $A(1, 2, 3), B(2, 3, 4), C(4, 5, 6)$: (A) $(3, 4, 5)$ (B) $(5, 6, 7)$ (C) $(1, 1, 1)$ (D) $(3, 3, 3)$
4. Triangle with vertices $(0, 0, 0), (3, 0, 0), (0, 4, 0)$ has perimeter: (A) 12 (B) 7 (C) 10 (D) 6

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Reflection of $(1, -2, 3)$ across x-axis is: (A) $(1, 2, -3)$ (B) $(-1, -2, 3)$ (C) $(1, 2, 3)$ (D) $(-1, 2, -3)$
2. Points $(1, 2, 3), (2, 4, 6), (3, 6, 9)$ are: (A) Collinear (B) Equilateral (C) Right-angled (D) Isosceles
3. Radius of sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z = 2$ is: (A) 4 (B) 2 (C) 16 (D) $\sqrt{12}$
4. Max distance between (x, y, z) with $x^2 + y^2 + z^2 = 1$ and $(3, 4, 0)$: (A) 4 (B) 5 (C) 6 (D) 7

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (C):** Q-IV with $z > 0$ is Octant IV. **Q2 (A):** $\sqrt{4 + 9 + 36} = 7$. **Q3 (C):** $x = 0, y = 0$. **Q4 (C):** $z = 0$.
Board Std	**Q1 (B):** $\sqrt{(-3)^2 + 7^2} = \sqrt{58}$. **Q2 (A):** $(x-7)^2 + 36 = (x+3)^2 + 16 \implies 20x = 60 \implies x = 3$. **Q3 (A):** $A + C - B = (3, 4, 5)$. **Q4 (A):** $3 + 4 + 5 = 12$.
Challenge	**Q1 (A):** Across x-axis, keep x and negate y, z. **Q2 (A):** $AB = \sqrt{14}, BC = \sqrt{14}, AC = 2\sqrt{14}$. **Q3 (A):** $\sqrt{1 + 4 + 9 + 2} = 4$. **Q4 (C):** Origin to $(3, 4, 0)$ is 5, plus radius $1 = 6$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: 3D axes, planes, octants. Day 2: Distance formula, collinearity, triangle/parallelogram problems.
- **How to Introduce the Concept:** Point to a corner of the classroom: floor edges are X, Y axes and the vertical wall corner is the Z-axis!
- **Difficult Areas & Remediation:** Confusing distance from XY-plane ($|z|$) with distance from Z-axis ($\sqrt{x^2+y^2}$).
- **Blackboard Teaching Examples:** Draw the 3D box from origin $(0,0,0)$ to $P(x,y,z)$ to derive $OP = \sqrt{x^2+y^2+z^2}$ via two right triangles.
- **Classroom Questions to Ask Students:** If a point is in the XZ-plane, which coordinate must be zero?
- **Common Student Doubts & Answers:** Students ask about 3D Section Formula; confirm it is deleted in 2026–27 Class 11.
- **Homework & Revision Strategy:** Assign 5 octant/projection MCQs and 5 distance/locus problems.

[TEACHER'S TIP] This short chapter is an easy 100% scorer—master it now to breeze through Class 12 3D Geometry.

UNIT IV: CALCULUS TYBA COACHING CENTRE

Chapter 12: Limits and Derivatives

Intuitive Limits, Standard Algebraic & Trig Limits, First Principles & Derivative Rules (2026–27)

A. Chapter Introduction & Examination Relevance

Calculus is the mathematics of change and motion. Chapter 12 introduces the twin pillars of differential calculus: the Limit of a function near a point a, and the Derivative (instantaneous rate of change / slope of tangent).

- **What You Will Learn:** Left-hand and right-hand limits (LHL, RHL), algebraic limit laws, $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$, $\lim_{x\to 0}\frac{\sin x}{x}=1$, $\lim_{x\to 0}\frac{1-\cos x}{x}=0$, derivative from first principles, and sum/difference/product/quotient rules.
- **Why This Chapter Matters:** Calculus is 8/80 marks in Class 11 CBSE (9 marks in RBSE) and explodes to 35/80 marks (44%) in Class 12 CBSE! Every hour invested in Chapter 12 pays off tenfold in Class 12.
- **Real-Life Applications:** Instantaneous velocity & acceleration, marginal cost/revenue in economics, optimization of profit/area, and physics rate laws.
- **Cross-Chapter Connections:** Uses functions from Chapter 2, trig identities from Chapter 3, and slope from Chapter 9; directly launches Class 12 Continuity, Differentiability, and Integrals.
- **Board & Competitive Relevance:** Unit IV carries 8/80 marks in CBSE (9M in RBSE). Expect 1M/2M limit evaluations, 3M piecewise LHL=RHL problems, and 3M/5M first-principles or quotient-rule derivatives.

B. Complete Concept Explanation & Derivations

1. Intuitive Idea of Limits, LHL & RHL

$\lim_{x\to a} f(x)=L$ means $f(x)$ approaches L as x gets arbitrarily close to a (from both left and right, without necessarily equaling $f(a)$). A limit exists iff both one-sided limits exist and are equal: LHL $=\lim_{x\to a^-} f(x)=\lim_{h\to 0} f(a-h)$ equals RHL $=\lim_{x\to a^+} f(x)=\lim_{h\to 0} f(a+h)$.

[CONCEPT MASTER – 1. Intuitive Idea of Limits, LHL & RHL] The indeterminate form $\frac{0}{0}$ does NOT mean 'undefined/no limit'—it signals that you must simplify (factor, rationalize, or use standard limits) before substituting!

2. Standard Algebraic & Trigonometric Limits

Two master limits govern Class 11: (1) $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$ (for all $n\in\mathbb{Q}$); (2) $\lim_{x\to 0}\frac{\sin x}{x}=1$ and $\lim_{x\to 0}\frac{\tan x}{x}=1$ (where x is in **radians**!). Also $\lim_{x\to 0}\frac{1-\cos x}{x}=0$, whereas $\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$.

[CONCEPT MASTER – 2. Standard Algebraic & Trigonometric Limits] Radians Warning: $\lim_{x\to 0}\frac{\sin x^\circ}{x}=\frac{\pi}{180}$, NOT 1! Always convert degrees to radians first.

3. Derivative from First Principles (Ab Initio)

Geometrically, $f'(x)$ is the slope of the tangent to $y=f(x)$ at x. By definition, $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. Using this, $(x^n)'=nx^{n-1}$, $(\sin x)'=\cos x$, $(\cos x)'=-\sin x$, $(\tan x)'=\sec^2 x$, and $(\text{constant})'=0$.

[CONCEPT MASTER – 3. Derivative from First Principles (Ab Initio)] In board exams, if a question says 'Find the derivative by first principles', using the shortcut formula directly gets 0 marks—you MUST write the $\lim_{h\to 0}$ steps!

4. Algebra of Derivatives (Product & Quotient Rules)

Linearity: $(af\pm bg)'=af'\pm bg'$. **Product Rule:** $(uv)'=u'v+uv'$. **Quotient Rule:** $\left(\frac{u}{v}\right)'=\frac{u'v-uv'}{v^2}$ $(v\neq 0)$.

[CONCEPT MASTER – 4. Algebra of Derivatives (Product & Quotient Rules)] Quotient Rule Memory Hook: 'Low d-High minus High d-Low, square the bottom and away we go!' Order in the numerator matters because of the minus sign.

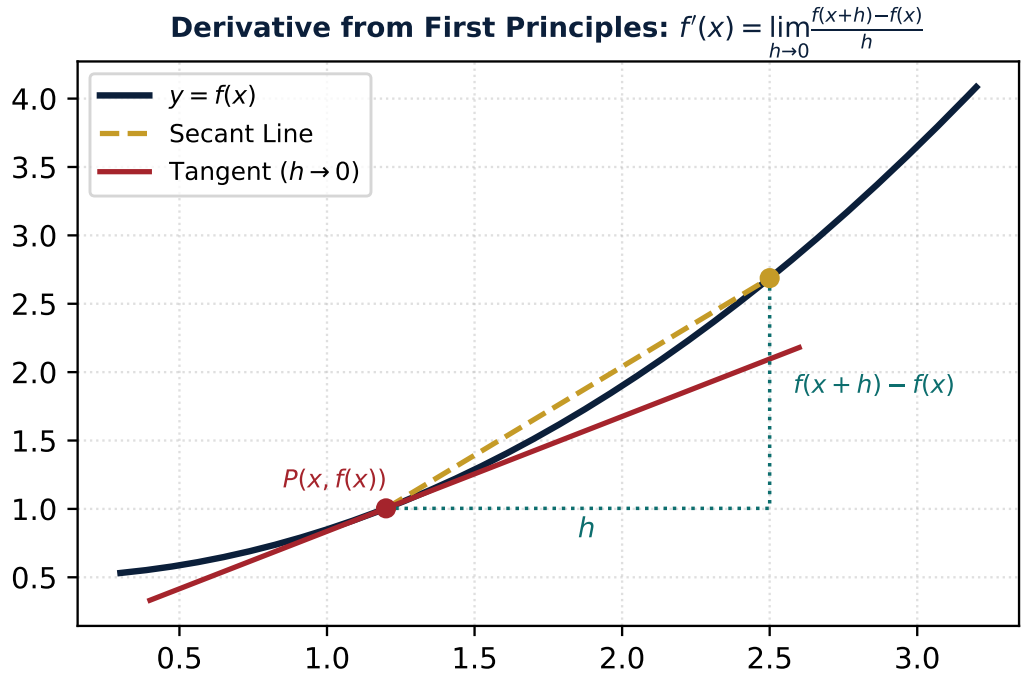


Figure 1: **Visual Model:** Secant line through $P(x, f(x))$ and $(x+h, f(x+h))$ approaching the tangent line as $h \to 0$ to define $f'(x)$.

C. Complete Formula Bank

[FORMULA BANK – 1. Existence of Limit] **Expression:** $\lim_{x\to a} f(x) = L \iff \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$
Symbols: $h \to 0^+$ in $f(a-h)$ and $f(a+h)$.
Conditions & When to Use: Mandatory for piecewise, modulus, and GIF functions.
Solved Mini-Example: $\lim_{x\to 0} \frac{|x|}{x}$ DNE since LHL $= -1 \neq 1 =$ RHL.
Common Mistake: Plugging $x = a$ into only one branch of a piecewise function.

[FORMULA BANK – 2. Standard Power Limit] **Expression:** $\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}$
Symbols: $n \in \mathbb{Q}, a > 0$ (or $n \in \mathbb{Z}$).
Conditions & When to Use: Form $\frac{0}{0}$ as $x \to a$.
Solved Mini-Example: $\lim_{x\to 2} \frac{x^5-32}{x-2} = 5(2)^4 = 80$.
Common Mistake: Forgetting to match a^n in the numerator.

[FORMULA BANK – 3. Standard Trig Limits] **Expression:** $\lim_{x\to 0} \frac{\sin x}{x} = 1$; $\lim_{x\to 0} \frac{\tan x}{x} = 1$; $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$
Symbols: x in radians.
Conditions & When to Use: The angle inside $\sin(\theta)$ and denominator θ must be identical and $\to 0$.
Solved Mini-Example: $\lim_{x\to 0} \frac{\sin 5x}{\sin 3x} = \frac{5}{3}$.
Common Mistake: Using $\frac{\sin x}{x} = 1$ as $x \to \infty$ (it is 0, not 1!).

[FORMULA BANK – 4. First Principles Definition] **Expression:** $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$
Symbols: h: small increment.
Conditions & When to Use: Limit must exist finitely.
Solved Mini-Example: For $f(x) = x^2$: $\lim_{h\to 0} \frac{(x+h)^2-x^2}{h} = \lim_{h\to 0}(2x+h) = 2x$.
Common Mistake: Setting $h = 0$ before canceling h from denominator.

[FORMULA BANK – 5. Standard Derivatives] **Expression:** $(x^n)' = nx^{n-1}$; $(\sin x)' = \cos x$; $(\cos x)' = -\sin x$; $(\tan x)' = \sec^2 x$; $(\sec x)' = \sec x \tan x$; $(\csc x)' = -\csc x \cot x$; $(\cot x)' = -\csc^2 x$
Symbols: x in radians.
Conditions & When to Use: Within domain.
Solved Mini-Example: $(\sqrt{x})' = \frac{1}{2\sqrt{x}}$; $(1/x)' = -\frac{1}{x^2}$.
Common Mistake: Forgetting the minus sign in $(\cos x)'$, $(\cot x)'$, $(\csc x)'$ (all 'co-' functions have minus!).

[FORMULA BANK – 6. Product & Quotient Rules] **Expression:** $(uv)' = u'v + uv'$; $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$
Symbols: u, v: differentiable functions.
Conditions & When to Use: $v(x) \neq 0$ for quotient rule.
Solved Mini-Example: $\left(\frac{x}{\sin x}\right)' = \frac{\sin x - x\cos x}{\sin^2 x}$.
Common Mistake: Reversing $u'v - uv'$ to $uv' - u'v$.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and JEE/CUET Foundation problem structures.

Pattern 1: Algebraic Limits ($0/0$ Factorization & Rationalization)

- **Identification Cue:** Evaluate $\lim_{x\to 1} \frac{x^3-1}{x-1}$ or $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$. | **Concept & Formula:** Cancel the $(x-a)$ zero factor.
- **Standard Step-by-Step:** Factor polynomials or multiply by conjugate $\sqrt{1+x}+1$, cancel $x-a$, substitute $x=a$.
- **Fastest Valid Method:** Use $\frac{x^n-a^n}{x-a} = na^{n-1}$ or L'Hôpital's Rule (for MCQ verification only!). | **Alternative:** Substitute $x = a+h$ with $h \to 0$.
- **Common Trap:** Stopping at $0/0$ and writing 'undefined'.
- **Practice Drill:** Ex. 2 & Ex. 5.

Pattern 2: Trigonometric Limits ($\frac{\sin ax}{bx}$, $1-\cos mx$)

- **Identification Cue:** Evaluate $\lim_{x\to 0} \frac{1-\cos 4x}{1-\cos 5x}$ or $\lim_{x\to \pi/2} \frac{\cot x}{\pi/2 - x}$. | **Concept & Formula:** Half-angle $1-\cos 2\theta = 2\sin^2\theta$ and substitution $y = \pi/2 - x$.
- **Standard Step-by-Step:** Convert $1-\cos$ to $2\sin^2$, balance $\frac{\sin\theta}{\theta}$, or shift $x \to a$ to $h \to 0$ via $x = a+h$.
- **Fastest Valid Method:** $\lim_{x\to 0} \frac{1-\cos ax}{1-\cos bx} = \frac{a^2}{b^2}$. | **Alternative:** L'Hôpital's rule for MCQ checks.
- **Common Trap:** Forgetting to shift $x \to \pi/2$ to $h \to 0$ before using $\frac{\sin\theta}{\theta} = 1$.
- **Practice Drill:** Ex. 6 & Ex. 10.

Pattern 3: Piecewise Limits (LHL = RHL)

- **Identification Cue:** Find k or evaluate $\lim_{x\to 0} f(x)$ for piecewise $f(x)$. | **Concept & Formula:** LHL = RHL.
- **Standard Step-by-Step:** Compute $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$ separately and equate.
- **Fastest Valid Method:** Plug a directly into left and right formulas when continuous. | **Alternative:** Use $x = a \pm h$.
- **Common Trap:** Evaluating only one branch.
- **Practice Drill:** Ex. 9 & Ex. 11.

Pattern 4: First-Principles & Product/Quotient Derivatives

- **Identification Cue:** Find derivative of $\sin(x+1)$ or $\frac{1}{x}$ from first principles, or differentiate $\frac{x+\cos x}{\tan x}$. | **Concept & Formula:** $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ and $(u/v)'$.
- **Standard Step-by-Step:** Use C–D formulas for trig first principles; apply quotient rule cleanly for rational trig.
- **Fastest Valid Method:** Check your first-principles answer using the standard derivative formula. | **Alternative:** Simplify trig expression before differentiating.
- **Common Trap:** Sign error in $\cos C - \cos D$ during first principles of $\cos x$.
- **Practice Drill:** Ex. 12 & Ex. 13.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. L'Hôpital's Rule (For MCQ Verification ONLY)] **Core Idea & Why It Works:** If $\lim_{x\to a}\frac{f(x)}{g(x)}$ is $\frac{0}{0}$ or $\frac{\infty}{\infty}$, it equals $\lim_{x\to a}\frac{f'(x)}{g'(x)}$ (differentiate top and bottom separately).
Valid Domain: All $0/0$ and ∞/∞ MCQs. | **Where It Fails:** Never use in CBSE/RBSE subjective answers (excluded from Class 11 syllabus); never apply when form is not $0/0$.
Easy Example: $\lim_{x\to 0}\frac{\sin 5x}{3x}=\lim_{x\to 0}\frac{5\cos 5x}{3}=\frac{5}{3}$.
Board-Level Example: Instant check for $\lim_{x\to 0}\frac{1-\cos 4x}{x^2}\to\frac{4\sin 4x}{2x}\to\frac{16\cos 4x}{2}=8$.
Advanced Example: Saves huge time on JEE/CUET MCQs.

[SMART SHORTCUT – 2. All 'Co-' Derivatives Have a Minus Sign] **Core Idea & Why It Works:** The derivatives of $\cos x, \cot x, \csc x$ ALL start with a negative sign ($-\sin x, -\csc^2 x, -\csc x\cot x$).
Valid Domain: All 6 trigonometric functions. | **Where It Fails:** Nothing.
Easy Example: $(\cos x)'=-\sin x$; $(\sin x)'=+\cos x$.
Board-Level Example: Eliminates sign slips in Product/Quotient rule problems.
Advanced Example: Matches Class 12 integration signs in reverse!

[SMART SHORTCUT – 3. Power-Limit Ratio $\lim_{x\to a}\frac{x^m-a^m}{x^n-a^n}=\frac{m}{n}a^{m-n}$] **Core Idea & Why It Works:** Divide numerator and denominator by $(x-a)$ to get $\frac{ma^{m-1}}{na^{n-1}}=\frac{m}{n}a^{m-n}$.
Valid Domain: All rational m, n ($n\neq 0, a>0$). | **Where It Fails:** If $a=0$ and $m\leq n$.
Easy Example: $\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}=\frac{15}{10}=\frac{3}{2}$.
Board-Level Example: Solves NCERT Ex. 12.1 Q9-style limits in 2 seconds.
Advanced Example: Works for fractional exponents like $x^{1/3}-a^{1/3}$.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Evaluate $\lim_{x\to 3}(x^2+4x-5)$.
Given & Required: Polynomial at $x=3$. | **Concept & Formula:** Direct substitution.
Detailed Solution: $3^2+4(3)-5=9+12-5=16$. **Final Answer:** 16
Fastest Alternative / Shortcut: Polynomials are continuous everywhere.

SOLVED EXAMPLE 2 [Beginner]
Question: Evaluate $\lim_{x\to 2}\frac{x^2-4}{x-2}$.
Given & Required: $0/0$ form. | **Concept & Formula:** Factorization.
Detailed Solution: $\lim_{x\to 2}\frac{(x-2)(x+2)}{x-2}=2+2=4$. **Final Answer:** 4
Fastest Alternative / Shortcut: $2(2)^{2-1}=4$.

SOLVED EXAMPLE 3 [Beginner]
Question: Evaluate $\lim_{x\to 0}\frac{\sin 4x}{\sin 2x}$.
Given & Required: $0/0$ trig form. | **Concept & Formula:** $\lim\frac{\sin\theta}{\theta}=1$.
Detailed Solution: $\lim_{x\to 0}\frac{\sin 4x}{4x}\cdot\frac{2x}{\sin 2x}\cdot 2=1\cdot 1\cdot 2=2$. **Final Answer:** 2
Fastest Alternative / Shortcut: Ratio of coefficients $4/2=2$.

SOLVED EXAMPLE 4 [Beginner]

Question: Find the derivative of $f(x) = 3x^4 - 5x^2 + 7x - 9$.
Given & Required: Polynomial. | **Concept & Formula:** $(x^n)' = nx^{n-1}$.
Detailed Solution: $f'(x) = 12x^3 - 10x + 7$. **Final Answer:** $12x^3 - 10x + 7$
Fastest Alternative / Shortcut: Constant 9 has derivative 0.

SOLVED EXAMPLE 5 [Intermediate]

Question: Evaluate $\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$.
Given & Required: $0/0$ radical. | **Concept & Formula:** Rationalize numerator.
Detailed Solution: $\lim_{x\to 0} \frac{(1+x)-1}{x(\sqrt{1+x}+1)} = \lim_{x\to 0} \frac{1}{\sqrt{1+x}+1} = \frac{1}{2}$. **Final Answer:** $\frac{1}{2}$
Fastest Alternative / Shortcut: Power limit with $n = 1/2, a = 1$: $\frac{1}{2}(1)^{-1/2} = \frac{1}{2}$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Evaluate $\lim_{x\to 1} \frac{x^{15}-1}{x^{10}-1}$.
Given & Required: $0/0$ power ratio. | **Concept & Formula:** $\frac{m}{n}a^{m-n}$.
Detailed Solution: Divide top and bottom by $(x-1)$: $\frac{15(1)^{14}}{10(1)^9} = \frac{3}{2}$. **Final Answer:** $\frac{3}{2}$
Fastest Alternative / Shortcut: $15/10 = 3/2$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Evaluate $\lim_{x\to 0} \frac{1-\cos x}{x^2}$.
Given & Required: $0/0$ trig form. | **Concept & Formula:** $1 - \cos x = 2\sin^2(x/2)$.
Detailed Solution: $\lim_{x\to 0} \frac{2\sin^2(x/2)}{x^2} = \frac{2}{4}\lim_{x\to 0}\left(\frac{\sin(x/2)}{x/2}\right)^2 = \frac{1}{2}$. **Final Answer:** $\frac{1}{2}$
Fastest Alternative / Shortcut: Standard result $\frac{1}{2}$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Differentiate $y = x^2 \sin x$.
Given & Required: Product of x^2 and $\sin x$. | **Concept & Formula:** Product rule.
Detailed Solution: $y' = (2x)\sin x + x^2(\cos x) = 2x\sin x + x^2\cos x$. **Final Answer:** $2x\sin x + x^2\cos x$
Fastest Alternative / Shortcut: $(uv)' = u'v + uv'$.

SOLVED EXAMPLE 9 [Board Standard]

Question: If $f(x) = \begin{cases} a + bx, & x < 1 \\ 4, & x = 1 \\ b - ax, & x > 1 \end{cases}$ and $\lim_{x\to 1} f(x) = f(1)$, find a and b.
Given & Required: LHL $=$ RHL $= 4$. | **Concept & Formula:** One-sided limits.
Detailed Solution: LHL $= a + b = 4$; RHL $= b - a = 4$. Adding gives $2b = 8 \implies b = 4$, so $a = 0$. **Final Answer:** $a = 0, b = 4$
Fastest Alternative / Shortcut: Equate both branches to 4.

SOLVED EXAMPLE 10 [Board Standard]

Question: Evaluate $\lim_{x\to \pi/2} \frac{\tan 2x}{x - \pi/2}$.
Given & Required: $x \to \pi/2$. | **Concept & Formula:** Shift $x = \frac{\pi}{2} + h$.
Detailed Solution: Let $x = \frac{\pi}{2} + h$ $(h \to 0)$. $\tan(2(\frac{\pi}{2} + h)) = \tan(\pi + 2h) = \tan 2h$. Limit $= \lim_{h\to 0} \frac{\tan 2h}{h} = 2$. **Final Answer:** 2
Fastest Alternative / Shortcut: Always shift $x \to a$ to $h \to 0$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Find the derivative of $f(x) = \sin x$ from first principles.

Given & Required: $f(x) = \sin x$. | **Concept & Formula:** C–D formula + $\frac{\sin h}{h} \to 1$.

Detailed Solution: $f'(x) = \lim_{h\to 0} \frac{\sin(x+h)-\sin x}{h} = \lim_{h\to 0} \frac{2\cos(x+h/2)\sin(h/2)}{h} = \lim_{h\to 0} \cos(x+\frac{h}{2}) \cdot \frac{\sin(h/2)}{h/2} = \cos x$. **Final Answer:** $\cos x$

Fastest Alternative / Shortcut: Pair 2 with h to make $h/2$.

SOLVED EXAMPLE 12 [Board Standard]

Question: Find the derivative of $f(x) = \frac{x+\cos x}{\tan x}$.

Given & Required: Quotient. | **Concept & Formula:** Quotient rule.

Detailed Solution: $f'(x) = \frac{(1-\sin x)\tan x-(x+\cos x)\sec^2 x}{\tan^2 x}$. **Final Answer:** $\frac{(1-\sin x)\tan x-(x+\cos x)\sec^2 x}{\tan^2 x}$

Fastest Alternative / Shortcut: Quotient rule directly.

SOLVED EXAMPLE 13 [Board Standard]

Question: Find the derivative of $f(x) = \frac{x^5-\cos x}{\sin x}$.

Given & Required: Quotient. | **Concept & Formula:** Quotient rule.

Detailed Solution: $f'(x) = \frac{(5x^4+\sin x)\sin x-(x^5-\cos x)\cos x}{\sin^2 x} = \frac{5x^4\sin x-x^5\cos x+1}{\sin^2 x}$ (using $\sin^2 x + \cos^2 x = 1$). **Final Answer:** $\frac{5x^4\sin x-x^5\cos x+1}{\sin^2 x}$

Fastest Alternative / Shortcut: Notice $\sin^2 x + \cos^2 x = 1$ simplifies the numerator!

SOLVED EXAMPLE 14 [Challenge]

Question: Evaluate $\lim_{x\to 0} \frac{\tan x-\sin x}{x^3}$.

Given & Required: $0/0$ form. | **Concept & Formula:** Factor $\tan x$ or $\sin x$.

Detailed Solution: $\frac{\sin x(1-\cos x)}{x^3\cos x} = \left(\frac{\sin x}{x}\right)\left(\frac{1-\cos x}{x^2}\right)\frac{1}{\cos x} = 1 \cdot \frac{1}{2} \cdot 1 = \frac{1}{2}$. **Final Answer:** $\frac{1}{2}$

Fastest Alternative / Shortcut: Memorize $\lim_{x\to 0} \frac{\tan x-\sin x}{x^3} = \frac{1}{2}$!

SOLVED EXAMPLE 15 [Challenge]

Question: Find the derivative of $\sqrt{\sin x}$ from first principles (or $f(x) = \frac{1}{x^2}$).

Given & Required: $f(x) = \frac{1}{x^2}$. | **Concept & Formula:** First principles.

Detailed Solution: $f'(x) = \lim_{h\to 0} \frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} = \lim_{h\to 0} \frac{x^2-(x+h)^2}{hx^2(x+h)^2} = \lim_{h\to 0} \frac{-2xh-h^2}{hx^2(x+h)^2} = -\frac{2}{x^3}$. **Final Answer:** $-\frac{2}{x^3}$

Fastest Alternative / Shortcut: Matches $(x^{-2})' = -2x^{-3}$.

SOLVED EXAMPLE 16 [Challenge]

Question: If $f(x) = \frac{x^n-a^n}{x-a}$ for $x \neq a$, what value of $f(a)$ makes $\lim_{x\to a} f(x) = f(a)$?

Given & Required: Removable discontinuity. | **Concept & Formula:** Standard power limit.

Detailed Solution: $f(a) = \lim_{x\to a} \frac{x^n-a^n}{x-a} = na^{n-1}$. **Final Answer:** na^{n-1}

Fastest Alternative / Shortcut: Direct standard limit.

G. Competency MCQ Mastery Section

Q1. $\lim_{x\to 0} \frac{\sin 7x}{3x}$ equals:

(A) 1 (B) 7/3 (C) 3/7 (D) 0

Answer: (B) $\frac{7}{3}\lim \frac{\sin 7x}{7x} = \frac{7}{3}$. *Fast Method:* Ratio of x-coefficients. *Distractor Note:* Inverting to 3/7.

Q2. $\lim_{x\to 2} \frac{x^4-16}{x-2}$ equals:

(A) 8 (B) 16 (C) 32 (D) 64

Answer: (C) $4(2)^3 = 32$. *Fast Method:* na^{n-1}. *Distractor Note:* Writing $4(2)^4 = 64$.

Q3. $\lim_{x\to 0} \frac{|x|}{x}$ is:

(A) 1 (B) -1 (C) 0 (D) Does not exist

Answer: (D) LHL $= -1$, RHL $= 1$; unequal. *Fast Method:* Signum jump at 0. *Distractor Note:* Picking 1 or 0.

Q4. Derivative of $\cos x$ with respect to x is:

(A) $\sin x$ (B) $-\sin x$ (C) $\sec^2 x$ (D) $-\cos x$

Answer: (B) Standard derivative. *Fast Method:* 'Co-' starts with minus. *Distractor Note:* Forgetting minus sign.

Q5. $\lim_{x\to 0} \frac{1-\cos 2x}{x^2}$ equals:

(A) 1 (B) 2 (C) $1/2$ (D) 4

Answer: (B) $\frac{2\sin^2 x}{x^2} \to 2$. *Fast Method:* $a^2/2 = 4/2 = 2$. *Distractor Note:* Answering $1/2$.

Q6. If $y = \frac{1}{x}$, then $\frac{dy}{dx}$ is:

(A) $\frac{1}{x^2}$ (B) $-\frac{1}{x^2}$ (C) $\ln x$ (D) -1

Answer: (B) $(x^{-1})' = -x^{-2} = -1/x^2$. *Fast Method:* Power rule $n = -1$. *Distractor Note:* Missing minus sign.

Q7. $\lim_{x\to 0} \frac{\sin x^\circ}{x}$ equals:

(A) 1 (B) π (C) $\frac{\pi}{180}$ (D) $\frac{180}{\pi}$

Answer: (C) $x^\circ = \frac{\pi x}{180}$ rad. *Fast Method:* Convert degrees to radians. *Distractor Note:* Picking 1.

Q8. If $f(x) = x\cos x$, then $f'(0)$ is:

(A) 0 (B) 1 (C) -1 (D) 2

Answer: (B) $f'(x) = \cos x - x\sin x \implies f'(0) = 1 - 0 = 1$. *Fast Method:* Plug $x = 0$ after product rule. *Distractor Note:* Differentiating 0 directly.

Q9. $\lim_{x\to\infty} \frac{\sin x}{x}$ equals:

(A) 1 (B) 0 (C) ∞ (D) Does not exist

Answer: (B) $|\sin x| \le 1$ divided by $\infty \to 0$. *Fast Method:* Bounded over ∞ is 0. *Distractor Note:* Confusing with $x \to 0$!

Q10. Derivative of $\tan x$ is:

(A) $\cot x$ (B) $\sec^2 x$ (C) $-\sec^2 x$ (D) $\sec x \tan x$

Answer: (B) Standard derivative. *Fast Method:* $(\tan x)' = \sec^2 x$. *Distractor Note:* Confusing with $(\sec x)'$.

Q11. $\lim_{x\to 1} \frac{x^3-1}{x^2-1}$ is:

(A) 1 (B) $3/2$ (C) $2/3$ (D) 3

Answer: (B) $\frac{m}{n} = \frac{3}{2}$. *Fast Method:* Ratio of exponents. *Distractor Note:* Saying $0/0$ DNE.

Q12. If $f(x) = 100x + 99x^2$, then $f'(0)$ is:

(A) 0 (B) 100 (C) 198 (D) 99

Answer: (B) $f'(x) = 100 + 198x \implies f'(0) = 100$. *Fast Method:* Coefficient of x is $f'(0)$! *Distractor Note:* Answering 0.

H. Special Exam & MCQ Strategies

- Always substitute $x = a$ mentally first: if you get a finite number (not $0/0$), you are already done!
- For $0/0$ trig limits, rewrite $1 - \cos 2\theta = 2\sin^2\theta$ and group every $\frac{\sin\theta}{\theta}$ pair.
- In subjective exams, NEVER use L'Hôpital's rule—always show factorization, rationalization, or standard limit steps.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Writing $\lim_{x\to\infty} \frac{\sin x}{x} = 1$. $\longrightarrow$ **Correction:** $\frac{\sin x}{x} \to 1$ ONLY as $x \to 0$; as $x \to \infty$, it is 0.

[COMMON MISTAKE] **Error:** Forgetting the minus sign in $(\cos x)' = -\sin x$ or in the Quotient Rule numerator. $\longrightarrow$ **Correction:** Keep $(u/v)' = \frac{u'v - uv'}{v^2}$ in strict order.

[COMMON MISTAKE] **Error:** Using shortcut derivative formulas when 'from first principles' is explicitly asked. $\longrightarrow$ **Correction:** Write $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ and evaluate the limit step by step.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$; $\lim_{x\to 0}\frac{\sin x}{x}=1$; $\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}$.

[QUICK REVISION] Key Theorems & Results: $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$; $(uv)'=u'v+uv'$; $(u/v)'=\frac{u'v-uv'}{v^2}$.

[QUICK REVISION] Shortcut Summary: All 'co-' derivatives have minus; $\frac{x^m-a^m}{x^n-a^n}\to\frac{m}{n}a^{m-n}$; L'Hôpital for MCQ check.

[QUICK REVISION] High-Frequency Exam Patterns: $0/0$ Algebraic limits, Trig limits, Piecewise LHL=RHL, First-principles & Quotient rule.

Last-Minute Revision Checklist:

- ☐ Check $0/0$ before simplifying
- ☐ Angles in radians
- ☐ LHL = RHL for piecewise
- ☐ Order in Quotient Rule

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. $\lim_{x\to 1}(x^3+x+2)$ is: (A) 2 (B) 3 (C) 4 (D) 5
2. $\lim_{x\to 0}\frac{\tan 3x}{x}$ is: (A) 1 (B) 3 (C) $1/3$ (D) 0
3. Derivative of x^5 is: (A) $5x^4$ (B) x^4 (C) $5x^5$ (D) $4x^3$
4. Derivative of $\sin x+\cos x$ is: (A) $\cos x+\sin x$ (B) $\cos x-\sin x$ (C) $-\cos x-\sin x$ (D) $\sin x-\cos x$

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. $\lim_{x\to 3}\frac{x^3-27}{x-3}$ is: (A) 9 (B) 18 (C) 27 (D) 81
2. $\lim_{x\to 0}\frac{1-\cos 2x}{1-\cos 6x}$ is: (A) $1/3$ (B) $1/9$ (C) 9 (D) 3
3. If $f(x)=x\sin x$, then $f'(\pi/2)$ is: (A) 0 (B) 1 (C) $\pi/2$ (D) -1
4. If $f(x)=\frac{x}{x+1}$, then $f'(1)$ is: (A) 1 (B) $1/2$ (C) $1/4$ (D) $-1/4$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. $\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}$ $(a,b\neq 0, a+b\neq 0)$ is: (A) 1 (B) a/b (C) b/a (D) 0
2. $\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}$ is: (A) 1 (B) $1/2$ (C) 2 (D) 0
3. If $f(x)=\lfloor x\rfloor$, then $\lim_{x\to 2^-}f(x)$ is: (A) 2 (B) 1 (C) 0 (D) DNE
4. If $y=\frac{\sin x+\cos x}{\sin x-\cos x}$, then $y'(0)$ is: (A) -2 (B) 2 (C) 1 (D) -1

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (C):** $1+1+2=4$. **Q2 (B):** $3\lim \frac{\tan 3x}{3x}=3$. **Q3 (A):** $nx^{n-1}=5x^4$. **Q4 (B):** $(\sin x)'+(\cos x)'=\cos x-\sin x$.
Board Std	**Q1 (C):** $3(3)^2=27$. **Q2 (B):** $\frac{2^2}{6^2}=\frac{4}{36}=\frac{1}{9}$. **Q3 (B):** $f'(x)=\sin x+x\cos x \implies 1+0=1$. **Q4 (C):** $f'(x)=\frac{1}{(x+1)^2} \implies f'(1)=1/4$.
Challenge	**Q1 (A):** Divide by x: $\frac{a+b}{a+b}=1$. **Q2 (B):** $\frac{1}{2}(1)^{-1/2}=1/2$. **Q3 (B):** For x slightly <2, $\lfloor x\rfloor=1$. **Q4 (A):** $y=\tan(x+\pi/4)$? Wait: $\frac{1+\tan x}{\tan x-1}=-\tan(\pi/4+x)$, derivative at 0 is $-\sec^2(\pi/4)=-2$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Intuitive limits, LHL/RHL. Day 2: Algebraic & trig limits. Day 3: First principles. Day 4: Product & quotient rules.
- **How to Introduce the Concept:** Ask: 'What is your speedometer reading at one exact instant ($\Delta t \to 0$)?' That is a derivative!
- **Difficult Areas & Remediation:** Shifting $x \to a$ to $h \to 0$ in trig limits and C–D steps in first principles.
- **Blackboard Teaching Examples:** Draw the secant-to-tangent animation with h shrinking to 0.
- **Classroom Questions to Ask Students:** Why is $\frac{0}{0}$ called indeterminate rather than zero or infinity?
- **Common Student Doubts & Answers:** Students ask why x must be in radians for $(\sin x)'=\cos x$; show $\sin x^\circ$ introduces $\frac{\pi}{180}$.
- **Homework & Revision Strategy:** Assign 6 limits, 3 first-principles derivations, and 5 product/quotient derivatives.

[TEACHER'S TIP] Emphasize that Class 12 Calculus starts directly where Class 11 Chapter 12 ends.

UNIT V: STATISTICS AND PROBABILITY TYBA COACHING CENTRE

Chapter 13: Statistics

Mean Deviation (About Mean & Median), Variance & Standard Deviation (2026–27)

A. Chapter Introduction & Examination Relevance

Central tendency (Mean, Median, Mode) tells us only the centre of data—two batches with the exact same average score can have wildly different consistency. Chapter 13 measures that spread (dispersion) using Mean Deviation, Variance, and Standard Deviation.

- **What You Will Learn:** Range, Mean Deviation about Mean ($\mathrm{MD}(\bar{x})$) and Median ($\mathrm{MD}(M)$) for ungrouped, discrete, and continuous grouped data; Variance (σ^2) and Standard Deviation (σ); and scaling/shifting invariance.
- **Why This Chapter Matters:** Statistics is a guaranteed 5-mark calculation problem in CBSE and RBSE Class 11 exams, plus fast 1M MCQs on Variance/SD scaling properties.
- **Real-Life Applications:** Quality control in manufacturing, stock-market volatility (standard deviation of returns), exam score normalization, and weather variability.
- **Cross-Chapter Connections:** Extends Class 10 Mean/Median into dispersion and connects to Class 12 Random Variable Variance.
- **Board & Competitive Relevance:** Unit V carries 12/80 marks. Chapter 13 contributes 5–7 marks via a 5M grouped-data Variance/SD or Mean Deviation table and 1M scaling MCQs. (Equal-mean CV comparison is excluded in 2026–27.)

B. Complete Concept Explanation & Derivations

1. Measures of Dispersion & Mean Deviation

Range $= L - S$ (largest minus smallest). **Mean Deviation** about A (Mean $\bar{x}$ or Median M) averages the absolute deviations: $\mathrm{MD}(A) = \frac{1}{N}\sum f_i|x_i - A|$, where $N = \sum f_i$ (or n for ungrouped data).

[CONCEPT MASTER – 1. Measures of Dispersion & Mean Deviation] Never forget the absolute value bars $|x_i - A|$! Without them, $\sum f_i(x_i - \bar{x})$ is ALWAYS zero.

2. Variance (σ^2) & Standard Deviation (σ)

Variance averages the squared deviations from the mean: $\sigma^2 = \frac{1}{N}\sum f_i(x_i - \bar{x})^2 = \frac{1}{N}\sum f_i x_i^2 - \bar{x}^2$. **Standard Deviation** is the positive square root $\sigma = \sqrt{\mathrm{Var}(X)}$.

[CONCEPT MASTER – 2. Variance (σ^2) & Standard Deviation (σ)] Computational shortcut: $\sigma^2 = \frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2$ avoids ugly decimal subtractions $(x_i - \bar{x})$!

3. Step-Deviation (Assumed Mean) Method for Grouped Data

For continuous class intervals of equal width h, take class marks $x_i = \frac{\text{lower}+\text{upper}}{2}$ and define $u_i = \frac{x_i - A}{h}$. Then $\bar{x} = A + h\frac{\sum f_i u_i}{N}$ and $\sigma^2 = h^2\left[\frac{\sum f_i u_i^2}{N} - \left(\frac{\sum f_i u_i}{N}\right)^2\right]$.

[CONCEPT MASTER – 3. Step-Deviation (Assumed Mean) Method for Grouped Data] Notice h^2 outside the bracket for Variance σ^2, which becomes $|h|$ outside the square root for Standard Deviation σ!

4. Effect of Origin Shift & Scale Change

If every observation x_i is transformed to $y_i = ax_i + b$: (1) Mean changes by both: $\bar{y} = a\bar{x} + b$; (2) Variance scales by a^2 only: $\mathrm{Var}(Y) = a^2\mathrm{Var}(X)$; (3) Standard Deviation and Mean Deviation scale by $|a|$ only: $\sigma_Y = |a|\sigma_X$ (adding/subtracting b does NOT change spread!).

[CONCEPT MASTER – 4. Effect of Origin Shift & Scale Change] Rationalised 2026–27 Scope: Analysis of frequency distributions with equal means (Coefficient of Variation comparison) is excluded from the 2026–27 NCERT syllabus.

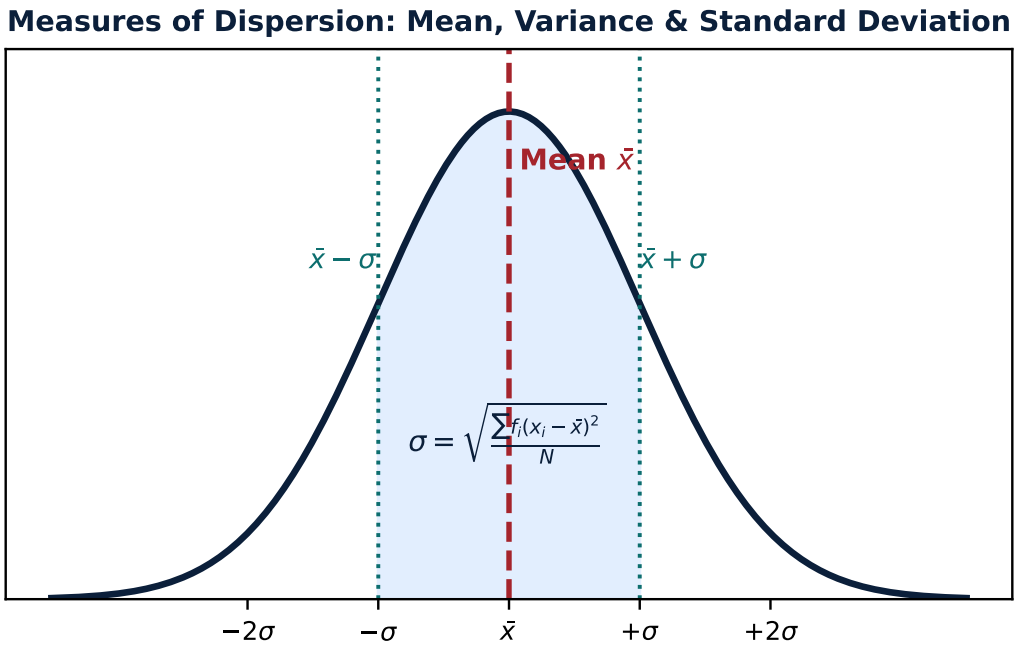


Figure 1: **Visual Model:** Symmetric distribution illustrating Mean $\bar{x}$ and Standard Deviation σ spread around the centre.

C. Complete Formula Bank

[FORMULA BANK – 1. Mean Deviation (Ungrouped / Grouped)] **Expression:** $\mathrm{MD}(A)=\frac{\sum f_i|x_i-A|}{N}$
Symbols: $A=\bar{x}$ or M; $N=\sum f_i$.
Conditions & When to Use: For ungrouped data, $f_i=1, N=n$.
Solved Mini-Example: Data $2,4,6$ ($\bar{x}=4$): $\mathrm{MD}=\frac{2+0+2}{3}=\frac{4}{3}$.
Common Mistake: Forgetting to multiply $|x_i-A|$ by f_i.

[FORMULA BANK – 2. Median for Continuous Grouped Data] **Expression:** $M=l+\left(\frac{\frac{N}{2}-cf}{f}\right)h$
Symbols: l: lower limit, cf: cumulative freq before median class, f: median class freq, h: width.
Conditions & When to Use: Used before computing $\mathrm{MD}(M)$.
Solved Mini-Example: Locate class where $cf\geq N/2$ first.
Common Mistake: Using cf of the median class itself instead of the preceding class.

[FORMULA BANK – 3. Variance (Direct & Shortcut)] **Expression:** $\sigma^2=\frac{1}{N}\sum f_i(x_i-\bar{x})^2=\frac{\sum f_i x_i^2}{N}-\bar{x}^2$
Symbols: $\bar{x}=\frac{\sum f_i x_i}{N}$.
Conditions & When to Use: Always $\sigma^2\geq 0$.
Solved Mini-Example: Data $1,2,3$: $\frac{1+4+9}{3}-2^2=\frac{14}{3}-4=\frac{2}{3}$.
Common Mistake: Forgetting to square $\bar{x}$ in $\bar{x}^2$.

[FORMULA BANK – 4. Step-Deviation Variance] **Expression:** $\sigma^2 = \frac{h^2}{N^2}\left[N\sum f_i u_i^2 - (\sum f_i u_i)^2\right]$
Symbols: $u_i = \frac{x_i - A}{h}$, h: class width.
Conditions & When to Use: Equal class width h.
Solved Mini-Example: Reduces 3-digit x_i to small integers $-2, -1, 0, 1, 2$.
Common Mistake: Multiplying by h instead of h^2 for variance.

[FORMULA BANK – 5. Standard Deviation] **Expression:** $\sigma = \sqrt{\text{Variance}}$
Symbols: $\sigma \geq 0$.
Conditions & When to Use: Same units as original data.
Solved Mini-Example: If $\sigma^2 = 16$, $\sigma = 4$.
Common Mistake: Stopping at Variance when SD is asked.

[FORMULA BANK – 6. First n Natural Numbers Variance] **Expression:** $\bar{x} = \frac{n+1}{2}$; $\sigma^2 = \frac{n^2-1}{12}$
Symbols: Data $1, 2, \ldots, n$.
Conditions & When to Use: Consecutive integers (step 1).
Solved Mini-Example: For $1, 2, \ldots, 7$: $\sigma^2 = \frac{49-1}{12} = 4, \sigma = 2$.
Common Mistake: Dividing by n instead of 12.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and JEE/CUET Foundation problem structures.

Pattern 1: Mean Deviation about Mean or Median (Ungrouped)

- **Identification Cue:** Find MD about median for $13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17$. | **Concept & Formula:** Sort data → find M → average $|x_i - M|$.
- **Standard Step-by-Step:** Sort ascending! If n is odd, $M = x_{(n+1)/2}$; if even, average of middle two.
- **Fastest Valid Method:** Check sum of ranks $= n$ before computing deviations. | **Alternative:** Use frequency table if values repeat.
- **Common Trap:** Forgetting to sort data before finding the median!
- **Practice Drill:** Ex. 1 & Ex. 5.

Pattern 2: Mean Deviation for Discrete / Continuous Data

- **Identification Cue:** Find MD about mean/median for grouped frequency table. | **Concept & Formula:** Class marks x_i, $f_i x_i$, $f_i|x_i - A|$.
- **Standard Step-by-Step:** Build columns: x_i, f_i, $f_i x_i$ (or cf), $|x_i - A|$, $f_i|x_i - A|$.
- **Fastest Valid Method:** Use assumed mean to find $\bar{x}$ first if x_i are large. | **Alternative:** Direct multiplication.
- **Common Trap:** Dividing by number of classes instead of $N = \sum f_i$.
- **Practice Drill:** Ex. 6 & Ex. 9.

Pattern 3: Variance & SD via Step-Deviation Table

- **Identification Cue:** Find Mean, Variance, and SD for grouped classes 0–10, 10–20, ... | **Concept & Formula:** $u_i = \frac{x_i - A}{h}$.
- **Standard Step-by-Step:** Pick middle x_i as A, build $u_i, f_i u_i, f_i u_i^2$, apply step-deviation formula.
- **Fastest Valid Method:** Step-deviation keeps numbers under 10, slashing arithmetic errors by 90%. | **Alternative:** Direct $\sum f_i(x_i - \bar{x})^2$ when $\bar{x}$ is an integer.
- **Common Trap:** Forgetting h^2 in σ^2 or taking h instead of h^2.
- **Practice Drill:** Ex. 10 & Ex. 11.

Pattern 4: Missing Observations & Incorrect-Observation Correction

- **Identification Cue:** Mean and SD of 100 items are 50 and 4; later one item was misread as 40 instead of 50. Find correct mean and SD. | **Concept & Formula:** Use $\sum x_i = n\bar{x}$ and $\sum x_i^2 = n(\sigma^2 + \bar{x}^2)$.
- **Standard Step-by-Step:** Compute old $\sum x_i$ and $\sum x_i^2$, subtract wrong values, add correct values, recompute new mean and variance.
- **Fastest Valid Method:** Never guess—always go through $\sum x_i^2 = n(\sigma^2 + \bar{x}^2)$. | **Alternative:** Direct equation solving for 2 missing observations.
- **Common Trap:** Adding $(x_{\text{new}} - x_{\text{old}})^2$ instead of $x_{\text{new}}^2 - x_{\text{old}}^2$ to $\sum x_i^2$!
- **Practice Drill:** Ex. 12 & Ex. 13.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Scaling & Shifting Rule ($y = ax + b$)] **Core Idea & Why It Works:** Adding b shifts Mean by $+b$ and leaves Variance/SD UNCHANGED. Multiplying by a scales Mean by a, SD by $|a|$, and Variance by a^2.

Valid Domain: All linear transformations of data. | **Where It Fails:** Non-linear transformations ($x^2, \log x$).

Easy Example: Add 5 to every item: Mean $+5$, SD unchanged.

Board-Level Example: Multiply every item by -3: $\sigma_{\text{new}} = 3\sigma_{\text{old}}$, $\text{Var}_{\text{new}} = 9\text{Var}_{\text{old}}$.

Advanced Example: Variance of $101, 102, \ldots, 110$ equals variance of $1, 2, \ldots, 10 = \frac{99}{12} = 8.25$!

[SMART SHORTCUT – 2. Variance of First n Consecutive Integers] **Core Idea & Why It Works:** For n numbers in A.P. with common difference d, Variance $= \frac{d^2(n^2-1)}{12}$.

Valid Domain: Any equally spaced A.P. of n terms. | **Where It Fails:** Repeated or unequally spaced data.

Easy Example: Variance of $2, 4, 6, 8, 10$ ($n = 5, d = 2$): $\frac{4(24)}{12} = 8$.

Board-Level Example: Solves NCERT Ex. 13.2 Q3 (first 10 multiples of 3) in 3 seconds: $\frac{9(99)}{12} = 74.25$.

Advanced Example: For $d = 1$, $\sigma^2 = \frac{n^2-1}{12}$.

[SMART SHORTCUT – 3. Zero-Variance Criterion] **Core Idea & Why It Works:** Variance $= 0$ (and $\text{MD} = 0, \sigma = 0$) if and only if ALL observations are identical.

Valid Domain: All datasets. | **Where It Fails:** Nothing.

Easy Example: SD of $7, 7, 7, 7, 7$ is 0.

Board-Level Example: Instant MCQ check.

Advanced Example: Variance is strictly positive whenever at least two values differ.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: Find the Mean Deviation about the mean for $4, 7, 8, 9, 10, 12, 13, 17$.

Given & Required: $n = 8$. | **Concept & Formula:** $\text{MD}(\bar{x})$.

Detailed Solution: $\bar{x} = \frac{80}{8} = 10$. Deviations $|x_i - 10|$: $6, 3, 2, 1, 0, 2, 3, 7$, sum $= 24$. $\text{MD} = \frac{24}{8} = 3$. **Final Answer:** 3

Fastest Alternative / Shortcut: Sum of $|x_i - 10|$ is 24.

SOLVED EXAMPLE 2 [Beginner]

Question: Find the Mean Deviation about the median for $3, 9, 5, 3, 12, 10, 18, 4, 7, 19, 21$.

Given & Required: $n = 11$ (odd). | **Concept & Formula:** Sort first!

Detailed Solution: Sorted: $3, 3, 4, 5, 7, 9, 10, 12, 18, 19, 21$. 6th term $M = 9$. Absolute deviations: $6, 6, 5, 4, 2, 0, 1, 3, 9, 10, 12$, sum $= 58$. $\text{MD}(M) = \frac{58}{11} = 5.27$. **Final Answer:** 5.27

Fastest Alternative / Shortcut: Always sort before picking M.

SOLVED EXAMPLE 3 [Beginner]

Question: Find the variance and standard deviation of $6, 7, 10, 12, 13, 4, 8, 12$.

Given & Required: $n = 8$. | **Concept & Formula:** $\sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n}$.

Detailed Solution: $\bar{x} = \frac{72}{8} = 9$. Squared deviations: $9, 4, 1, 9, 16, 25, 1, 9$, sum $= 74$. $\sigma^2 = \frac{74}{8} = 9.25$; $\sigma = \sqrt{9.25} \approx 3.04$. **Final Answer:** $\sigma^2 = 9.25$, $\sigma \approx 3.04$

Fastest Alternative / Shortcut: Integer $\bar{x} = 9$ makes direct method easy.

SOLVED EXAMPLE 4 [Beginner]

Question: If the variance of a dataset is 36 and each observation is doubled, find the new standard deviation.

Given & Required: $\sigma_{\text{old}} = \sqrt{36} = 6$, $a = 2$. | **Concept & Formula:** $\sigma_{\text{new}} = |a|\sigma_{\text{old}}$.

Detailed Solution: $\sigma_{\text{new}} = 2 \times 6 = 12$. **Final Answer:** 12

Fastest Alternative / Shortcut: Scale SD by 2 directly.

SOLVED EXAMPLE 5 [Intermediate]

Question: Find the variance of the first n natural numbers.

Given & Required: $1, 2, \ldots, n$. | **Concept & Formula:** $\frac{\sum x_i^2}{n} - \bar{x}^2$.

Detailed Solution: $\bar{x} = \frac{n+1}{2}$. $\sigma^2 = \frac{n(n+1)(2n+1)}{6n} - \frac{(n+1)^2}{4} = \frac{n+1}{12}[2(2n+1) - 3(n+1)] = \frac{n^2-1}{12}$. **Final Answer:** $\frac{n^2-1}{12}$

Fastest Alternative / Shortcut: Classic NCERT derivation.

SOLVED EXAMPLE 6 [Intermediate]

Question: Find the mean and variance for the first 10 multiples of 3.

Given & Required: $3, 6, 9, \ldots, 30$ ($n = 10, d = 3$). | **Concept & Formula:** A.P. scaling.

Detailed Solution: $\bar{x} = 3\left(\frac{11}{2}\right) = 16.5$. $\sigma^2 = 3^2\left(\frac{10^2-1}{12}\right) = \frac{9 \times 99}{12} = 74.25$. **Final Answer: Mean** $= 16.5$, $\sigma^2 = 74.25$

Fastest Alternative / Shortcut: Scale 1..10 by 3.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find the Mean Deviation about the mean for: $x_i : 5, 10, 15, 20, 25$; $f_i : 7, 4, 6, 3, 5$.

Given & Required: $N = 25$. | **Concept & Formula:** Discrete grouped MD.

Detailed Solution: $\sum f_i x_i = 35+40+90+60+125 = 350 \implies \bar{x} = \frac{350}{25} = 14$. $\sum f_i|x_i - 14| = 7(9)+4(4)+6(1)+3(6)+5(11) = 158$. MD $= \frac{158}{25} = 6.32$. **Final Answer:** 6.32

Fastest Alternative / Shortcut: Check $N = 25$.

SOLVED EXAMPLE 8 [Intermediate]

Question: The variance of 20 observations is 5. If each observation is multiplied by 2, find the new variance.

Given & Required: $\sigma^2 = 5, a = 2$. | **Concept & Formula:** $\text{Var}(aX) = a^2\text{Var}(X)$.

Detailed Solution: $\text{Var}_{\text{new}} = 2^2 \times 5 = 20$. **Final Answer:** 20

Fastest Alternative / Shortcut: Multiply variance by $a^2 = 4$.

SOLVED EXAMPLE 9 [Board Standard]

Question: Calculate Mean Deviation about the median for continuous data: Classes 0–10, 10–20, 20–30, 30–40, 40–50, 50–60 with frequencies $6, 7, 15, 16, 4, 2$.

Given & Required: $N = 50, h = 10$. | **Concept & Formula:** Grouped median + MD(M).

Detailed Solution: $N/2 = 25$; cf: $6, 13, 28, \cdots \implies$ Median class 20–30. $M = 20 + \frac{25-13}{15} \times 10 = 28$. Midpoints $5, 15, 25, 35, 45, 55$; $\sum f_i|x_i - 28| = 6(23) + 7(13) + 15(3) + 16(7) + 4(17) + 2(27) = 508$. MD($M$) $= \frac{508}{50} = 10.16$. **Final Answer:** 10.16

Fastest Alternative / Shortcut: cf before median class is 13.

SOLVED EXAMPLE 10 [Board Standard]

Question: Find the mean, variance, and standard deviation using shortcut method for: Classes 30–40, 40–50, 50–60, 60–70, 70–80, 80–90, 90–100 with f_i : 3, 7, 12, 15, 8, 3, 2.

Given & Required: $N = 50, A = 65, h = 10$. | **Concept & Formula:** Step-deviation.

Detailed Solution: u_i: $-3, -2, -1, 0, 1, 2, 3$. $\sum f_i u_i = -9-14-12+0+8+6+6 = -15$. $\sum f_i u_i^2 = 27+28+12+0+8+12+18 = 105$. $\bar{x} = 65 + 10(-\frac{15}{50}) = 62$. $\sigma^2 = 100[\frac{105}{50} - (-0.3)^2] = 100(2.1 - 0.09) = 201$. $\sigma = \sqrt{201} \approx 14.18$. **Final Answer: Mean** $= 62$**,** $\sigma^2 = 201$**,** $\sigma \approx 14.18$

Fastest Alternative / Shortcut: Centered $A = 65$ cancels most terms!

SOLVED EXAMPLE 11 [Board Standard]

Question: The mean and variance of 8 observations are 9 and 9.25. If six of the observations are $6, 7, 10, 12, 12, 13$, find the remaining two.

Given & Required: $n = 8, \bar{x} = 9, \sigma^2 = 9.25$. | **Concept & Formula:** $\sum x_i$ and $\sum x_i^2$.

Detailed Solution: $\sum x_i = 72$; sum of $6 = 60 \implies x + y = 12$. $\sum x_i^2 = 8(9.25 + 81) = 722$; sum of 6 squares $= 638 \implies x^2 + y^2 = 84$. Then $2xy = 144 - 84 = 60 \implies xy = 30$? Wait: $(x - y)^2 = 2(84) - 144 = 24$? Wait: $36 + 49 + 100 + 144 + 144 + 169 = 642$, so $x^2 + y^2 = 722 - 642 = 80 \implies 2xy = 144 - 80 = 64 \implies xy = 32 \implies 4, 8$. **Final Answer:** 4 **and** 8

Fastest Alternative / Shortcut: Check $4 + 8 = 12$ and $4^2 + 8^2 = 80$.

SOLVED EXAMPLE 12 [Board Standard]

Question: The mean and standard deviation of 100 observations were calculated as 40 and 5.1, respectively, by a student who took by mistake 50 instead of 40 for one observation. Find the correct mean and SD.

Given & Required: $n = 100$, wrong $\bar{x} = 40, \sigma = 5.1$. | **Concept & Formula:** Adjust $\sum x_i$ and $\sum x_i^2$.

Detailed Solution: Wrong $\sum x_i = 4000 \implies$ Correct $\sum x_i = 4000 - 50 + 40 = 3990 \implies \bar{x}_{\text{new}} = 39.9$. Wrong $\sum x_i^2 = 100(26.01 + 1600) = 162601 \implies$ Correct $\sum x_i^2 = 162601 - 2500 + 1600 = 161701$. Correct $\sigma^2 = 1617.01 - (39.9)^2 = 1617.01 - 1592.01 = 25 \implies \sigma = 5$. **Final Answer: Mean** $= 39.9$**,** $\sigma = 5$

Fastest Alternative / Shortcut: Clean integer $\sigma = 5$ confirms accuracy!

SOLVED EXAMPLE 13 [Board Standard]

Question: Given $\bar{x} = 18$, $\sum(x_i - 18)^2$? Wait: if for $n = 10$, $\sum x_i = 120$ and $\sum x_i^2 = 1690$, find SD.

Given & Required: $n = 10, \sum x_i = 120, \sum x_i^2 = 1690$. | **Concept & Formula:** Shortcut variance.

Detailed Solution: $\bar{x} = 12$. $\sigma^2 = \frac{1690}{10} - 12^2 = 169 - 144 = 25 \implies \sigma = 5$. **Final Answer:** $\sigma = 5$

Fastest Alternative / Shortcut: $169 - 144 = 25$.

SOLVED EXAMPLE 14 [Challenge]

Question: If the standard deviation of $x_1, x_2, \ldots, x_n$ is 3.5, find the standard deviation of $-2x_1 - 3, -2x_2 - 3, \ldots, -2x_n - 3$.

Given & Required: $\sigma_X = 3.5, Y = -2X - 3$. | **Concept & Formula:** $\sigma_Y = |a|\sigma_X$.

Detailed Solution: $\sigma_Y = |-2| \times 3.5 = 7$. **Final Answer:** 7

Fastest Alternative / Shortcut: SD is always non-negative ($|-2| = 2$).

SOLVED EXAMPLE 15 [Challenge]

Question: The mean of 5 observations is 4.4 and their variance is 8.24. If three observations are $1, 2, 6$, find the other two.

Given & Required: $n = 5, \bar{x} = 4.4, \sigma^2 = 8.24$. | **Concept & Formula:** $\sum x_i, \sum x_i^2$.

Detailed Solution: $\sum x_i = 22 \implies x + y = 13$. $\sum x_i^2 = 5(8.24 + 19.36) = 138$. $1 + 4 + 36 = 41 \implies x^2 + y^2 = 97$. $(x - y)^2 = 2(97) - 169 = 25 \implies x - y = 5 \implies 9, 4$. **Final Answer:** 4 **and** 9

Fastest Alternative / Shortcut: $9 + 4 = 13, 81 + 16 = 97$.

SOLVED EXAMPLE 16 [Challenge]

Question: Find the mean deviation about the mean for the first $2n+1$ terms of an A.P. $a, a+d, \ldots, a+2nd$.

Given & Required: Symmetric A.P. of $2n+1$ terms. | **Concept & Formula:** Mean = middle term $a+nd$.

Detailed Solution: Deviations are $nd, (n-1)d, \ldots, d, 0, d, \ldots, nd$. Sum $= 2d\frac{n(n+1)}{2} = n(n+1)d$. MD $= \frac{n(n+1)d}{2n+1}$. **Final Answer:** $\frac{n(n+1)d}{2n+1}$

Fastest Alternative / Shortcut: Test $n=1$ $(a, a+d, a+2d)$: $\frac{2d}{3}$.

G. Competency MCQ Mastery Section

Q1. If the variance of data is 121, the standard deviation is:

(A) ± 11 (B) 11 (C) 121 (D) 11^4

Answer: (B) SD is always the non-negative square root. *Fast Method:* $\sqrt{121} = 11$. *Distractor Note:* Picking ± 11.

Q2. If each observation is increased by 10, the variance:

(A) Increases by 10 (B) Increases by 100 (C) Remains unchanged (D) Multiplies by 10

Answer: (C) Shifting does not change spread. *Fast Method:* b cancels in $x_i - \bar{x}$. *Distractor Note:* Adding 10 or 100.

Q3. Variance of $5, 5, 5, 5, 5$ is:

(A) 5 (B) 25 (C) 0 (D) 1

Answer: (C) No variation among identical values. *Fast Method:* All equal $\implies \sigma^2 = 0$. *Distractor Note:* Answering 5.

Q4. Variance of the first 7 natural numbers $1, 2, \ldots, 7$ is:

(A) 2 (B) 4 (C) 8 (D) 12

Answer: (B) $\frac{7^2-1}{12} = \frac{48}{12} = 4$. *Fast Method:* $(n^2-1)/12$. *Distractor Note:* Taking SD $= 2$.

Q5. If $\sigma_X = 4$, then $\mathrm{Var}(3X+5)$ is:

(A) 12 (B) 17 (C) 144 (D) 36

Answer: (C) $\sigma_Y = 3 \times 4 = 12$, so $\mathrm{Var}(Y) = 12^2 = 144$. *Fast Method:* $a^2\sigma_X^2 = 9 \times 16 = 144$. *Distractor Note:* Answering 12 (that's SD) or $3 \times 16 = 48$.

Q6. Range of the data $14, 29, 8, 35, 19, 22$ is:

(A) 27 (B) 21 (C) 8 (D) 35

Answer: (A) $35 - 8 = 27$. *Fast Method:* Max − Min. *Distractor Note:* Subtracting adjacent ends $22 - 14$.

Q7. Sum of deviations of observations from their arithmetic mean $\sum(x_i - \bar{x})$ is always:

(A) Positive (B) Negative (C) Zero (D) Minimum

Answer: (C) By definition of mean, $\sum x_i = n\bar{x}$. *Fast Method:* Always 0. *Distractor Note:* Confusing with $\sum |x_i - M|$ being minimum.

Q8. Mean deviation is minimum when taken about the:

(A) Mean (B) Median (C) Mode (D) Origin

Answer: (B) $\sum |x_i - A|$ is minimized at $A =$ Median. *Fast Method:* MD minimum at Median; squared dev min at Mean. *Distractor Note:* Picking Mean.

Q9. If $\sum_{i=1}^{10} x_i = 60$ and $\sum_{i=1}^{10} x_i^2 = 400$, variance is:

(A) 4 (B) 2 (C) 16 (D) 36

Answer: (A) $\frac{400}{10} - 6^2 = 40 - 36 = 4$. *Fast Method:* $\frac{\sum x^2}{n} - \bar{x}^2$. *Distractor Note:* Forgetting to square $\bar{x} = 6$.

Q10. Mean deviation about mean for $3, 5, 7$ is:

(A) 0 (B) $4/3$ (C) 2 (D) $8/3$

Answer: (B) $\bar{x} = 5$; $\frac{2+0+2}{3} = \frac{4}{3}$. *Fast Method:* Sum $|x_i - 5| = 4$. *Distractor Note:* Dividing by 2.

Q11. If $u_i = \frac{x_i - 25}{10}$ and $\sigma_u = 3$, then σ_x is:

(A) 0.3 (B) 3 (C) 30 (D) 55

Answer: (C) $x_i = 10u_i + 25 \implies \sigma_x = 10\sigma_u = 30$. *Fast Method:* Multiply by $h = 10$. *Distractor Note:* Adding 25.

Q12. Which measure of dispersion is least affected by extreme values?

(A) Range (B) Mean Deviation about Median (C) Variance (D) Standard Deviation

Answer: (B) Median and absolute deviations resist outliers better than squared deviations or range. *Fast Method:* Range uses only extremes. *Distractor Note:* Picking Range.

H. Special Exam & MCQ Strategies

- Always choose A (assumed mean) as the middle class mark with the highest frequency—it keeps $\sum f_i u_i$ tiny.
- In 5-mark grouped-data questions, draw neat vertical columns with ruled borders and verify $N = \sum f_i$ before computing.
- Double-check whether the question asks for Variance (σ^2) or Standard Deviation (σ) at the end!

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Forgetting to sort observations before finding the Median. $\longrightarrow$ **Correction:** Always arrange data in ascending order first.

[COMMON MISTAKE] **Error:** Multiplying by h instead of h^2 in the step-deviation formula for Variance. $\longrightarrow$ **Correction:** Variance uses h^2; Standard Deviation uses $|h|$.

[COMMON MISTAKE] **Error:** Adding $(x_{\text{new}} - x_{\text{old}})^2$ when correcting $\sum x_i^2$. $\longrightarrow$ **Correction:** Update $\sum x_i^2$ by subtracting x_{old}^2 and adding x_{new}^2.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $\text{MD}(A) = \frac{\sum f_i|x_i - A|}{N}$; $\sigma^2 = \frac{\sum f_i x_i^2}{N} - \bar{x}^2$; Step-dev: $\sigma^2 = h^2[\frac{\sum f_i u_i^2}{N} - (\frac{\sum f_i u_i}{N})^2]$.

[QUICK REVISION] Key Theorems & Results: First n natural numbers: $\sigma^2 = \frac{n^2-1}{12}$; $y = ax + b \implies \sigma_y = |a|\sigma_x$, $\text{Var}(y) = a^2\text{Var}(x)$.

[QUICK REVISION] Shortcut Summary: Zero variance $\iff$ all items equal; $\sum(x_i - \bar{x}) = 0$; MD min about Median.

[QUICK REVISION] High-Frequency Exam Patterns: MD about Mean/Median, Step-deviation Variance/SD, 2 missing observations, Misread observation correction.

Last-Minute Revision Checklist:

- ☐ Sort data for Median
- ☐ Absolute value bars in MD
- ☐ h^2 for Variance, h for SD
- ☐ Square $\bar{x}$ in shortcut

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Range of $5, 12, 3, 19, 8$ is: (A) 14 (B) 16 (C) 19 (D) 3
2. If $\sigma = 7$, variance is: (A) 49 (B) $\sqrt{7}$ (C) 14 (D) 7
3. Mean deviation about mean of $2, 4, 6, 8$ is: (A) 2 (B) 4 (C) 1 (D) 2.5
4. SD of $10, 10, 10, 10$ is: (A) 10 (B) 0 (C) 1 (D) 100

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. Variance of first 5 natural numbers is: (A) 2 (B) 4 (C) 2.5 (D) 1
2. If $\text{Var}(X) = 9$, then $\text{Var}(2X - 7)$ is: (A) 18 (B) 11 (C) 36 (D) 29
3. For $n = 5$, $\sum x_i = 20, \sum x_i^2 = 100$. Variance is: (A) 4 (B) 2 (C) 16 (D) 20
4. Mean of 5 numbers is 6. If 10 is added to each, new mean and SD change by: (A) $+10, 0$ (B) $+10, +10$ (C) $0, +10$ (D) $0, 0$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Variance of $4, 8, 12, 16, 20$ is: (A) 16 (B) 32 (C) 8 (D) 4
2. Mean of 10 items is 12, SD is 3. Sum of squares $\sum x_i^2$ is: (A) 1530 (B) 1440 (C) 1470 (D) 1500
3. If $\text{Var}(X) = 4$, then $\text{Var}(-3X + 4)$ is: (A) -36 (B) 36 (C) 12 (D) 40
4. Mean deviation about median for $1, 2, 3, 4, 5, 6, 7$ is: (A) 2 (B) $12/7$ (C) $10/7$ (D) 1

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $19-3=16$. **Q2 (A):** $7^2=49$. **Q3 (A):** $\bar{x}=5$; $\frac{3+1+1+3}{4}=2$. **Q4 (B):** All values equal.
Board Std	**Q1 (A):** $(25-1)/12=2$. **Q2 (C):** $2^2\times 9=36$. **Q3 (A):** $100/5-4^2=20-16=4$. **Q4 (A):** Mean shifts by $+10$; SD is unchanged.
Challenge	**Q1 (B):** $d=4, n=5 \implies \frac{16(24)}{12}=32$. **Q2 (A):** $10(9+144)=1530$. **Q3 (B):** $(-3)^2\times 4=36$. **Q4 (B):** $M=4$; $(3+2+1+0+1+2+3)/7=12/7$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Range & MD about Mean/Median. Day 2: Variance & SD (ungrouped). Day 3: Step-deviation grouped data & correction problems.
- **How to Introduce the Concept:** Compare two batsmen with the same average 50: $(50,50,50)$ vs $(0,50,100)$ to motivate dispersion!
- **Difficult Areas & Remediation:** Forgetting to sort for Median and using $x^2_{\text{new}}-x^2_{\text{old}}$ in correction problems.
- **Blackboard Teaching Examples:** Walk through one complete 7-column step-deviation table neatly on the board.
- **Classroom Questions to Ask Students:** Why do we square $(x_i-\bar{x})$ in Variance instead of just averaging $(x_i-\bar{x})$?
- **Common Student Doubts & Answers:** Students ask why CV / equal-mean analysis isn't included; confirm it was rationalised out in NCERT 2026–27.
- **Homework & Revision Strategy:** Assign 2 MD tables, 2 step-deviation tables, and 2 observation-correction problems.

[TEACHER'S TIP] Neat table columns = guaranteed 5/5 marks in CBSE/RBSE Statistics!

UNIT V: STATISTICS AND PROBABILITY TYBA COACHING CENTRE

Chapter 14: Probability

Sample Space, Events, Mutually Exclusive & Exhaustive Events & Axiomatic Probability (2026–27)

A. Chapter Introduction & Examination Relevance

Probability quantifies uncertainty rigorously. Moving beyond Class 10's simple empirical counts, Chapter 14 introduces Kolmogorov's axiomatic probability, algebra of events using sets (Ch. 1), and counting techniques (Ch. 6).

- **What You Will Learn:** Random experiments, sample space S, events (sure, impossible, simple, compound), 'A or B' ($A \cup B$), 'A and B' ($A \cap B$), 'A but not B' ($A - B$), mutually exclusive and exhaustive events, axiomatic probability, and $P(A \cup B)$ addition law.
- **Why This Chapter Matters:** Probability is the direct bridge to Class 12 Conditional Probability, Bayes' Theorem, and Random Variables (8/80 marks in Class 12).
- **Real-Life Applications:** Weather forecasting, insurance risk pricing, medical test reliability, machine learning predictions, and quality-control sampling.
- **Cross-Chapter Connections:** Directly combines Chapter 1 (Set operations & De Morgan's Laws) and Chapter 6 (Combinations ${}^nC_r$ for cards/balls).
- **Board & Competitive Relevance:** Unit V carries 12/80 marks. Chapter 14 contributes 5–7 marks via 1M MCQs on $P(A \cup B)$ / mutually exclusive events and 3M/5M card, dice, and committee probability problems.

B. Complete Concept Explanation & Derivations

1. Sample Space & Types of Events (Set Language)

The **sample space** S is the set of all possible outcomes of a random experiment. Any subset $E \subseteq S$ is an **event**. Using Chapter 1 set algebra: 'A or B' is $A \cup B$; 'A and B' is $A \cap B$; 'not A' is $A' = S - A$; 'A but not B' is $A - B = A \cap B'$; 'neither A nor B' is $(A \cup B)' = A' \cap B'$.

[CONCEPT MASTER – 1. Sample Space & Types of Events (Set Language)] Two events A and B are **Mutually Exclusive** if $A \cap B = \emptyset$. Events $E_1, \ldots, E_k$ are **Exhaustive** if $E_1 \cup \cdots \cup E_k = S$.

2. Axiomatic Definition of Probability

For a finite sample space $S = \{\omega_1, \ldots, \omega_n\}$: (1) $0 \le P(\omega_i) \le 1$; (2) $\sum P(\omega_i) = P(S) = 1$; (3) $P(A) = \sum_{\omega_i \in A} P(\omega_i)$. For **equally likely** outcomes, $P(A) = \frac{n(A)}{n(S)}$.

[CONCEPT MASTER – 2. Axiomatic Definition of Probability] Always verify $0 \le P(E) \le 1$. If your calculation gives $P(E) > 1$ or $P(E) < 0$, an arithmetic or counting error occurred!

3. Addition Theorem & Complementary Events

For any two events $A, B \subseteq S$: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. If A and B are mutually exclusive ($P(A \cap B) = 0$), then $P(A \cup B) = P(A) + P(B)$. Also $P(A') = 1 - P(A)$ and $P(A - B) = P(A) - P(A \cap B)$.

[CONCEPT MASTER – 3. Addition Theorem & Complementary Events] Complement Trick: Whenever a problem asks for 'at least one', compute $1 - P(\text{none})$!

4. Combinatorial Probability (${}^nC_r$ with Cards, Balls & Committees)

When selecting r items at random from N items without replacement, the sample space has $n(S) = {}^NC_r$ equally likely outcomes. For example, drawing 2 cards from a 52-card deck gives $n(S) = {}^{52}C_2 = 1326$.

[CONCEPT MASTER – 4. Combinatorial Probability (${}^nC_r$ with Cards, Balls & Committees)] Remember standard 52-card deck counts: 4 suits (13 each), 26 red / 26 black, 12 face cards (J, Q, K of each suit), 4 Aces.

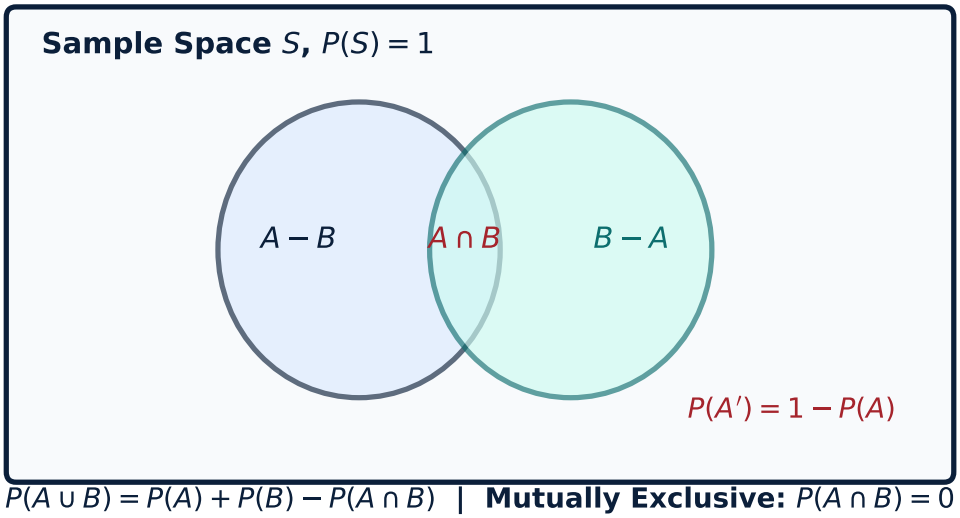


Figure 1: **Visual Model:** Sample space S ($P(S) = 1$) illustrating $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, complement $P(A') = 1 - P(A)$, and disjoint events.

C. Complete Formula Bank

[FORMULA BANK – 1. Equally Likely Probability] **Expression:** $P(E) = \frac{n(E)}{n(S)}$
Symbols: $n(E)$: favourable outcomes, $n(S)$: total outcomes.
Conditions & When to Use: Outcomes in S must be equally likely; $0 \leq P(E) \leq 1$.
Solved Mini-Example: Getting sum 7 on 2 dice: $\frac{6}{36} = \frac{1}{6}$.
Common Mistake: Assuming unequal outcomes (like sum of 2 dice 2..12) are equally likely.

[FORMULA BANK – 2. Addition Law of Probability] **Expression:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Symbols: $A, B \subseteq S$.
Conditions & When to Use: Universal for all events.
Solved Mini-Example: $P(A) = 0.5, P(B) = 0.4, P(A \cap B) = 0.2 \implies P(A \cup B) = 0.7$.
Common Mistake: Forgetting to subtract $P(A \cap B)$ when events overlap.

[FORMULA BANK – 3. Complementary & De Morgan Probability] **Expression:** $P(A') = 1 - P(A)$; $P(A' \cap B') = 1 - P(A \cup B)$
Symbols: A': complement of A.
Conditions & When to Use: Always valid.
Solved Mini-Example: $P(\text{neither } A \text{ nor } B) = 1 - P(A \cup B)$.
Common Mistake: Writing $P(A' \cap B') = 1 - P(A \cap B)$.

[FORMULA BANK – 4. Mutually Exclusive & Exhaustive] **Expression:** ME: $P(A \cap B) = 0$; Exhaustive: $P(A \cup B) = 1$
Symbols: A, B: events.
Conditions & When to Use: If both ME and Exhaustive, $P(A) + P(B) = 1$.
Solved Mini-Example: Even and Odd on a die are both ME and Exhaustive.
Common Mistake: Confusing 'mutually exclusive' with 'independent' (which is Class 12).

[FORMULA BANK – 5. Difference Event Probability] **Expression:** $P(A-B)=P(A\cap B')=P(A)-P(A\cap B)$
Symbols: $A-B$: A occurs and B does not.
Conditions & When to Use: Any events A, B.
Solved Mini-Example: If $P(A)=0.6, P(A\cap B)=0.25$, then $P(A \text{ only})=0.35$.
Common Mistake: Computing $P(A)-P(B)$.

[FORMULA BANK – 6. Exactly One of A or B] **Expression:** $P(\text{exactly one})=P(A)+P(B)-2P(A\cap B)$
Symbols: Symmetric difference $A\triangle B$.
Conditions & When to Use: Any events A, B.
Solved Mini-Example: $P(A\cup B)-P(A\cap B)$.
Common Mistake: Subtracting $P(A\cap B)$ only once.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE, RBSE, and JEE/CUET Foundation problem structures.

Pattern 1: Dice, Coins & Sample Space Events

- **Identification Cue:** Two dice are thrown or three coins are tossed; find $P(\text{at least 2 heads})$ or $P(\text{sum } \geq 10)$. | **Concept & Formula:** List S (2^n or $6^2=36$) and count $n(E)$.
- **Standard Step-by-Step:** Write S or count systematically by outcome sums, then divide $n(E)/n(S)$.
- **Fastest Valid Method:** Use 2-dice sum triangle: Sum 2..12 has counts $1,2,3,4,5,6,5,4,3,2,1$! | **Alternative:** Complement $1-P(E')$.
- **Common Trap:** Missing (a,b) vs (b,a) distinct pairs in 2 dice.
- **Practice Drill:** Ex. 1 & Ex. 5.

Pattern 2: Venn / Addition Law & De Morgan Problems

- **Identification Cue:** Given $P(A), P(B), P(A\cap B)$ (or $P(A\cup B)$), find $P(\text{neither})$, $P(A \text{ only})$, $P(\text{not } A \text{ or not } B)$. | **Concept & Formula:** Set-probability identities.
- **Standard Step-by-Step:** Use $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ and De Morgan's Laws.
- **Fastest Valid Method:** Draw a 2-circle Venn diagram with probabilities inside each region. | **Alternative:** Algebraic substitution.
- **Common Trap:** Mixing up $P(A'\cup B')=1-P(A\cap B)$ with $P(A'\cap B')=1-P(A\cup B)$.
- **Practice Drill:** Ex. 7 & Ex. 9.

Pattern 3: Combinatorial Probability (Cards, Balls, Committees)

- **Identification Cue:** 4 cards drawn from 52, or 2 balls from 5 red and 3 blue; find $P(\text{at least 1 red})$. | **Concept & Formula:** $\frac{{}^mC_r\times\cdots}{{}^NC_k}$ and $1-P(\text{none})$.
- **Standard Step-by-Step:** Compute $n(S)={}^NC_k$; for 'at least 1', subtract $P(0 \text{ of that type})$ from 1.
- **Fastest Valid Method:** $1-\frac{{}^{N-m}C_k}{{}^NC_k}$ saves multi-case addition. | **Alternative:** Sum disjoint cases.
- **Common Trap:** Forgetting to multiply/divide combinations properly.
- **Practice Drill:** Ex. 10 & Ex. 12.

Pattern 4: Lottery / Word-Arrangement / Leap-Year Probability

- **Identification Cue:** 53 Sundays in a leap year, or letters of ASSASSINATION arranged so all S's are together. | **Concept & Formula:** Total outcomes vs favourable.
- **Standard Step-by-Step:** For leap year (366 days = 52 weeks + 2 extra days), list the 7 consecutive day-pairs.
- **Fastest Valid Method:** Leap year: $2/7$; Non-leap year: $1/7$. | **Alternative:** Permutations ratio.
- **Common Trap:** Writing $2/366$ for 53 Sundays in a leap year!
- **Practice Drill:** Ex. 13 & Ex. 15.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Two-Dice Sum Frequency Pyramid] **Core Idea & Why It Works:** For 2 fair dice ($n(S) = 36$), the number of ways to get sum $k \in \{2..12\}$ is $6 - |k - 7|$: $1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1$.
Valid Domain: All 2-dice sum problems. | **Where It Fails:** 3 dice or product of dice.
Easy Example: Sum $= 9 \implies 6 - |9 - 7| = 4$ ways $\implies 4/36 = 1/9$.
Board-Level Example: Sum ≥ 10 $(10, 11, 12)$: $3 + 2 + 1 = 6$ ways $\implies 1/6$.
Advanced Example: Sum ≤ 4: $1 + 2 + 3 = 6$ ways $\implies 1/6$.

[SMART SHORTCUT – 2. 'At Least One' = $1 - P$(None)] **Core Idea & Why It Works:** Whenever 'at least one' appears, compute the single complementary case 'zero' and subtract from 1.
Valid Domain: All finite probability problems. | **Where It Fails:** Nothing.
Easy Example: 3 coins, at least 1 head: $1 - P(\text{TTT}) = 1 - \frac{1}{8} = \frac{7}{8}$.
Board-Level Example: Lottery: 10 prizes, 25 blanks; $P(\text{at least 1 prize in 2 tickets}) = 1 - \frac{^{25}C_2}{^{35}C_2}$.
Advanced Example: Cuts 4-case sums down to 1 line!

[SMART SHORTCUT – 3. 53 Sundays Shortcut] **Core Idea & Why It Works:** Non-leap year $(365 = 52 \times 7 + 1)$: $P(53 \text{ of any day}) = \frac{1}{7}$. Leap year $(366 = 52 \times 7 + 2)$: $P(53 \text{ of any day}) = \frac{2}{7}$.
Valid Domain: Any single weekday. | **Where It Fails:** If asked for '53 Sundays AND 53 Mondays' in a leap year (then $\frac{1}{7}$).
Easy Example: 53 Fridays in leap year $= \frac{2}{7}$.
Board-Level Example: 53 Sundays in non-leap year $= \frac{1}{7}$.
Advanced Example: 53 Sundays OR 53 Mondays in leap year $= \frac{3}{7}$.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: A coin is tossed three times. Find the probability of getting at least 2 heads.
Given & Required: $n(S) = 2^3 = 8$. | **Concept & Formula:** Listing outcomes.
Detailed Solution: Favourable: {HHH, HHT, HTH, THH} (4 outcomes). $P = \frac{4}{8} = \frac{1}{2}$. **Final Answer:** $\frac{1}{2}$
Fastest Alternative / Shortcut: By symmetry, ≥ 2 heads and ≥ 2 tails split 8 outcomes 4-and-4!

SOLVED EXAMPLE 2 [Beginner]
Question: A die is thrown. Find the probability of getting a prime number.
Given & Required: $S = \{1, 2, 3, 4, 5, 6\}$. | **Concept & Formula:** $E = \{2, 3, 5\}$.
Detailed Solution: $P(E) = \frac{3}{6} = \frac{1}{2}$. **Final Answer:** $\frac{1}{2}$
Fastest Alternative / Shortcut: 1 is NOT prime.

SOLVED EXAMPLE 3 [Beginner]
Question: If $P(E) = 0.05$, what is the probability of 'not E'?
Given & Required: $P(E) = 0.05$. | **Concept & Formula:** $P(E') = 1 - P(E)$.
Detailed Solution: $P(E') = 1 - 0.05 = 0.95$. **Final Answer:** 0.95
Fastest Alternative / Shortcut: $1 - 0.05$.

SOLVED EXAMPLE 4 [Beginner]

Question: One card is drawn from a well-shuffled deck of 52 cards. Find the probability that it is a face card.

Given & Required: 12 face cards (4J, 4Q, 4K). | **Concept & Formula:** $n(E)/n(S)$.

Detailed Solution: $P = \frac{12}{52} = \frac{3}{13}$. **Final Answer:** $\frac{3}{13}$

Fastest Alternative / Shortcut: 3 face cards per 13-card suit.

SOLVED EXAMPLE 5 [Intermediate]

Question: Two dice are thrown simultaneously. Find the probability of getting a sum of 8 or a doublet.

Given & Required: $n(S) = 36$. | **Concept & Formula:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Detailed Solution: Sum 8: $(2,6), (3,5), (4,4), (5,3), (6,2)$ (5). Doublets: 6. Overlap: $(4,4)$ (1). Total $= 5 + 6 - 1 = 10$. $P = \frac{10}{36} = \frac{5}{18}$. **Final Answer:** $\frac{5}{18}$

Fastest Alternative / Shortcut: Subtract $(4,4)$ once!

SOLVED EXAMPLE 6 [Intermediate]

Question: If $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$, find $P(A$ or $B)$ if A and B are mutually exclusive.

Given & Required: $P(A \cap B) = 0$. | **Concept & Formula:** Disjoint addition.

Detailed Solution: $P(A \cup B) = \frac{3}{5} + \frac{1}{5} = \frac{4}{5}$. **Final Answer:** $\frac{4}{5}$

Fastest Alternative / Shortcut: Direct sum for ME events.

SOLVED EXAMPLE 7 [Intermediate]

Question: In a class of 60 students, 30 opted for NCC, 32 for NSS, and 24 for both. Find the probability that a student opted for neither NCC nor NSS.

Given & Required: $n(U) = 60, n(A) = 30, n(B) = 32, n(A \cap B) = 24$. | **Concept & Formula:** $P(A' \cap B') = 1 - P(A \cup B)$.

Detailed Solution: $n(A \cup B) = 30 + 32 - 24 = 38$. Neither $= 60 - 38 = 22$. $P = \frac{22}{60} = \frac{11}{30}$. **Final Answer:** $\frac{11}{30}$

Fastest Alternative / Shortcut: Venn count $60 - 38 = 22$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Check whether $P(A) = 0.5, P(B) = 0.7, P(A \cap B) = 0.6$ is consistently defined.

Given & Required: Three probabilities. | **Concept & Formula:** $P(A \cap B) \leq \min(P(A), P(B))$.

Detailed Solution: Impossible, because $P(A \cap B) = 0.6 > 0.5 = P(A)$. **Final Answer: Not consistent**

Fastest Alternative / Shortcut: Intersection cannot exceed either event.

SOLVED EXAMPLE 9 [Board Standard]

Question: If A and B are events such that $P(A) = 0.42$, $P(B) = 0.48$, and $P(A$ and $B) = 0.16$, find (i) P(not A), (ii) $P(A$ or $B)$, (iii) P(neither A nor B).

Given & Required: $P(A) = 0.42, P(B) = 0.48, P(A \cap B) = 0.16$. | **Concept & Formula:** Addition & De Morgan laws.

Detailed Solution: (i) $1 - 0.42 = 0.58$; (ii) $0.42 + 0.48 - 0.16 = 0.74$; (iii) $1 - 0.74 = 0.26$. **Final Answer:** $0.58, 0.74, 0.26$

Fastest Alternative / Shortcut: Compute $P(A \cup B)$ first.

SOLVED EXAMPLE 10 [Board Standard]

Question: A bag contains 9 discs of which 4 are red, 3 are blue, and 2 are yellow. A disc is drawn at random. Find the probability that it is (i) red or blue, (ii) not blue.

Given & Required: Total $= 9$. | **Concept & Formula:** Disjoint events.

Detailed Solution: (i) $\frac{4+3}{9} = \frac{7}{9}$; (ii) $1 - \frac{3}{9} = \frac{6}{9} = \frac{2}{3}$. **Final Answer:** $\frac{7}{9}$ **and** $\frac{2}{3}$

Fastest Alternative / Shortcut: Direct counts.

SOLVED EXAMPLE 11 [Board Standard]

Question: 4 cards are drawn from a 52-card deck. Find the probability of obtaining 3 diamonds and 1 spade.

Given & Required: 13 diamonds, 13 spades. | **Concept & Formula:** ${}^nC_r$ ratio.

Detailed Solution: $P = \frac{{}^{13}C_3 \times {}^{13}C_1}{{}^{52}C_4} = \frac{286 \times 13}{270725} = \frac{286}{20825}$. **Final Answer:** $\frac{286}{20825}$

Fastest Alternative / Shortcut: Cancel 13 with 52 in ${}^{52}C_4$.

SOLVED EXAMPLE 12 [Board Standard]

Question: A committee of two persons is selected from 2 men and 2 women. Find the probability that the committee has (i) no man, (ii) 1 man, (iii) 2 men.

Given & Required: 2M, 2W, select 2. | **Concept & Formula:** Combinations.

Detailed Solution: $n(S) = {}^4C_2 = 6$. (i) $\frac{{}^2C_2}{6} = \frac{1}{6}$; (ii) $\frac{{}^2C_1 . {}^2C_1}{6} = \frac{4}{6} = \frac{2}{3}$; (iii) $\frac{{}^2C_2}{6} = \frac{1}{6}$. **Final Answer:** $\frac{1}{6}, \frac{2}{3}, \frac{1}{6}$

Fastest Alternative / Shortcut: Note $\frac{1}{6} + \frac{4}{6} + \frac{1}{6} = 1$ (exhaustive!).

SOLVED EXAMPLE 13 [Board Standard]

Question: In a lottery, a person chooses 6 different natural numbers from 1 to 20; if these match the 6 drawn numbers, he wins. Find the probability of winning.

Given & Required: Order doesn't matter. | **Concept & Formula:** ${}^{20}C_6$.

Detailed Solution: $n(S) = {}^{20}C_6 = 38760$. Only 1 winning combination, so $P = \frac{1}{38760}$. **Final Answer:** $\frac{1}{38760}$

Fastest Alternative / Shortcut: $1/{}^{20}C_6$.

SOLVED EXAMPLE 14 [Challenge]

Question: If 4-digit numbers greater than 5000 are randomly formed from digits $0, 1, 3, 5, 7$ without repetition, find the probability of forming a number divisible by 5.

Given & Required: Digits $\{0, 1, 3, 5, 7\}$, > 5000, no repeat. | **Concept & Formula:** Restricted counting.

Detailed Solution: First digit $\in \{5, 7\}$ (2 choices), remaining 3 from 4 digits $= 2 \times {}^4P_3 = 48$ numbers. Divisible by 5: (a) starts with 5, ends with 0: $1 \times 3 \times 2 \times 1 = 6$; (b) starts with 7, ends with 0 or 5: $1 \times 3 \times 2 \times 2 = 12$. Total $= 18$. $P = \frac{18}{48} = \frac{3}{8}$. **Final Answer:** $\frac{3}{8}$

Fastest Alternative / Shortcut: Split by first digit 5 vs 7.

SOLVED EXAMPLE 15 [Challenge]

Question: Find the probability that when a hand of 7 cards is drawn from a 52-card deck, it contains all Kings.

Given & Required: 4 Kings, 48 others, draw 7. | **Concept & Formula:** Select 4 Kings and 3 others.

Detailed Solution: $P = \frac{{}^4C_4 \times {}^{48}C_3}{{}^{52}C_7} = \frac{1}{7735}$. **Final Answer:** $\frac{1}{7735}$

Fastest Alternative / Shortcut: Cancel factorials cleanly.

SOLVED EXAMPLE 16 [Challenge]

Question: Out of 100 students, two sections of 40 and 60 are formed. If you and your friend are among the 100, find the probability that you both enter the same section.

Given & Required: 100 students $\to$ 40 and 60. | **Concept & Formula:** Both in 40 OR both in 60.

Detailed Solution: $P = \frac{{}^{98}C_{38} + {}^{98}C_{58}}{{}^{100}C_{40}} = \frac{40 \times 39 + 60 \times 59}{100 \times 99} = \frac{1560 + 3540}{9900} = \frac{5100}{9900} = \frac{17}{33}$. **Final Answer:** $\frac{17}{33}$

Fastest Alternative / Shortcut: $\frac{40}{100} \cdot \frac{39}{99} + \frac{60}{100} \cdot \frac{59}{99} = \frac{17}{33}$.

G. Competency MCQ Mastery Section

Q1. If $P(A) = 0.4, P(B) = 0.5, P(A \cap B) = 0.2$, then $P(A \cup B)$ is:

(A) 0.7 (B) 0.9 (C) 0.3 (D) 1.1

Answer: (A) $0.4 + 0.5 - 0.2 = 0.7$. *Fast Method:* Addition law. *Distractor Note:* Adding $0.4 + 0.5 = 0.9$.

Q2. If A and B are mutually exclusive events with $P(A) = 0.3, P(B) = 0.4$, then $P(A \cap B)$ is:

(A) 0.12 (B) 0.7 (C) 0 (D) 0.1

Answer: (C) Mutually exclusive means $A \cap B = \emptyset \implies P = 0$. *Fast Method:* Disjoint = 0 overlap. *Distractor Note:* Multiplying 0.3×0.4 (that's independent!).

Q3. Probability of getting 53 Sundays in a leap year is:

(A) $1/7$ (B) $2/7$ (C) $53/366$ (D) $1/366$

Answer: (B) 2 extra days out of 7 pairs contain Sunday. *Fast Method:* 2 extra days $\implies 2/7$. *Distractor Note:* Picking $1/7$ (non-leap).

Q4. Probability of getting a sum of 9 on two dice is:

(A) $1/9$ (B) $1/6$ (C) $5/36$ (D) $1/12$

Answer: (A) 4 ways: $(3,6),(4,5),(5,4),(6,3)$ out of 36. *Fast Method:* $(6-|9-7|)/36 = 4/36$. *Distractor Note:* Missing reverse pairs.

Q5. If $P(A) = 0.6, P(B) = 0.5, P(A \cup B) = 0.8$, then P(neither A nor B) is:

(A) 0.2 (B) 0.3 (C) 0.1 (D) 0.7

Answer: (A) $1 - P(A \cup B) = 1 - 0.8 = 0.2$. *Fast Method:* $1 - 0.8$. *Distractor Note:* Computing $P(A \cap B) = 0.3$.

Q6. Number of outcomes when 4 coins are tossed simultaneously is:

(A) 8 (B) 12 (C) 16 (D) 32

Answer: (C) $2^4 = 16$. *Fast Method:* 2^n. *Distractor Note:* Multiplying $4 \times 2 = 8$.

Q7. Probability of drawing a red king from a 52-card deck is:

(A) $1/13$ (B) $1/26$ (C) $1/52$ (D) $2/13$

Answer: (B) 2 red kings out of $52 = 1/26$. *Fast Method:* Hearts King + Diamonds King. *Distractor Note:* Using 4 kings $(1/13)$.

Q8. If $P(A) = 0.35$, then $P(A')$ is:

(A) 0.65 (B) 0.35 (C) 0.75 (D) 1.35

Answer: (A) $1 - 0.35 = 0.65$. *Fast Method:* Complement. *Distractor Note:* Arithmetic slip.

Q9. Probability of getting at least one tail when 2 coins are tossed is:

(A) $1/4$ (B) $1/2$ (C) $3/4$ (D) 1

Answer: (C) $1 - P(\text{HH}) = 1 - 1/4 = 3/4$. *Fast Method:* Complement of HH. *Distractor Note:* Picking $1/2$.

Q10. For an impossible event $\emptyset$, $P(\emptyset)$ is:

(A) 1 (B) 0 (C) -1 (D) Undefined

Answer: (B) Axiomatic probability of $\emptyset$ is 0. *Fast Method:* 0. *Distractor Note:* Picking 1 (sure event).

Q11. If $P(A) = 0.5, P(B) = 0.4, P(A \cap B) = 0.1$, then $P(A - B)$ is:

(A) 0.4 (B) 0.1 (C) 0.3 (D) 0.8

Answer: (A) $P(A) - P(A \cap B) = 0.5 - 0.1 = 0.4$. *Fast Method:* A only $= 0.5 - 0.1$. *Distractor Note:* Doing $P(A) - P(B) = 0.1$.

Q12. If E_1, E_2 are mutually exclusive and exhaustive and $P(E_1) = 2P(E_2)$, then $P(E_1)$ is:

(A) $1/3$ (B) $2/3$ (C) $1/2$ (D) $3/4$

Answer: (B) $2p + p = 1 \implies p = 1/3 \implies P(E_1) = 2/3$. *Fast Method:* Ratio $2:1$ of total 1. *Distractor Note:* Picking $1/3$ $(P(E_2))$.

H. Special Exam & MCQ Strategies

- Translate English event phrases directly into Set symbols: 'or' $\to \cup$, 'and' $\to \cap$, 'not' $\to'$, 'neither-nor' $\to (A \cup B)'$.
- Always check $0 \leq P(E) \leq 1$ after every calculation as a sanity check.

- Use $1 - P(\text{none})$ whenever 'at least one' appears in coin, card, or ball problems.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Confusing Mutually Exclusive ($P(A \cap B) = 0$) with Independent ($P(A \cap B) = P(A)P(B)$). $\longrightarrow$ **Correction:** In Class 11, Mutually Exclusive ALWAYS means $A \cap B = \emptyset$, so $P(A \cap B) = 0$.

[COMMON MISTAKE] **Error:** Writing $P(A' \cap B') = 1 - P(A \cap B)$. $\longrightarrow$ **Correction:** By De Morgan's Law, $A' \cap B' = (A \cup B)'$, so $P(A' \cap B') = 1 - P(A \cup B)$.

[COMMON MISTAKE] **Error:** Treating unordered card/committee selections with ${}^nP_r$ instead of ${}^nC_r$. $\longrightarrow$ **Correction:** Drawing cards or selecting balls simultaneously uses combinations ${}^nC_r$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $P(E) = \frac{n(E)}{n(S)}$; $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; $P(A') = 1 - P(A)$; $P(A - B) = P(A) - P(A \cap B)$.

[QUICK REVISION] Key Theorems & Results: ME: $P(A \cap B) = 0$; Exhaustive: $P(A \cup B) = 1$; De Morgan: $P(A' \cap B') = 1 - P(A \cup B)$.

[QUICK REVISION] Shortcut Summary: 2-dice sum k: $6 - |k - 7|$ ways; 'At least 1' $= 1 - P(\text{none})$; Leap year 53 Sundays $= 2/7$.

[QUICK REVISION] High-Frequency Exam Patterns: Dice/coin sample space, Venn $P(A \cup B)$ & De Morgan, Card/ball ${}^nC_r$ selection, Leap-year/digit probability.

Last-Minute Revision Checklist:

- ☐ $0 \leq P(E) \leq 1$
- ☐ ME means $P(A \cap B) = 0$
- ☐ De Morgan $A' \cap B' = (A \cup B)'$
- ☐ Complement for 'at least 1'

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. If $P(A) = 3/8$, then $P(A')$ is: (A) $3/8$ (B) $5/8$ (C) 1 (D) 0
2. Probability of getting a head on a fair coin is: (A) 0 (B) 1 (C) $1/2$ (D) $1/4$
3. Probability of drawing an Ace from 52 cards is: (A) $1/52$ (B) $1/13$ (C) $1/4$ (D) $4/13$
4. Sum of probabilities of all elementary events in S is: (A) 0 (B) 1 (C) $1/2$ (D) n

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. If $P(A) = 0.5, P(B) = 0.3, P(A \cap B) = 0.1$, find $P(A \cup B)$: (A) 0.7 (B) 0.8 (C) 0.9 (D) 0.6
2. Probability of sum 7 on two dice is: (A) $1/6$ (B) $1/12$ (C) $5/36$ (D) $7/36$
3. 2 balls drawn from 3 red and 4 black. $P(\text{both red})$: (A) $1/7$ (B) $3/7$ (C) $2/7$ (D) $6/49$
4. Probability of 53 Mondays in a non-leap year: (A) $1/7$ (B) $2/7$ (C) $1/365$ (D) $53/365$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Letters of ASSASSINATION are arranged. $P(\text{all S's together})$: (A) $2/143$ (B) $1/143$ (C) $4/143$ (D) $6/143$
2. If $P(A \cup B) = 0.7, P(A \cap B) = 0.2$, then $P(A') + P(B')$ is: (A) 0.9 (B) 1.1 (C) 1.3 (D) 0.5
3. 3 vertices of a regular hexagon are chosen. $P(\text{equilateral triangle})$: (A) $1/10$ (B) $1/5$ (C) $3/10$ (D) $1/20$
4. If A, B are ME events with $P(A) = 1/3, P(B) = 1/4$, then $P(A' \cap B')$ is: (A) $5/12$ (B) $7/12$ (C) $1/12$ (D) $11/12$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $1-3/8=5/8$. **Q2 (C):** $1/2$. **Q3 (B):** $4/52=1/13$. **Q4 (B):** $P(S)=1$.
Board Std	**Q1 (A):** $0.5+0.3-0.1=0.7$. **Q2 (A):** $6/36=1/6$. **Q3 (A):** ${}^3C_2/{}^7C_2=3/21=1/7$. **Q4 (A):** 1 extra day $\implies 1/7$.
Challenge	**Q1 (A):** 13 letters (4 S). Tie 4 S's $\implies 10!/13!\times 4!=\frac{4!}{13\times12\times11}=\frac{2}{143}$. **Q2 (B):** $2-(P(A)+P(B))=2-(0.7+0.2)=1.1$. **Q3 (A):** 2 equilateral triangles out of ${}^6C_3=20 \implies 2/20=1/10$. **Q4 (A):** $1-(1/3+1/4)=1-7/12=5/12$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Sample space, events as sets. Day 2: Axioms & $P(A\cup B)$. Day 3: Combinatorial & word/leap-year problems.
- **How to Introduce the Concept:** Ask: 'If you roll 2 dice, are sums 2 and 7 equally likely?' Let them list ways to get 7 vs 2!
- **Difficult Areas & Remediation:** De Morgan's laws in probability and distinguishing ME from independent.
- **Blackboard Teaching Examples:** Draw the 2-dice 6×6 grid and circle the diagonal of sum $=7$.
- **Classroom Questions to Ask Students:** Why can't $P(A\cap B)$ ever be greater than $P(A)$?
- **Common Student Doubts & Answers:** Students ask about Conditional Probability and Bayes' Theorem; clarify those start in Class 12 Chapter 13.
- **Homework & Revision Strategy:** Assign 4 dice/coin problems, 4 Venn/De Morgan problems, and 4 card/ball ${}^nC_r$ problems.

[TEACHER'S TIP] Remind students that Probability = Chapter 1 (Sets) + Chapter 6 (Counting)!

BACK MATTER: COMPLETE CLASS 11 REVISION TYBA COACHING CENTRE

Chapter BM: Master Formula Index, Shortcut Bank & 80-Mark Sample Paper

TYBA Coaching Centre, Ajmer | CBSE (Code 041) & RBSE (2026–27) Final Exam Readiness

1. Master Formula Index (All 14 Chapters at a Glance)

Chapter	Essential Master Formulas
01. Sets	$n(P(A)) = 2^n$; $n(A \cup B) = n(A) + n(B) - n(A \cap B)$; $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$.
02. Relations & Functions	$n(A \times B) = pq$; Total Relations $= 2^{pq}$; $\lfloor x \rfloor \le x < \lfloor x \rfloor + 1$; $\text{sgn}(x) = \lvert x \rvert / x$ $(x \neq 0)$.
03. Trig Functions	$l = r\theta$ (θ in rad); $\sin 2x = 2\sin x \cos x$; $\cos 2x = 1 - 2\sin^2 x = 2\cos^2 x - 1$; $\sin 3x = 3s - 4s^3$; $\cos 3x = 4c^3 - 3c$; C–D sums.
04. Complex Numbers	$i^2 = -1, i^4 = 1$; $\lvert z \rvert = \sqrt{a^2 + b^2}$; $z\bar{z} = \lvert z \rvert^2$; $z^{-1} = \frac{\bar{z}}{\lvert z \rvert^2}$; $\lvert z_1 z_2 \rvert = \lvert z_1 \rvert \lvert z_2 \rvert$.
05. Linear Inequalities	Negative scale flips $<\leftrightarrow>$; $\lvert x - a \rvert < r \iff (a - r, a + r)$; $\lvert x - a \rvert > r \iff (-\infty, a - r) \cup (a + r, \infty)$.
06. Perm. & Comb.	${}^nP_r = \frac{n!}{(n-r)!}$; ${}^nC_r = \frac{n!}{r!(n-r)!}$; ${}^nC_a = {}^nC_b \implies a = b$ or $a + b = n$; ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$.
07. Binomial Theorem	$(a+b)^n = \sum_{r=0}^{n} {}^nC_r a^{n-r} b^r$; $\sum {}^nC_r = 2^n$; Even/Odd sum $= 2^{n-1}$; $(1+x)^n > 1 + nx$ $(x > 0, n \ge 2)$.
08. Sequences & Series	$a_n = ar^{n-1}$; $S_n = \frac{a(r^n - 1)}{r - 1}$; $S_\infty = \frac{a}{1-r}$ $(\lvert r \rvert < 1)$; $G = \sqrt{ab}$; $AM \ge GM$; $x^2 - 2Ax + G^2 = 0$.
09. Straight Lines	$m = \frac{y_2 - y_1}{x_2 - x_1}$; $\tan\theta = \left\lvert \frac{m_2 - m_1}{1 + m_1 m_2} \right\rvert$; $\frac{x}{a} + \frac{y}{b} = 1$; $d = \frac{\lvert Ax_1 + By_1 + C \rvert}{\sqrt{A^2 + B^2}}$; $d_\parallel = \frac{\lvert C_1 - C_2 \rvert}{\sqrt{A^2 + B^2}}$.
10. Conic Sections	Circle: $(x-h)^2 + (y-k)^2 = r^2$; Parabola: $y^2 = \pm 4ax$; Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(c^2 = a^2 - b^2)$; Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $(c^2 = a^2 + b^2)$; LR $= \frac{2b^2}{a}$.
11. 3D Geometry	$PQ = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$; From XY-plane: $\lvert z \rvert$; From X-axis: $\sqrt{y^2 + z^2}$; $G = \frac{A+B+C}{3}$.
12. Limits & Deriv.	$\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$; $\lim_{x \to 0} \frac{\sin x}{x} = 1$; $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$; $(uv)' = u'v + uv'$; $(u/v)' = \frac{u'v - uv'}{v^2}$.
13. Statistics	$\text{MD}(A) = \frac{\sum f_i \lvert x_i - A \rvert}{N}$; $\sigma^2 = \frac{\sum f_i x_i^2}{N} - \bar{x}^2$; $\sigma_y = \lvert a \rvert \sigma_x$ for $y = ax + b$; $\sigma^2(1..n) = \frac{n^2 - 1}{12}$.
14. Probability	$P(E) = \frac{n(E)}{n(S)}$; $P(A \cup B) = P(A) + P(B) - P(A \cap B)$; $P(A') = 1 - P(A)$; ME: $P(A \cap B) = 0$.

2. Master Smart Shortcut Index (High-Speed Exam Techniques)

- **Ch. 1–2:** Min $n(A \cup B) = \max(n(A), n(B))$; $\lfloor x \rfloor + \lfloor -x \rfloor = -1$ $(x \notin \mathbb{Z})$; $x = g(y)$ inversion for rational range.
- **Ch. 3–4:** $a\sin x + b\cos x \in [-\sqrt{a^2 + b^2}, \sqrt{a^2 + b^2}]$; $\frac{1+i}{1-i} = i$; $\sum_{k=n}^{n+3} i^k = 0$; $\lvert z_1 / z_2 \rvert = \lvert z_1 \rvert / \lvert z_2 \rvert$ directly.
- **Ch. 5–7:** Move x to keep coeff $+$; n-gon diagonals $= \frac{n(n-3)}{2}$; Set variables $= 1$ for sum of binomial coefficients.
- **Ch. 8–11:** Repdigit sum $d + dd + \cdots = \frac{d}{81}[10^{n+1} - 9n - 10]$; Foot/image ratio; Octant $= Q + (4$ if $z < 0)$; $D = A + C - B$.
- **Ch. 12–14:** $\lim \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} a^{m-n}$; L'Hôpital (MCQ check only); 2-dice sum k ways $= 6 - \lvert k - 7 \rvert$; Leap-year 53 Sundays $= 2/7$.

3. Full-Length CBSE/RBSE Sample Question Paper (80 Marks • 3 Hours)

General Instructions: All questions are compulsory. Section A: Q1–Q20 (1M each); Section B: Q21–Q25 (2M each); Section C: Q26–Q31 (3M each); Section D: Q32–Q35 (5M each); Section E: Q36–Q38 (4M Case-Based each).

Section A: Multiple Choice & Assertion-Reason Questions ($20 \times 1 = 20$ Marks)

1. If $A = \{x \in \mathbb{Z} : x^2 \leq 4\}$, then $n(P(A))$ is: (A) 8 (B) 16 (C) 32 (D) 64
2. Domain of $f(x) = \sqrt{16 - x^2}$ is: (A) $(-4, 4)$ (B) $[-4, 4]$ (C) $[0, 4]$ (D) $\mathbb{R}$
3. Value of $\sin \frac{31\pi}{3}$ is: (A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{2}}$
4. Multiplicative inverse of $3 - 4i$ is: (A) $\frac{3+4i}{5}$ (B) $\frac{3+4i}{25}$ (C) $\frac{3-4i}{25}$ (D) $3 + 4i$
5. Value of $\left(\frac{1+i}{1-i}\right)^4$ is: (A) 1 (B) -1 (C) i (D) $-i$
6. Solution of $-3x + 17 < -13$ for $x \in \mathbb{R}$ is: (A) $(-\infty, 10)$ (B) $(10, \infty)$ (C) $[10, \infty)$ (D) $(-10, 10)$
7. If ${}^nC_9 = {}^nC_8$, then ${}^nC_2$ equals: (A) 17 (B) 136 (C) 153 (D) 72
8. Number of diagonals in a decagon ($n = 10$) is: (A) 35 (B) 45 (C) 20 (D) 30
9. Sum of all coefficients in $(2x - y)^6$ is: (A) 1 (B) 64 (C) 0 (D) -1
10. Sum of the infinite G.P. $1, \frac{1}{3}, \frac{1}{9}, \ldots$ is: (A) $2/3$ (B) $3/2$ (C) 3 (D) $4/3$
11. Distance between parallel lines $3x - 4y + 7 = 0$ and $3x - 4y - 3 = 0$ is: (A) 1 (B) 2 (C) 5 (D) 10
12. Eccentricity of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is: (A) $3/5$ (B) $4/5$ (C) $5/3$ (D) $3/4$
13. Focus of the parabola $x^2 = -12y$ is: (A) $(-3, 0)$ (B) $(0, -3)$ (C) $(0, 3)$ (D) $(3, 0)$
14. The point $(-2, -3, -5)$ lies in Octant: (A) III (B) V (C) VII (D) VIII
15. $\lim_{x \to 0} \frac{\sin 5x}{\tan 3x}$ equals: (A) $3/5$ (B) $5/3$ (C) 1 (D) 0
16. Derivative of $f(x) = x \cos x$ at $x = 0$ is: (A) 0 (B) 1 (C) -1 (D) 2
17. Variance of the first 7 natural numbers is: (A) 2 (B) 4 (C) 8 (D) 7
18. Probability of getting 53 Sundays in a leap year is: (A) $1/7$ (B) $2/7$ (C) $1/366$ (D) $2/365$
19. **Assertion (A):** $(A \cup B)' = A' \cap B'$. **Reason (R):** De Morgan's Law reverses $\cup$ to $\cap$ under complement. (A) Both true, R explains A (B) Both true, R doesn't explain A (C) A true, R false (D) A false, R true.
20. **Assertion (A):** $\lim_{x \to 0} \frac{|x|}{x}$ does not exist. **Reason (R):** LHL $= -1 \neq 1 =$ RHL. (A) Both true, R explains A (B) Both true, R doesn't explain A (C) A true, R false (D) A false, R true.

Section B: Very Short Answer Questions ($5 \times 2 = 10$ Marks)

21. Let $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3\}$, $B = \{3, 4, 5\}$. Verify $(A \cup B)' = A' \cap B'$.
22. Find the domain and range of $f(x) = \sqrt{9 - x^2}$.
23. Express $(1 - i)^4$ in $a + ib$ form.
24. Find the equation of the line perpendicular to $x - 7y + 5 = 0$ having x-intercept 3.
25. Find $\lim_{x \to 1} \frac{x^{15} - 1}{x^{10} - 1}$.

Section C: Short Answer Questions ($6 \times 3 = 18$ Marks)

26. If $\cos x = -\frac{3}{5}$ and x lies in Quadrant III, find $\sin x$ and $\tan x$.
27. Solve the system: $5(2x - 7) - 3(2x + 3) \leq 0$ and $2x + 19 \leq 6x + 47$.
28. Find the number of arrangements of the letters of DAUGHTER so that all vowels always come together.
29. Show that $9^{n+1} - 8n - 9$ is divisible by 64 for all $n \in \mathbb{N}$.
30. Find the foci, vertices, eccentricity, and latus rectum of $9x^2 + 4y^2 = 36$.
31. Find the derivative of $f(x) = \sin x$ from first principles.

Section D: Long Answer Questions ($4 \times 5 = 20$ Marks)

32. Prove that $\cos^2 x + \cos^2(x + \frac{\pi}{3}) + \cos^2(x - \frac{\pi}{3}) = \frac{3}{2}$.
33. Find the sum of n terms of the series $8 + 88 + 888 + \cdots$
34. Find the equation of the circle passing through $(4, 1)$ and $(6, 5)$ with centre on $4x + y = 16$.
35. Find the Mean, Variance, and Standard Deviation of the grouped data: Classes $30–40, 40–50, 50–60, 60–70, 70–80, 80–90, 90–100$ with frequencies $3, 7, 12, 15, 8, 3, 2$.

Section E: Case-Based Questions ($3 \times 4 = 12$ Marks)

36. **Survey Analytics (Sets):** In a survey of 100 students, 40 study Maths, 45 Physics, 35 Chemistry, 15 M&P, 12 P&C, 10 M&C, and 5 all three. Find: (i) $n(M \cup P \cup C)$ (2M), (ii) students studying none (1M), (iii) students studying only Maths (1M).
37. **Committee Selection (P&C):** A committee of 7 is formed from 9 boys and 4 girls. Find the number of ways if the committee has: (i) exactly 3 girls (2M), (ii) at least 3 girls (2M).
38. **Card & Event Probability:** Two cards are drawn from a 52-card deck, and in another experiment $P(A) = 0.42, P(B) = 0.48, P(A \cap B) = 0.16$. Find: (i) $P(A \cup B)$ (1M), (ii) $P(\text{neither } A \text{ nor } B)$ (1M), (iii) $P(\text{both drawn cards are Aces})$ (2M).

4. Sample Paper Answer Key (Quick Verification)

Section	Verified Answers
Section A	1(C), 2(B), 3(B), 4(B), 5(A), 6(B), 7(B), 8(A), 9(A), 10(B), 11(B), 12(A), 13(B), 14(C), 15(B), 16(B), 17(B), 18(B), 19(A), 20(A).
Section B	21. $\{1, 6\}$; 22. Dom $[-3, 3]$, Range $[0, 3]$; 23. $-4 + 0i$; 24. $7x + y - 21 = 0$; 25. $3/2$.
Section C	26. $-4/5, 4/3$; 27. $[-7, 11]$; 28. 4320; 29. $(1 + 8)^{n+1}$ expansion; 30. Foci $(0, \pm\sqrt{5})$, Vertices $(0, \pm 3)$, $e = \sqrt{5}/3$, LR $= 8/3$; 31. $\cos x$.
Section D	32. $3/2$; 33. $\frac{80}{81}(10^n - 1) - \frac{8n}{9}$; 34. $(x - 3)^2 + (y - 4)^2 = 10$; 35. Mean $= 62$, $\sigma^2 = 201$, $\sigma \approx 14.18$.
Section E	36. (i) 88, (ii) 12, (iii) 20; 37. (i) 504, (ii) 588; 38. (i) 0.74, (ii) 0.26, (iii) $\frac{{}^4C_2}{{}^{52}C_2} = \frac{1}{221}$.