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CBSE (STANDARD & BASIC) + RBSE + FOUNDATION | 2026–27 EDITION

CLASS 10 MATHEMATICS

THE ULTIMATE COMPLETE MASTER GUIDE

All 14 Rationalised NCERT Chapters • Concepts, Derivations, Formula Banks, Shortcuts, 224+ Solved Examples, MCQs, 3-Tier Tests & Teacher's Guides

Feature	Textbook Specification
Curriculum Alignment	Official 2026–27 Rationalised NCERT / CBSE (041 & 241) & RBSE Class 10 Syllabus
Chapter Coverage	All 14 Chapters across 7 Official Units (Number Systems to Probability)
Pedagogical Structure	Uniform 12-Section Architecture (Sections A through L) in every chapter
Visual Learning	14 Custom High-Resolution Vector Mathematical Diagrams
Practice Volume	224+ Fully Solved Examples, 140+ Competency MCQs, 42 Three-Tier Chapter Tests
Faculty Support	Complete Section L Classroom Teaching Guide in every chapter

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Prepared for classroom instruction, board exam mastery, and competitive foundation preparation.

1. Official 2026–27 Class 10 Mathematics Exam Pattern (CBSE & RBSE)

This textbook follows the latest rationalised NCERT Class 10 Mathematics curriculum (14 chapters across 7 units), which serves as the common foundation for both **CBSE Class 10 (Code 041 Standard & Code 241 Basic)** and **RBSE Class 10 (Rajasthan Board)**. Every theory paper carries **80 Marks (3 Hours)** plus **20 Marks Internal/Sessional Assessment** for a total of 100 Marks.

Unit	Unit Name	CBSE (80M)	RBSE (80M)	Chapters Included
I	Number Systems	06	04	Ch. 1: Real Numbers
II	Algebra	20	18	Ch. 2: Polynomials; Ch. 3: Linear Equations; Ch. 4: Quadratic Eq.; Ch.
III	Coordinate Geometry	06	07	Ch. 7: Coordinate Geometry
IV	Geometry	15	10	Ch. 6: Triangles; Ch. 10: Circles
V	Trigonometry	12	13	Ch. 8: Intro to Trigonometry; Ch. 9: Applications of Trigonometry
VI	Mensuration	10	11	Ch. 11: Areas Related to Circles; Ch. 12: Surface Areas & Volumes
VII	Statistics & Probability	11	17	Ch. 13: Statistics; Ch. 14: Probability
Total	**All 7 Units (14 Chapters)**	**80 Marks**	**80 Marks**	**+ 20 Marks Internal / Sessional = 100 Marks**

Verified Sources: CBSE Academic Curriculum 2026–27 (041/241) and RBSE Class 10 Maths Syllabus 2026–27.

[BOARD & EXAM ALERT] CBSE papers include Competency-Based MCQs, Assertion-Reason, and Case-Study questions (50% weightage), whereas RBSE papers feature a balanced mix of MCQs, Fill-in-the-Blanks, Very Short, Short, and Long Answer questions with internal choices. Sections F, G, and K in this book train students for both formats.

2. Complete 14-Chapter Syllabus Mapping & Deleted / Rationalised Topics

Ch.	Chapter Title	Official Examinable Subtopics (2026–27)	Excluded / Rationalised Out
1	Real Numbers	Fundamental Theorem of Arithmetic, prime factorisation, HCF & LCM, irrationality proofs ($\sqrt{2}, \sqrt{3}, \sqrt{5}$)	Euclid's Division Lemma; terminating/non-terminating decimal theorem
2	Polynomials	Geometrical meaning of zeros; relationship between zeros and coefficients of quadratic polynomials	Division algorithm for polynomials
3	Pair of Linear Equations	Graphical solution, consistency conditions ($a_1/a_2, b_1/b_2, c_1/c_2$), substitution & elimination	Cross-multiplication method; equations reducible to linear form
4	Quadratic Equations	Standard form $ax^2 + bx + c = 0$, factorisation, quadratic formula, discriminant & nature of roots	Completing the square method
5	Arithmetic Progressions	n-th term a_n, sum of first n terms S_n, daily-life word problems	NIL (Full chapter retained)
6	Triangles	Similarity criteria (AAA/AA, SAS, SSS), Basic Proportionality Theorem (Thales) & converse	Area of similar triangles theorem; Pythagoras theorem proof (application retained in basics)
7	Coordinate Geometry	Distance formula, internal section formula, midpoint formula	Area of a triangle using coordinates
8	Intro to Trigonometry	Trig ratios of acute angles, standard values ($0°$–$90°$), three fundamental identities	Trigonometric ratios of complementary angles
9	Applications of Trig.	Heights and distances, angle of elevation & depression (max two right triangles, $30°, 45°, 60°$)	NIL (Full chapter retained)
10	Circles	Tangent at any point perpendicular to radius; lengths of tangents from external point are equal	NIL (Full chapter retained)
11	Areas Related to Circles	Area of sector and segment of a circle; arc length; central angles $60°, 90°, 120°$	Areas of combinations of plane figures
12	Surface Areas & Volumes	Surface areas and volumes of combinations of two solids (cube, cuboid, sphere, hemisphere, cylinder, cone)	Frustum of a cone; conversion of solid from one shape to another
13	Statistics	Mean (direct, assumed mean, step-deviation), Median, and Mode of grouped data	Cumulative frequency graph (Ogive)
14	Probability	Classical definition of probability; simple problems on coins, dice, cards, and complements	NIL (Full chapter retained)

Note: The standalone chapter *Constructions* (formerly Chapter 11) is completely deleted from the rationalised NCERT/CBSE/RBSE syllabus.

3. 12-Section Mastery Architecture (Inside Every Chapter)

Every chapter in this textbook follows a uniform 12-section pedagogical structure:

- **Section A (Introduction & Relevance):** Learning goals, real-life uses, cross-chapter links, and board weightage.
- **Section B (Complete Concepts & Derivations):** Ground-up definitions, step-by-step proofs, and a custom vector diagram.
- **Section C (Complete Formula Bank):** Every formula with symbols, validity conditions, solved example, and mistake warning.
- **Section D (Question Type Classification):** High-frequency patterns with identification cues and fastest valid methods.
- **Section E (Smart Shortcut Techniques):** Mathematically valid shortcuts with domain limits and 3-level examples.
- **Section F (16 Fully Solved Examples):** Graded from Beginner and Intermediate to Board Standard and Challenge Level.
- **Section G (Competency MCQ Mastery):** Exam-grade MCQs with answer keys, fast tricks, and distractor analysis.
- **Section H (Special Exam Strategies) & Section I (Common Mistakes):** Time management, step-marking rules, and error fixes.
- **Section J (One-Page Quick Revision Sheet):** Consolidated formulas, theorems, and last-minute checklist.
- **Section K (Three-Tier Chapter Tests):** Basic, Board Standard, and Challenge tests with separate solutions.
- **Section L (Teacher's Classroom Guide):** Lecture sequence, blackboard examples, student doubts, and homework plan.

[TEACHER'S TIP] Students aiming for 95+ in CBSE/RBSE Boards should master Sections B, C, F, and J first, then time themselves on Section K. Faculty can use Section L directly for daily 45-minute lesson planning.

UNIT I: NUMBER SYSTEMS TYBA COACHING CENTRE

Chapter 1: Real Numbers

Fundamental Theorem of Arithmetic, Prime Factorisation, HCF/LCM & Irrationality Proofs

A. Chapter Introduction & Board Examination Relevance

Real Numbers form the bedrock of Class 10 Mathematics. Under the 2026–27 rationalised NCERT/CBSE/RBSE syllabus, this chapter focuses on the Fundamental Theorem of Arithmetic, prime factorisation for finding HCF and LCM, and rigorous contradiction proofs of irrationality for numbers such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, and $a + b\sqrt{p}$.

- **What You Will Learn:** Prime factorisation of composite integers, uniqueness of prime decomposition, computing HCF and LCM via prime powers, applying $\text{HCF}(a,b) \times \text{LCM}(a,b) = a \times b$, checking whether a^n can end with digit 0, and proving irrationality by contradiction.
- **Why This Chapter Matters:** Carries 6 marks in CBSE and 4 marks in RBSE, with an almost guaranteed 3-mark or 5-mark proof question (e.g., Prove $\sqrt{5}$ is irrational) plus 1–2 MCQs on HCF/LCM and prime exponents.
- **Real-Life Applications:** RSA public-key cryptography relies on the difficulty of factoring large composites into primes; gear-ratio synchronization, traffic-light cycle alignment, and bell-ringing schedules use LCM; tiling and equal-batch packaging use HCF.
- **Cross-Chapter Connections:** Connects directly to Polynomials (rational vs. irrational zeros), Quadratic Equations (surd roots via discriminant), and Class 11 Sets & Number Theory.
- **Board & Foundation Relevance:** High-scoring, predictable chapter. Deleted topics: Euclid's Division Lemma and terminating/non-terminating decimal expansion theorems are excluded from board exams.

B. Complete Concept Explanation & Derivations

1. Classification of Real Numbers & Prime vs. Composite Integers

Every number on the continuous number line is a **real number** ($\mathbb{R}$), partitioned into **rational numbers** ($\mathbb{Q} = \{p/q : p, q \in \mathbb{Z}, q \neq 0\}$) and **irrational numbers** ($\mathbb{R} \setminus \mathbb{Q}$, non-terminating and non-repeating decimals). A natural number $p > 1$ is **prime** if its only positive divisors are 1 and p. A natural number $n > 1$ that has more than two positive divisors is **composite**. The number 1 is neither prime nor composite.

[CONCEPT MASTER – 1. Classification of Real Numbers & Prime vs. Composite Integers] The only even prime number is 2. Every prime number greater than 3 is of the form $6k \pm 1$ for some integer $k \geq 1$, though not every $6k \pm 1$ is prime (e.g., 25).

2. Fundamental Theorem of Arithmetic (FTA)

Theorem: Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur. Formally, any integer $n > 1$ has a unique canonical prime representation $n = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$ where $p_1 < p_2 < \cdots < p_k$ are primes and $a_i \geq 1$.

[CONCEPT MASTER – 2. Fundamental Theorem of Arithmetic (FTA)] If a prime p divides a^2 (where a is a positive integer), then p must divide a. This single lemma powers every irrationality proof in Class 10.

3. HCF and LCM by Prime Factorisation

Write each positive integer in canonical prime-power form. The **Highest Common Factor (HCF)** is the product of the *smallest* power of each *common* prime factor. The **Least Common Multiple (LCM)** is the product of the *greatest* power of *each* prime factor involved in the numbers. Always remember: $\text{HCF}(a,b)$ always divides $\text{LCM}(a,b)$.

[CONCEPT MASTER – 3. HCF and LCM by Prime Factorisation] For two positive integers a and b, $\text{HCF}(a,b) \times \text{LCM}(a,b) = a \times b$. Warning: This identity does NOT hold for three or more numbers!

4. Ending with Digit Zero (a^n Problems)

Any positive integer ending with the digit 0 must be divisible by $10 = 2 \times 5$. By the uniqueness of the Fundamental Theorem of Arithmetic, a^n can end with the digit 0 for some natural number n *if and only if* the prime factorisation of a contains both 2 and 5. For instance, $6^n = (2 \times 3)^n = 2^n \times 3^n$ lacks the prime 5, so 6^n can never end with 0.

[CONCEPT MASTER – 4. Ending with Digit Zero (a^n Problems)] Similarly, a^n ends with 5 only if 5 divides a and 2 does not divide a.

5. Proving Irrationality by Contradiction

To prove $\sqrt{p}$ (for prime p, such as $2, 3, 5$) is irrational: (1) Assume $\sqrt{p}$ is rational, so $\sqrt{p} = a/b$ where a, b are coprime integers ($\text{HCF}(a, b) = 1$) and $b \neq 0$. (2) Square both sides: $pb^2 = a^2$, so $p \mid a^2 \implies p \mid a$. (3) Let $a = pk$; substituting gives $pb^2 = p^2k^2 \implies b^2 = pk^2$, so $p \mid b^2 \implies p \mid b$. (4) Thus p divides both a and b, contradicting $\text{HCF}(a, b) = 1$. Hence $\sqrt{p}$ is irrational.

[CONCEPT MASTER – 5. Proving Irrationality by Contradiction] For composite expressions like $3 + 2\sqrt{5}$, isolate $\sqrt{5} = \frac{a-3b}{2b}$ (rational) and state the contradiction with the known fact that $\sqrt{5}$ is irrational.

Prime Factorisation Tree & Fundamental Theorem of Arithmetic

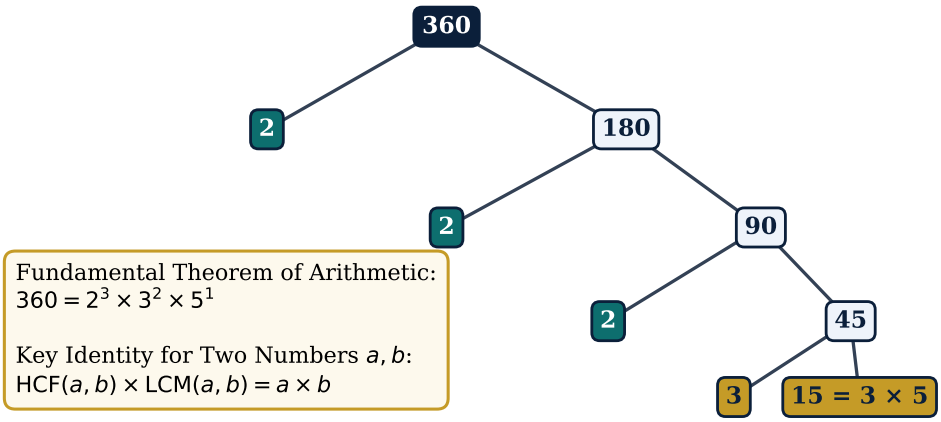


Figure 1: **Visual Model:** Prime factorisation tree of $360 = 2^3 \times 3^2 \times 5^1$ and the Fundamental Theorem of Arithmetic.

C. Complete Formula Bank

[FORMULA BANK – 1. Canonical Prime Factorisation] **Expression:** $n = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$
Symbols: p_i: distinct primes; $a_i \in \mathbb{N}$
Conditions & When to Use: Valid for every integer $n > 1$
Solved Mini-Example: $140 = 2^2 \times 5^1 \times 7^1$
Common Mistake: Leaving 4 or 9 as factors instead of 2^2 or 3^2.

[FORMULA BANK – 2. HCF via Prime Powers] **Expression:** $\text{HCF}(a, b) = \prod p_i^{\min(x_i, y_i)}$
Symbols: x_i, y_i: exponents of common prime p_i
Conditions & When to Use: Include only primes common to all numbers; if none, $\text{HCF} = 1$
Solved Mini-Example: For $2^3 \cdot 3^2$ and $2^2 \cdot 3^3$, $\text{HCF} = 2^2 \cdot 3^2 = 36$
Common Mistake: Taking the maximum exponent instead of minimum.

[FORMULA BANK – 3. LCM via Prime Powers] **Expression:** $\text{LCM}(a,b) = \prod p_i^{\max(x_i,y_i)}$
Symbols: x_i, y_i: exponents of all primes appearing in a or b
Conditions & When to Use: Include every prime factor present in at least one number
Solved Mini-Example: For $2^3 \cdot 3^2$ and $2^2 \cdot 5$, $\text{LCM} = 2^3 \cdot 3^2 \cdot 5 = 360$
Common Mistake: Omitting non-common primes like 5.

[FORMULA BANK – 4. Two-Number HCF–LCM Product Rule] **Expression:** $\text{HCF}(a,b) \times \text{LCM}(a,b) = a \times b$
Symbols: $a, b \in \mathbb{N}$
Conditions & When to Use: Strictly for 2 positive integers only
Solved Mini-Example: If $\text{HCF}(26,91) = 13$, $\text{LCM} = \frac{26\times 91}{13} = 182$
Common Mistake: Applying $\text{HCF} \times \text{LCM} = a \cdot b \cdot c$ to 3 numbers.

[FORMULA BANK – 5. Prime Divisibility Lemma] **Expression:** $p \mid a^2 \implies p \mid a$
Symbols: p is prime, $a \in \mathbb{Z}^+$
Conditions & When to Use: Holds only when p is prime (fails for composite $p = 4$, e.g. $4 \mid 6^2$ but $4 \nmid 6$)
Solved Mini-Example: $5 \mid a^2 \implies a = 5k$
Common Mistake: Assuming $4 \mid a^2 \implies 4 \mid a$.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE Standard/Basic, RBSE Board, and Foundation MCQ/Case-Study problem structures.

Pattern 1: HCF & LCM of Integers / Algebraic Monomials

- **Identification Cue:** Find HCF and LCM of $p^a q^b$ and $p^c q^d$, or verify $\text{HCF} \times \text{LCM} = a \times b$. | **Concept & Formula:** Min/max exponents of prime factors
- **Standard Step-by-Step:** Factorise both numbers completely, pick min powers for HCF and max powers for LCM.
- **Fastest Valid Method:** For $x = a^3b^2, y = ab^3$, read $\text{HCF} = ab^2$, $\text{LCM} = a^3b^3$ directly. | **Alternative:** Use $\text{LCM} = (a \times b)/\text{HCF}$.
- **Common Trap:** Confusing a^3 with $3a$ in algebraic factorisation.
- **Practice Drill:** Find HCF and LCM of 96 and 404, and verify the product rule.

Pattern 2: Word Problems: Bells, Tracks, Packaging & Tiling

- **Identification Cue:** Keywords: 'minimum time together / next ring' (LCM) vs. 'maximum size / equal rows' (HCF). | **Concept & Formula:** HCF vs. LCM selection
- **Standard Step-by-Step:** Identify whether a common multiple or common divisor is needed, then factorise.
- **Fastest Valid Method:** Minimum/together = LCM; Maximum/largest equal = HCF. | **Alternative:** List multiples for small numbers to double-check.
- **Common Trap:** Finding HCF when the question asks for simultaneous recurrence.
- **Practice Drill:** Three bells toll at intervals of $12, 15, 18$ min. When do they toll together next?

Pattern 3: Digit-Ending Problems ($4^n, 6^n, 12^n$)

- **Identification Cue:** Check whether k^n can end with digit 0 or 5 for any $n \in \mathbb{N}$. | **Concept & Formula:** Uniqueness of Fundamental Theorem of Arithmetic
- **Standard Step-by-Step:** Factorise base k and check presence of both 2 and 5.
- **Fastest Valid Method:** No 5 in base $\implies$ never ends in 0. | **Alternative:** Compute $k^1, k^2, k^3 \pmod{10}$ to observe cyclicity.
- **Common Trap:** Forgetting to cite the uniqueness of FTA in board exams.
- **Practice Drill:** Show that 12^n cannot end with 0 or 5 for any $n \in \mathbb{N}$.

Pattern 4: Composite Number Justification

- **Identification Cue:** Explain why $7 \times 11 \times 13 + 13$ is a composite number. | **Concept & Formula:** Factoring out common term to show > 2 divisors
- **Standard Step-by-Step:** Factor out 13: $13(7 \times 11 + 1) = 13 \times 78$, a product of factors > 1.
- **Fastest Valid Method:** Spot common factor c in $A \cdot c + c = c(A + 1)$. | **Alternative:** Evaluate full product if numbers are small.
- **Common Trap:** Expanding the whole product unnecessarily and risking arithmetic errors.
- **Practice Drill:** Explain why $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ is composite.

Pattern 5: Irrationality Proofs ($\sqrt{p}$ and $a + b\sqrt{p}$)

- **Identification Cue:** Prove $\sqrt{3}$ or $5 - 2\sqrt{3}$ is irrational. | **Concept & Formula:** Proof by contradiction + prime divisibility lemma
- **Standard Step-by-Step:** Assume rational p/q with $\gcd(p, q) = 1$, square (for $\sqrt{p}$) or isolate radical (for $a + b\sqrt{p}$).
- **Fastest Valid Method:** For $a + b\sqrt{p}$, never square! Simply isolate $\sqrt{p} = \frac{r-as}{bs}$ in 3 lines. | **Alternative:** Verify irrationality numerically only for intuition, never as proof.
- **Common Trap:** Squaring $3 + 2\sqrt{5} = p/q$ creates messy cross-terms.
- **Practice Drill:** Prove that $2 + \sqrt{3}$ is irrational given $\sqrt{3}$ is irrational.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. 3-Line Radical Isolation for $a + b\sqrt{p}$] **Core Idea & Why It Works:** If $a + b\sqrt{p} = r$ (rational) with $b \neq 0$, then $\sqrt{p} = (r - a)/b$ is rational, contradicting $\sqrt{p} \notin \mathbb{Q}$.

Valid Domain: All linear radical expressions $a \pm b\sqrt{p}$ | **Where It Fails:** Sums of two radicals like $\sqrt{2} + \sqrt{3}$ (needs squaring once)

Easy Example: Prove $3 + \sqrt{2} \notin \mathbb{Q}$: $\sqrt{2} = (p/q) - 3 \in \mathbb{Q}$, contradiction.

Board-Level Example: Prove $5 - 2\sqrt{3} \notin \mathbb{Q}$: $\sqrt{3} = (5q - p)/(2q) \in \mathbb{Q}$, contradiction.

Advanced Example: Prove $\frac{1}{\sqrt{2}} \notin \mathbb{Q}$: $\sqrt{2} = q/p \in \mathbb{Q}$, contradiction.

[SMART SHORTCUT – 2. LCM-HCF Divisibility Filter for MCQs] **Core Idea & Why It Works:** For any two integers, HCF MUST divide LCM exactly.

Valid Domain: All 'Which cannot be the HCF/LCM?' MCQs | **Where It Fails:** When multiple options divide LCM (then check $a \times b$)

Easy Example: If LCM $= 120$, can HCF $= 18$? No, $18 \nmid 120$.

Board-Level Example: Two numbers have HCF $= 15$; which cannot be LCM? $150, 180, 200, 300 \to 200$ ($15 \nmid 200$).

Advanced Example: Find k if $\text{HCF}(65, 117) = 65m - 117$: HCF $= 13 \implies 65m = 130 \implies m = 2$.

[SMART SHORTCUT – 3. Exponent Sum Trick for HCF $\times$ LCM] **Core Idea & Why It Works:** For $a = p^x q^y$ and $b = p^u q^v$, $\text{HCF} \cdot \text{LCM} = p^{x+u} q^{y+v}$ directly.

Valid Domain: Algebraic prime-power MCQs | **Where It Fails:** None

Easy Example: $a = x^2 y, b = xy^3 \implies \text{HCF} = xy, \text{LCM} = x^2 y^3$.

Board-Level Example: If $p = a^2 b^3, q = a^3 b$, $\text{LCM}(p, q)/\text{HCF}(p, q) = a^{3-2} b^{3-1} = ab^2$.

Advanced Example: Product of exponents in $\text{LCM}(2^4 \cdot 3^2, 2^2 \cdot 3^5)$ is $4 + 5 = 9$.

[SMART SHORTCUT – 4. Trailing Zeros Count via Power of 5] **Core Idea & Why It Works:** In $n!$, the number of trailing zeros equals $\lfloor n/5 \rfloor + \lfloor n/25 \rfloor + \ldots$.

Valid Domain: Foundation/Olympiad trailing-zero MCQs | **Where It Fails:** Non-factorial products (count min of 2s and 5s directly)

Easy Example: Trailing zeros in $2^5 \times 5^3 \times 7^2$ is $\min(5, 3) = 3$.

Board-Level Example: Zeros in $25 \times 16 \times 15 = (5^2)(2^4)(3 \cdot 5) = 2^4 \cdot 5^3 \cdot 3 \to 3$ zeros.

Advanced Example: Trailing zeros in $50! = 10 + 2 = 12$.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: Express 156 as a product of its prime factors.

Given & Required: $n = 156$ | **Concept & Formula:** FTA Prime Factorisation

Detailed Solution: $156 = 2 \times 78 = 2 \times 2 \times 39 = 2^2 \times 3 \times 13$. **Final Answer:** $2^2 \times 3 \times 13$

Fastest Alternative / Shortcut: Split $156 = 12 \times 13 = (2^2 \cdot 3) \cdot 13$.

SOLVED EXAMPLE 2 [Beginner]

Question: Find the HCF of 96 and 404 by prime factorisation, and hence find their LCM.

Given & Required: $a = 96, b = 404$ | **Concept & Formula:** $\text{HCF} \times \text{LCM} = a \times b$

Detailed Solution: $96 = 2^5 \times 3$, $404 = 2^2 \times 101$. Common min power: $\text{HCF} = 2^2 = 4$. $\text{LCM} = \frac{96 \times 404}{4} = 96 \times 101 = 9696$. **Final Answer:** **HCF** $= 4$, **LCM** $= 9696$

Fastest Alternative / Shortcut: Note $404 = 4 \times 101$, and 101 is prime, so $\text{HCF} = 4$ immediately.

SOLVED EXAMPLE 3 [Beginner]

Question: Find the HCF and LCM of $12, 15$, and 21 by prime factorisation.

Given & Required: $12, 15, 21$ | **Concept & Formula:** 3-Number Prime Powers

Detailed Solution: $12 = 2^2 \cdot 3$, $15 = 3 \cdot 5$, $21 = 3 \cdot 7$. $\text{HCF} = 3^1 = 3$. $\text{LCM} = 2^2 \cdot 3 \cdot 5 \cdot 7 = 420$. **Final Answer:** **HCF** $= 3$, **LCM** $= 420$

Fastest Alternative / Shortcut: Common prime is only 3.

SOLVED EXAMPLE 4 [Beginner]

Question: If $\text{HCF}(306, 657) = 9$, find $\text{LCM}(306, 657)$.

Given & Required: $a = 306, b = 657, \text{HCF} = 9$ | **Concept & Formula:** Product Rule

Detailed Solution: $\text{LCM} = \frac{306 \times 657}{9} = 34 \times 657 = 22338$. **Final Answer:** 22338

Fastest Alternative / Shortcut: Divide $306/9 = 34$ first, then 34×657.

SOLVED EXAMPLE 5 [Intermediate]

Question: Check whether 6^n can end with the digit 0 for any natural number n.

Given & Required: $6^n, n \in \mathbb{N}$ | **Concept & Formula:** Uniqueness of FTA

Detailed Solution: $6^n = (2 \times 3)^n = 2^n \times 3^n$. For a number to end in 0, it must be divisible by $10 = 2 \times 5$. Since 5 is not a prime factor of 6, 6^n never ends in 0. **Final Answer:** **No, never**

Fastest Alternative / Shortcut: Check if $5 \mid 6$; since no, impossible.

SOLVED EXAMPLE 6 [Intermediate]

Question: Explain why $7 \times 11 \times 13 + 13$ is a composite number.

Given & Required: $N = 7 \times 11 \times 13 + 13$ | **Concept & Formula:** Definition of Composite Number

Detailed Solution: $N = 13(7 \times 11 + 1) = 13 \times 78 = 13 \times 2 \times 3 \times 13$. Since N has prime factors other than 1 and itself, it is composite. **Final Answer:** **Composite (**13×78**)**

Fastest Alternative / Shortcut: Factor out 13 immediately.

SOLVED EXAMPLE 7 [Intermediate]

Question: There is a circular path around a sports field. Sonia takes 18 minutes and Ravi takes 12 minutes to drive one round. Starting together, after how many minutes will they meet again at the starting point?

Given & Required: 18 min, 12 min | **Concept & Formula:** LCM Application

Detailed Solution: They meet after $\text{LCM}(18, 12)$ min. $18 = 2 \cdot 3^2$, $12 = 2^2 \cdot 3 \implies \text{LCM} = 2^2 \cdot 3^2 = 36$ min. **Final Answer:** 36 **minutes**

Fastest Alternative / Shortcut: Multiples of 18: $18, 36$ (divisible by 12).

SOLVED EXAMPLE 8 [Intermediate]

Question: Find the largest number which divides 70 and 125, leaving remainders 5 and 8 respectively.

Given & Required: 70 (rem 5), 125 (rem 8) | **Concept & Formula:** HCF of Adjusted Numbers

Detailed Solution: Required number divides $70-5=65$ and $125-8=117$. $65=5\times13$, $117=3^2\times13$. $\text{HCF}(65,117)=13$. **Final Answer:** 13

Fastest Alternative / Shortcut: Subtract remainders first, then take HCF.

SOLVED EXAMPLE 9 [Board Standard]

Question: Prove that $\sqrt{5}$ is irrational.

Given & Required: $\sqrt{5}$ | **Concept & Formula:** Proof by Contradiction

Detailed Solution: Assume $\sqrt{5}=a/b$ with $\gcd(a,b)=1, b\neq0$. Squaring: $5b^2=a^2 \implies 5\mid a^2 \implies 5\mid a$. Put $a=5c \implies 5b^2=25c^2 \implies b^2=5c^2 \implies 5\mid b$. Thus 5 divides both a and b, contradicting $\gcd(a,b)=1$. Hence $\sqrt{5}$ is irrational. **Final Answer: Proved by contradiction**

Fastest Alternative / Shortcut: Follow the 4-step standard board proof template.

SOLVED EXAMPLE 10 [Board Standard]

Question: Prove that $3+2\sqrt{5}$ is irrational, given that $\sqrt{5}$ is irrational.

Given & Required: $3+2\sqrt{5}$ | **Concept & Formula:** Radical Isolation

Detailed Solution: Suppose $3+2\sqrt{5}=a/b$ $(a,b\in\mathbb{Z},b\neq0)$. Then $\sqrt{5}=\frac{a-3b}{2b}$. Since a,b are integers, $\frac{a-3b}{2b}$ is rational, contradicting that $\sqrt{5}$ is irrational. Hence $3+2\sqrt{5}$ is irrational. **Final Answer: Proved**

Fastest Alternative / Shortcut: Isolate $\sqrt{5}$ without squaring.

SOLVED EXAMPLE 11 [Board Standard]

Question: Two positive integers p and q are written as $p=x^3y^2$ and $q=xy^3$, where x,y are prime numbers. Find $\text{HCF}(p,q)$ and $\text{LCM}(p,q)$.

Given & Required: $p=x^3y^2, q=xy^3$ | **Concept & Formula:** Prime Power Exponents

Detailed Solution: $\text{HCF}(p,q)=x^{\min(3,1)}y^{\min(2,3)}=xy^2$. $\text{LCM}(p,q)=x^{\max(3,1)}y^{\max(2,3)}=x^3y^3$. **Final Answer: HCF** $=xy^2$**, LCM** $=x^3y^3$

Fastest Alternative / Shortcut: Pick smaller exponents for HCF, larger for LCM.

SOLVED EXAMPLE 12 [Board Standard]

Question: Three sets of English, Hindi, and Mathematics books have 336, 240, and 96 books respectively. They must be stacked by subject with the same number of books in each stack. Find the minimum number of stacks.

Given & Required: 336, 240, 96 | **Concept & Formula:** HCF for Stack Size

Detailed Solution: Max books per stack $=\text{HCF}(336,240,96)$. $336=2^4\cdot3\cdot7$, $240=2^4\cdot3\cdot5$, $96=2^5\cdot3 \implies \text{HCF}=2^4\cdot3=48$. Total stacks $=\frac{336}{48}+\frac{240}{48}+\frac{96}{48}=7+5+2=14$. **Final Answer:** 14 **stacks**

Fastest Alternative / Shortcut: Find HCF $=48$, then sum quotients.

SOLVED EXAMPLE 13 [Board Standard]

Question: If $\text{LCM}(x,18)=36$ and $\text{HCF}(x,18)=2$, find x.

Given & Required: $b=18, \text{LCM}=36, \text{HCF}=2$ | **Concept & Formula:** Product Rule

Detailed Solution: $x\times18=36\times2 \implies x=\frac{72}{18}=4$. **Final Answer:** $x=4$

Fastest Alternative / Shortcut: $x=(36\times2)/18=4$.

SOLVED EXAMPLE 14 [Challenge]

Question: The ratio of two numbers is 3 : 4 and their HCF is 4. Find their LCM.

Given & Required: Ratio 3 : 4, HCF $=4$ | **Concept & Formula:** Coprime Ratio Property

Detailed Solution: Let numbers be $3k, 4k$. Since $\gcd(3,4)=1$, HCF $=k=4$. Numbers are 12 and 16. LCM $=3\times4\times4=48$. **Final Answer:** 48

Fastest Alternative / Shortcut: For coprime ratio $m:n$ with HCF h, LCM $=m\cdot n\cdot h$.

SOLVED EXAMPLE 15 [Challenge]

Question: Prove that $\sqrt{2}+\sqrt{3}$ is irrational.

Given & Required: $\sqrt{2}+\sqrt{3}$ | **Concept & Formula:** Squaring Contradiction

Detailed Solution: Let $\sqrt{2}+\sqrt{3}=r\in\mathbb{Q}$. Squaring: $5+2\sqrt{6}=r^2 \implies \sqrt{6}=\frac{r^2-5}{2}\in\mathbb{Q}$, contradicting $\sqrt{6}\notin\mathbb{Q}$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Square once to reduce to a single radical $\sqrt{6}$.

SOLVED EXAMPLE 16 [Challenge]

Question: Find the smallest number which when divided by 28 and 32 leaves remainders 8 and 12 respectively.

Given & Required: 28 (rem 8), 32 (rem 12) | **Concept & Formula:** Constant Difference LCM

Detailed Solution: Notice $28-8=20$ and $32-12=20$. Required number $=\text{LCM}(28,32)-20=224-20=204$. **Final Answer:** 204

Fastest Alternative / Shortcut: $\text{LCM}(28,32)-20=224-20=204$.

G. Competency MCQ Mastery Section

Q1. The total number of factors of a prime number is:

(A) 1 (B) 0 (C) 2 (D) 3

Answer: (C) A prime number $p>1$ has exactly two factors: 1 and p. *Fast Method:* Definition of prime. *Distractor Note:* 1 is not prime; primes have 2 factors.

Q2. If $a=2^3\times 3$ and $b=2\times 3\times 5$, then $\text{LCM}(a,b)$ is:

(A) 6 (B) 30 (C) 120 (D) 360

Answer: (C) $\text{LCM}=2^3\times 3\times 5=8\times 15=120$. *Fast Method:* Max powers of $2,3,5$. *Distractor Note:* 6 is the HCF.

Q3. The HCF of the smallest composite number and the smallest prime number is:

(A) 1 (B) 2 (C) 4 (D) 6

Answer: (B) Smallest prime $=2$, smallest composite $=4$. $\text{HCF}(2,4)=2$. *Fast Method:* $\gcd(2,4)=2$. *Distractor Note:* Do not take 1 as prime.

Q4. If $\text{HCF}(26,169)=13$, then $\text{LCM}(26,169)$ is:

(A) 26 (B) 52 (C) 338 (D) 13

Answer: (C) $(26\times 169)/13=2\times 169=338$. *Fast Method:* $2\times 169=338$. *Distractor Note:* Dividing wrong factor.

Q5. Which of the following cannot be the HCF of two numbers whose LCM is 240?

(A) 12 (B) 16 (C) 35 (D) 40

Answer: (C) HCF must divide LCM. 35 does not divide 240. *Fast Method:* Check divisibility of 240. *Distractor Note:* All other options divide 240.

Q6. For any natural number n, 8^n cannot end with the digit:

(A) 2 (B) 4 (C) 6 (D) 0

Answer: (D) $8^n=2^{3n}$ has no factor of 5, so it cannot end in 0. *Fast Method:* Missing factor 5. *Distractor Note:* $8^1=8, 8^2=64, 8^3=512, 8^4=4096$.

Q7. If p_1 and p_2 are two distinct odd prime numbers, then $p_1^2-p_2^2$ is:

(A) An even number (B) An odd number (C) An odd prime (D) A prime number

Answer: (A) Odd squared is odd; odd $-$ odd $=$ even. *Fast Method:* Test $5^2-3^2=16$ (even). *Distractor Note:* Never odd.

Q8. The exponent of 2 in the prime factorisation of 144 is:

(A) 2 (B) 3 (C) 4 (D) 5

Answer: (C) $144=16\times 9=2^4\times 3^2$. *Fast Method:* $16=2^4$. *Distractor Note:* Confusing 2^4 with 2^3.

Q9. The product of a non-zero rational number and an irrational number is:

(A) Always irrational (B) Always rational (C) Rational or irrational (D) One

Answer: (A) If $r \cdot x = q$, then $x = q/r \in \mathbb{Q}$ ($r \neq 0$), contradiction. *Fast Method:* Non-zero rational $\times$ irrational = irrational. *Distractor Note:* Only $0 \times \sqrt{2} = 0$ is rational.

Q10. Assertion (A): $\text{HCF}(a,b) \times \text{LCM}(a,b) = a \times b$. Reason (R): If $\text{HCF}(12,18) = 6$, then $\text{LCM}(12,18) = 36$.

(A) Both A & R true, R explains A (B) Both A & R true, R does not explain A (C) A true, R false (D) A false, R true

Answer: (B) Both statements are true, but R is a numerical instance, not the proof of A. *Fast Method:* Instance $\neq$ general reason. *Distractor Note:* Choosing (A) when R is just an example.

Q11. If $d = \text{HCF}(45, 105)$, then d equals:

(A) 5 (B) 15 (C) 45 (D) 315

Answer: (B) $45 = 3^2 \cdot 5, 105 = 3 \cdot 5 \cdot 7 \implies \text{HCF} = 15$. *Fast Method:* $15 \times 3 = 45, 15 \times 7 = 105$. *Distractor Note:* 315 is LCM.

Q12. If $x = 2^3 \times 3 \times 5^2$, $y = 2^2 \times 3^3$, then $\text{HCF}(x,y)$ is:

(A) 12 (B) 108 (C) 6 (D) 36

Answer: (A) $\text{HCF} = 2^2 \times 3^1 = 12$. *Fast Method:* $4 \times 3 = 12$. *Distractor Note:* Taking 3^3 instead of 3^1.

H. Special Board & MCQ Examination Strategies

- Always write the prime factorisation of every composite number in exponential form before picking HCF or LCM.
- For board proofs of $a + b\sqrt{p}$, never square both sides; isolate $\sqrt{p}$ directly in 3 clean lines.
- In word problems, look for 'simultaneous / together / minimum' (LCM) vs. 'equal division / largest container / maximum' (HCF).
- Double-check two-number HCF and LCM answers using $\text{HCF} \times \text{LCM} = a \times b$.
- In Assertion-Reason MCQs, check if Reason is a general theorem (explains A) or merely a numerical example (does not explain A).

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Applying $\text{HCF}(a,b,c) \times \text{LCM}(a,b,c) = a \cdot b \cdot c$ to three numbers. $\longrightarrow$ **Correction:** That product formula works strictly for TWO positive integers.

[COMMON MISTAKE] **Error:** Assuming $4 \mid a^2 \implies 4 \mid a$ during contradiction proofs. $\longrightarrow$ **Correction:** Only a PRIME p satisfies $p \mid a^2 \implies p \mid a$.

[COMMON MISTAKE] **Error:** Treating 1 as a prime number. $\longrightarrow$ **Correction:** 1 is neither prime nor composite; the smallest prime is 2.

[COMMON MISTAKE] **Error:** Forgetting to state $\text{HCF}(a,b) = 1$ (coprime) at the start of the $\sqrt{p}$ proof. $\longrightarrow$ **Correction:** Always assume $\sqrt{p} = a/b$ in lowest terms where a, b are coprime.

[COMMON MISTAKE] **Error:** Subtracting remainders after taking HCF/LCM. $\longrightarrow$ **Correction:** For 'divides a, b leaving remainders r_1, r_2', subtract r_1, r_2 FIRST, then take HCF.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $n = p_1^{a_1} \cdots p_k^{a_k}$; $\text{HCF} = \prod p_i^{\min}$; $\text{LCM} = \prod p_i^{\max}$; $\text{HCF}(a,b) \times \text{LCM}(a,b) = a \times b$.

[QUICK REVISION] Key Theorems & Results: Prime divisibility: $p \mid a^2 \implies p \mid a$ (p prime). $\text{HCF}(a,b)$ always divides $\text{LCM}(a,b)$.

[QUICK REVISION] Shortcut Summary: Radical isolation for $a + b\sqrt{p}$; divisibility filter for HCF/LCM MCQs.

[QUICK REVISION] High-Frequency Board Patterns: 3-mark irrationality proof ($\sqrt{2}, \sqrt{3}, \sqrt{5}$ or $a + b\sqrt{p}$); 6^n ending in 0; HCF/LCM word problems.

Last-Minute Revision Checklist:

- ☐ Can write canonical prime factorisation and compute HCF/LCM for 2 or 3 numbers.
- ☐ Know that $\text{HCF} \times \text{LCM} = a \times b$ holds only for 2 numbers.
- ☐ Can write the 4-step contradiction proof for $\sqrt{p}$ without skipping $\gcd(a,b) = 1$.
- ☐ Can prove $a + b\sqrt{p}$ irrational by isolating $\sqrt{p}$ in 3 lines.
- ☐ Know that a^n ends in 0 iff both 2 and 5 divide a.

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Prime factorisation of 140 is: (A) $2^2 \cdot 5 \cdot 7$ (B) $2 \cdot 5^2 \cdot 7$ (C) $2^2 \cdot 35$ (D) $4 \cdot 5 \cdot 7$
2. $\text{HCF}(12, 18)$ is: (A) 2 (B) 3 (C) 6 (D) 36
3. $\text{LCM}(8, 12)$ is: (A) 4 (B) 24 (C) 48 (D) 96
4. Smallest composite number is: (A) 1 (B) 2 (C) 3 (D) 4
5. If $\text{HCF}(a,b) = 1$, then $\text{LCM}(a,b)$ is: (A) $a + b$ (B) a/b (C) ab (D) 1

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. If $\text{HCF}(99, 153) = 9$, then $\text{LCM}(99, 153)$ is: (A) 1683 (B) 15147 (C) 990 (D) 1530
2. Exponent of 3 in 3780 is: (A) 1 (B) 2 (C) 3 (D) 4
3. Largest number dividing 245 and 1029 leaving remainder 5 in each case: (A) 15 (B) 16 (C) 24 (D) 32
4. Two bells ring every 20 and 25 min. If they ring at 9:00 AM, next together at: (A) 10:00 AM (B) 10:20 AM (C) 10:40 AM (D) 11:00 AM
5. Which number is irrational? (A) $\sqrt{4}$ (B) 3.14 (C) $\sqrt{12}/\sqrt{3}$ (D) $2 + \sqrt{3}$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If n is a natural number, then $9^{2n} - 4^{2n}$ is always divisible by: (A) 5 only (B) 13 only (C) Both 5 and 13 (D) None
2. If $\text{HCF}(65, 117) = 65m - 117$, then m equals: (A) 1 (B) 2 (C) 3 (D) 4
3. Least number divisible by all integers from 1 to 10 (inclusive) is: (A) 100 (B) 504 (C) 2520 (D) 5040
4. If $a = 2^3 \cdot 3^x$ and $b = 2^2 \cdot 3 \cdot 5$, and $\text{LCM}(a,b) = 1080$, then x is: (A) 1 (B) 2 (C) 3 (D) 4
5. Ratio of LCM to HCF of 28 and 42 is: (A) 6 : 1 (B) 3 : 2 (C) 2 : 3 (D) 1 : 6

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (A):** $140 = 4 \times 35 = 2^2 \times 5 \times 7$. **Q2 (C):** $12 = 2^2 \cdot 3, 18 = 2 \cdot 3^2 \implies \text{HCF} = 6$. **Q3 (B):** $2^3 \cdot 3 = 24$. **Q4 (D):** $4 = 2 \times 2$. **Q5 (C):** $1 \times \text{LCM} = ab$.
Board Std	**Q1 (A):** $(99 \times 153)/9 = 11 \times 153 = 1683$. **Q2 (C):** $3780 = 2^2 \cdot 3^3 \cdot 5 \cdot 7$. **Q3 (B):** $\text{HCF}(240, 1024) = 16$. **Q4 (C):** $\text{LCM}(20, 25) = 100$ min $= 1$ hr 40 min. **Q5 (D):** $\sqrt{12}/\sqrt{3} = 2 \in \mathbb{Q}$; $2 + \sqrt{3} \notin \mathbb{Q}$.
Challenge	**Q1 (C):** $a^{2n} - b^{2n}$ is divisible by both $a - b = 5$ and $a + b = 13$. **Q2 (B):** $\text{HCF} = 13 \implies 65m = 130 \implies m = 2$. **Q3 (C):** $2^3 \cdot 3^2 \cdot 5 \cdot 7 = 2520$. **Q4 (C):** $1080 = 2^3 \cdot 3^3 \cdot 5 \implies x = 3$. **Q5 (A):** HCF$= 14$, LCM$= 84$, ratio $= 84/14 = 6 : 1$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: FTA & Prime Factor Trees; Day 2: HCF/LCM & Word Problems; Day 3: a^n digit-0 & Irrationality Proofs; Day 4: Revision & 3-Tier Test.
- **How to Introduce the Concept:** Ask students: 'Every molecule is built from unique atoms; similarly, every composite integer has a unique prime-atom DNA.'
- **Difficult Areas & Remediation:** Students struggle with why $p \mid a^2 \implies p \mid a$ holds only for primes, and why $a + b\sqrt{p}$ doesn't need squaring.
- **Blackboard Teaching Examples:** Contrast $\sqrt{5}$ proof (requires squaring) side-by-side with $3 + 2\sqrt{5}$ proof (only rearrangement).
- **Classroom Questions to Ask Students:** Ask: 'Can two numbers have HCF $= 12$ and LCM $= 100$?' (No, $12 \nmid 100$.)
- **Common Student Doubts & Answers:** Why isn't 1 prime? Because FTA uniqueness would fail ($6 = 2 \cdot 3 = 1 \cdot 2 \cdot 3$).
- **Homework & Revision Strategy:** Assign 5 prime-power HCF/LCM problems, 2 word problems, and 3 contradiction proofs.

[TEACHER'S TIP] Remind students to explicitly write 'Euclid's Division Lemma is deleted' so they don't waste time on outdated guides.

UNIT II: ALGEBRA TYBA COACHING CENTRE

Chapter 2: Polynomials

Geometrical Meaning of Zeros, Quadratic Zero-Coefficient Relations & Forming Polynomials

A. Chapter Introduction & Board Examination Relevance

Polynomials bridge arithmetic and algebra. In the 2026–27 rationalised Class 10 syllabus, this chapter focuses on the degree of a polynomial, geometrical interpretation of zeros as x-intercepts of $y = p(x)$, and the exact algebraic relationships between zeros and coefficients of quadratic polynomials (and forming a polynomial from given zeros).

- **What You Will Learn:** Identifying polynomial degrees, reading the number of real zeros from graphs, computing $\alpha + \beta = -b/a$ and $\alpha\beta = c/a$, evaluating symmetric expressions like $\alpha^2 + \beta^2$ or $1/\alpha + 1/\beta$, and constructing $k[x^2 - (\alpha + \beta)x + \alpha\beta]$.
- **Why This Chapter Matters:** Part of the 20-mark (CBSE) / 18-mark (RBSE) Algebra unit. Expect 3–5 marks directly from zero-coefficient problems and parabola graph MCQs.
- **Real-Life Applications:** Projectile trajectories, parabolic satellite dishes, suspension bridge cables, and revenue-cost profit curves in business.
- **Cross-Chapter Connections:** Directly prepares students for Quadratic Equations (Chapter 4) and Coordinate Geometry.
- **Board & Foundation Relevance:** Division algorithm for polynomials is deleted in the 2026–27 rationalised syllabus; focus 100% on graphs and zero-coefficient relations.

B. Complete Concept Explanation & Derivations

1. Polynomials, Degree, and Zeros

An algebraic expression $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ with non-negative integer exponents and $a_n \neq 0$ is a polynomial of **degree** n. A real number α is a **zero** of $p(x)$ if $p(\alpha) = 0$. Linear ($n = 1$) has at most 1 zero; Quadratic ($n = 2$) has at most 2 zeros; Cubic ($n = 3$) has at most 3 zeros.

[CONCEPT MASTER – 1. Polynomials, Degree, and Zeros] Expressions with $\sqrt{x}$ or $1/x$ (negative/fractional powers of x) are NOT polynomials.

2. Geometrical Meaning of Zeros

The real zeros of $y = p(x)$ are the x-coordinates of the points where the graph $y = p(x)$ intersects (or touches) the x-axis. For a quadratic polynomial $ax^2 + bx + c$ ($a \neq 0$), the graph is a **parabola**: upward-opening if $a > 0$, downward-opening if $a < 0$.

[CONCEPT MASTER – 2. Geometrical Meaning of Zeros] Count only x-axis intersections (never y-axis intersections!) to find the number of real zeros from a graph.

3. Zeros & Coefficients of a Quadratic Polynomial

If α and β are the zeros of $p(x) = ax^2 + bx + c$ ($a \neq 0$), then $(x - \alpha)$ and $(x - \beta)$ are factors, so $ax^2 + bx + c = k(x - \alpha)(x - \beta) = k[x^2 - (\alpha + \beta)x + \alpha\beta]$. Comparing coefficients gives: **Sum of zeros** $\alpha + \beta = -\frac{b}{a}$ and **Product of zeros** $\alpha\beta = \frac{c}{a}$.

[CONCEPT MASTER – 3. Zeros & Coefficients of a Quadratic Polynomial] When sum $S = \alpha + \beta$ and product $P = \alpha\beta$ are given, the family of quadratic polynomials is $k(x^2 - Sx + P)$ for any non-zero real k.

4. Symmetric Functions of Zeros

Board and MCQ questions frequently ask for expressions in α and β without finding the actual zeros: $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$; $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$; $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$; and $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$.

[CONCEPT MASTER – 4. Symmetric Functions of Zeros] Always express the target quantity in terms of $(\alpha+\beta)$ and $\alpha\beta$ before substituting numbers.

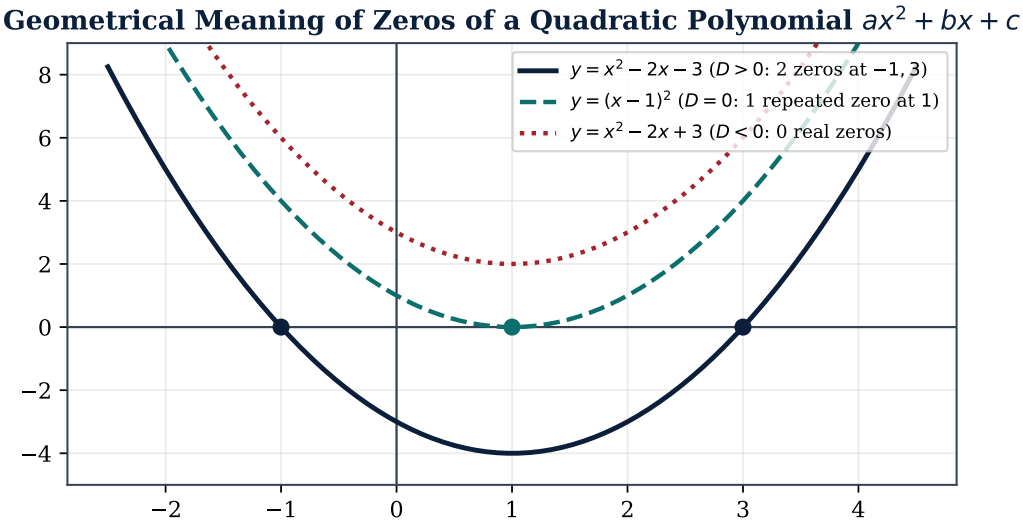


Figure 1: **Visual Model:** Parabolas showing 2 distinct real zeros ($D>0$), 1 repeated zero ($D=0$), and 0 real zeros ($D<0$).

C. Complete Formula Bank

[FORMULA BANK – 1. Zero of Linear Polynomial] **Expression:** $p(x)=ax+b \implies \alpha=-\frac{b}{a}$
Symbols: $a\neq 0$
Conditions & When to Use: Degree 1 polynomial
Solved Mini-Example: $3x-6=0 \implies x=2$
Common Mistake: Writing $+b/a$ instead of $-b/a$.

[FORMULA BANK – 2. Sum & Product of Quadratic Zeros] **Expression:** $\alpha+\beta=-\frac{b}{a}, \quad \alpha\beta=\frac{c}{a}$
Symbols: $p(x)=ax^2+bx+c$, $a\neq 0$
Conditions & When to Use: Quadratic polynomial
Solved Mini-Example: For $2x^2-5x+3$: $\alpha+\beta=5/2, \alpha\beta=3/2$
Common Mistake: Forgetting the minus sign in $-b/a$.

[FORMULA BANK – 3. Forming a Quadratic Polynomial] **Expression:** $p(x)=k\left[x^2-(\alpha+\beta)x+\alpha\beta\right]$
Symbols: $k\neq 0$ real constant
Conditions & When to Use: Given sum and product (or two zeros)
Solved Mini-Example: Sum $=4$, Product $=1 \implies x^2-4x+1$
Common Mistake: Confusing given zeros α,β with given sum S and product P.

[FORMULA BANK – 4. Sum of Squares of Zeros] **Expression:** $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$
Symbols: α,β zeros
Conditions & When to Use: Any quadratic
Solved Mini-Example: If $\alpha+\beta=5, \alpha\beta=6$, $\alpha^2+\beta^2=25-12=13$
Common Mistake: Writing $(\alpha+\beta)^2$ without $-2\alpha\beta$.

[FORMULA BANK – 5. Reciprocal Sum of Zeros] **Expression:** $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=-\frac{b}{c}$
Symbols: $\alpha,\beta\neq 0$ ($c\neq 0$)
Conditions & When to Use: Constant term $c\neq 0$
Solved Mini-Example: For x^2-5x+6: $1/\alpha+1/\beta=5/6$
Common Mistake: Computing individual roots when $-b/c$ gives it in 1 second.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE Standard/Basic, RBSE Board, and Foundation MCQ/Case-Study problem structures.

Pattern 1: Reading Zeros from Graphs

- **Identification Cue:** Given graph of $y=p(x)$, find the number of zeros. | **Concept & Formula:** x-axis intersections
- **Standard Step-by-Step:** Count distinct points where the curve meets or touches the x-axis.
- **Fastest Valid Method:** Count x-intercepts only. | **Alternative:** Ignore y-intercepts.
- **Common Trap:** Counting y-axis crossing as a zero.
- **Practice Drill:** A parabola touches the x-axis at $(3,0)$ only. How many real zeros does it have?

Pattern 2: Finding Zeros & Verifying Relations

- **Identification Cue:** Find zeros of $x^2+7x+10$ or $4u^2+8u$ or t^2-15 and verify. | **Concept & Formula:** Factorisation + $-b/a, c/a$ check
- **Standard Step-by-Step:** Factorise, list α,β, compute $\alpha+\beta$ and $\alpha\beta$, compare with $-b/a$ and c/a.
- **Fastest Valid Method:** For $t^2-15=0$, immediately $t=\pm\sqrt{15}$, sum $=0$, product $=-15$. | **Alternative:** Use quadratic formula if splitting is hard.
- **Common Trap:** Missing $b=0$ in t^2-15 or $c=0$ in $4u^2+8u$.
- **Practice Drill:** Find zeros of $6x^2-3-7x$ and verify the relation.

Pattern 3: Parameter k from Zero Conditions

- **Identification Cue:** One zero is negative of other / reciprocal of other / $\alpha^2+\beta^2=K$. | **Concept & Formula:** $\alpha+\beta=0$ or $\alpha\beta=1$
- **Standard Step-by-Step:** Translate condition into a,b,c equation and solve for k.
- **Fastest Valid Method:** Negative of other $\implies b=0$; Reciprocal $\implies c=a$. | **Alternative:** Substitute $x=\alpha$ if one zero is given numerically.
- **Common Trap:** Setting $c=0$ instead of $b=0$ for opposite-sign zeros.
- **Practice Drill:** If one zero of $(k^2+4)x^2+13x+4k$ is reciprocal of the other, find k.

Pattern 4: Forming Polynomials with Transformed Zeros

- **Identification Cue:** Form a polynomial whose zeros are $2\alpha,2\beta$ or $1/\alpha,1/\beta$. | **Concept & Formula:** New Sum S' & New Product P'
- **Standard Step-by-Step:** Find $\alpha+\beta,\alpha\beta$, compute S',P', write $x^2-S'x+P'$.
- **Fastest Valid Method:** For $1/\alpha,1/\beta$, simply reverse coefficients: cx^2+bx+a. | **Alternative:** Substitute $x\to 1/x$ directly.
- **Common Trap:** Forgetting to clear fractions with k.
- **Practice Drill:** If α,β are zeros of x^2-5x+4, find polynomial with zeros $1/\alpha,1/\beta$.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Opposite & Reciprocal Zero Instant Rules] **Core Idea & Why It Works:** Opposite zeros ($\beta = -\alpha$) $\iff$ $b = 0$. Reciprocal zeros ($\beta = 1/\alpha$) $\iff$ $c = a$.
Valid Domain: All parameter-k board MCQs | **Where It Fails:** When $a = 0$ (not quadratic)
Easy Example: $x^2 - (k-2)x + 5$ has opposite zeros $\implies k - 2 = 0 \implies k = 2$.
Board-Level Example: $(k^2+4)x^2 + 13x + 4k$ has reciprocal zeros $\implies k^2 + 4 = 4k \implies (k-2)^2 = 0 \implies k = 2$.
Advanced Example: Zero product $= 0 \implies c = 0$.

[SMART SHORTCUT – 2. Reciprocal-Zero Coefficient Reversal] **Core Idea & Why It Works:** The quadratic with zeros $1/\alpha, 1/\beta$ for $ax^2 + bx + c$ is $cx^2 + bx + a$ ($c \neq 0$).
Valid Domain: All reciprocal-zero formation questions | **Where It Fails:** If $c = 0$ (0 has no reciprocal)
Easy Example: Zeros of $2x^2 - 5x + 3$ flipped $\to 3x^2 - 5x + 2$.
Board-Level Example: $x^2 - 2x - 8 \to -8x^2 - 2x + 1$ or $8x^2 + 2x - 1$.
Advanced Example: Sum of reciprocals $= -b/c$ in one glance.

[SMART SHORTCUT – 3. Sum-to-Difference Identity] **Core Idea & Why It Works:** $|\alpha - \beta| = \frac{\sqrt{b^2 - 4ac}}{|a|}$.
Valid Domain: Finding $|\alpha - \beta|$ without solving roots | **Where It Fails:** Complex roots ($b^2 < 4ac$)
Easy Example: For $x^2 - 5x + 6$: $|\alpha - \beta| = \sqrt{25 - 24}/1 = 1$.
Board-Level Example: If $\alpha - \beta = 1$ for $x^2 - 5x + k$: $\sqrt{25 - 4k} = 1 \implies k = 6$.
Advanced Example: Quickly check $\alpha^2 - \beta^2 = (\alpha + \beta)(\alpha - \beta)$.

[SMART SHORTCUT – 4. Sign of a, b, c from Parabola Graph] **Core Idea & Why It Works:** Opens up $\implies a > 0$; y-intercept gives c; vertex $x = -b/(2a)$ gives sign of b.
Valid Domain: Competency graph MCQs | **Where It Fails:** None
Easy Example: U-shape crossing y-axis below origin $\implies a > 0, c < 0$.
Board-Level Example: Vertex in Q1 with $a > 0 \implies -b/(2a) > 0 \implies b < 0$.
Advanced Example: Zero count from $D = b^2 - 4ac$.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Find the zeros of $p(x) = x^2 - 2x - 8$ and verify the relationship between zeros and coefficients.
Given & Required: $a = 1, b = -2, c = -8$ | **Concept & Formula:** Splitting Middle Term
Detailed Solution: $x^2 - 4x + 2x - 8 = (x-4)(x+2) = 0 \implies \alpha = 4, \beta = -2$. Sum $= 4 + (-2) = 2 = -(-2)/1$; Product $= 4(-2) = -8 = -8/1$. **Final Answer:** $4, -2$
Fastest Alternative / Shortcut: Factors of -8 summing to -2 are $-4, +2$; flip signs to get zeros $4, -2$.

SOLVED EXAMPLE 2 [Beginner]
Question: Find a quadratic polynomial whose sum and product of zeros are $1/4$ and -1 respectively.
Given & Required: $S = 1/4, P = -1$ | **Concept & Formula:** $k(x^2 - Sx + P)$
Detailed Solution: $k(x^2 - \frac{1}{4}x - 1)$. Taking $k = 4$ gives $4x^2 - x - 4$. **Final Answer:** $4x^2 - x - 4$
Fastest Alternative / Shortcut: Multiply through by denominator 4.

SOLVED EXAMPLE 3 [Beginner]
Question: Find the zeros of $4u^2 + 8u$.
Given & Required: $a = 4, b = 8, c = 0$ | **Concept & Formula:** Common Factor
Detailed Solution: $4u(u+2) = 0 \implies u = 0, -2$. Sum $= -2 = -8/4$, Product $= 0 = 0/4$. **Final Answer:** $0, -2$
Fastest Alternative / Shortcut: Constant $c = 0 \implies$ one zero is always 0.

SOLVED EXAMPLE 4 [Beginner]

Question: Find the zeros of $t^2 - 15$.

Given & Required: $a = 1, b = 0, c = -15$ | **Concept & Formula:** $a^2 - b^2$

Detailed Solution: $(t - \sqrt{15})(t + \sqrt{15}) = 0 \implies t = \pm\sqrt{15}$. **Final Answer:** $\sqrt{15}, -\sqrt{15}$

Fastest Alternative / Shortcut: $t^2 = 15 \implies t = \pm\sqrt{15}$.

SOLVED EXAMPLE 5 [Intermediate]

Question: Find a quadratic polynomial whose zeros are $3 + \sqrt{2}$ and $3 - \sqrt{2}$.

Given & Required: $\alpha = 3 + \sqrt{2}, \beta = 3 - \sqrt{2}$ | **Concept & Formula:** Constructing from Conjugate Zeros

Detailed Solution: $S = 6$, $P = 9 - 2 = 7$. Polynomial: $x^2 - 6x + 7$. **Final Answer:** $x^2 - 6x + 7$

Fastest Alternative / Shortcut: Use $S = 2(3) = 6$ and $P = 3^2 - 2 = 7$.

SOLVED EXAMPLE 6 [Intermediate]

Question: If 1 is a zero of $p(x) = ax^2 - 3(a - 1)x - 1$, find a.

Given & Required: $p(1) = 0$ | **Concept & Formula:** Zero Definition

Detailed Solution: $a(1)^2 - 3(a-1)(1) - 1 = 0 \implies a - 3a + 3 - 1 = 0 \implies -2a + 2 = 0 \implies a = 1$. **Final Answer:** $a = 1$

Fastest Alternative / Shortcut: Sum of coefficients $= 0$ when $x = 1$ is a zero.

SOLVED EXAMPLE 7 [Intermediate]

Question: If α, β are zeros of $x^2 - 6x + k$ and $3\alpha + 2\beta = 20$, find k.

Given & Required: $\alpha + \beta = 6, 3\alpha + 2\beta = 20$ | **Concept & Formula:** Simultaneous Equations

Detailed Solution: $\alpha + (2\alpha + 2\beta) = 20 \implies \alpha + 12 = 20 \implies \alpha = 8$, so $\beta = -2$. Thus $k = \alpha\beta = -16$. **Final Answer:** $k = -16$

Fastest Alternative / Shortcut: Write $3\alpha + 2\beta = \alpha + 2(\alpha + \beta) = 20$.

SOLVED EXAMPLE 8 [Intermediate]

Question: If α, β are zeros of $4x^2 - 5x - 1$, find $\alpha^2\beta + \alpha\beta^2$.

Given & Required: $\alpha + \beta = 5/4, \alpha\beta = -1/4$ | **Concept & Formula:** Factoring Symmetric Expression

Detailed Solution: $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha + \beta) = (-\frac{1}{4})(\frac{5}{4}) = -\frac{5}{16}$. **Final Answer:** $-5/16$

Fastest Alternative / Shortcut: Factor out $\alpha\beta$ first.

SOLVED EXAMPLE 9 [Board Standard]

Question: Find the zeros of $6x^2 - 3 - 7x$ and verify the relationship between zeros and coefficients.

Given & Required: $6x^2 - 7x - 3$ | **Concept & Formula:** Standard Ordering & Factorisation

Detailed Solution: Reorder: $6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3 = (3x + 1)(2x - 3) = 0 \implies \alpha = -1/3, \beta = 3/2$. Sum $= 7/6 = -(-7)/6$; Product $= -1/2 = -3/6$. **Final Answer:** $-1/3, 3/2$

Fastest Alternative / Shortcut: Always sort in descending powers of x first!

SOLVED EXAMPLE 10 [Board Standard]

Question: If α, β are zeros of $2x^2 + 5x + k$ satisfying $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, find k.

Given & Required: $\alpha + \beta = -5/2, \alpha\beta = k/2$ | **Concept & Formula:** Symmetric Identity

Detailed Solution: $(\alpha + \beta)^2 - \alpha\beta = \frac{21}{4} \implies \frac{25}{4} - \frac{k}{2} = \frac{21}{4} \implies \frac{k}{2} = 1 \implies k = 2$. **Final Answer:** $k = 2$

Fastest Alternative / Shortcut: Note $\alpha^2 + \beta^2 + \alpha\beta = (\alpha + \beta)^2 - \alpha\beta$.

SOLVED EXAMPLE 11 [Board Standard]

Question: If one zero of $3x^2 - 8x + 2k + 1$ is seven times the other, find k.

Given & Required: $\beta = 7\alpha$ | **Concept & Formula:** Sum & Product

Detailed Solution: $\alpha + 7\alpha = 8/3 \implies 8\alpha = 8/3 \implies \alpha = 1/3, \beta = 7/3$. Product $= (1/3)(7/3) = 7/9 = (2k+1)/3 \implies 2k + 1 = 7/3 \implies 2k = 4/3 \implies k = 2/3$. **Final Answer:** $k = 2/3$

Fastest Alternative / Shortcut: Use $\alpha + m\alpha = -b/a$ to find α immediately.

SOLVED EXAMPLE 12 [Board Standard]

Question: If α, β are zeros of $x^2 - p(x+1) - c$, show that $(\alpha+1)(\beta+1) = 1 - c$.

Given & Required: $x^2 - px - (p + c)$ | **Concept & Formula:** Expanding & Substituting

Detailed Solution: $\alpha + \beta = p, \alpha\beta = -(p + c)$. $(\alpha+1)(\beta+1) = \alpha\beta + (\alpha+\beta) + 1 = -p - c + p + 1 = 1 - c$. **Final Answer:** $1 - c$

Fastest Alternative / Shortcut: Rewrite in standard form $x^2 - px - (p + c)$ first.

SOLVED EXAMPLE 13 [Board Standard]

Question: Find a quadratic polynomial whose zeros are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$, where α, β are zeros of $x^2 - 5x + 6$.

Given & Required: $x^2 - 5x + 6$ | **Concept & Formula:** Transformed Zeros

Detailed Solution: $\alpha + \beta = 5, \alpha\beta = 6$. $S' = \frac{1}{\alpha} + \frac{1}{\beta} = 5/6$, $P' = 1/6$. Polynomial: $6x^2 - 5x + 1$. **Final Answer:** $6x^2 - 5x + 1$

Fastest Alternative / Shortcut: Reverse coefficients: $6x^2 - 5x + 1$.

SOLVED EXAMPLE 14 [Challenge]

Question: If α, β are zeros of $x^2 - 4x + 1$, find $\alpha^3 + \beta^3$.

Given & Required: $\alpha + \beta = 4, \alpha\beta = 1$ | **Concept & Formula:** Cubic Symmetric Identity

Detailed Solution: $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = 4^3 - 3(1)(4) = 64 - 12 = 52$. **Final Answer:** 52

Fastest Alternative / Shortcut: $S^3 - 3PS$.

SOLVED EXAMPLE 15 [Challenge]

Question: If α, β are zeros of $2x^2 - 4x + 5$, find $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$.

Given & Required: $\alpha + \beta = 2, \alpha\beta = 5/2$ | **Concept & Formula:** Reciprocal Squares

Detailed Solution: $\frac{\alpha^2+\beta^2}{(\alpha\beta)^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2} = \frac{4-5}{25/4} = -\frac{4}{25}$. **Final Answer:** $-4/25$

Fastest Alternative / Shortcut: Combine over common denominator $(\alpha\beta)^2$.

SOLVED EXAMPLE 16 [Challenge]

Question: For what value of k does $x^2 - (k+6)x + 2(2k-1)$ have sum of zeros equal to half their product?

Given & Required: $S = P/2$ | **Concept & Formula:** Zero Relation Equation

Detailed Solution: $k + 6 = \frac{1}{2} \cdot 2(2k-1) = 2k - 1 \implies k = 7$. **Final Answer:** $k = 7$

Fastest Alternative / Shortcut: $k + 6 = 2k - 1 \implies k = 7$.

G. Competency MCQ Mastery Section

Q1. The degree of the polynomial $5x^3 - 4x^2 + 7x - \sqrt{2}$ is:

(A) 1 (B) 2 (C) 3 (D) 0

Answer: (C) Highest power of x is 3. *Fast Method:* Read highest exponent. *Distractor Note:* Constant $\sqrt{2}$ does not stop it from being a polynomial.

Q2. If the graph of $y = p(x)$ does not intersect the x-axis at all, then $p(x)$ has:

(A) 1 zero (B) 2 zeros (C) No real zeros (D) Infinitely many zeros

Answer: (C) No x-intercepts means 0 real zeros. *Fast Method:* Count x-axis crossings. *Distractor Note:* Do not count y-intercept.

Q3. The sum of zeros of $3x^2 - 7x + 5$ is:

(A) $-7/3$ (B) $7/3$ (C) $5/3$ (D) $-5/3$

Answer: (B) $\alpha + \beta = -(-7)/3 = 7/3$. *Fast Method:* $-b/a$. *Distractor Note:* Missing negative sign.

Q4. A quadratic polynomial whose zeros are -3 and 4 is:

(A) $x^2 - x - 12$ (B) $x^2 + x + 12$ (C) $x^2 - x + 12$ (D) $2x^2 + 2x - 24$

Answer: (A) $S = 1, P = -12 \implies x^2 - x - 12$. *Fast Method:* Check $p(-3) = 0$. *Distractor Note:* Using -3 and 4 as S and P.

Q5. If one zero of $x^2 + 3x + k$ is 2, then k is:

(A) 10 (B) -10 (C) 5 (D) -5

Answer: (B) $2^2 + 3(2) + k = 0 \implies k = -10$. *Fast Method:* Put $x = 2$. *Distractor Note:* Sign error.

Q6. If the zeros of $ax^2 + bx + c$ ($c \neq 0$) are equal, then:

(A) c and a have opposite signs (B) c and b have opposite signs (C) c and a have the same sign (D) c and b have the same sign

Answer: (C) Equal roots $\implies b^2 = 4ac \geq 0 \implies ac > 0$, so a and c share the same sign. *Fast Method:* $b^2 = 4ac \implies ac > 0$. *Distractor Note:* Guessing b instead of a, c.

Q7. The number of polynomials having zeros -2 and 5 is:

(A) 1 (B) 2 (C) 3 (D) More than 3 (infinitely many)

Answer: (D) $k(x^2 - 3x - 10)$ works for every $k \neq 0$. *Fast Method:* Infinitely many due to k. *Distractor Note:* Picking 1.

Q8. If α, β are zeros of $x^2 - 1$, then $\alpha + \beta$ is:

(A) 2 (B) 1 (C) -1 (D) 0

Answer: (D) $b = 0 \implies -b/a = 0$. *Fast Method:* Zeros are $+1, -1$. *Distractor Note:* Confusing sum with product (-1).

Q9. If α, β are zeros of $2x^2 - 7x + 3$, then $\alpha^2 + \beta^2$ equals:

(A) $37/4$ (B) $49/4$ (C) $25/4$ (D) $13/4$

Answer: (A) $(7/2)^2 - 2(3/2) = 49/4 - 3 = 37/4$. *Fast Method:* $S^2 - 2P$. *Distractor Note:* Subtracting P instead of $2P$.

Q10. Assertion (A): $x^2 + 4x + 5$ has two real zeros. Reason (R): A quadratic polynomial has at most two zeros.

(A) Both A & R true, R explains A (B) Both A & R true, R does not explain A (C) A true, R false (D) A false, R true

Answer: (D) $D = 16 - 20 = -4 < 0$, so A is false. R is true. *Fast Method:* Check $D = b^2 - 4ac$. *Distractor Note:* Assuming every quadratic has 2 real zeros.

Q11. If one zero of $3x^2 + 8x + k$ is reciprocal of the other, then k is:

(A) 3 (B) -3 (C) $1/3$ (D) $-1/3$

Answer: (A) Product $= k/3 = 1 \implies k = 3$. *Fast Method:* $c = a \implies k = 3$. *Distractor Note:* Setting $k = 1$.

Q12. If α, β are zeros of $x^2 + px + q$, then the polynomial with zeros $1/\alpha, 1/\beta$ is:

(A) $qx^2 + px + 1$ (B) $px^2 + qx + 1$ (C) $x^2 + qx + p$ (D) $qx^2 - px + 1$

Answer: (A) Reverse coefficients $1, p, q \to qx^2 + px + 1$. *Fast Method:* Coefficient reversal. *Distractor Note:* Flipping sign of p.

H. Special Board & MCQ Examination Strategies

- Always arrange the polynomial in standard form $ax^2 + bx + c$ before reading a, b, c.
- Read carefully whether a problem gives individual zeros (α, β) or already gives their Sum (S) and Product (P).
- In graph MCQs, count only intersections/tangencies with the x-axis.
- Remember the two instant shortcuts: opposite zeros $\implies b = 0$; reciprocal zeros $\implies c = a$.
- Never spend time solving for ugly radical roots when the question only asks for $\alpha^2 + \beta^2$ or $1/\alpha + 1/\beta$.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Using $x^2 + Sx + P$ instead of $x^2 - Sx + P$. $\longrightarrow$ **Correction:** The coefficient of x is ALWAYS $-(\alpha + \beta)$.

[COMMON MISTAKE] **Error:** Treating given Sum and Product as α and β. $\longrightarrow$ **Correction:** If Sum $= 1/4, Product = -1$, plug directly into $x^2 - Sx + P$, do not add them again.

[COMMON MISTAKE] **Error:** Forgetting the negative sign in $\alpha + \beta = -b/a$. $\longrightarrow$ **Correction:** For $x^2 - 5x + 6$, $b = -5$, so $\alpha + \beta = -(-5)/1 = +5$.

[COMMON MISTAKE] **Error:** Counting the y-axis intersection as a zero on a graph. $\longrightarrow$ **Correction:** Only x-axis intersections are zeros of $y = p(x)$.

[COMMON MISTAKE] **Error:** Writing $(\alpha + \beta)^2 = \alpha^2 + \beta^2$. $\longrightarrow$ **Correction:** $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $\alpha+\beta=-b/a$; $\alpha\beta=c/a$; $p(x)=k[x^2-(\alpha+\beta)x+\alpha\beta]$; $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$.

[QUICK REVISION] Key Theorems & Results: Opposite zeros $\iff$ $b=0$; Reciprocal zeros $\iff$ $c=a$; Zero at $x=0 \iff c=0$.

[QUICK REVISION] Shortcut Summary: Coefficient reversal cx^2+bx+a for reciprocal zeros; $|\alpha-\beta|=\sqrt{b^2-4ac}/|a|$.

[QUICK REVISION] High-Frequency Board Patterns: Graph zero counting; zero-coefficient verification; finding k from $\alpha^2+\beta^2$ or reciprocal zeros.

Last-Minute Revision Checklist:

- ☐ Can identify polynomials vs. non-polynomials.
- ☐ Can read real zero counts from $y=p(x)$ graphs.
- ☐ Can verify $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$ cleanly.
- ☐ Can build $k(x^2-Sx+P)$ from either (S,P) or (α,β).
- ☐ Know that Division Algorithm for polynomials is deleted in 2026–27.

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Zeros of x^2-9 are: (A) $3,3$ (B) $-3,-3$ (C) $3,-3$ (D) $9,-9$
2. Product of zeros of $2x^2+3x-6$ is: (A) -3 (B) 3 (C) $-3/2$ (D) $3/2$
3. Polynomial with sum 0 and product $\sqrt{5}$: (A) $x^2-\sqrt{5}$ (B) $x^2+\sqrt{5}$ (C) $x^2+\sqrt{5}x$ (D) $x^2-\sqrt{5}x$
4. Degree of constant non-zero polynomial 7 is: (A) 1 (B) 7 (C) 0 (D) Undefined
5. If $p(x)=x^2-5x+6$, then $p(2)$ equals: (A) 0 (B) 2 (C) 4 (D) -2

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. If α,β are zeros of x^2-4x+3, then $1/\alpha+1/\beta$ is: (A) $3/4$ (B) $4/3$ (C) $-4/3$ (D) 4
2. If -2 is a zero of $3x^2+4x+2k$, then k is: (A) 2 (B) -2 (C) 4 (D) -4
3. Quadratic polynomial with zeros $-2/3$ and $2/3$ is: (A) $9x^2-4$ (B) $9x^2+4$ (C) $3x^2-2$ (D) $4x^2-9$
4. If α,β are zeros of x^2-5x+k with $\alpha-\beta=1$, k is: (A) 4 (B) 5 (C) 6 (D) 8
5. If $a+b+c=0$ for ax^2+bx+c, one zero is definitely: (A) 0 (B) 1 (C) -1 (D) a

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If α,β are zeros of $2x^2+5x+1$, then $\alpha+\beta+\alpha\beta$ equals: (A) -2 (B) 2 (C) 3 (D) -3
2. If zeros of $x^2+(a+1)x+b$ are 2 and -3, then (a,b) is: (A) $(0,-6)$ (B) $(2,-6)$ (C) $(5,-1)$ (D) $(-7,-1)$
3. If α,β are zeros of x^2-x-4, value of $\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta$ is: (A) $15/4$ (B) $-15/4$ (C) 4 (D) 15
4. If $p(x)=ax^2+bx+c$ has equal zeros, then c equals: (A) $-b/2a$ (B) $b/2a$ (C) $b^2/4a$ (D) $-b^2/4a$
5. Maximum number of points where $y=x^2-4x+5$ cuts the x-axis: (A) 0 (B) 1 (C) 2 (D) 3

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (C):** $(x-3)(x+3)=0$. **Q2 (A):** $c/a=-6/2=-3$. **Q3 (B):** $x^2-0x+\sqrt{5}$. **Q4 (C):** $7x^0 \implies$ degree 0. **Q5 (A):** $4-10+6=0$.
Board Std	**Q1 (B):** $(\alpha+\beta)/(\alpha\beta)=4/3$. **Q2 (B):** $12-8+2k=0 \implies k=-2$. **Q3 (A):** $x^2-4/9 \to 9x^2-4$. **Q4 (C):** $\alpha=3,\beta=2 \implies k=6$. **Q5 (B):** $p(1)=a+b+c=0$.
Challenge	**Q1 (A):** $-5/2+1/2=-2$. **Q2 (A):** $2-3=-1=-(a+1) \implies a=0$; $b=2(-3)=-6$. **Q3 (A):** $-1/4-(-4)=15/4$. **Q4 (C):** $b^2-4ac=0 \implies c=b^2/(4a)$. **Q5 (A):** $D=16-20<0 \implies 0$ real zeros.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Degree & Graphs; Day 2: Factorisation & Zero-Coefficient Verification; Day 3: Symmetric Expressions & Forming Polynomials; Day 4: Competency MCQs & Test.
- **How to Introduce the Concept:** Show a cricket ball trajectory and ask: 'Where does the ball hit the ground? Those two points are the zeros of the parabola!'
- **Difficult Areas & Remediation:** Students mix up 'zeros are α,β' vs. 'sum and product are S,P'. Drill this contrast with 4 rapid examples.
- **Blackboard Teaching Examples:** Put standard form ax^2+bx+c, $S=-b/a$, $P=c/a$ in a permanent corner box on the board.
- **Classroom Questions to Ask Students:** Ask: 'If $x^2+1=0$ has no real zeros, what does its graph look like?' (Floats entirely above the x-axis.)
- **Common Student Doubts & Answers:** Why write k in $k(x^2-Sx+P)$? Because multiplying a polynomial by 2 or 5 stretches the curve vertically without moving its x-intercepts.
- **Homework & Revision Strategy:** 10 verification & formation problems + 5 symmetric-expression MCQs.

[TEACHER'S TIP] Remind students that Polynomial Division Algorithm is excluded in 2026–27.

UNIT II: ALGEBRA TYBA COACHING CENTRE

Chapter 3: Pair of Linear Equations in Two Variables

Graphical Method, Consistency Conditions, Substitution, Elimination & Word Problems

A. Chapter Introduction & Board Examination Relevance

A pair of linear equations in two variables models two simultaneous constraints in real life. In the 2026–27 syllabus, students must master the graphical method, the algebraic ratio conditions ($a_1/a_2, b_1/b_2, c_1/c_2$) for intersecting/parallel/coincident lines, and solving algebraically by Substitution and Elimination.

- **What You Will Learn:** Plotting linear equations, determining consistency (unique, infinite, or no solution) via coefficient ratios, solving systems by Substitution and Elimination, and translating age, digit, fraction, boat-stream, and fixed-charge word problems.
- **Why This Chapter Matters:** A staple 4–6 mark contributor in both CBSE and RBSE boards, regularly appearing as a 3-mark algebraic/word problem or a 5-mark graphical solution (plus Case Study).
- **Real-Life Applications:** Break-even analysis in economics, taxi fare structures (fixed + per-km charge), mixture problems, and upstream/downstream navigation.
- **Cross-Chapter Connections:** Connects to Coordinate Geometry (intersection points and triangle vertices formed with axes) and Linear Programming in Class 12.
- **Board & Foundation Relevance:** Cross-multiplication method and equations reducible to linear form (e.g., $1/x + 1/y$) are deleted in the 2026–27 syllabus.

B. Complete Concept Explanation & Derivations

1. Standard Form & Graphical Interpretation

The general form of a pair of linear equations in x and y is $a_1x+b_1y+c_1 = 0$ and $a_2x+b_2y+c_2 = 0$, where $a_1^2+b_1^2 \neq 0$ and $a_2^2 + b_2^2 \neq 0$. Each equation represents a straight line in the Cartesian plane. Two lines in a plane can either intersect at one point, be parallel, or coincide.

[CONCEPT MASTER – 1. Standard Form & Graphical Interpretation] A system is called **consistent** if it has at least one solution (intersecting or coincident), and **inconsistent** if it has no solution (parallel).

2. Algebraic Conditions for Number of Solutions

Compare the ratios of coefficients: (1) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \implies$ Intersecting lines, **Unique solution** (Consistent). (2) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \implies$ Coincident lines, **Infinitely many solutions** (Dependent & Consistent). (3) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \implies$ Parallel lines, **No solution** (Inconsistent).

[CONCEPT MASTER – 2. Algebraic Conditions for Number of Solutions] Always bring BOTH equations to the same side ($= 0$ or $= c$) before comparing c_1/c_2!

3. Substitution & Elimination Methods

Substitution: Express one variable (say y) in terms of x from one equation and substitute into the other. **Elimination:** Multiply one or both equations by suitable non-zero constants so the coefficients of x (or y) become numerically equal, then add or subtract to eliminate that variable.

[CONCEPT MASTER – 3. Substitution & Elimination Methods] When coefficients of x and y are swapped (e.g., $37x + 43y = 123$ and $43x + 37y = 117$), NEVER multiply by 37 and 43! Add the two equations and subtract them to get instant $x + y$ and $x - y$.

C. Complete Formula Bank

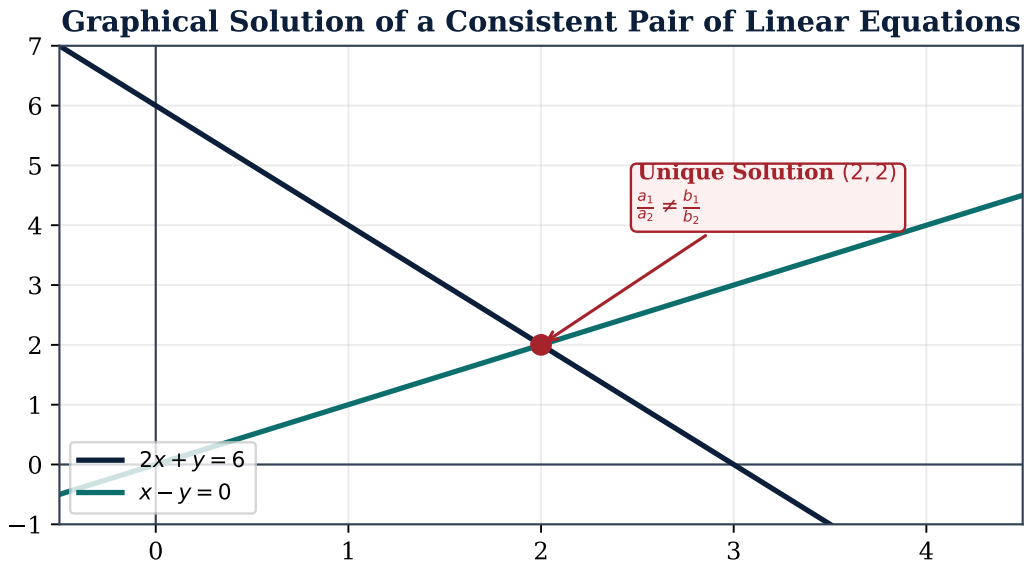


Figure 1: **Visual Model:** Graphical intersection of two consistent lines $2x + y = 6$ and $x - y = 0$ at the unique solution $(2, 2)$.

[FORMULA BANK – 1. Unique Solution Condition] **Expression:** $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$
Symbols: a_i, b_i: coefficients of x, y
Conditions & When to Use: Intersecting lines (1 solution)
Solved Mini-Example: $x + 2y = 5, 3x + y = 4$: $1/3 \neq 2/1$
Common Mistake: Checking c_1/c_2 when $a_1/a_2 \neq b_1/b_2$ (unnecessary).

[FORMULA BANK – 2. Infinitely Many Solutions] **Expression:** $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$
Symbols: c_i: constant terms on same side
Conditions & When to Use: Coincident lines (dependent)
Solved Mini-Example: $2x + 3y = 7, 4x + 6y = 14$
Common Mistake: Forgetting $\pm$ when solving $k^2 = 36$ and not verifying c_1/c_2.

[FORMULA BANK – 3. No Solution (Parallel Lines)] **Expression:** $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
Symbols: a_i, b_i, c_i in matching standard form
Conditions & When to Use: Parallel lines (inconsistent)
Solved Mini-Example: $2x - y = 4, 4x - 2y = 5$
Common Mistake: Writing c_1 on LHS in Eq 1 and on RHS in Eq 2.

[FORMULA BANK – 4. Two-Digit Number Representation] **Expression:** $N = 10x + y$, Reversed $= 10y + x$
Symbols: x: tens digit (1–9), y: units digit (0–9)
Conditions & When to Use: Digit word problems
Solved Mini-Example: Difference $(10x + y) - (10y + x) = 9(x - y)$
Common Mistake: Writing the number as xy instead of $10x + y$.

[FORMULA BANK – 5. Boat & Stream Speeds] **Expression:** $v_{\text{down}} = u + v, \quad v_{\text{up}} = u - v$
Symbols: u: boat speed in still water, v: stream speed $(u > v)$
Conditions & When to Use: Time = Distance/Speed
Solved Mini-Example: Downstream 20 km in 2 hr $\implies u + v = 10$
Common Mistake: Subtracting speeds as $v - u$ upstream.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE Standard/Basic, RBSE Board, and Foundation MCQ/Case-Study problem structures.

Pattern 1: Parameter k for Unique / Infinite / No Solution

- **Identification Cue:** Find k for which the system has no solution / infinitely many solutions. | **Concept & Formula:** Ratio comparison
- **Standard Step-by-Step:** Set up $\frac{a_1}{a_2} = \frac{b_1}{b_2}$ and check $\frac{c_1}{c_2}$.
- **Fastest Valid Method:** Cross-multiply $a_1 b_2 = a_2 b_1$, then verify c_1/c_2 for each root. | **Alternative:** Compare slopes $-a_1/b_1$ and $-a_2/b_2$.
- **Common Trap:** Accepting both $k = +6$ and $k = -6$ when one makes the lines coincident instead of parallel.
- **Practice Drill:** Find k for no solution: $3x + y = 1$, $(2k-1)x + (k-1)y = 2k + 1$.

Pattern 2: Graphical Solution & Triangle Area with Axes

- **Identification Cue:** Solve graphically and shade/find area of triangle formed with x-axis (or y-axis). | **Concept & Formula:** Plotting 2 points per line + $\frac{1}{2} \times$ base $\times$ height
- **Standard Step-by-Step:** Find intercepts $(x, 0)$ and $(0, y)$, plot lines, locate intersection (x_0, y_0).
- **Fastest Valid Method:** Height of triangle with x-axis is simply $|y_0|$! | **Alternative:** Verify intersection algebraically first.
- **Common Trap:** Reading triangle base from wrong axis.
- **Practice Drill:** Solve $x - y + 1 = 0$ and $3x + 2y - 12 = 0$ graphically and shade triangle with x-axis.

Pattern 3: Swapped-Coefficient Linear Systems

- **Identification Cue:** $ax + by = c$ and $bx + ay = d$ with large a, b. | **Concept & Formula:** Add & Subtract Trick
- **Standard Step-by-Step:** Add equations $\rightarrow$ divide by $(a + b)$; subtract $\rightarrow$ divide by $(a - b)$.
- **Fastest Valid Method:** Instant reduction to $x + y = p$ and $x - y = q$. | **Alternative:** Standard elimination (slow).
- **Common Trap:** Arithmetic errors if multiplying large primes.
- **Practice Drill:** Solve $152x - 378y = -74$ and $-378x + 152y = -604$.

Pattern 4: Word Problems (Ages, Fractions, Fixed Charges)

- **Identification Cue:** '5 years ago...', 'numerator & denominator...', 'fixed charge + per km...' | **Concept & Formula:** Two-variable linear modeling
- **Standard Step-by-Step:** Define x, y clearly with units, form 2 equations, solve by elimination.
- **Fastest Valid Method:** Check final (x, y) back against English sentences. | **Alternative:** Use options in MCQs.
- **Common Trap:** Forgetting to add/subtract years from BOTH people in age problems.
- **Practice Drill:** A fraction becomes $9/11$ if 2 is added to both numerator and denominator...

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Add-Subtract Trick for Swapped Coefficients] **Core Idea & Why It Works:** For $Ax + By = C$ and $Bx + Ay = D$, adding gives $(A + B)(x + y) = C + D$; subtracting gives $(A - B)(x - y) = C - D$.

Valid Domain: Whenever x, y coefficients are interchanged | **Where It Fails:** Non-swapped coefficients

Easy Example: $21x + 47y = 110, 47x + 21y = 162 \implies 68(x + y) = 272 \implies x + y = 4$.

Board-Level Example: $152x - 378y = -74, -378x + 152y = -604 \implies x + y = 3, x - y = 1 \implies (2, 1)$.

Advanced Example: Directly find $x + y$ without solving x and y individually!

[SMART SHORTCUT – 2. Option-Plug Verification in MCQs] **Core Idea & Why It Works:** Substitute the 4 coordinate pairs directly into the simpler equation first.

Valid Domain: All 'Solve the system' MCQs | **Where It Fails:** When options ask for $x + y$ or k instead of (x, y)

Easy Example: Plug $(2, 1)$ into $x + 2y = 4 \rightarrow 2 + 2 = 4$ (true).

Board-Level Example: Check second equation only for options that pass the first.

Advanced Example: Eliminates 3 options in under 15 seconds.

[SMART SHORTCUT – 3. Axis-Triangle Height = $|y_0|$ or $|x_0|$] **Core Idea & Why It Works:** The triangle bounded by two lines and the x-axis has height $|y_0|$ (intersection y-coordinate).
Valid Domain: All graphical triangle-area questions | **Where It Fails:** If lines are parallel to the axis
Easy Example: Intersection $(2,3)$, x-intercepts $(-1,0)$ and $(4,0)$ $\implies$ Area $= \frac{1}{2}(5)(3) = 7.5$.
Board-Level Example: Triangle with y-axis has base $= |c_1/b_1 - c_2/b_2|$ and height $= |x_0|$.
Advanced Example: Double-check graphical shading.

[SMART SHORTCUT – 4. Sign Check for c_1/c_2] **Core Idea & Why It Works:** Always write both equations as $a_ix + b_iy = c_i$ OR $a_ix + b_iy + c_i = 0$ before comparing.
Valid Domain: All consistency MCQs | **Where It Fails:** None
Easy Example: $2x + 3y = 5$ and $4x + 6y - 10 = 0 \to$ move -10 to RHS $= 10$, so $5/10 = 1/2$ (coincident!).
Board-Level Example: Prevents the #1 trap of misreading $+10$ vs -10.
Advanced Example: Saves marks in Assertion-Reason questions.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 **[Beginner]**
Question: Solve by substitution: $x + y = 14$ and $x - y = 4$.
Given & Required: $x + y = 14, x - y = 4$ | **Concept & Formula:** Substitution
Detailed Solution: From Eq 2, $x = y + 4$. Sub into Eq 1: $(y + 4) + y = 14 \implies 2y = 10 \implies y = 5, x = 9$. **Final Answer:** $x = 9, y = 5$
Fastest Alternative / Shortcut: Add directly: $2x = 18 \implies x = 9$.

SOLVED EXAMPLE 2 **[Beginner]**
Question: Determine whether $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$ intersect, are parallel, or coincide.
Given & Required: $a_1 = 5, b_1 = -4; a_2 = 7, b_2 = 6$ | **Concept & Formula:** Ratio Test
Detailed Solution: $\frac{a_1}{a_2} = \frac{5}{7}$, $\frac{b_1}{b_2} = \frac{-4}{6} = -\frac{2}{3}$. Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, they intersect at one point. **Final Answer: Intersecting (Unique solution)**
Fastest Alternative / Shortcut: Ratios have opposite signs, so unequal immediately.

SOLVED EXAMPLE 3 **[Beginner]**
Question: Solve by elimination: $3x + 4y = 10$ and $2x - 2y = 2$.
Given & Required: Two linear equations | **Concept & Formula:** Elimination
Detailed Solution: Multiply Eq 2 by 2: $4x - 4y = 4$. Add to Eq 1: $7x = 14 \implies x = 2$. Sub into Eq 2: $4 - 2y = 2 \implies y = 1$. **Final Answer:** $x = 2, y = 1$
Fastest Alternative / Shortcut: Divide Eq 2 by 2: $x - y = 1$, then add $3(x - y) = 3$ to Eq 1.

SOLVED EXAMPLE 4 **[Beginner]**
Question: Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find its dimensions.
Given & Required: $l + w = 36, l - w = 4$ | **Concept & Formula:** Linear System
Detailed Solution: Add: $2l = 40 \implies l = 20$ m, $w = 16$ m. **Final Answer:** $l = 20$ **m,** $w = 16$ **m**
Fastest Alternative / Shortcut: Sum $= 36$, diff $= 4 \implies (36 \pm 4)/2 = 20, 16$.

SOLVED EXAMPLE 5 **[Intermediate]**
Question: For what value of k will $kx + 3y - (k - 3) = 0$ and $12x + ky - k = 0$ have infinitely many solutions?
Given & Required: Infinite solutions | **Concept & Formula:** $a_1/a_2 = b_1/b_2 = c_1/c_2$
Detailed Solution: $\frac{k}{12} = \frac{3}{k} \implies k^2 = 36 \implies k = \pm 6$. Check c_1/c_2: $\frac{3}{k} = \frac{k-3}{k} \implies k - 3 = 3 \implies k = 6$. ($k = -6$ gives $-1/2 \neq 3/2$). **Final Answer:** $k = 6$
Fastest Alternative / Shortcut: Test both $+6$ and -6 in $b_1/b_2 = c_1/c_2$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. Find their present ages.

Given & Required: Nuri $= x$, Sonu $= y$ | **Concept & Formula:** Age Word Problem

Detailed Solution: $x - 5 = 3(y - 5) \implies x - 3y = -10$. $x + 10 = 2(y + 10) \implies x - 2y = 10$. Subtract: $y = 20$, $x = 50$. **Final Answer:** **Nuri** $= 50$ **yr, Sonu** $= 20$ **yr**

Fastest Alternative / Shortcut: Subtract equations directly to eliminate x.

SOLVED EXAMPLE 7 [Intermediate]

Question: The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of digits. Find the number.

Given & Required: $x + y = 9$, $9(10x + y) = 2(10y + x)$ | **Concept & Formula:** Digit Problem

Detailed Solution: $90x + 9y = 20y + 2x \implies 88x = 11y \implies y = 8x$. With $x + y = 9$, $9x = 9 \implies x = 1, y = 8$. Number $= 18$. **Final Answer:** 18

Fastest Alternative / Shortcut: Check $9 \times 18 = 162 = 2 \times 81$.

SOLVED EXAMPLE 8 [Intermediate]

Question: A fraction becomes $1/3$ when 1 is subtracted from the numerator and becomes $1/4$ when 8 is added to its denominator. Find the fraction.

Given & Required: $\frac{x-1}{y} = \frac{1}{3}, \frac{x}{y+8} = \frac{1}{4}$ | **Concept & Formula:** Fraction Problem

Detailed Solution: $3x - y = 3$ and $4x - y = 8$. Subtract: $x = 5$, then $y = 12$. Fraction $= 5/12$. **Final Answer:** $5/12$

Fastest Alternative / Shortcut: Verify: $4/12 = 1/3$ and $5/20 = 1/4$.

SOLVED EXAMPLE 9 [Board Standard]

Question: Find k so that $3x + y = 1$ and $(2k - 1)x + (k - 1)y = 2k + 1$ has no solution.

Given & Required: No solution | **Concept & Formula:** $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Detailed Solution: $\frac{3}{2k-1} = \frac{1}{k-1} \implies 3k - 3 = 2k - 1 \implies k = 2$. Check c_1/c_2: $\frac{1}{2(2)+1} = \frac{1}{5} \neq \frac{1}{1}$. Valid! **Final Answer:** $k = 2$

Fastest Alternative / Shortcut: Solve $3(k - 1) = 2k - 1 \implies k = 2$.

SOLVED EXAMPLE 10 [Board Standard]

Question: A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs. 27 for a book kept for 7 days, while Susy paid Rs. 21 for 5 days. Find the fixed charge and daily charge.

Given & Required: Fixed x (3 days), extra y/day | **Concept & Formula:** Fixed + Variable Cost

Detailed Solution: Saritha: $x + 4y = 27$. Susy: $x + 2y = 21$. Subtract: $2y = 6 \implies y = 3, x = 15$. **Final Answer:** **Fixed** $=$ **Rs.** 15**, Extra** $=$ **Rs.** 3**/day**

Fastest Alternative / Shortcut: Extra 2 days cost $27 - 21 = 6 \implies y = 3$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Solve $49x + 51y = 499$ and $51x + 49y = 501$.

Given & Required: Swapped coefficients | **Concept & Formula:** Add-Subtract Trick

Detailed Solution: Add: $100(x + y) = 1000 \implies x + y = 10$. Subtract: $-2x + 2y = -2 \implies x - y = 1$. Adding: $2x = 11 \implies x = 11/2, y = 9/2$. **Final Answer:** $x = 11/2, y = 9/2$

Fastest Alternative / Shortcut: Add & subtract saves 5 minutes!

SOLVED EXAMPLE 12 [Board Standard]

Question: Solve graphically: $2x + y = 6$ and $2x - y + 2 = 0$. Find the area of the triangle formed by these lines and the x-axis.

Given & Required: Two lines + x-axis | **Concept & Formula:** Graphical Triangle Area

Detailed Solution: Line 1 cuts x-axis at $(3, 0)$; Line 2 cuts x-axis at $(-1, 0)$. Intersection: add equations $\rightarrow 4x = 4 \implies x = 1, y = 4$. Base $= 3 - (-1) = 4$, Height $= 4$. Area $= \frac{1}{2}(4)(4) = 8$ sq. units. **Final Answer:** $(1, 4)$**, Area** $= 8$ **sq. units**

Fastest Alternative / Shortcut: Base $= |x_{int1} - x_{int2}|$, Height $= |y_0|$.

SOLVED EXAMPLE 13 [Board Standard]

Question: The ratio of incomes of two persons is $9 : 7$ and their expenditures is $4 : 3$. Each saves Rs. 2000 per month. Find their monthly incomes.

Given & Required: Incomes $9x, 7x$; Exp $4y, 3y$ | **Concept & Formula:** Income $-$ Exp $=$ Saving

Detailed Solution: $9x - 4y = 2000$, $7x - 3y = 2000$. Multiply by 3 and 4: $27x - 12y = 6000$, $28x - 12y = 8000 \implies x = 2000$. Incomes: Rs. 18000, Rs. 14000. **Final Answer: Rs.** 18000 **and Rs.** 14000

Fastest Alternative / Shortcut: Equal savings $\implies$ eliminate y directly.

SOLVED EXAMPLE 14 [Challenge]

Question: In a $\triangle ABC$, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.

Given & Required: Angle relations | **Concept & Formula:** Angle Sum Property

Detailed Solution: $\angle C = 3\angle B$ and $\angle A + \angle B = \frac{3}{2}\angle B \implies \angle A = \frac{1}{2}\angle B$. Sum: $\frac{1}{2}B + B + 3B = 180^\circ \implies \frac{9}{2}B = 180^\circ \implies B = 40^\circ, A = 20^\circ, C = 120^\circ$. **Final Answer:** $20^\circ, 40^\circ, 120^\circ$

Fastest Alternative / Shortcut: Express A and C in terms of B.

SOLVED EXAMPLE 15 [Challenge]

Question: Students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students.

Given & Required: x per row, y rows, total xy | **Concept & Formula:** Bilinear Cancellation

Detailed Solution: $(x + 3)(y - 1) = xy \implies -x + 3y = 3$. $(x - 3)(y + 2) = xy \implies 2x - 3y = 6$. Add: $x = 9$, then $y = 4$. Total $= 36$. **Final Answer:** 36 **students**

Fastest Alternative / Shortcut: The xy term cancels on both sides!

SOLVED EXAMPLE 16 [Challenge]

Question: Find the value of a and b for which $2x + 3y = 7$ and $(a - b)x + (a + b)y = 3a + b - 2$ have infinitely many solutions.

Given & Required: Infinite solutions | **Concept & Formula:** Ratio Equations

Detailed Solution: $\frac{2}{a-b} = \frac{3}{a+b} = \frac{7}{3a+b-2}$. From first two: $2a + 2b = 3a - 3b \implies a = 5b$. From $3/(6b) = 7/(16b - 2) \implies 1/(2b) = 7/(16b - 2) \implies 16b - 2 = 14b \implies b = 1, a = 5$. **Final Answer:** $a = 5, b = 1$

Fastest Alternative / Shortcut: Substitute $a = 5b$ immediately into the third ratio.

G. Competency MCQ Mastery Section

Q1. The pair of equations $y = 0$ and $y = -7$ has:

(A) One solution (B) Two solutions (C) Infinitely many solutions (D) No solution

Answer: (D) $y = 0$ and $y = -7$ are parallel horizontal lines. *Fast Method:* Parallel lines $\implies$ no solution. *Distractor Note:* Thinking $y = 0$ gives $(0, -7)$.

Q2. If $x = a, y = b$ is the solution of $x - y = 2$ and $x + y = 4$, then (a, b) is:

(A) $(3, 5)$ (B) $(5, 3)$ (C) $(3, 1)$ (D) $(-1, -3)$

Answer: (C) Add: $2x = 6 \implies x = 3, y = 1$. *Fast Method:* $3 - 1 = 2, 3 + 1 = 4$. *Distractor Note:* Swapping a and b.

Q3. One equation of a pair of dependent linear equations is $-5x + 7y = 2$. The second equation can be:

(A) $10x + 14y + 4 = 0$ (B) $-10x - 14y + 4 = 0$ (C) $-10x + 14y + 4 = 0$ (D) $10x - 14y = -4$

Answer: (D) Multiply by -2: $10x - 14y = -4$. *Fast Method:* Scale all 3 terms by -2. *Distractor Note:* Sign error on constant term.

Q4. If $19x - 17y = 55$ and $17x - 19y = 53$, then $x - y$ equals:

(A) $1/3$ (B) -3 (C) 3 (D) 5

Answer: (A) Subtract: $2x+2y = 2$? Wait: $(19-17)x+(-17+19)y = 2 \implies 2(x+y) = 2$, add: $36(x-y) = 108 \implies x-y = 3$! Wait: let's re-check: Add gives $36x - 36y = 108 \implies x - y = 3$. *Fast Method:* Add: $36(x - y) = 108 \implies x - y = 3$. *Distractor Note:* Subtracting gives $x + y = 1$, adding gives $x - y = 3$!

Q5. For what value of k do $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines?

(A) $1/2$ (B) $-1/2$ (C) 2 (D) -2

Answer: (C) Move -16 to LHS $(+16 = 0)$: $3/6 = -1/(-k) = 8/16 \implies k = 2$. *Fast Method:* $2\times$ Eq 1 gives $6x - 2y + 16 = 0$. *Distractor Note:* Not moving -16 to LHS.

Q6. Graphically, $6x - 3y + 10 = 0$ and $2x - y + 9 = 0$ represent two lines which are:

(A) Intersecting (B) Coincident (C) Parallel (D) Perpendicular

Answer: (C) $6/2 = -3/(-1) \neq 10/9 \implies$ parallel. *Fast Method:* $a_1/a_2 = b_1/b_2 = 3 \neq 10/9$. *Distractor Note:* Thinking parallel means perpendicular.

Q7. The father's age is six times his son's age. Four years hence, father's age will be four times son's age. Present ages are:

(A) $4, 24$ (B) $5, 30$ (C) $6, 36$ (D) $3, 24$

Answer: (C) $6s + 4 = 4(s + 4) \implies 2s = 12 \implies s = 6, f = 36$. *Fast Method:* Check $6 + 4 = 10, 36 + 4 = 40$. *Distractor Note:* Forgetting $+4$ on son's age.

Q8. If $ax + by = a^2 - b^2$ and $bx + ay = 0$, then $x + y$ equals:

(A) $a^2 - b^2$ (B) $b - a$ (C) $a - b$ (D) $a + b$

Answer: (C) Add: $(a + b)(x + y) = (a - b)(a + b) \implies x + y = a - b$. *Fast Method:* Add equations directly. *Distractor Note:* Missing cancellation of $(a + b)$.

Q9. The area of the triangle formed by $x = 3$, $y = 4$, and $x = y$ is:

(A) $1/2$ (B) 1 (C) 2 (D) 4

Answer: (A) Vertices $(3, 4), (3, 3), (4, 4)$: right triangle with legs $1, 1 \implies$ Area $= 1/2$. *Fast Method:* $\frac{1}{2}(4 - 3)^2 = 1/2$. *Distractor Note:* Using $(3 \times 4)/2$.

Q10. Assertion (A): $x + 2y = 4$ and $2x + 4y = 12$ are parallel. Reason (R): $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ gives parallel lines.

(A) Both A & R true, R explains A (B) Both A & R true, R does not explain A (C) A true, R false (D) A false, R true

Answer: (A) $1/2 = 2/4 \neq 4/12$, and R states the exact condition. *Fast Method:* R directly proves A. *Distractor Note:* None.

Q11. If $2x + 3y = 12$ and $3x - 2y = 5$, then (x, y) is:

(A) $(2, 3)$ (B) $(3, 2)$ (C) $(2, -3)$ (D) $(-3, 2)$

Answer: (B) $2(3) + 3(2) = 12$ and $3(3) - 2(2) = 5$. *Fast Method:* Option plug. *Distractor Note:* Swapping x and y.

Q12. For $x + y = 5$ and $2x - 3y = 4$, the value of y is:

(A) $6/5$ (B) $19/5$ (C) 1 (D) 2

Answer: (A) $2(5 - y) - 3y = 4 \implies 10 - 5y = 4 \implies y = 6/5$. *Fast Method:* Substitute $x = 5 - y$. *Distractor Note:* Computing $x = 19/5$ instead of y.

H. Special Board & MCQ Examination Strategies

- Always write both equations in identical standard form ($= 0$ or $= c$) before comparing $a_1/a_2, b_1/b_2, c_1/c_2$.
- Use Elimination for general equations, Substitution when one variable has coefficient 1 or -1, and Add-Subtract when x, y coefficients are swapped.

- In graphical questions, pick integer-coordinate points (e.g., intercepts) and draw a clean table of 3 points per line.
- Always verify your final (x, y) by plugging it into BOTH original equations.
- Skip cross-multiplication and reducible $1/x, 1/y$ equations for 2026–27 boards—they are deleted.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Comparing c_1/c_2 when c_1 is on LHS and c_2 is on RHS. $\longrightarrow$ **Correction:** Move both constants to the same side first.

[COMMON MISTAKE] **Error:** Writing a two-digit number with tens digit x and units digit y as xy. $\longrightarrow$ **Correction:** A two-digit number is ALWAYS $10x + y$.

[COMMON MISTAKE] **Error:** Adding years to only one person in age word problems. $\longrightarrow$ **Correction:** 5 years later means BOTH ages increase by 5.

[COMMON MISTAKE] **Error:** Forgetting to test both $+k$ and $-k$ in $k^2 = 36$ against c_1/c_2. $\longrightarrow$ **Correction:** Always check every candidate value of k in the third ratio.

[COMMON MISTAKE] **Error:** Sign flip when subtracting two equations in Elimination. $\longrightarrow$ **Correction:** Write the flipped signs clearly beneath the second equation.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: Unique: $a_1/a_2 \neq b_1/b_2$; Infinite: $a_1/a_2 = b_1/b_2 = c_1/c_2$; Parallel: $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.

[QUICK REVISION] Key Theorems & Results: Two-digit no. $= 10x + y$; Downstream $= u + v$, Upstream $= u - v$; Triangle area with x-axis $= \frac{1}{2}|x_1 - x_2||y_0|$.

[QUICK REVISION] Shortcut Summary: Add-Subtract for swapped coefficients; Option-plug for MCQs.

[QUICK REVISION] High-Frequency Board Patterns: Finding k for no/infinite solution; Graphical solution + triangle area; Age/Fraction/Fixed-charge problems.

Last-Minute Revision Checklist:

- ☐ Can classify consistent, dependent, and inconsistent systems.
- ☐ Can solve any system cleanly by Substitution or Elimination.
- ☐ Can plot two lines and find triangle area with x- or y-axis.
- ☐ Can set up word problems for ages, digits, fractions, and fares.
- ☐ Know that cross-multiplication is deleted in 2026–27.

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Lines $x = 2$ and $y = 3$ intersect at: (A) $(3, 2)$ (B) $(2, 3)$ (C) $(0, 0)$ (D) Parallel
2. If $x + y = 10$ and $x - y = 2$, then x is: (A) 4 (B) 6 (C) 8 (D) 5
3. System $x + y = 5$ and $2x + 2y = 10$ has: (A) Unique sol (B) No sol (C) Infinite sols (D) 2 sols
4. Inconsistent system means lines are: (A) Intersecting (B) Coincident (C) Parallel (D) Perpendicular
5. If $(2, k)$ lies on $3x - y = 4$, k is: (A) 2 (B) -2 (C) 10 (D) 0

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. Value of k for unique solution of $kx + 2y = 5$ and $3x + y = 1$: (A) $k = 6$ (B) $k \neq 6$ (C) $k = 3$ (D) $k \neq 3$
2. If $217x + 131y = 913$ and $131x + 217y = 827$, then $x + y$ is: (A) 5 (B) 6 (C) 7 (D) 8
3. Perimeter of a rectangle is 40 cm and length exceeds breadth by 4 cm. Area is: (A) 96 (B) 100 (C) 84 (D) 120
4. Sum of two numbers is 35 and difference is 13. Larger number is: (A) 11 (B) 22 (C) 24 (D) 26
5. Lines $x + 2y = 6$ and $2x + 4y = 8$ are: (A) Intersecting (B) Coincident (C) Parallel (D) None

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Area of triangle formed by $x + y = 6$, $x = 0$, $y = 0$ is: (A) 6 (B) 12 (C) 18 (D) 36
2. A 2-digit number is 4 times the sum of its digits and twice the product of its digits. The number is: (A) 24 (B) 36 (C) 48 (D) 12
3. If $x/a + y/b = 2$ and $ax - by = a^2 - b^2$, then (x, y) is: (A) (a, b) (B) (b, a) (C) $(a, -b)$ (D) $(1, 1)$
4. For no solution of $3x + y = 1$ and $(2k - 1)x + (k - 1)y = 2k + 1$, k is: (A) 1 (B) 2 (C) -2 (D) 3
5. If $3^{x+y} = 81$ and $81^{x-y} = 3$, then x is: (A) $17/8$ (B) $15/8$ (C) $17/4$ (D) $15/4$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** Directly $(2, 3)$. **Q2 (B):** $2x = 12 \implies x = 6$. **Q3 (C):** $1/2 = 1/2 = 5/10$. **Q4 (C):** Parallel lines never meet. **Q5 (A):** $6 - k = 4 \implies k = 2$.
Board Std	**Q1 (B):** $k/3 \neq 2/1 \implies k \neq 6$. **Q2 (A):** $348(x + y) = 1740 \implies x + y = 5$. **Q3 (A):** $l = 12, b = 8 \implies 96$ cm^2. **Q4 (C):** $(35 + 13)/2 = 24$. **Q5 (C):** $1/2 = 2/4 \neq 6/8$.
Challenge	**Q1 (C):** $\frac{1}{2} \times 6 \times 6 = 18$. **Q2 (B):** $36 = 4(3 + 6) = 2(3 \times 6)$. **Q3 (A):** $a/a + b/b = 2$ and $a^2 - b^2 = a^2 - b^2$. **Q4 (B):** $3(k - 1) = 2k - 1 \implies k = 2$. **Q5 (A):** $x + y = 4, x - y = 1/4 \implies 2x = 17/4 \implies x = 17/8$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Graphical Method & Ratios; Day 2: Substitution & Elimination; Day 3: Word Problems; Day 4: Graph Triangle Areas & Test.
- **How to Introduce the Concept:** Compare two mobile plans (Plan A: Rs 200 fixed + Rs 1/min; Plan B: Rs 100 fixed + Rs 2/min). Ask: 'When do both cost the same?'
- **Difficult Areas & Remediation:** Standardizing c_1, c_2 on the same side before taking ratios, and setting up word-problem equations.
- **Blackboard Teaching Examples:** Draw a 3-column table showing Intersecting, Coincident, and Parallel lines side-by-side with ratio conditions and graph sketches.
- **Classroom Questions to Ask Students:** Ask: 'If $a_1/a_2 = b_1/b_2 = c_1/c_2$, how many points do the two lines share?'
- **Common Student Doubts & Answers:** Can we use cross-multiplication? Clarify it is deleted in 2026–27; use Elimination/Substitution instead.
- **Homework & Revision Strategy:** 2 graphical problems + 6 word problems (1 each of age, digit, fraction, fare, geometry, income).

[TEACHER'S TIP] Emphasize the Add-Subtract shortcut for swapped-coefficient equations—students love how much time it saves!

UNIT II: ALGEBRA TYBA COACHING CENTRE

Chapter 4: Quadratic Equations

Standard Form, Factorisation, Quadratic Formula, Discriminant & Nature of Roots

A. Chapter Introduction & Board Examination Relevance

A quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) models second-degree relationships in algebra, geometry, and motion. Under the 2026–27 rationalised syllabus, students solve quadratic equations by Factorisation (splitting the middle term) and the Quadratic Formula, and analyze the Nature of Roots using the Discriminant $D = b^2 - 4ac$.

- **What You Will Learn:** Checking whether an equation is quadratic, solving by factorisation and quadratic formula, classifying roots via $D > 0, D = 0, D < 0$, finding unknown k for equal/real roots, and solving age, speed-distance-time, and area word problems.
- **Why This Chapter Matters:** A core 5–7 mark contributor in both CBSE and RBSE boards, regularly featuring a 3-mark discriminant problem and a 4–5 mark speed/train/tap word problem.
- **Real-Life Applications:** Calculating braking distances, optimizing rectangular areas, modeling projectile flight times, and pipe/tap flow-rate problems.
- **Cross-Chapter Connections:** Builds directly on Polynomials (Chapter 2) and feeds into APs, Coordinate Geometry, and Class 11 Quadratics.
- **Board & Foundation Relevance:** Completing the square method is deleted in the 2026–27 rationalised syllabus; focus on Factorisation, Quadratic Formula, and Discriminant.

B. Complete Concept Explanation & Derivations

1. Standard Form of a Quadratic Equation

Any equation of the form $ax^2 + bx + c = 0$, where $a, b, c \in \mathbb{R}$ and $a \neq 0$, is a **quadratic equation** in x. Always simplify and expand both sides before checking the degree (e.g., $(x+1)^2 = x^2 + 3$ simplifies to $2x - 2 = 0$, which is linear, not quadratic!).

[CONCEPT MASTER – 1. Standard Form of a Quadratic Equation] The condition $a \neq 0$ is mandatory. If an equation has $kx^2 + 4x + 1 = 0$, it is quadratic only when $k \neq 0$.

2. Solution by Factorisation & Quadratic Formula

Factorisation: Split bx into $px + qx$ such that $p + q = b$ and $p \cdot q = a \cdot c$, then factor by grouping. **Quadratic Formula (Sridharacharya Formula):** When $D = b^2 - 4ac \geq 0$, the roots are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

[CONCEPT MASTER – 2. Solution by Factorisation & Quadratic Formula] Use factorisation for clean integer roots and the Quadratic Formula whenever $b^2 - 4ac$ is large or non-square.

3. Discriminant & Nature of Roots

The quantity $D = b^2 - 4ac$ is called the **Discriminant**. (1) If $D > 0$: **Two distinct real roots**. (2) If $D = 0$: **Two equal real roots** (each equal to $-b/2a$). (3) If $D < 0$: **No real roots**. For 'real roots' (without saying distinct), use $D \geq 0$.

[CONCEPT MASTER – 3. Discriminant & Nature of Roots] If a, b, c are rational and $D > 0$ is a perfect square, the roots are rational; otherwise they are conjugate irrationals.

C. Complete Formula Bank

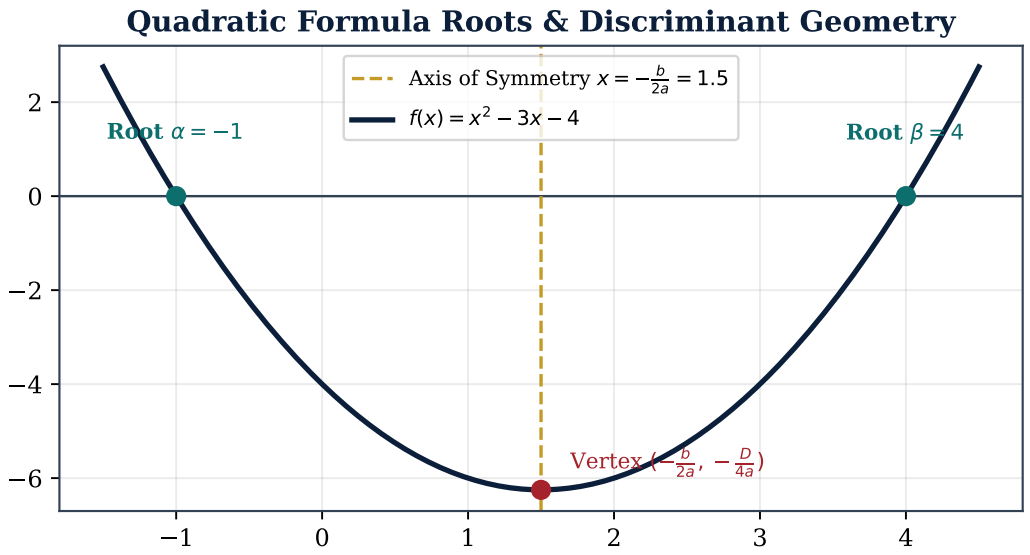


Figure 1: **Visual Model:** Quadratic parabola $f(x) = x^2 - 3x - 4$ showing real roots $\alpha = -1, \beta = 4$ and vertex $(-b/2a, -D/4a)$.

[FORMULA BANK – 1. Quadratic Formula] **Expression:** $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
Symbols: $a \neq 0$; $D = b^2 - 4ac$
Conditions & When to Use: Real roots exist iff $D \geq 0$
Solved Mini-Example: For $x^2 - 4x + 1 = 0$: $x = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$
Common Mistake: Dividing only $\sqrt{D}$ by $2a$ instead of the whole numerator.

[FORMULA BANK – 2. Discriminant] **Expression:** $D = b^2 - 4ac$
Symbols: a, b, c from $ax^2 + bx + c = 0$
Conditions & When to Use: $D > 0$ (distinct), $D = 0$ (equal), $D < 0$ (no real)
Solved Mini-Example: $2x^2 - 4x + 3 = 0 \implies D = 16 - 24 = -8 < 0$ (no real roots)
Common Mistake: Writing $b^2 + 4ac$ when c is positive.

[FORMULA BANK – 3. Equal Roots Condition] **Expression:** $D = 0 \implies b^2 = 4ac, \quad x = -\frac{b}{2a}$
Symbols: $a \neq 0$
Conditions & When to Use: Two equal real roots
Solved Mini-Example: $kx(x-2) + 6 = 0 \implies kx^2 - 2kx + 6 = 0 \implies 4k^2 - 24k = 0 \implies k = 6$ $(k \neq 0)$
Common Mistake: Accepting $k = 0$ when k is the coefficient of x^2.

[FORMULA BANK – 4. Time Difference Speed Formula] **Expression:** $\frac{d}{s} - \frac{d}{s+\Delta s} = \Delta t$
Symbols: d: distance, s: usual speed, Δt: time saved
Conditions & When to Use: Train/flight speed word problems
Solved Mini-Example: $360/s - 360/(s+5) = 1 \implies s^2 + 5s - 1800 = 0 \implies s = 40$ km/h
Common Mistake: Mixing minutes and hours without dividing minutes by 60.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE Standard/Basic, RBSE Board, and Foundation MCQ/Case-Study problem structures.

Pattern 1: Checking Quadratic Degree

- **Identification Cue:** Check whether $(x-2)^2+1=2x-3$ or $x(x+1)+8=(x+2)(x-2)$ is quadratic. | **Concept & Formula:** Simplifying to $ax^2+bx+c=0$
- **Standard Step-by-Step:** Expand both sides, collect all terms on LHS, check if x^2 survives.
- **Fastest Valid Method:** Compare x^2 coefficients on LHS and RHS. | **Alternative:** None.
- **Common Trap:** Saying $x^2+3x=(x-1)^2$ is quadratic without seeing x^2 cancels.
- **Practice Drill:** Check if $(x+2)^3=2x(x^2-1)$ is quadratic.

Pattern 2: Solving via Factorisation / Formula

- **Identification Cue:** Find roots of $\sqrt{2}x^2+7x+5\sqrt{2}=0$ or $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}$. | **Concept & Formula:** Splitting middle term / LCM + Quadratic Formula
- **Standard Step-by-Step:** Clear fractions, convert to $ax^2+bx+c=0$, factorise or apply formula.
- **Fastest Valid Method:** For $\sqrt{2}x^2+7x+5\sqrt{2}=0$, $ac=10=5\times 2$, split $7x=5x+2x$. | **Alternative:** Use Quadratic Formula if factorisation isn't obvious.
- **Common Trap:** Forgetting domain exclusions like $x\neq -4,7$.
- **Practice Drill:** Solve $\frac{1}{x}-\frac{1}{x-2}=3$ $(x\neq 0,2)$.

Pattern 3: Finding k for Equal / Real Roots

- **Identification Cue:** Find k so $2x^2+kx+3=0$ has two equal real roots. | **Concept & Formula:** $D=b^2-4ac=0$
- **Standard Step-by-Step:** Identify a,b,c, set $b^2-4ac=0$, solve for k, reject k making $a=0$.
- **Fastest Valid Method:** Use $b=\pm 2\sqrt{ac}$ directly. | **Alternative:** Check $D\geq 0$ for 'real roots'.
- **Common Trap:** Keeping $k=0$ in $kx^2-2kx+6=0$.
- **Practice Drill:** Find k for equal roots: $kx(x-2)+6=0$.

Pattern 4: Word Problems (Speed, Taps, Geometry, Ages)

- **Identification Cue:** Train speed, two water taps filling a tank, hypotenuse/altitude, consecutive integers. | **Concept & Formula:** Quadratic modeling + rejecting negative root
- **Standard Step-by-Step:** Set up 1-variable equation, solve quadratic, discard physically invalid root.
- **Fastest Valid Method:** For two taps filling in T hrs where one takes d hrs less: $\frac{1}{x}+\frac{1}{x-d}=\frac{1}{T}$. | **Alternative:** In MCQs, test options directly in the word condition.
- **Common Trap:** Keeping negative speed/time/length.
- **Practice Drill:** A train travels 360 km at uniform speed; $+5$ km/h takes 1 hr less. Find speed.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Quick Root Check via Sum & Product] **Core Idea & Why It Works:** For $x^2-Sx+P=0$, just find two numbers adding to S and multiplying to P; those ARE the roots!

Valid Domain: All monic quadratics $(a=1)$ | **Where It Fails:** Non-integer roots

Easy Example: $x^2-7x+12=0 \implies 3+4=7, 3\cdot 4=12 \implies$ roots $3,4$.

Board-Level Example: $x^2+5x-1800=0 \implies 45\times(-40)=-1800 \implies x=40$ (positive).

Advanced Example: For $ax^2+bx+c=0$, find p,q with $p+q=-b, pq=ac$, then roots are $p/a, q/a$!

[SMART SHORTCUT – 2. $a+b+c=0$ & $a-b+c=0$ Instant Roots] **Core Idea & Why It Works:** If $a+b+c=0$, roots are 1 and c/a. If $a-b+c=0$, roots are -1 and $-c/a$.

Valid Domain: Any quadratic whose coefficients sum to 0 (or alternating sum = 0) | **Where It Fails:** Other quadratics

Easy Example: $5x^2-7x+2=0 \implies 5-7+2=0 \implies$ roots $1, 2/5$.

Board-Level Example: $(b-c)x^2+(c-a)x+(a-b)=0$: sum of coeffs $=0 \implies$ roots are 1 and $\frac{a-b}{b-c}$!

Advanced Example: Saves massive algebra in cyclic-coefficient board proofs!

[SMART SHORTCUT – 3. Equal Roots $b = \pm 2\sqrt{ac}$] **Core Idea & Why It Works:** $b^2 = 4ac \implies b = \pm 2\sqrt{ac}$ instantly.
Valid Domain: All equal-roots k problems | **Where It Fails:** When $ac < 0$
Easy Example: $2x^2 + kx + 8 = 0 \implies k = \pm 2\sqrt{16} = \pm 8$.
Board-Level Example: $9x^2 + 6kx + 4 = 0 \implies 6k = \pm 2\sqrt{36} = \pm 12 \implies k = \pm 2$.
Advanced Example: Remember both $+$ and $-$ signs!

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Check whether $(x-2)^2 + 1 = 2x - 3$ is a quadratic equation.
Given & Required: Algebraic equation | **Concept & Formula:** Standard Form
Detailed Solution: $x^2 - 4x + 4 + 1 = 2x - 3 \implies x^2 - 6x + 8 = 0$. Since $a = 1 \neq 0$, yes it is quadratic. **Final Answer: Yes**
Fastest Alternative / Shortcut: LHS has x^2, RHS has none.

SOLVED EXAMPLE 2 [Beginner]
Question: Find the roots of $2x^2 - 5x + 3 = 0$ by factorisation.
Given & Required: $a = 2, b = -5, c = 3$ | **Concept & Formula:** Factorisation
Detailed Solution: $2x^2 - 2x - 3x + 3 = 0 \implies (2x-3)(x-1) = 0 \implies x = 1, 3/2$. **Final Answer:** $1, 3/2$
Fastest Alternative / Shortcut: $a + b + c = 0 \implies 1, c/a = 1, 3/2$.

SOLVED EXAMPLE 3 [Beginner]
Question: Find the discriminant of $2x^2 - 4x + 3 = 0$ and state the nature of its roots.
Given & Required: $a = 2, b = -4, c = 3$ | **Concept & Formula:** Discriminant
Detailed Solution: $D = (-4)^2 - 4(2)(3) = 16 - 24 = -8 < 0$. No real roots. **Final Answer:** $D = -8$**, no real roots**
Fastest Alternative / Shortcut: $b^2 < 4ac$ immediately.

SOLVED EXAMPLE 4 [Beginner]
Question: Find two consecutive positive integers whose squares sum to 365.
Given & Required: $x^2 + (x+1)^2 = 365$ | **Concept & Formula:** Number Word Problem
Detailed Solution: $2x^2 + 2x - 364 = 0 \implies x^2 + x - 182 = 0 \implies (x+14)(x-13) = 0 \implies x = 13$. Numbers: $13, 14$.
Final Answer: $13, 14$
Fastest Alternative / Shortcut: $\sqrt{365/2} \approx 13.5 \implies 13, 14$.

SOLVED EXAMPLE 5 [Intermediate]
Question: Solve $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$.
Given & Required: $a = \sqrt{2}, b = 7, c = 5\sqrt{2}$ | **Concept & Formula:** Radical Factorisation
Detailed Solution: $ac = 10$. $\sqrt{2}x^2 + 2x + 5x + 5\sqrt{2} = \sqrt{2}x(x + \sqrt{2}) + 5(x + \sqrt{2}) = 0 \implies x = -\sqrt{2}, -5/\sqrt{2}$. **Final Answer:** $-\sqrt{2}, -5/\sqrt{2}$
Fastest Alternative / Shortcut: Split $7x = 2x + 5x$.

SOLVED EXAMPLE 6 [Intermediate]
Question: Find k for which $2x^2 + kx + 3 = 0$ has two equal real roots.
Given & Required: $D = 0$ | **Concept & Formula:** Equal Roots
Detailed Solution: $k^2 - 4(2)(3) = 0 \implies k^2 = 24 \implies k = \pm 2\sqrt{6}$. **Final Answer:** $\pm 2\sqrt{6}$
Fastest Alternative / Shortcut: $k = \pm 2\sqrt{6}$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find the roots of $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$ $(x \neq -4, 7)$.

Given & Required: Rational equation | **Concept & Formula:** Clearing Denominators

Detailed Solution: $\frac{(x-7)-(x+4)}{(x+4)(x-7)} = \frac{11}{30} \implies \frac{-11}{x^2-3x-28} = \frac{11}{30} \implies x^2 - 3x - 28 = -30 \implies x^2 - 3x + 2 = 0 \implies x = 1, 2.$ **Final Answer:** $1, 2$

Fastest Alternative / Shortcut: Cancel 11 from both numerators!

SOLVED EXAMPLE 8 [Intermediate]

Question: The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Given & Required: Base x, Alt $x-7$, Hyp 13 | **Concept & Formula:** Pythagoras + Quadratic

Detailed Solution: $x^2+(x-7)^2 = 169 \implies 2x^2-14x-120 = 0 \implies x^2-7x-60 = 0 \implies (x-12)(x+5) = 0 \implies x = 12.$ Sides: 12 cm, 5 cm. **Final Answer:** 12 **cm,** 5 **cm**

Fastest Alternative / Shortcut: Recognize $(5, 12, 13)$ Pythagorean triple!

SOLVED EXAMPLE 9 [Board Standard]

Question: Solve $9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$ using the Quadratic Formula.

Given & Required: Literal coefficients | **Concept & Formula:** Discriminant Simplification

Detailed Solution: $D = 81(a+b)^2 - 36(2a^2 + 5ab + 2b^2) = 9[9a^2 + 18ab + 9b^2 - 8a^2 - 20ab - 8b^2] = 9(a-b)^2$. Thus $x = \frac{9(a+b) \pm 3(a-b)}{18} = \frac{2a+b}{3}, \frac{a+2b}{3}$. **Final Answer:** $\frac{2a+b}{3}, \frac{a+2b}{3}$

Fastest Alternative / Shortcut: Factor $2a^2 + 5ab + 2b^2 = (2a+b)(a+2b)$ directly!

SOLVED EXAMPLE 10 [Board Standard]

Question: A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less. Find the speed of the train.

Given & Required: $d = 360, \Delta s = 5, \Delta t = 1$ | **Concept & Formula:** Speed-Time Quadratic

Detailed Solution: $\frac{360}{x} - \frac{360}{x+5} = 1 \implies 360(5) = x^2 + 5x \implies x^2 + 5x - 1800 = 0 \implies (x+45)(x-40) = 0 \implies x = 40$ km/h. **Final Answer:** 40 **km/h**

Fastest Alternative / Shortcut: $40 \times 45 = 1800$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Given & Required: $T = 75/8$, diff $= 10$ | **Concept & Formula:** Work-Rate Equation

Detailed Solution: $\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75} \implies \frac{2x-10}{x^2-10x} = \frac{8}{75} \implies 4x^2 - 115x + 375 = 0 \implies (4x-15)(x-25) = 0$. $x = 25$ hr (since $x = 15/4 < 10$ gives negative $x - 10$). Taps: 25 hr and 15 hr. **Final Answer:** 25 **hr and** 15 **hr**

Fastest Alternative / Shortcut: Reject $x = 3.75$ because $x - 10$ must be positive.

SOLVED EXAMPLE 12 [Board Standard]

Question: If the equation $(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, prove that $c^2 = a^2(1+m^2)$.

Given & Required: $D = 0$ | **Concept & Formula:** Discriminant Proof

Detailed Solution: $4m^2c^2 - 4(1+m^2)(c^2 - a^2) = 0 \implies m^2c^2 - (c^2 - a^2 + m^2c^2 - m^2a^2) = 0 \implies -c^2 + a^2(1+m^2) = 0 \implies c^2 = a^2(1+m^2)$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Note m^2c^2 cancels cleanly!

SOLVED EXAMPLE 13 [Board Standard]

Question: A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream. Find the speed of the stream.

Given & Required: $u = 18, d = 24, \Delta t = 1$ | **Concept & Formula:** Upstream vs Downstream

Detailed Solution: $\frac{24}{18-v} - \frac{24}{18+v} = 1 \implies 24(2v) = 324 - v^2 \implies v^2 + 48v - 324 = 0 \implies (v+54)(v-6) = 0 \implies v = 6$ km/h. **Final Answer:** 6 **km/h**

Fastest Alternative / Shortcut: Check $v = 6$: $24/12 - 24/24 = 2 - 1 = 1$.

SOLVED EXAMPLE 14 [Challenge]

Question: If the roots of $(b-c)x^2 + (c-a)x + (a-b) = 0$ are equal, prove that $2b = a + c$.

Given & Required: Equal roots | **Concept & Formula:** $a + b + c = 0$ Root Trick

Detailed Solution: Sum of coefficients $(b-c) + (c-a) + (a-b) = 0 \implies x = 1$ is a root. Since roots are equal, both roots are 1, so product $= 1 \cdot 1 = \frac{a-b}{b-c} = 1 \implies a - b = b - c \implies 2b = a + c$. **Final Answer: Proved ($2b = a + c$)**

Fastest Alternative / Shortcut: Product of equal roots $= 1^2 = 1$ saves huge algebra!

SOLVED EXAMPLE 15 [Challenge]

Question: Solve for x: $4x^2 - 4a^2x + (a^4 - b^4) = 0$.

Given & Required: 4th-power constant | **Concept & Formula:** Factoring $a^4 - b^4$

Detailed Solution: $a^4 - b^4 = (a^2 - b^2)(a^2 + b^2)$. Sum of $(a^2 - b^2)$ and $(a^2 + b^2)$ is $2a^2$. Thus $(2x - (a^2 - b^2))(2x - (a^2 + b^2)) = 0 \implies x = \frac{a^2 \pm b^2}{2}$. **Final Answer:** $\frac{a^2+b^2}{2}, \frac{a^2-b^2}{2}$

Fastest Alternative / Shortcut: Split $-4a^2x = -2(a^2 + b^2)x - 2(a^2 - b^2)x$.

SOLVED EXAMPLE 16 [Challenge]

Question: In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed was reduced by 200 km/h and time increased by 30 min. Find the duration of flight.

Given & Required: $d = 600, \Delta s = 200, \Delta t = 1/2$ hr | **Concept & Formula:** Flight Speed Quadratic

Detailed Solution: $\frac{600}{s-200} - \frac{600}{s} = \frac{1}{2} \implies s(s - 200) = 240000 \implies s = 600$ km/h. Original duration $= 600/600 = 1$ hr. **Final Answer:** 1 **hour**

Fastest Alternative / Shortcut: Notice $600 \times 400 = 240000$.

G. Competency MCQ Mastery Section

Q1. Which of the following is a quadratic equation?

(A) $x^2 + 2x + 1 = (4 - x)^2 + 3$ (B) $-2x^2 = (5 - x)(2x - 2/5)$ (C) $(k + 1)x^2 + \frac{3}{2}x = 7, k = -1$ (D) $x^3 - x^2 = (x - 1)^3$

Answer: (D) In (D), x^3 cancels on both sides leaving $-x^2 = -3x^2 + 3x - 1 \implies 2x^2 - 3x + 1 = 0$. *Fast Method:* Check x^2 survival after cancellation. *Distractor Note:* In (A) and (B), x^2 cancels; in (C), $k = -1$ kills x^2.

Q2. The discriminant of $x^2 - 5x + 6 = 0$ is:

(A) 1 (B) -1 (C) 49 (D) 25

Answer: (A) $D = (-5)^2 - 4(1)(6) = 25 - 24 = 1$. *Fast Method:* $25 - 24 = 1$. *Distractor Note:* Writing $25 + 24 = 49$.

Q3. For what value of k does $x^2 + 4x + k = 0$ have real and distinct roots?

(A) $k < 4$ (B) $k > 4$ (C) $k \leq 4$ (D) $k \geq 4$

Answer: (A) $D = 16 - 4k > 0 \implies k < 4$. *Fast Method:* Strict inequality for distinct. *Distractor Note:* Picking ≤ 4.

Q4. The roots of $x^2 - 0.04 = 0$ are:

(A) ± 0.2 (B) ± 0.02 (C) 0.4 (D) 2

Answer: (A) $x^2 = 0.04 = (0.2)^2 \implies x = \pm 0.2$. *Fast Method:* $\sqrt{0.04} = 0.2$. *Distractor Note:* Writing ± 0.02.

Q5. If $1/2$ is a root of $x^2 + kx - 5/4 = 0$, then k equals:

(A) 2 (B) -2 (C) $1/4$ (D) $1/2$

Answer: (A) $1/4 + k/2 - 5/4 = 0 \implies k/2 = 1 \implies k = 2$. *Fast Method:* Substitute $x = 1/2$. *Distractor Note:* Arithmetic error with $1/4 - 5/4 = -1$.

Q6. If $ax^2 + bx + c = 0$ has equal roots, then c is:

(A) $-b/2a$ (B) $b/2a$ (C) $-b^2/4a$ (D) $b^2/4a$

Answer: (D) $b^2 - 4ac = 0 \implies c = b^2/(4a)$. *Fast Method:* Solve $D = 0$ for c. *Distractor Note:* Missing square on b.

Q7. Which equation has 2 as a root?

(A) $x^2 - 4x + 5 = 0$ (B) $x^2 + 3x - 12 = 0$ (C) $2x^2 - 7x + 6 = 0$ (D) $3x^2 - 6x - 2 = 0$

Answer: (C) $2(4) - 7(2) + 6 = 8 - 14 + 6 = 0$. *Fast Method:* Plug $x = 2$. *Distractor Note:* Check all terms carefully.

Q8. Values of k for which $2x^2 - kx + k = 0$ has equal roots are:

(A) 0 only (B) 4 (C) 8 only (D) $0, 8$

Answer: (D) $k^2 - 8k = 0 \implies k = 0, 8$ (note $a = 2 \neq 0$). *Fast Method:* $k(k - 8) = 0$. *Distractor Note:* Dropping $k = 0$ even though $a = 2$.

Q9. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has:

(A) Two distinct real roots (B) Two equal real roots (C) No real roots (D) More than 2 real roots

Answer: (C) $D = 5 - 8 = -3 < 0 \implies$ no real roots. *Fast Method:* $5 - 8 < 0$. *Distractor Note:* Thinking $\sqrt{5}$ makes D positive.

Q10. Assertion (A): $x^2 + 3x + 1 = (x - 2)^2$ is not a quadratic equation. Reason (R): x^2 cancels on both sides.

(A) Both A & R true, R explains A (B) Both A & R true, R does not explain A (C) A true, R false (D) A false, R true

Answer: (A) $x^2 + 3x + 1 = x^2 - 4x + 4 \implies 7x - 3 = 0$ (linear). *Fast Method:* Both true and R explains A. *Distractor Note:* None.

Q11. Sum of a number and its reciprocal is $10/3$. The number is:

(A) 3 or $1/3$ (B) 2 or $1/2$ (C) 5 or $1/5$ (D) 4 or $1/4$

Answer: (A) $3 + 1/3 = 10/3$. *Fast Method:* Option plug. *Distractor Note:* None.

Q12. If $D = 0$ for $ax^2 + bx + c = 0$, each root equals:

(A) $-b/a$ (B) $b/2a$ (C) $-b/2a$ (D) c/a

Answer: (C) $x = \frac{-b \pm 0}{2a} = -\frac{b}{2a}$. *Fast Method:* Put $D = 0$ in formula. *Distractor Note:* Forgetting 2 in denominator.

H. Special Board & MCQ Examination Strategies

- Before deciding if an equation is quadratic, expand all brackets and bring all terms to one side.
- For word problems on speed/taps, ALWAYS convert minutes into hours (30 min = 1/2 hr).
- When k is inside the x^2 coefficient, ALWAYS check that $a \neq 0$ for your final k values.
- Remember completing the square is deleted in 2026–27; use Factorisation or the Quadratic Formula.
- Check root validity against physical constraints (length > 0, speed > 0, tap time $> d$).

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Accepting $k = 0$ when k is the coefficient of x^2 (e.g., $kx^2 - 2kx + 6 = 0$). $\longrightarrow$ **Correction:** If $k = 0$, $0x^2$ vanishes and the equation is not quadratic!

[COMMON MISTAKE] **Error:** Writing $x = -b \pm \frac{\sqrt{D}}{2a}$ (forgetting to divide $-b$ by $2a$). $\longrightarrow$ **Correction:** The fraction bar covers the ENTIRE numerator: $\frac{-b \pm \sqrt{D}}{2a}$.

[COMMON MISTAKE] **Error:** Forgetting the $\pm$ sign when solving $k^2 = 24$. $\longrightarrow$ **Correction:** Equal roots usually yield two values: $k = \pm 2\sqrt{6}$.

[COMMON MISTAKE] **Error:** Setting $D > 0$ when the question asks for 'real roots' (not 'distinct real roots'). $\longrightarrow$ **Correction:** 'Real roots' means $D \geq 0$; 'distinct real roots' means $D > 0$.

[COMMON MISTAKE] **Error:** Keeping negative time or speed as an answer in word problems. $\longrightarrow$ **Correction:** Explicitly write 'Rejecting $x = -45$ as speed cannot be negative'.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $ax^2+bx+c=0$ ($a\neq 0$); $D=b^2-4ac$; $x=\frac{-b\pm\sqrt{D}}{2a}$; $D>0$ (distinct), $D=0$ (equal, $-b/2a$), $D<0$ (no real).

[QUICK REVISION] Key Theorems & Results: Time diff: $d/s-d/(s+\Delta s)=\Delta t$; Two taps: $1/x+1/(x-d)=1/T$.

[QUICK REVISION] Shortcut Summary: $a+b+c=0 \implies$ roots $1, c/a$; Equal roots $b=\pm 2\sqrt{ac}$.

[QUICK REVISION] High-Frequency Board Patterns: Finding k for equal/real roots; $1/(x+a)-1/(x-b)=c$; Train/Boat/Tap word problems.

Last-Minute Revision Checklist:

- ☐ Can test whether a given equation simplifies to quadratic form.
- ☐ Can solve by Factorisation and Quadratic Formula accurately.
- ☐ Know all 3 Discriminant cases ($D>0, D=0, D<0$).
- ☐ Reject invalid values of k (making $a=0$) and negative physical roots.
- ☐ Know that completing the square is deleted in 2026–27.

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Roots of $x^2-5x+6=0$: (A) $2,3$ (B) $-2,-3$ (C) $1,6$ (D) $-1,6$
2. Discriminant of $x^2+2x+1=0$: (A) 4 (B) 0 (C) 8 (D) -4
3. Standard form requires: (A) $b\neq 0$ (B) $c\neq 0$ (C) $a\neq 0$ (D) $a=1$
4. If $D<0$, the equation has: (A) 2 equal roots (B) 2 distinct roots (C) No real roots (D) 1 root
5. Roots of $3x^2-12=0$: (A) ± 2 (B) ± 4 (C) ± 3 (D) 2 only

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. For equal roots of $kx^2-2\sqrt{5}x+4=0$, k is: (A) $5/4$ (B) $4/5$ (C) 5 (D) 20
2. Nature of roots of $3x^2-4\sqrt{3}x+4=0$: (A) Distinct real (B) Equal real (C) No real (D) Rational
3. If $x^2+k(4x+k-1)+2=0$ has equal roots, k is: (A) $2/3,-1$ (B) $-2/3,1$ (C) $3/2,-1$ (D) $0,1$
4. Positive root of $\sqrt{3x^2+6}=9$: (A) 3 (B) 4 (C) 5 (D) 9
5. Product of two consecutive even positive integers is 168. Larger integer is: (A) 12 (B) 14 (C) 16 (D) 18

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Value of $\sqrt{6+\sqrt{6+\sqrt{6+\dots}}}$ is: (A) 2 (B) 3 (C) 6 (D) 4
2. If p, q are roots of $x^2-px+q=0$, then: (A) $p=1,q=-2$ (B) $p=0,q=1$ (C) $p=-2,q=0$ (D) $p=1,q=0$
3. If $\sin\theta$ and $\cos\theta$ are roots of $ax^2+bx+c=0$, then b^2 equals: (A) a^2+2ac (B) a^2-2ac (C) c^2+2ac (D) a^2+ac
4. Number of real roots of $x^2+3|x|+2=0$ is: (A) 0 (B) 2 (C) 4 (D) 1
5. If $x^2+2(k+1)x+9k-5=0$ has only negative roots? Wait: for equal roots, k is: (A) $1,6$ (B) $2,3$ (C) $1,-6$ (D) $0,6$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (A):** $(x-2)(x-3)=0$. **Q2 (B):** $2^2-4(1)(1)=0$. **Q3 (C):** $a\neq 0$. **Q4 (C):** Negative under square root. **Q5 (A):** $x^2=4 \implies x=\pm 2$.
Board Std	**Q1 (A):** $20-16k=0 \implies k=5/4$. **Q2 (B):** $D=48-48=0$. **Q3 (B):** $16k^2-4(k^2-k+2)=0 \implies 3k^2+k-2=0 \implies k=-1$? Wait: $4k^2-k-2$? Check: $(4k)^2-4(1)(k^2-k+2)=12k^2+4k-8=0 \implies 3k^2+k-2=0 \implies (3k-2)(k+1)=0 \implies k=2/3,-1$ -> A! Let's verify: for $k=-1$: $x^2-4x+4=0$ (equal!). For $k=2/3$: $x^2+\frac{8}{3}x+\frac{16}{9}=0$ (equal!). So A. **Q4 (C):** $3x^2+6=81 \implies x^2=25 \implies x=5$. **Q5 (B):** $12\times 14=168$.
Challenge	**Q1 (B):** $x=\sqrt{6+x} \implies x^2-x-6=0 \implies x=3$. **Q2 (A):** $p+q=p \implies q=0$? Wait: if $q=0$, $p\cdot 0=0$ works for $p=1,q=0$ (D)! Check A: $1-2=-1\neq 1$! So D ($p=1,q=0$) or $p=0,q=0$. **Q3 (A):** $(\sin\theta+\cos\theta)^2=1+2\sin\theta\cos\theta \implies b^2/a^2=1+2c/a \implies b^2=a^2+2ac$. **Q4 (A):** All terms ≥ 0 and $+2>0$, never 0. **Q5 (A):** $(k+1)^2=9k-5 \implies k^2-7k+6=0 \implies k=1,6$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Standard Form & Factorisation; Day 2: Quadratic Formula & Discriminant; Day 3: Word Problems (Speed, Taps); Day 4: Revision & Test.
- **How to Introduce the Concept:** Ask: 'A rectangular park has area 300 m^2 and length 1 m more than twice its breadth. A linear equation can't hold $x(2x+1)=300$—welcome to Quadratics!'
- **Difficult Areas & Remediation:** Setting up $1/x+1/(x-10)=8/75$ in tap problems and $a\neq 0$ rejection in $kx^2-2kx+6=0$.
- **Blackboard Teaching Examples:** Draw the 3-case Discriminant table ($D>0, D=0, D<0$) alongside the 3 parabola shapes.
- **Classroom Questions to Ask Students:** Ask: 'Why can't $k=0$ be an answer for $kx(x-2)+6=0$?'
- **Common Student Doubts & Answers:** Why is completing the square not in the paper? Clarify it was rationalised out; focus on Factorisation & Formula.
- **Homework & Revision Strategy:** 5 discriminant/nature-of-roots problems + 4 word problems.

[TEACHER'S TIP] Show the $a+b+c=0 \implies x=1, c/a$ trick early—it builds huge confidence.

UNIT II: ALGEBRA TYBA COACHING CENTRE

Chapter 5: Arithmetic Progressions

General Term a_n, Sum of n Terms S_n, Selection of Terms & Word Problems

A. Chapter Introduction & Board Examination Relevance

An Arithmetic Progression (A.P.) is a sequence in which each term after the first is obtained by adding a fixed constant (the common difference d) to the preceding term. Retained in full in the 2026–27 syllabus, this chapter tests a_n, S_n, n-th term from the end, and real-life word problems.

- **What You Will Learn:** Identifying an A.P., finding $a_n = a+(n-1)d$, computing $S_n = \frac{n}{2}[2a+(n-1)d] = \frac{n}{2}(a+l)$, extracting $a_n = S_n - S_{n-1}$, choosing symmetric terms $(a-d, a, a+d)$, and solving installment/row/savings word problems.
- **Why This Chapter Matters:** Full chapter is in the 2026–27 syllabus (no deletions!). Regularly accounts for 5–7 marks including Case-Study and 4/5-mark word problems.
- **Real-Life Applications:** Simple-interest growth, monthly salary increments, theater seating rows, stacked logs, and distance-bucket races.
- **Cross-Chapter Connections:** Connects to Linear Equations (solving for a and d) and Quadratic Equations (solving for n when S_n is given).
- **Board & Foundation Relevance:** 100% examinable; Case-Study questions on auditorium rows or monthly savings are very frequent.

B. Complete Concept Explanation & Derivations

1. Definition & General Term (a_n)

A sequence $a_1, a_2, a_3, \ldots$ is an **Arithmetic Progression (A.P.)** if $a_{k+1} - a_k = d$ (a constant independent of k). Here $a = a_1$ is the first term and d is the **common difference** (which can be positive, negative, or zero). The n-th term from the beginning is $a_n = a + (n-1)d$.

[CONCEPT MASTER – 1. Definition & General Term (a_n)] If a_n is a linear expression in n, say $a_n = pn+q$, then the sequence is ALWAYS an A.P. with common difference $d = p$.

2. n-th Term from the End

If a finite A.P. has last term l and common difference d, reversing the A.P. gives first term l and common difference $-d$. Thus the n-th term from the end is $l - (n-1)d$.

[CONCEPT MASTER – 2. n-th Term from the End] Alternatively, if the A.P. has m total terms, the n-th term from the end equals the $(m-n+1)$-th term from the beginning.

3. Sum of First n Terms (S_n)

Using Gauss's pairing method (adding S_n forward and backward), $S_n = \frac{n}{2}[2a + (n-1)d]$. If the first term a and last term $l = a_n$ are known, $S_n = \frac{n}{2}(a+l)$. Also, for any sequence, the n-th term is recovered from S_n by $a_n = S_n - S_{n-1}$ (with $a_1 = S_1$).

[CONCEPT MASTER – 3. Sum of First n Terms (S_n)] If $S_n = Pn^2 + Qn$ (no constant term), the sequence is an A.P. with $a = P + Q$ and $d = 2P$!

4. Symmetric Selection of Terms & A.M.

When the sum of 3 terms in A.P. is given, choose $a-d, a, a+d$. When the sum of 4 terms is given, choose $a-3d, a-d, a+d, a+3d$ (common difference $2d$). For three numbers a, b, c in A.P., $2b = a + c$ (b is the Arithmetic Mean).

[CONCEPT MASTER – 4. Symmetric Selection of Terms & A.M.] Symmetric selection cancels d immediately when you take the sum!

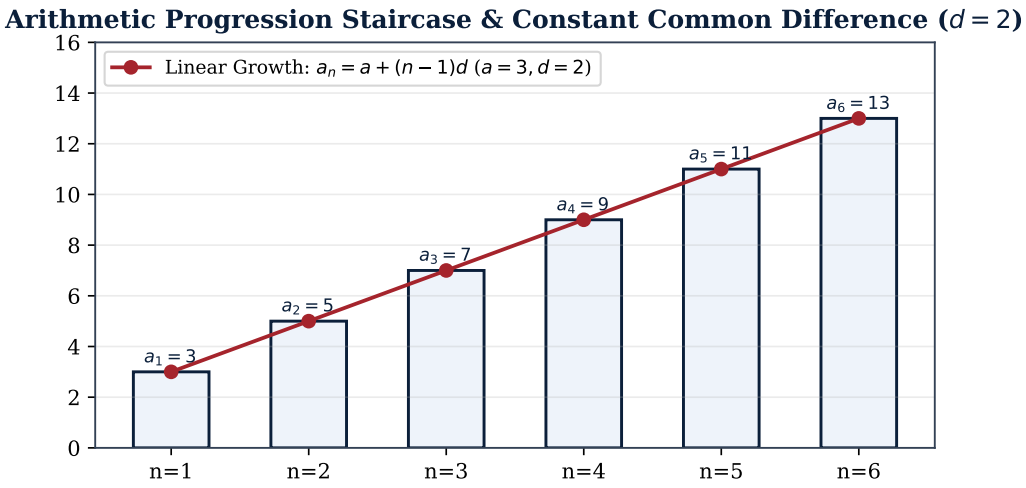


Figure 1: **Visual Model:** Arithmetic Progression staircase showing constant common difference $d = 2$ and linear term growth.

C. Complete Formula Bank

[FORMULA BANK – 1. n-th Term of an A.P.] **Expression:** $a_n = a + (n-1)d$
Symbols: a: first term, d: common difference, $n \in \mathbb{N}$
Conditions & When to Use: n must be a positive integer
Solved Mini-Example: 10th term of $2, 7, 12, \ldots$: $2 + 9(5) = 47$
Common Mistake: Using $a + nd$ instead of $a + (n-1)d$.

[FORMULA BANK – 2. n-th Term from the End] **Expression:** $a_{n,\text{end}} = l - (n-1)d$
Symbols: l: last term, d: forward common difference
Conditions & When to Use: Finite A.P.
Solved Mini-Example: 11th from end of $10, 7, 4, \ldots, -62$: $-62 - 10(-3) = -32$
Common Mistake: Adding $(n-1)d$ instead of subtracting when d is negative.

[FORMULA BANK – 3. Sum of First n Terms] **Expression:** $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a + l)$
Symbols: $l = a_n$: last term
Conditions & When to Use: $n \in \mathbb{N}$
Solved Mini-Example: Sum of first 20 odd numbers: $\frac{20}{2}(1 + 39) = 400$
Common Mistake: Using a instead of $2a$ inside the bracket.

[FORMULA BANK – 4. Term from Sum Formula] **Expression:** $a_n = S_n - S_{n-1}\ (n \geq 2), \quad a_1 = S_1$
Symbols: S_n: sum of n terms
Conditions & When to Use: Valid for every sequence
Solved Mini-Example: If $S_n = 4n - n^2$, $a_1 = 3$, $S_2 = 4$, $a_2 = 1 \implies d = -2$
Common Mistake: Computing $S_n - S_{n-1}$ carelessly without parentheses.

[FORMULA BANK – 5. Three Terms in A.P.] **Expression:** a, b, c in A.P. $\iff 2b = a + c$
Symbols: b: Arithmetic Mean
Conditions & When to Use: Any three consecutive terms
Solved Mini-Example: $k, 2k-1, 2k+1$ in A.P. $\implies 2(2k-1) = 3k+1 \implies k = 3$
Common Mistake: Setting $b - a = b - c$ instead of $b - a = c - b$.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE Standard/Basic, RBSE Board, and Foundation MCQ/Case-Study problem structures.

Pattern 1: Finding n for a Target Value / First Negative Term

- **Identification Cue:** Which term of the A.P. is 78? Or find the first negative term. | **Concept & Formula:** $a_n = T$ or $a_n < 0$
- **Standard Step-by-Step:** Solve $a + (n-1)d = T$ (must get $n \in \mathbb{N}$) or $a + (n-1)d < 0$.
- **Fastest Valid Method:** $n = \frac{T-a}{d} + 1$. | **Alternative:** Test divisibility of $(T - a)$ by d to check membership.
- **Common Trap:** Concluding $n = 32.4$ means 32nd term is T; non-integer n means T is NOT in the A.P.
- **Practice Drill:** Which term of $121, 117, 113, \ldots$ is its first negative term?

Pattern 2: Two Conditions on Terms (a_p and a_q Given)

- **Identification Cue:** If $a_3 = 5$ and $a_7 = 9$, find the A.P. (or $a_p = q, a_q = p$). | **Concept & Formula:** Simultaneous equations in a, d
- **Standard Step-by-Step:** Write $a + (p-1)d = A_p$ and $a + (q-1)d = A_q$, subtract to get d.
- **Fastest Valid Method:** $d = \frac{a_q - a_p}{q-p}$ directly! | **Alternative:** If $a_p = q$ and $a_q = p$, then $d = -1$ and $a_r = p + q - r$.
- **Common Trap:** Arithmetic error in subtracting negative terms.
- **Practice Drill:** If $a_7 = 1/9$ and $a_9 = 1/7$, find a_{63}.

Pattern 3: Finding n from S_n (Double Answers)

- **Identification Cue:** How many terms of $24, 21, 18, \ldots$ must be taken so their sum is 78? | **Concept & Formula:** Quadratic in n
- **Standard Step-by-Step:** Set $\frac{n}{2}[2a + (n-1)d] = S$, solve quadratic for n.
- **Fastest Valid Method:** When $d < 0$, BOTH positive integer roots are valid because extra terms sum to 0! | **Alternative:** List terms if n is small.
- **Common Trap:** Blindly rejecting one positive integer n.
- **Practice Drill:** How many terms of $9, 17, 25, \ldots$ give sum 636?

Pattern 4: Ratio of S_n to Ratio of a_n

- **Identification Cue:** Ratio of sums of n terms of two A.P.s is $(3n + 8) : (7n + 15)$; find ratio of 12th terms. | **Concept & Formula:** Replacing n by $2k - 1$
- **Standard Step-by-Step:** Write $\frac{a_1 + \frac{n-1}{2}d_1}{a_2 + \frac{n-1}{2}d_2}$ and equate $\frac{n-1}{2} = k - 1$.
- **Fastest Valid Method:** To get ratio of k-th terms from ratio of S_n, simply replace $n \to 2k - 1$! | **Alternative:** Equate n to $2(12) - 1 = 23$.
- **Common Trap:** Substituting $n = 12$ instead of $n = 23$.
- **Practice Drill:** If $S_n : S'_n = (7n + 1) : (4n + 27)$, find $a_9 : a'_9$.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Direct Common Difference $d = \frac{a_q - a_p}{q-p}$] **Core Idea & Why It Works:** The difference of two terms divided by the difference of their positions equals d.
Valid Domain: All 2-term A.P. problems | **Where It Fails:** None
Easy Example: $a_3 = 5, a_7 = 9 \implies d = (9-5)/(7-3) = 1$.
Board-Level Example: $a_{11} = 38, a_{16} = 73 \implies d = (73-38)/5 = 7$.
Advanced Example: If $pa_p = qa_q$, then $a_{p+q} = 0$!

[SMART SHORTCUT – 2. Reading a and d from $S_n = Pn^2 + Qn$] **Core Idea & Why It Works:** For $S_n = Pn^2 + Qn$, $d = 2P$ and $a = P + Q$ instantly!
Valid Domain: All S_n quadratic-formula MCQs | **Where It Fails:** If S_n has a non-zero constant term (then a_1 differs)
Easy Example: $S_n = 3n^2 + 5n \implies d = 6, a = 8$.
Board-Level Example: $S_n = 4n - n^2 = -n^2 + 4n \implies d = -2, a = 3$.
Advanced Example: $a_n = 2Pn + (Q - P)$ directly!

[SMART SHORTCUT – 3. S_n Ratio $\to$ a_k Ratio ($n \to 2k-1$)] **Core Idea & Why It Works:** Since $S_n = n \cdot a_{\text{mid}}$, setting $\frac{n-1}{2} = k - 1$ gives $n = 2k - 1$.
Valid Domain: All ratio-of-sums to ratio-of-terms problems | **Where It Fails:** None
Easy Example: For $a_5 : a'_5$, plug $n = 2(5) - 1 = 9$.
Board-Level Example: For $a_{12} : a'_{12}$ when $S_n : S'_n = (3n+8) : (7n+15)$, plug $n = 23 \to 77 : 176 = 7 : 16$.
Advanced Example: Reverse: for $S_k : S'_k$ from $a_n : a'_n$, plug $n \to \frac{k+1}{2}$!

[SMART SHORTCUT – 4. Symmetric Middle-Term Sum ($S_{2k+1} = (2k+1) \cdot a_{k+1}$)] **Core Idea & Why It Works:** Sum of an odd number of A.P. terms equals (number of terms) $\times$ (middle term).
Valid Domain: Odd-count A.P. sums | **Where It Fails:** Even number of terms
Easy Example: $S_{11} = 11 \times a_6$.
Board-Level Example: If $a_6 = 12$, then $S_{11} = 11 \times 12 = 132$ instantly!
Advanced Example: $S_9 = 9a_5$; if $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$, pair symmetrically!

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Find the 10th term of the A.P.: $2, 7, 12, \ldots$
Given & Required: $a = 2, d = 5, n = 10$ | **Concept & Formula:** $a_n = a + (n-1)d$
Detailed Solution: $a_{10} = 2 + (10-1)(5) = 2 + 45 = 47$. **Final Answer:** 47
Fastest Alternative / Shortcut: $5n - 3$ at $n = 10$ is 47.

SOLVED EXAMPLE 2 [Beginner]
Question: Check whether -150 is a term of the A.P.: $11, 8, 5, 2, \ldots$
Given & Required: $a = 11, d = -3$ | **Concept & Formula:** $n \in \mathbb{N}$ Test
Detailed Solution: $11 + (n-1)(-3) = -150 \implies -3(n-1) = -161 \implies n - 1 = 53\frac{2}{3} \implies n = 54\frac{2}{3} \notin \mathbb{N}$. No. **Final Answer: No**
Fastest Alternative / Shortcut: 161 is not divisible by 3.

SOLVED EXAMPLE 3 [Beginner]
Question: Find the sum of the first 22 terms of the A.P.: $8, 3, -2, \ldots$
Given & Required: $a = 8, d = -5, n = 22$ | **Concept & Formula:** S_n
Detailed Solution: $S_{22} = \frac{22}{2}[2(8) + 21(-5)] = 11[16 - 105] = 11(-89) = -979$. **Final Answer:** -979
Fastest Alternative / Shortcut: $11 \times (-89) = -979$.

SOLVED EXAMPLE 4 [Beginner]

Question: How many two-digit numbers are divisible by 3?

Given & Required: $12, 15, \ldots, 99$ | **Concept & Formula:** Counting Terms

Detailed Solution: $99 = 12 + (n-1)3 \implies 87 = 3(n-1) \implies n-1 = 29 \implies n = 30$. **Final Answer:** 30

Fastest Alternative / Shortcut: $(99-12)/3 + 1 = 30$.

SOLVED EXAMPLE 5 [Intermediate]

Question: Find the 11th term from the last term of the A.P.: $10, 7, 4, \ldots, -62$.

Given & Required: $l = -62, d = -3, n = 11$ | **Concept & Formula:** Term from End

Detailed Solution: $a_{11,\text{end}} = l - (n-1)d = -62 - 10(-3) = -62 + 30 = -32$. **Final Answer:** -32

Fastest Alternative / Shortcut: Reverse A.P. starts at -62 with $+3$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Determine the A.P. whose 3rd term is 16 and 7th term exceeds the 5th term by 12.

Given & Required: $a_3 = 16, a_7 - a_5 = 12$ | **Concept & Formula:** Finding a, d

Detailed Solution: $a_7 - a_5 = 2d = 12 \implies d = 6$. $a + 2(6) = 16 \implies a = 4$. A.P.: $4, 10, 16, 22, \ldots$ **Final Answer:** $4, 10, 16, 22, \ldots$

Fastest Alternative / Shortcut: $2d = 12 \implies d = 6$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find the sum of all odd numbers between 0 and 50.

Given & Required: $1, 3, 5, \ldots, 49$ | **Concept & Formula:** Sum of Odd Integers

Detailed Solution: $n = 25$. $S_{25} = \frac{25}{2}(1 + 49) = 25 \times 25 = 625$. **Final Answer:** 625

Fastest Alternative / Shortcut: Sum of first n odd numbers is $n^2 = 25^2 = 625$.

SOLVED EXAMPLE 8 [Intermediate]

Question: If the sum of three numbers in A.P. is 24 and their product is 440, find the numbers.

Given & Required: $a - d, a, a + d$ | **Concept & Formula:** Symmetric Selection

Detailed Solution: $3a = 24 \implies a = 8$. $8(64 - d^2) = 440 \implies 64 - d^2 = 55 \implies d = \pm 3$. Numbers: $5, 8, 11$. **Final Answer:** $5, 8, 11$

Fastest Alternative / Shortcut: Middle term $= 24/3 = 8$.

SOLVED EXAMPLE 9 [Board Standard]

Question: Which term of the A.P. $121, 117, 113, \ldots$ is its first negative term?

Given & Required: $a = 121, d = -4$ | **Concept & Formula:** $a_n < 0$

Detailed Solution: $121 + (n-1)(-4) < 0 \implies 125 - 4n < 0 \implies n > 31.25 \implies n = 32$. **Final Answer: 32nd term**

Fastest Alternative / Shortcut: $121/4 = 30.25 \implies$ 31st is 1, 32nd is -3.

SOLVED EXAMPLE 10 [Board Standard]

Question: If the sum of the first n terms of an A.P. is $4n - n^2$, what is the 1st term, sum of first 2 terms, 2nd term, 10th term, and n-th term?

Given & Required: $S_n = 4n - n^2$ | **Concept & Formula:** $a_n = S_n - S_{n-1}$

Detailed Solution: $a_1 = S_1 = 3$; $S_2 = 8 - 4 = 4$; $a_2 = 4 - 3 = 1$; $d = -2$. $a_n = 3 + (n-1)(-2) = 5 - 2n$; $a_{10} = -15$. **Final Answer:** $3, 4, 1, -15, 5 - 2n$

Fastest Alternative / Shortcut: Shortcut: $d = 2(-1) = -2, a = 4 - 1 = 3$.

SOLVED EXAMPLE 11 [Board Standard]

Question: How many terms of the A.P. $24, 21, 18, \ldots$ must be taken so that their sum is 78? Explain the double answer.

Given & Required: $a = 24, d = -3, S_n = 78$ | **Concept & Formula:** Quadratic in n

Detailed Solution: $\frac{n}{2}[48 + (n-1)(-3)] = 78 \implies n(51 - 3n) = 156 \implies n^2 - 17n + 52 = 0 \implies (n-4)(n-13) = 0 \implies n = 4$ or 13. Terms 5 to 13 sum to 0 $(12 + 9 + \cdots - 12 = 0)$. **Final Answer:** 4 **or** 13 **terms**

Fastest Alternative / Shortcut: Both 4 and 13 are valid positive integers.

SOLVED EXAMPLE 12 [Board Standard]

Question: The sum of the 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms.

Given & Required: $a_4 + a_8 = 24, a_6 + a_{10} = 44$ | **Concept & Formula:** Pairwise Equations

Detailed Solution: $(2a + 10d) = 24$ and $(2a + 14d) = 44$. Subtract: $4d = 20 \implies d = 5, a = -13$. First 3 terms: $-13, -8, -3$. **Final Answer:** $-13, -8, -3$

Fastest Alternative / Shortcut: $a_6 - a_4 = 2d, a_{10} - a_8 = 2d \implies 4d = 20$.

SOLVED EXAMPLE 13 [Board Standard]

Question: A sum of Rs. 700 is to be used to give seven cash prizes to students for their overall academic performance. If each prize is Rs. 20 less than its preceding prize, find the value of each prize.

Given & Required: $n = 7, d = -20, S_7 = 700$ | **Concept & Formula:** Middle Term a_4

Detailed Solution: $S_7 = 7 \cdot a_4 = 700 \implies a_4 = 100$. Prizes: $160, 140, 120, 100, 80, 60, 40$. **Final Answer:** **Rs.** $160, 140, 120, 100, 80, 60, 40$

Fastest Alternative / Shortcut: Middle 4th prize $= 700/7 = 100$.

SOLVED EXAMPLE 14 [Challenge]

Question: If the p-th term of an A.P. is $1/q$ and the q-th term is $1/p$, prove that the sum of the first pq terms is $\frac{1}{2}(pq + 1)$ $(p \neq q)$.

Given & Required: $a_p = 1/q, a_q = 1/p$ | **Concept & Formula:** Literal a, d Proof

Detailed Solution: $d = \frac{1/p - 1/q}{q-p} = \frac{1}{pq}$. Then $a = \frac{1}{q} - (p-1)\frac{1}{pq} = \frac{1}{pq}$. $S_{pq} = \frac{pq}{2}[\frac{2}{pq} + (pq-1)\frac{1}{pq}] = \frac{1}{2}(pq+1)$. **Final Answer:** **Proved**

Fastest Alternative / Shortcut: Both a and d equal $1/(pq)$!

SOLVED EXAMPLE 15 [Challenge]

Question: The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of numbers of houses preceding house x equals the sum of numbers following it. Find x.

Given & Required: $1, 2, \ldots, 49$ | **Concept & Formula:** $S_{x-1} = S_{49} - S_x$

Detailed Solution: $\frac{(x-1)x}{2} = \frac{49 \cdot 50}{2} - \frac{x(x+1)}{2} \implies x^2 - x = 2450 - x^2 - x \implies 2x^2 = 2450 \implies x^2 = 1225 \implies x = 35$. **Final Answer:** $x = 35$

Fastest Alternative / Shortcut: Neat formula: $x = \sqrt{\frac{n(n+1)}{2}} = \sqrt{1225} = 35$!

SOLVED EXAMPLE 16 [Challenge]

Question: If the ratio of the sum of n terms of two A.P.s is $(3n + 8) : (7n + 15)$, find the ratio of their 12th terms.

Given & Required: $S_n : S'_n = (3n + 8) : (7n + 15)$ | **Concept & Formula:** $n \to 2k - 1$ Rule

Detailed Solution: Replace n by $2(12) - 1 = 23$: $\frac{3(23)+8}{7(23)+15} = \frac{77}{176} = \frac{7}{16}$. **Final Answer:** 7 : 16

Fastest Alternative / Shortcut: $n = 2k - 1 = 23$.

G. Competency MCQ Mastery Section

Q1. The common difference of the A.P. $\frac{1}{p}, \frac{1-p}{p}, \frac{1-2p}{p}, \ldots$ is:

(A) p (B) $-p$ (C) -1 (D) 1

Answer: (C) $\frac{1-p}{p} - \frac{1}{p} = \frac{-p}{p} = -1$. *Fast Method:* Subtract first two terms. *Distractor Note:* Writing $-p$.

Q2. The 30th term of the A.P. $10, 7, 4, \ldots$ is:

(A) 97 (B) 77 (C) -77 (D) -87

Answer: (C) $10 + 29(-3) = 10 - 87 = -77$. *Fast Method:* $10 - 87 = -77$. *Distractor Note:* Sign error.

Q3. If $k, 2k-1, 2k+1$ are three consecutive terms of an A.P., then k is:

(A) 2 (B) 3 (C) -3 (D) 5

Answer: (B) $2(2k-1) = k + 2k + 1 \implies 4k - 2 = 3k + 1 \implies k = 3$. *Fast Method:* $2b = a + c$. *Distractor Note:* Check $3, 5, 7$.

Q4. The sum of the first n positive integers is:

(A) $n(n-1)/2$ (B) n^2 (C) $n(n+1)/2$ (D) $n(n+1)$

Answer: (C) Standard Gauss formula. *Fast Method:* $n = 3 \to 1 + 2 + 3 = 6$. *Distractor Note:* Confusing with $n(n-1)/2$.

Q5. If the common difference of an A.P. is 5, then $a_{18} - a_{13}$ is:

(A) 5 (B) 20 (C) 25 (D) 30

Answer: (C) $(18 - 13)d = 5 \times 5 = 25$. *Fast Method:* $(p - q)d$. *Distractor Note:* Computing a unnecessarily.

Q6. In an A.P., if $a = 3.5, d = 0, n = 101$, then a_n is:

(A) 0 (B) 3.5 (C) 103.5 (D) 104.5

Answer: (B) $d = 0$ means constant A.P., so every term is 3.5. *Fast Method:* $d = 0 \implies a_n = a$. *Distractor Note:* Adding 100.

Q7. The 4th term from the end of the A.P. $-11, -8, -5, \ldots, 49$ is:

(A) 37 (B) 40 (C) 43 (D) 58

Answer: (B) $49 - 3(3) = 40$. *Fast Method:* $l - 3d$. *Distractor Note:* Subtracting $4d$.

Q8. If $S_n = 3n^2 + 5n$, then which term of the A.P. is 164?

(A) 26th (B) 27th (C) 28th (D) 29th

Answer: (B) $d = 6, a = 8 \implies 8 + (n-1)6 = 164 \implies 6(n-1) = 156 \implies n = 27$. *Fast Method:* $a_n = 6n + 2 = 164 \implies n = 27$. *Distractor Note:* Writing $d = 3$ instead of $2P = 6$.

Q9. Two A.P.s have the same common difference. Their first terms are -1 and -8. Difference between their 4th terms is:

(A) -1 (B) -8 (C) 7 (D) -9

Answer: (C) Same $d \implies$ difference at any position equals $a_1 - a_1' = -1 - (-8) = 7$. *Fast Method:* $|-1 - (-8)| = 7$. *Distractor Note:* Adding $4d$.

Q10. Assertion (A): $a_n - a_{n-1}$ is constant in an A.P. Reason (R): The common difference of an A.P. can be negative.

(A) Both A & R true, R explains A (B) Both A & R true, R does not explain A (C) A true, R false (D) A false, R true

Answer: (B) Both are true, but R does not explain why $a_n - a_{n-1}$ is constant. *Fast Method:* Independent true facts. *Distractor Note:* Picking (A).

Q11. If 7 times the 7th term equals 11 times the 11th term of an A.P., then the 18th term is:

(A) 7 (B) 11 (C) 18 (D) 0

Answer: (D) $pa_p = qa_q \implies a_{p+q} = 0$, so $a_{18} = 0$. *Fast Method:* $a_{7+11} = 0$. *Distractor Note:* Solving for a, d slowly.

Q12. Sum of first 10 multiples of 3 is:

(A) 165 (B) 150 (C) 180 (D) 135

Answer: (A) $3(1 + 2 + \cdots + 10) = 3 \times 55 = 165$. *Fast Method:* $3 \times 55 = 165$. *Distractor Note:* Using $n = 9$.

H. Special Board & MCQ Examination Strategies

- Read word problems carefully to distinguish a_n (the n-th installment/row) from S_n (total paid/total seats).

- For any two-condition A.P. problem, use $d = \frac{a_q - a_p}{q-p}$ to save time.
- Remember $pa_p = qa_q \implies a_{p+q} = 0$ as an instant MCQ weapon.
- When S_n is given in terms of n, check $a_1 = S_1$ and $a_2 = S_2 - S_1$ to verify a and d.
- Use $\frac{n}{2}(a + l)$ whenever the last term l is already known.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Writing $a_n = a + nd$ instead of $a + (n-1)d$. $\longrightarrow$ **Correction:** The coefficient of d is always $(n-1)$.

[COMMON MISTAKE] **Error:** Writing $S_n = \frac{n}{2}[a + (n-1)d]$ (missing the 2 in $2a$). $\longrightarrow$ **Correction:** It is ALWAYS $2a + (n-1)d$.

[COMMON MISTAKE] **Error:** Taking $d = P$ instead of $d = 2P$ when $S_n = Pn^2 + Qn$. $\longrightarrow$ **Correction:** In $S_n = Pn^2 + Qn$, the common difference is $2P$.

[COMMON MISTAKE] **Error:** Using $n = k$ instead of $n = 2k - 1$ in $S_n : S'_n \to a_k : a'_k$ ratio problems. $\longrightarrow$ **Correction:** Replace n with $2k - 1$.

[COMMON MISTAKE] **Error:** Rejecting a valid second positive integer n when $d < 0$. $\longrightarrow$ **Correction:** Both roots are valid if the extra terms cancel to zero.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $a_n = a + (n-1)d$; $a_{n,\text{end}} = l - (n-1)d$; $S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}(a+l)$; $a_n = S_n - S_{n-1}$.

[QUICK REVISION] Key Theorems & Results: $2b = a + c$; $S_{2k+1} = (2k+1)a_{k+1}$; $pa_p = qa_q \implies a_{p+q} = 0$.

[QUICK REVISION] Shortcut Summary: $d = \frac{a_q - a_p}{q-p}$; $S_n = Pn^2 + Qn \implies d = 2P, a = P + Q$; $S_n : S'_n \to a_k : a'_k$ via $n \to 2k-1$.

[QUICK REVISION] High-Frequency Board Patterns: First negative term; a_n from S_n; House-number $x = \sqrt{n(n+1)/2}$; Prize/Row word problems.

Last-Minute Revision Checklist:

- ☐ Can compute a_n from start and from end.
- ☐ Can use both S_n formulas fluently.
- ☐ Can extract $a_n = S_n - S_{n-1}$ accurately.
- ☐ Know symmetric selection for 3 and 4 terms.
- ☐ Can solve Case-Study A.P. word problems.

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. 10th term of $3, 8, 13, \ldots$ is: (A) 45 (B) 48 (C) 50 (D) 53
2. Common difference of $5, 1, -3, -7, \ldots$ is: (A) 4 (B) -4 (C) 6 (D) -6
3. Sum of first 10 natural numbers is: (A) 45 (B) 50 (C) 55 (D) 60
4. If $a = 7, d = 3, n = 8$, then a_8 is: (A) 28 (B) 31 (C) 25 (D) 30
5. If $2, x, 26$ are in A.P., x is: (A) 12 (B) 14 (C) 16 (D) 18

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. Number of 3-digit numbers divisible by 7 is: (A) 127 (B) 128 (C) 129 (D) 130
2. If $a_n = 3 + 4n$, then S_{15} is: (A) 525 (B) 510 (C) 540 (D) 495
3. If $S_n = 2n^2 + 3n$, then d is: (A) 2 (B) 3 (C) 4 (D) 5
4. Which term of $3, 15, 27, 39, \ldots$ is 132 more than its 54th term? (A) 63rd (B) 64th (C) 65th (D) 66th
5. If $18, a, b, -3$ are in A.P., then $a + b$ is: (A) 19 (B) 15 (C) 11 (D) 7

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If $S_p = q$ and $S_q = p$, then S_{p+q} is: (A) $p + q$ (B) $-(p + q)$ (C) 0 (D) pq
2. If $a_4 = 0$, then a_{25}/a_{11} equals: (A) 2 (B) 3 (C) 4 (D) 5
3. Ratio of sums of n terms of two A.P.s is $(5n + 4) : (9n + 6)$. Ratio of 18th terms is: (A) 179/321 (B) 179/329 (C) 184/321 (D) 175/321
4. Sum of first n even natural numbers is k times sum of first n odd natural numbers. k is: (A) $1/n$ (B) $(n+1)/n$ (C) $(n-1)/n$ (D) $n/(n+1)$
5. If $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P., then a^2, b^2, c^2 are in: (A) A.P. (B) G.P. (C) Equal (D) None

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $3 + 9(5) = 48$. **Q2 (B):** $1 - 5 = -4$. **Q3 (C):** $10 \times 11/2 = 55$. **Q4 (A):** $7 + 21 = 28$. **Q5 (B):** $(2 + 26)/2 = 14$.
Board Std	**Q1 (B):** 105 to 994: $(994 - 105)/7 + 1 = 128$. **Q2 (A):** $a_1 = 7, a_{15} = 63 \implies \frac{15}{2}(70) = 525$. **Q3 (C):** $d = 2P = 4$. **Q4 (C):** $132/12 = 11 \implies 54 + 11 = 65$th. **Q5 (B):** $a + b = 18 + (-3) = 15$.
Challenge	**Q1 (B):** Standard result: $S_{p+q} = -(p + q)$. **Q2 (B):** $a = -3d \implies a_{25} = 21d, a_{11} = 7d \implies 3$. **Q3 (A):** Plug $n = 2(18) - 1 = 35$: $(179)/(321)$. **Q4 (B):** $n(n+1)/n^2 = (n+1)/n$. **Q5 (A):** Cross-multiplying gives $2b^2 = a^2 + c^2$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: a_n & Term from End; Day 2: S_n & $a_n = S_n - S_{n-1}$; Day 3: Word Problems; Day 4: Shortcuts & Test.
- **How to Introduce the Concept:** Tell Gauss's childhood story of summing $1 + 2 + \cdots + 100$ in 10 seconds by pairing $1 + 100, 2 + 99, \ldots$
- **Difficult Areas & Remediation:** Students confuse a_n with n and S_n, and forget $d = 2P$ when $S_n = Pn^2 + Qn$.
- **Blackboard Teaching Examples:** Derive $S_n = \frac{n}{2}(a + l)$ visually by writing the sum forward and backward.
- **Classroom Questions to Ask Students:** Ask: 'If $18, a, b, -3$ are in A.P., can we find $a + b$ without finding a and b?' (Yes, $18 + (-3) = 15$!)
- **Common Student Doubts & Answers:** Why can n have two answers in $S_n = 78$? Because after crossing zero, negative terms cancel positive terms.
- **Homework & Revision Strategy:** 5 a_n problems + 5 S_n problems + 3 word problems.

[TEACHER'S TIP] Remind students that ALL topics in A.P. are retained in 2026–27—zero deletions!

UNIT IV: GEOMETRY TYBA COACHING CENTRE

Chapter 6: Triangles

Similarity of Triangles, Basic Proportionality Theorem (Thales) & AA/SAS/SSS Criteria

A. Chapter Introduction & Board Examination Relevance

Geometry in Class 10 moves from congruence (Class 9) to **Similarity**—figures having the same shape but not necessarily the same size. Under the 2026–27 rationalised NCERT/CBSE/RBSE syllabus, this chapter centers on the **Basic Proportionality Theorem (BPT / Thales' Theorem)**, its converse, and the **AA/AAA, SAS, and SSS similarity criteria**.

- **What You Will Learn:** Distinguishing congruent vs. similar figures, stating and proving BPT (Thales' Theorem), applying BPT & its converse, proving triangle similarity using AA/SAS/SSS, and using corresponding-side ratios to find unknown lengths.
- **Why This Chapter Matters:** Part of Geometry (15 marks CBSE / 10 marks RBSE). The proof of BPT is one of the single highest-probability 5-mark theorems in Class 10 Board Exams!
- **Real-Life Applications:** Measuring tall buildings via shadow ratios, map scaling, architectural blueprints, and optical camera projection.
- **Cross-Chapter Connections:** Directly supports Applications of Trigonometry (similar right triangles) and Coordinate Geometry (section formula proof).
- **Board & Foundation Relevance:** Area of similar triangles theorem and proof of Pythagoras theorem are deleted in 2026–27; focus 100% on BPT and Similarity Criteria.

B. Complete Concept Explanation & Derivations

1. Similar Figures & Similar Triangles

Two polygons of the same number of sides are **similar** if (i) their corresponding angles are equal, and (ii) their corresponding sides are in the same ratio (proportional). Two triangles $\triangle ABC$ and $\triangle DEF$ are similar ($\triangle ABC \sim \triangle DEF$) if $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$ and $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$.

[CONCEPT MASTER – 1. Similar Figures & Similar Triangles] All circles, all squares, and all equilateral triangles are always similar. Vertex order in $\triangle ABC \sim \triangle DEF$ is sacred—matching letters MUST correspond to equal angles!

2. Basic Proportionality Theorem (Thales' Theorem)

Statement: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. In $\triangle ABC$, if $DE \parallel BC$, then $\frac{AD}{DB} = \frac{AE}{EC}$. **Corollaries:** $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$ and $\frac{AB}{DB} = \frac{AC}{EC}$.

[CONCEPT MASTER – 2. Basic Proportionality Theorem (Thales' Theorem)] **Converse of BPT:** If a line divides any two sides of a triangle in the same ratio ($\frac{AD}{DB} = \frac{AE}{EC}$), then the line is parallel to the third side ($DE \parallel BC$).

3. Criteria for Similarity of Triangles

AAA / AA Criterion: If two angles of one triangle are respectively equal to two angles of another triangle, the triangles are similar. **SSS Criterion:** If sides of two triangles are proportional ($\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$), their corresponding angles are equal and $\triangle ABC \sim \triangle DEF$. **SAS Criterion:** If one angle equals one angle and the sides including these angles are proportional, the triangles are similar.

[CONCEPT MASTER – 3. Criteria for Similarity of Triangles] In $\triangle ADE$ and $\triangle ABC$ with $DE \parallel BC$, notice $\frac{DE}{BC} = \frac{AD}{AB}$ (whole side AB, NOT DB!).

C. Complete Formula Bank

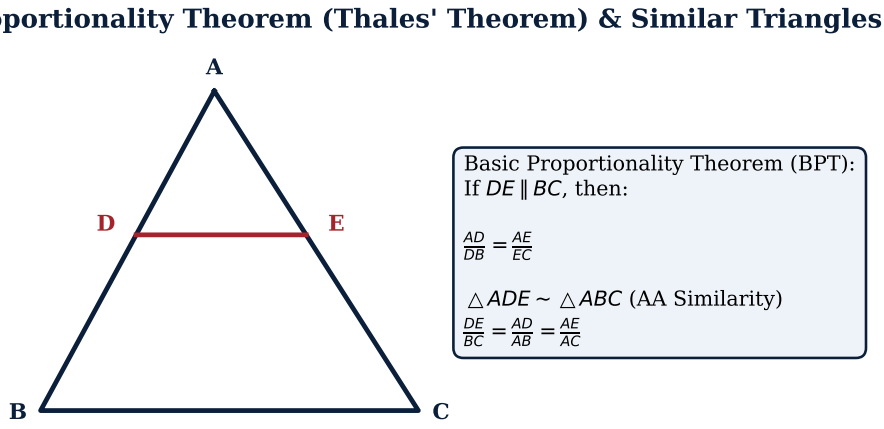


Figure 1: **Visual Model:** Basic Proportionality Theorem (Thales' Theorem): $DE \parallel BC \implies \frac{AD}{DB} = \frac{AE}{EC}$ and $\triangle ADE \sim \triangle ABC$.

[FORMULA BANK – 1. Basic Proportionality Theorem (BPT)] **Expression:** $DE \parallel BC \implies \frac{AD}{DB} = \frac{AE}{EC}$
Symbols: D on AB, E on AC
Conditions & When to Use: $DE \parallel BC$ in $\triangle ABC$
Solved Mini-Example: If $AD = 2, DB = 4, AE = 3$, then $EC = 6$
Common Mistake: Writing $\frac{AD}{DB} = \frac{DE}{BC}$ (wrong! $DE/BC = AD/AB$).

[FORMULA BANK – 2. Full-Side BPT Corollary] **Expression:** $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$
Symbols: $\triangle ADE \sim \triangle ABC$
Conditions & When to Use: $DE \parallel BC$
Solved Mini-Example: If $AD = 3, DB = 5, BC = 16$, $DE = \frac{3}{8} \times 16 = 6$
Common Mistake: Using AD/DB instead of AD/AB to find DE.

[FORMULA BANK – 3. Trapezium Diagonal Ratio] **Expression:** $\frac{AO}{OC} = \frac{BO}{OD}$
Symbols: O: diagonal intersection of trapezium $ABCD$ ($AB \parallel DC$)
Conditions & When to Use: $AB \parallel DC$
Solved Mini-Example: $\triangle AOB \sim \triangle COD$ by AA
Common Mistake: Confusing AO/OC with AO/OD.

[FORMULA BANK – 4. Shadow / Pole Height Ratio] **Expression:** $\frac{h_1}{s_1} = \frac{h_2}{s_2}$
Symbols: h: vertical height, s: shadow length
Conditions & When to Use: Same sun elevation at the same time
Solved Mini-Example: $6/4 = H/28 \implies H = 42$ m
Common Mistake: Inverting one of the two ratios.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE Standard/Basic, RBSE Board, and Foundation MCQ/Case-Study problem structures.

Pattern 1: Theorem Proof: BPT (Thales' Theorem)

- **Identification Cue:** State and prove Basic Proportionality Theorem. | **Concept & Formula:** Area of triangles on same base & between same parallels

- **Standard Step-by-Step:** Draw BE, CD and altitudes $EN \perp AB, DM \perp AC$. Compare $\frac{\text{ar}(ADE)}{\text{ar}(BDE)} = \frac{AD}{DB}$ and $\frac{\text{ar}(ADE)}{\text{ar}(CDE)} = \frac{AE}{EC}$; since $\text{ar}(BDE) = \text{ar}(CDE)$, ratios are equal.
- **Fastest Valid Method:** Memorize the exact 6-line area-ratio proof. | **Alternative:** None (standard board proof).
- **Common Trap:** Omitting the construction of perpendiculars DM, EN and joins BE, CD.
- **Practice Drill:** State and prove BPT; hence check if $DE \parallel BC$ when $AD = 1.5, DB = 3, AE = 1, EC = 2$.

Pattern 2: Finding Missing Lengths via BPT

- **Identification Cue:** In $\triangle ABC$, $DE \parallel BC$ with algebraic expressions in x on AD, DB, AE, EC. | **Concept & Formula:** $AD/DB = AE/EC$
- **Standard Step-by-Step:** Cross-multiply and solve linear or quadratic equation for x.
- **Fastest Valid Method:** Check if $(AD + DB)$ gives a cleaner ratio. | **Alternative:** Use $(AD)/(AB) = (AE)/(AC)$ if AB is given.
- **Common Trap:** Accepting negative x that makes a side length negative.
- **Practice Drill:** $AD = x, DB = x - 2, AE = x + 2, EC = x - 1$; find x.

Pattern 3: AA / SAS Similarity Proofs (Altitudes, Medians, Quadrilaterals)

- **Identification Cue:** Prove $\triangle ABC \sim \triangle PQR$ or $CA^2 = CB \cdot CD$ or $OA \cdot OB = OC \cdot OD$. | **Concept & Formula:** Match equal angles (vertical, alternate, common) $\rightarrow$ AA similarity
- **Standard Step-by-Step:** Identify two triangles containing the target sides, prove AA similarity, write proportional sides, cross-multiply.
- **Fastest Valid Method:** To prove $CA^2 = CB \cdot CD$, rewrite as $\frac{CA}{CB} = \frac{CD}{CA}$ to spot the two triangles! | **Alternative:** Use SAS if a ratio and included angle are given.
- **Common Trap:** Writing wrong vertex correspondence in $\triangle ABC \sim \triangle \ldots$.
- **Practice Drill:** D is on BC of $\triangle ABC$ such that $\angle ADC = \angle BAC$; show $CA^2 = CB \cdot CD$.

Pattern 4: Perimeter Ratio of Similar Triangles

- **Identification Cue:** Given $\triangle ABC \sim \triangle DEF$, find perimeter ratio. | **Concept & Formula:** $\frac{\text{Perim}(ABC)}{\text{Perim}(DEF)} = \frac{AB}{DE}$
- **Standard Step-by-Step:** Perimeter ratio equals corresponding side ratio (and median/altitude ratio).
- **Fastest Valid Method:** Direct equality with side ratio. | **Alternative:** Compute individual sides if needed.
- **Common Trap:** Squaring the side ratio (squaring is for area, not perimeter!).
- **Practice Drill:** If $\triangle ABC \sim \triangle DEF$, $AB = 4, DE = 6$, Perimeter$(ABC) = 18$, find Perimeter(DEF).

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Product-to-Proportion Trick ($CA^2 = CB \cdot CD$)] **Core Idea & Why It Works:** Rewrite any target product $P \cdot Q = R \cdot S$ as $\frac{P}{R} = \frac{S}{Q}$ to instantly see which two triangles to prove similar.
Valid Domain: All board similarity proofs | **Where It Fails:** None
Easy Example: $OA \cdot OB = OC \cdot OD \rightarrow \frac{OA}{OC} = \frac{OD}{OB}$ ($\triangle OAD \sim \triangle OCB$).
Board-Level Example: $CA^2 = CB \cdot CD \rightarrow \frac{CA}{CB} = \frac{CD}{CA}$ ($\triangle BAC \sim \triangle ADC$).
Advanced Example: Works for $PA \cdot PB = PC \cdot PD$ in circles too!

[SMART SHORTCUT – 2. Ladder / Lamp-Post Ratio $DE = \frac{AD}{AB}BC$] **Core Idea & Why It Works:** When $DE \parallel BC$, NEVER use AD/DB to find DE; ALWAYS use $AD/(AD + DB)$.
Valid Domain: All parallel-base triangle questions | **Where It Fails:** When DE is not parallel to BC
Easy Example: $AD = 2, DB = 3, BC = 10 \implies DE = \frac{2}{5}(10) = 4$.
Board-Level Example: Saves 90% of students from the $2/3 \times 10$ trap!
Advanced Example: Applies to nested similar triangles.

[SMART SHORTCUT – 3. Perimeter & Median = Side Ratio (1:1 Power)] **Core Idea & Why It Works:** If $\triangle_1 \sim \triangle_2$ with side ratio $a : b$, then ratio of perimeters = altitudes = medians = angle bisectors = $a : b$.
Valid Domain: All similarity ratio MCQs | **Where It Fails:** Area ratio (which is $a^2 : b^2$, though deleted from CBSE proofs)
Easy Example: Sides $2 : 3 \implies$ Perimeters $2 : 3$.
Board-Level Example: Medians $3 : 5 \implies$ Perimeters $3 : 5$.
Advanced Example: Instant 5-second MCQ solver.

[SMART SHORTCUT – 4. Vertex Matching by Angle Order] **Core Idea & Why It Works:** Write equal angles vertically: $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F \implies \triangle ABC \sim \triangle DEF$.
Valid Domain: All vertex-order MCQs | **Where It Fails:** None
Easy Example: If $\angle A = \angle F$ and $\angle B = \angle E$, then $\triangle ABC \sim \triangle FED$.
Board-Level Example: Prevents choosing wrong option like $\triangle DEF$ vs $\triangle FED$.
Advanced Example: Guarantees correct side ratios.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: In $\triangle ABC$, $DE \parallel BC$. If $AD = 1.5$ cm, $DB = 3$ cm, $AE = 1$ cm, find EC.
Given & Required: $DE \parallel BC$ | **Concept & Formula:** BPT
Detailed Solution: $\frac{1.5}{3} = \frac{1}{EC} \implies \frac{1}{2} = \frac{1}{EC} \implies EC = 2$ cm. **Final Answer:** 2 **cm**
Fastest Alternative / Shortcut: $DB = 2 \cdot AD \implies EC = 2 \cdot AE = 2$.

SOLVED EXAMPLE 2 [Beginner]
Question: Check if $EF \parallel QR$ in $\triangle PQR$ if $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm, $FR = 2.4$ cm.
Given & Required: Four segment lengths | **Concept & Formula:** Converse of BPT
Detailed Solution: $\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$; $\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$. Since $1.3 \neq 1.5$, $EF \nparallel QR$. **Final Answer: No, not parallel**
Fastest Alternative / Shortcut: Compare ratios.

SOLVED EXAMPLE 3 [Beginner]
Question: A vertical pole of length 6 m casts a shadow 4 m long on the ground, and at the same time a tower casts a shadow 28 m long. Find the height of the tower.
Given & Required: $h_1 = 6, s_1 = 4, s_2 = 28$ | **Concept & Formula:** AA Similarity
Detailed Solution: Right triangles are similar: $\frac{6}{4} = \frac{H}{28} \implies H = 42$ m. **Final Answer:** 42 **m**
Fastest Alternative / Shortcut: $28/4 = 7 \implies 6 \times 7 = 42$.

SOLVED EXAMPLE 4 [Beginner]
Question: If $\triangle ABC \sim \triangle QRP$ with $\angle A = 50°, \angle B = 60°$, find $\angle P$.
Given & Required: $\triangle ABC \sim \triangle QRP$ | **Concept & Formula:** Corresponding Angles
Detailed Solution: $\angle C = 180° - (50° + 60°) = 70°$. Since C corresponds to P, $\angle P = 70°$. **Final Answer:** $70°$
Fastest Alternative / Shortcut: Third letter $C \leftrightarrow P$.

SOLVED EXAMPLE 5 [Intermediate]
Question: In $\triangle ABC$, $DE \parallel BC$ with $AD = x, DB = x - 2, AE = x + 2, EC = x - 1$. Find x.
Given & Required: $DE \parallel BC$ | **Concept & Formula:** BPT Algebraic
Detailed Solution: $\frac{x}{x-2} = \frac{x+2}{x-1} \implies x(x-1) = (x+2)(x-2) \implies x^2 - x = x^2 - 4 \implies x = 4$. **Final Answer:** $x = 4$
Fastest Alternative / Shortcut: x^2 cancels cleanly!

SOLVED EXAMPLE 6 [Intermediate]

Question: In $\triangle ABC$, $DE \parallel BC$. If $AD = 4$ cm, $DB = 6$ cm, $BC = 15$ cm, find DE.

Given & Required: $AD = 4, DB = 6, BC = 15$ | **Concept & Formula:** Full-Side Similarity

Detailed Solution: $\triangle ADE \sim \triangle ABC \implies \frac{DE}{BC} = \frac{AD}{AB} = \frac{4}{10} \implies DE = 0.4 \times 15 = 6$ cm. **Final Answer:** 6 **cm**

Fastest Alternative / Shortcut: $AB = 4 + 6 = 10$, so $DE = \frac{4}{10}(15) = 6$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Diagonals of a trapezium $ABCD$ with $AB \parallel DC$ intersect at O. Show that $\frac{OA}{OC} = \frac{OB}{OD}$.

Given & Required: Trapezium $ABCD, AB \parallel DC$ | **Concept & Formula:** AA Similarity

Detailed Solution: In $\triangle AOB$ and $\triangle COD$: $\angle OAB = \angle OCD$ (alt. int.), $\angle OBA = \angle ODC$ (alt. int.). By AA, $\triangle AOB \sim \triangle COD \implies \frac{OA}{OC} = \frac{OB}{OD}$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Alternate interior angles give AA immediately.

SOLVED EXAMPLE 8 [Intermediate]

Question: A girl of height 90 cm walks away from the base of a lamp-post at 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.

Given & Required: Girl $= 0.9$ m, Lamp $= 3.6$ m, dist $= 4.8$ m | **Concept & Formula:** Similar Right Triangles

Detailed Solution: Distance walked $= 1.2 \times 4 = 4.8$ m. Let shadow $= x$. $\frac{3.6}{0.9} = \frac{4.8+x}{x} \implies 4x = 4.8 + x \implies 3x = 4.8 \implies x = 1.6$ m. **Final Answer:** 1.6 **m**

Fastest Alternative / Shortcut: Height ratio $4:1 \implies (4.8 + x)/x = 4 \implies x = 1.6$.

SOLVED EXAMPLE 9 [Board Standard]

Question: State and prove the Basic Proportionality Theorem (Thales' Theorem).

Given & Required: $\triangle ABC, DE \parallel BC$ | **Concept & Formula:** BPT Proof

Detailed Solution: Join BE, CD; draw $DM \perp AC, EN \perp AB$. $\frac{\text{ar}(ADE)}{\text{ar}(BDE)} = \frac{\frac{1}{2}AD \cdot EN}{\frac{1}{2}DB \cdot EN} = \frac{AD}{DB}$. Similarly $\frac{\text{ar}(ADE)}{\text{ar}(CDE)} = \frac{AE}{EC}$. Since $\triangle BDE$ and $\triangle CDE$ share base DE between parallels $DE \parallel BC$, $\text{ar}(BDE) = \text{ar}(CDE)$. Hence $\frac{AD}{DB} = \frac{AE}{EC}$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Always include the figure and construction lines.

SOLVED EXAMPLE 10 [Board Standard]

Question: D is a point on side BC of $\triangle ABC$ such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$.

Given & Required: $\angle ADC = \angle BAC$ | **Concept & Formula:** AA Similarity

Detailed Solution: In $\triangle ABC$ and $\triangle DAC$: $\angle BAC = \angle ADC$ (given), $\angle C = \angle C$ (common). By AA, $\triangle ABC \sim \triangle DAC \implies \frac{CB}{CA} = \frac{CA}{CD} \implies CA^2 = CB \cdot CD$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Write $\angle C$ common + given angle.

SOLVED EXAMPLE 11 [Board Standard]

Question: Sides AB, BC and median AD of $\triangle ABC$ are proportional to sides PQ, QR and median PM of $\triangle PQR$. Show $\triangle ABC \sim \triangle PQR$.

Given & Required: $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$ | **Concept & Formula:** SSS then SAS

Detailed Solution: $\frac{BC}{QR} = \frac{2BD}{2QM} = \frac{BD}{QM} = \frac{AB}{PQ} = \frac{AD}{PM} \implies \triangle ABD \sim \triangle PQM$ (SSS) $\implies \angle B = \angle Q$. Now in $\triangle ABC, \triangle PQR$: $\frac{AB}{PQ} = \frac{BC}{QR}$ and $\angle B = \angle Q \implies \triangle ABC \sim \triangle PQR$ (SAS). **Final Answer: Proved**

Fastest Alternative / Shortcut: Halve BC and QR to get BD and QM.

SOLVED EXAMPLE 12 [Board Standard]

Question: If AD and PM are medians of similar triangles $\triangle ABC$ and $\triangle PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$.

Given & Required: $\triangle ABC \sim \triangle PQR$ | **Concept & Formula:** SAS Similarity

Detailed Solution: $\angle B = \angle Q$ and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{2BD}{2QM} = \frac{BD}{QM} \implies \triangle ABD \sim \triangle PQM$ (SAS) $\implies \frac{AB}{PQ} = \frac{AD}{PM}$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Converse flow of Ex 11.

SOLVED EXAMPLE 13 [Board Standard]

Question: In $\triangle ABC$, E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove $\triangle ABD \sim \triangle ECF$.

Given & Required: $AB = AC, AD \perp BC, EF \perp AC$ | **Concept & Formula:** AA Similarity

Detailed Solution: $AB = AC \implies \angle B = \angle C$. Also $\angle ADB = \angle EFC = 90°$. By AA, $\triangle ABD \sim \triangle ECF$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Base angles of isosceles $\triangle$ are equal.

SOLVED EXAMPLE 14 [Challenge]

Question: In a trapezium $ABCD$ ($AB \parallel DC$), E and F are points on non-parallel sides AD, BC such that $EF \parallel AB$. Show that $\frac{AE}{ED} = \frac{BF}{FC}$.

Given & Required: $AB \parallel EF \parallel DC$ | **Concept & Formula:** Joining Diagonal + BPT

Detailed Solution: Join AC meeting EF at G. In $\triangle ADC$, $EG \parallel DC \implies \frac{AE}{ED} = \frac{AG}{GC}$. In $\triangle CBA$, $GF \parallel AB \implies \frac{AG}{GC} = \frac{BF}{FC}$. Equating gives $\frac{AE}{ED} = \frac{BF}{FC}$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Use diagonal AC as a bridge between two triangles.

SOLVED EXAMPLE 15 [Challenge]

Question: If two poles of heights a and b stand vertically on a ground c apart, find the height of the intersection of lines joining the top of each to the foot of the other.

Given & Required: Poles a, b at distance c | **Concept & Formula:** Crossed-Ladder Similarity

Detailed Solution: By similar triangles, $\frac{h}{a} + \frac{h}{b} = 1 \implies h = \frac{ab}{a+b}$ (independent of c!). **Final Answer:** $\frac{ab}{a+b}$

Fastest Alternative / Shortcut: Harmonic formula $\frac{ab}{a+b}$.

SOLVED EXAMPLE 16 [Challenge]

Question: Through the midpoint M of side CD of a parallelogram $ABCD$, the line BM is drawn intersecting AC at L and AD produced at E. Prove $EL = 2BL$.

Given & Required: Parallelogram $ABCD$, M midpoint of CD | **Concept & Formula:** Congruence + AA Similarity

Detailed Solution: $\triangle BMC \cong \triangle EMD$ (ASA) $\implies ED = BC = AD \implies AE = 2BC$. Since $\triangle ALE \sim \triangle CLB$ (AA), $\frac{EL}{BL} = \frac{AE}{BC} = 2 \implies EL = 2BL$. **Final Answer: Proved**

Fastest Alternative / Shortcut: $AE = AD + DE = 2BC$.

G. Competency MCQ Mastery Section

Q1. If in $\triangle ABC$ and $\triangle DEF$, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar when:

(A) $\angle B = \angle E$ (B) $\angle A = \angle D$ (C) $\angle B = \angle D$ (D) $\angle A = \angle F$

Answer: (C) Included angle between AB, BC is $\angle B$; between DE, FD is $\angle D$. *Fast Method:* Common letter in numerator/denominator. *Distractor Note:* Picking $\angle B = \angle E$ without checking FD.

Q2. In $\triangle ABC$, $DE \parallel BC$ with $AD = 2, BD = 3, AE = 3.6$, then AC equals:

(A) 5.4 (B) 9.0 (C) 7.2 (D) 6.0

Answer: (B) $\frac{2}{5} = \frac{3.6}{AC} \implies AC = 9.0$. *Fast Method:* $EC = 5.4 \implies AC = 3.6 + 5.4 = 9$. *Distractor Note:* Answering $EC = 5.4$ instead of AC.

Q3. Which of the following is NOT a similarity criterion for two triangles?

(A) AAA (B) SAS (C) SSS (D) ASA

Answer: (D) ASA is a congruence criterion (AA already suffices for similarity). *Fast Method:* ASA has a fixed side. *Distractor Note:* Confusing congruence with similarity.

Q4. Two triangles are similar if their corresponding:

(A) Angles are equal (B) Sides are proportional (C) Both (A) and (B) (D) None

Answer: (C) Definition of similarity. *Fast Method:* Definition. *Distractor Note:* None.

Q5. In $\triangle ABC$, $DE \parallel BC$, $AD = 3, DB = 4, BC = 14$, then DE is:

(A) 6 (B) 10.5 (C) 8 (D) 7

Answer: (A) $DE/14 = 3/(3+4) = 3/7 \implies DE = 6$. *Fast Method:* Use $AD/AB = 3/7$. *Distractor Note:* Using $3/4$ gives 10.5.

Q6. If $\triangle ABC \sim \triangle PQR$, $AB = 6, BC = 8, PQ = 3$, then QR is:

(A) 4 (B) 6 (C) 16 (D) 2

Answer: (A) $6/3 = 8/QR \implies QR = 4$. *Fast Method:* Scale factor $1/2$. *Distractor Note:* None.

Q7. All _____ triangles are always similar:

(A) Isosceles (B) Right-angled (C) Equilateral (D) Scalene

Answer: (C) All angles are $60°$. *Fast Method:* Equilateral. *Distractor Note:* Isosceles need not have equal angles.

Q8. A 5 m pole casts a 2 m shadow. At the same time, a tree casts a 10 m shadow. Tree height is:

(A) 20 m (B) 25 m (C) 15 m (D) 4 m

Answer: (B) $5/2 = H/10 \implies H = 25$ m. *Fast Method:* $5 \times 5 = 25$. *Distractor Note:* None.

Q9. Perimeters of two similar triangles are 30 cm and 20 cm. If one side of first is 12 cm, corresponding side of second is:

(A) 8 cm (B) 9 cm (C) 18 cm (D) 6 cm

Answer: (A) $30/20 = 12/x \implies x = 8$ cm. *Fast Method:* Ratio $3:2$. *Distractor Note:* Squaring the ratio.

Q10. Assertion (A): If $DE \parallel BC$ in $\triangle ABC$, then $\frac{AD}{DB} = \frac{AE}{EC}$. Reason (R): A line parallel to one side divides the other two sides in the same ratio.

(A) Both A & R true, R explains A (B) Both A & R true, R does not explain A (C) A true, R false (D) A false, R true

Answer: (A) R is the statement of BPT. *Fast Method:* BPT. *Distractor Note:* None.

Q11. In trapezium $ABCD$ ($AB \parallel CD$), diagonals meet at O. If $OA = 3x-1, OB = 5x-3, OC = 2x+1, OD = 6x-5$, then x is:

(A) 2 (B) 3 (C) $1/2$ (D) 4

Answer: (A) Test $x = 2$: $OA = 5, OC = 5 \implies 1$; $OB = 7, OD = 7 \implies 1$. *Fast Method:* Option plug $x = 2$. *Distractor Note:* Solving quadratic slowly.

Q12. If $\triangle ABC \sim \triangle DEF$, $\angle A = 47°, \angle E = 83°$, then $\angle C$ is:

(A) $50°$ (B) $60°$ (C) $70°$ (D) $80°$

Answer: (A) $\angle B = \angle E = 83° \implies \angle C = 180 - 47 - 83 = 50°$. *Fast Method:* $180 - 130 = 50°$. *Distractor Note:* None.

H. Special Board & MCQ Examination Strategies

- Memorize the exact proof of BPT (Thales' Theorem)—it is one of the most frequently asked 5-mark questions.
- Always check vertex correspondence by matching equal angles before writing side ratios.
- When asked for DE with $DE \parallel BC$, use $\frac{AD}{AB} = \frac{DE}{BC}$, NOT $\frac{AD}{DB}$.
- Rewrite target products ($CA^2 = CB \cdot CD$) as proportions ($CA/CB = CD/CA$) to immediately identify the two triangles.
- Skip Area of Similar Triangles theorem proofs—they are deleted in the 2026–27 syllabus.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Writing $\frac{AD}{DB} = \frac{DE}{BC}$ when $DE \parallel BC$. $\longrightarrow$ **Correction:** DE/BC equals AD/AB, NOT AD/DB.

[COMMON MISTAKE] **Error:** Ignoring vertex order in similarity statements. $\longrightarrow$ **Correction:** If $\angle A = \angle D, \angle B = \angle F$, write $\triangle ABC \sim \triangle DFE$, not $\triangle DEF$.

[COMMON MISTAKE] **Error:** Squaring the side ratio for perimeter ratio. $\longrightarrow$ **Correction:** Perimeter ratio $=$ side ratio $(a : b)$.

[COMMON MISTAKE] **Error:** Writing SAS when the equal angle is NOT included between the two proportional sides. $\longrightarrow$ **Correction:** The angle must lie strictly between the two proportional sides.

[COMMON MISTAKE] **Error:** Omitting the area-equality step in the BPT proof. $\longrightarrow$ **Correction:** Explicitly state that $\triangle BDE$ and $\triangle CDE$ lie on the same base DE between parallels $DE \parallel BC$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: BPT: $\frac{AD}{DB} = \frac{AE}{EC}$; Corollary: $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$; Trapezium: $\frac{AO}{OC} = \frac{BO}{OD}$.

[QUICK REVISION] Key Theorems & Results: Similarity Criteria: AA/AAA, SAS, SSS. Perim ratio = Side ratio = Median ratio = Altitude ratio.

[QUICK REVISION] Shortcut Summary: Product-to-proportion ($CA^2 = CB \cdot CD \to CA/CB = CD/CA$); Crossed poles $h = ab/(a+b)$.

[QUICK REVISION] High-Frequency Board Patterns: BPT statement & proof; Algebraic BPT; $CA^2 = CB \cdot CD$; Lamp-post shadow problem.

Last-Minute Revision Checklist:

- ☐ Can state and prove BPT cleanly with diagram and construction.
- ☐ Can apply converse of BPT to test parallel lines.
- ☐ Can write exact vertex correspondence for AA/SAS/SSS.
- ☐ Never confuse AD/DB with AD/AB when finding DE.
- ☐ Know Area of Similar Triangles theorem is deleted in 2026–27.

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. In $\triangle ABC$, $DE \parallel BC$. If $AD = 2, DB = 4, AE = 3$, then EC is: (A) 4 (B) 5 (C) 6 (D) 8
2. All circles are: (A) Congruent (B) Similar (C) Equal (D) None
3. If $\triangle ABC \sim \triangle DEF$ and $AB/DE = 2/3$, then Perim(ABC)/Perim(DEF) is: (A) $4/9$ (B) $2/3$ (C) $3/2$ (D) $\sqrt{2}/\sqrt{3}$
4. AA similarity needs how many pairs of equal angles? (A) 1 (B) 2 (C) 3 (D) 0
5. If $AD/DB = AE/EC$, then by converse of BPT: (A) $DE \perp BC$ (B) $DE = BC$ (C) $DE \parallel BC$ (D) $AD = AE$

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. In $\triangle ABC$, $DE \parallel BC$, $AD = x, DB = x-2, AE = x+2, EC = x-1$. x equals: (A) 2 (B) 3 (C) 4 (D) 5
2. If $\triangle ABC \sim \triangle EDF$, which is NOT true? (A) $BC \cdot EF = AC \cdot FD$ (B) $AB \cdot EF = AC \cdot DE$ (C) $BC \cdot DE = AB \cdot EF$ (D) $BC \cdot DE = AB \cdot FD$
3. In $\triangle ABC$, $DE \parallel BC$. $AD = 2, DB = 3, DE = 4$, then BC is: (A) 6 (B) 8 (C) 10 (D) 12
4. A 1.5 m boy stands 3 m from a lamp-post and casts a 1.5 m shadow. Lamp height is: (A) 3 m (B) 4.5 m (C) 6 m (D) 4 m
5. Diagonals of trapezium $ABCD$ ($AB \parallel CD$) meet at O. If $AB = 2CD$, then $AO : OC$ is: (A) 1 : 2 (B) 2 : 1 (C) 4 : 1 (D) 1 : 4

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Two poles of heights 6 m and 12 m stand on a plane. Height of intersection of lines joining tops to opposite feet: (A) 3 m (B) 4 m (C) 4.5 m (D) 5 m
2. In $\triangle ABC$, $\angle BAC = 90°, AD \perp BC$. If $BD = 4, CD = 9$, then AD is: (A) 5 (B) 6 (C) 6.5 (D) 36
3. If D, E, F are midpoints of sides of $\triangle ABC$, then Perim(DEF) : Perim(ABC) is: (A) 1 : 4 (B) 1 : 2 (C) 1 : 3 (D) 2 : 3

4. In $\triangle ABC$, $AB = 6\sqrt{3}, AC = 12, BC = 6$. Angle B is: (A) 120° (B) 60° (C) 90° (D) 45°
5. If $\triangle ABC \sim \triangle PQR$ with $AB = 5, PQ = 10, QR = 12$, and Perim$(ABC) = 15$, then AC is: (A) 4 (B) 5 (C) 6 (D) 3

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (C):** $2/4 = 3/EC \implies EC = 6$. **Q2 (B):** Same shape. **Q3 (B):** Same as side ratio. **Q4 (B):** Two equal angles imply third is equal. **Q5 (C):** Converse of BPT.
Board Std	**Q1 (C):** $x(x-1) = x^2 - 4 \implies x = 4$. **Q2 (C):** $AB/ED = BC/DF = AC/EF \implies$ (C) is false. **Q3 (C):** $4/BC = 2/5 \implies BC = 10$. **Q4 (B):** $H/1.5 = (3+1.5)/1.5 = 3 \implies H = 4.5$ m. **Q5 (B):** $AO/OC = AB/CD = 2:1$.
Challenge	**Q1 (B):** $\frac{6\times12}{6+12} = 4$ m. **Q2 (B):** $AD^2 = BD \cdot CD = 36 \implies AD = 6$. **Q3 (B):** Each side of $\triangle DEF$ is half the opposite side. **Q4 (C):** $AB^2 + BC^2 = 108 + 36 = 144 = AC^2 \implies 90^\circ$. **Q5 (A):** $BC = 6, AB = 5 \implies AC = 15 - 11 = 4$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Similarity vs Congruence & BPT Proof; Day 2: BPT & Converse Numericals; Day 3: AA/SAS/SSS Proofs; Day 4: Revision & Test.
- **How to Introduce the Concept:** Hold up two passport-size and postcard-size copies of the same photo: 'Same shape, different size—that is Similarity!'
- **Difficult Areas & Remediation:** Vertex ordering ($\triangle ABC \sim \triangle FED$) and distinguishing AD/DB from AD/AB.
- **Blackboard Teaching Examples:** Draw the BPT proof diagram in large scale with color-coded altitudes EN and DM.
- **Classroom Questions to Ask Students:** Ask: 'Why is $\triangle BDE$ equal in area to $\triangle CDE$?' (Same base DE, same parallels $DE \parallel BC$.)
- **Common Student Doubts & Answers:** Is Pythagoras theorem proof in the 2026–27 exam? No, it is deleted, though basic right-triangle calculation may still appear in Trigonometry/Circles.
- **Homework & Revision Strategy:** Write BPT proof twice + 6 numericals + 4 similarity proofs.

[TEACHER'S TIP] Teach the Product-to-Proportion trick ($CA^2 = CB \cdot CD \to CA/CB = CD/CA$)—it turns hard geometry proofs into 3-line routines.

UNIT III: COORDINATE GEOMETRY TYBA COACHING CENTRE

Chapter 7: Coordinate Geometry

Distance Formula, Collinearity, Quadrilateral Identification, Section & Midpoint Formulas

A. Chapter Introduction & Board Examination Relevance

Coordinate Geometry translates geometric shapes into algebraic equations on the Cartesian plane. Under the 2026–27 rationalised syllabus, this chapter covers the **Distance Formula**, identifying types of triangles and quadrilaterals, equidistant points, the **Internal Section Formula**, and the **Midpoint Formula**.

- **What You Will Learn:** Computing distance between two points, distance from the origin, checking collinearity via $AB + BC = AC$, classifying triangles/quadrilaterals, finding points on x- or y-axis equidistant from two points, dividing segments in ratio $m_1 : m_2$, and finding unknown parallelogram vertices via midpoint equality.
- **Why This Chapter Matters:** Standalone Unit III carrying 6 marks in CBSE and 7 marks in RBSE—one of the highest-scoring, lowest-risk units in the entire syllabus.
- **Real-Life Applications:** GPS distance tracking, computer graphics rendering, drone path planning, and center-of-mass / midpoint calculations.
- **Cross-Chapter Connections:** Connects to Linear Equations (graphical coordinates), Triangles (midpoints & medians), and Class 11 Straight Lines & Conics.
- **Board & Foundation Relevance:** Area of a triangle using coordinates is deleted in the 2026–27 rationalised syllabus; focus 100% on Distance and Section formulas.

B. Complete Concept Explanation & Derivations

1. Cartesian Axes & Distance Formula

Any point on the x-axis has form $(x, 0)$ and on the y-axis has form $(0, y)$. By the Pythagoras theorem on $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$, the distance between $A(x_1, y_1)$ and $B(x_2, y_2)$ is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Distance of $P(x, y)$ from origin $O(0, 0)$ is $\sqrt{x^2 + y^2}$.

[CONCEPT MASTER – 1. Cartesian Axes & Distance Formula] Distance from x-axis is $|y|$ (not x!), and distance from y-axis is $|x|$ (not y!).

2. Collinearity & Shape Classification

Three points A, B, C are **collinear** if the sum of the two smaller distances equals the largest distance (e.g., $AB + BC = AC$). For quadrilaterals $ABCD$ (in order): **Square**: 4 equal sides & equal diagonals; **Rhombus**: 4 equal sides & unequal diagonals; **Rectangle**: opposite sides equal & equal diagonals; **Parallelogram**: opposite sides equal & unequal diagonals.

[CONCEPT MASTER – 2. Collinearity & Shape Classification] Always check diagonals in addition to sides when distinguishing a Square from a Rhombus, or a Rectangle from a Parallelogram.

3. Internal Section Formula & Midpoint

The coordinates of $P(x, y)$ dividing the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m_1 : m_2$ are $x = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$ and $y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$. For the **Midpoint** $(1 : 1)$, $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

[CONCEPT MASTER – 3. Internal Section Formula & Midpoint] When finding an unknown ratio, ALWAYS assume $k : 1$! This reduces two unknowns (m_1, m_2) to a single variable k.

C. Complete Formula Bank

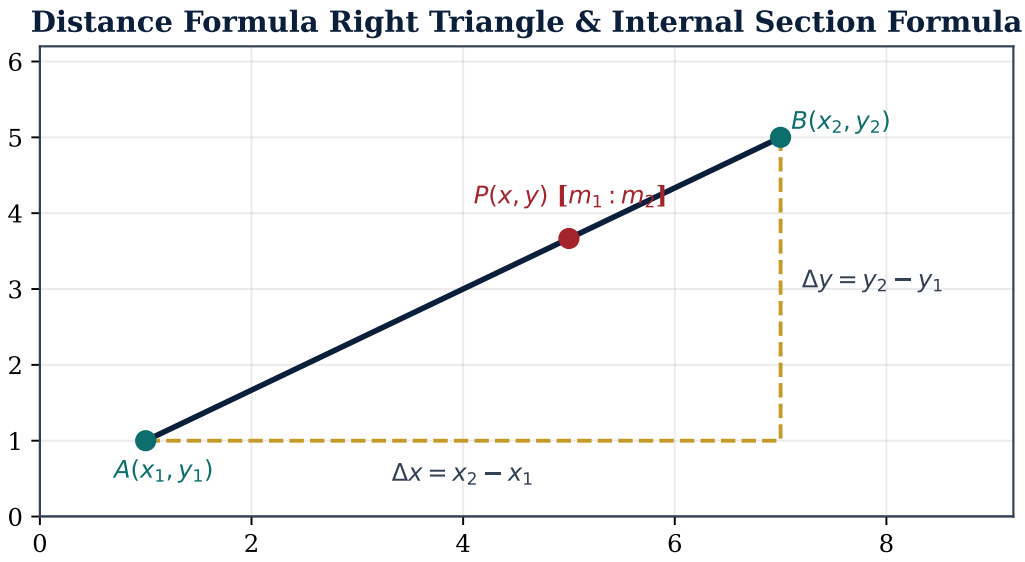


Figure 1: **Visual Model:** Distance formula right triangle $(\Delta x, \Delta y)$ and internal section point $P(x, y)$ dividing AB in ratio $m_1 : m_2$.

[FORMULA BANK – 1. Distance Formula] **Expression:** $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Symbols: $A(x_1, y_1), B(x_2, y_2)$
Conditions & When to Use: Always non-negative
Solved Mini-Example: Dist between $(2, 3)$ and $(5, 7)$ is $\sqrt{3^2 + 4^2} = 5$
Common Mistake: Subtracting $(x_2 - x_1)^2 - (y_2 - y_1)^2$ instead of adding.

[FORMULA BANK – 2. Distance from Origin] **Expression:** $OP = \sqrt{x^2 + y^2}$
Symbols: $P(x, y)$, $O(0, 0)$
Conditions & When to Use: Any point
Solved Mini-Example: Dist of $(-6, 8)$ from origin is $\sqrt{36 + 64} = 10$
Common Mistake: Writing $(-6)^2 = -36$.

[FORMULA BANK – 3. Internal Section Formula] **Expression:** $P = \left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$
Symbols: Ratio $m_1 : m_2$ from A to B
Conditions & When to Use: Internal division $(m_1, m_2 > 0)$
Solved Mini-Example: $2 : 1$ from $(1, 1)$ to $(7, 4) \to (5, 3)$
Common Mistake: Crossing m_1 with x_1 instead of x_2.

[FORMULA BANK – 4. Midpoint Formula] **Expression:** $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
Symbols: $m_1 = m_2 = 1$
Conditions & When to Use: Midpoint of AB
Solved Mini-Example: Midpoint of $(4, -2)$ and $(6, 8)$ is $(5, 3)$
Common Mistake: Subtracting coordinates instead of adding.

[FORMULA BANK – 5. 4th Vertex of Parallelogram] **Expression:** $D = A + C - B$
Symbols: A, B, C, D in cyclic order
Conditions & When to Use: Diagonals AC and BD bisect each other
Solved Mini-Example: $A(1, 2), B(4, 3), C(6, 6) \implies D = (1 + 6 - 4, 2 + 6 - 3) = (3, 5)$
Common Mistake: Equating adjacent vertices instead of opposite diagonals.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE Standard/Basic, RBSE Board, and Foundation MCQ/Case-Study problem structures.

Pattern 1: Equidistant Point on an Axis

- **Identification Cue:** Find a point on the x-axis (or y-axis) equidistant from A and B. | **Concept & Formula:** $PA^2 = PB^2$ with $(x, 0)$ or $(0, y)$
- **Standard Step-by-Step:** Let $P = (x, 0)$, set $PA^2 = PB^2$, expand and cancel x^2 to solve linear equation.
- **Fastest Valid Method:** Use $PA^2 = PB^2$ directly (never write square roots!). | **Alternative:** Midpoint perpendicular bisector.
- **Common Trap:** Taking $(0, x)$ for x-axis instead of $(x, 0)$.
- **Practice Drill:** Find a point on the x-axis equidistant from $(2, -5)$ and $(-2, 9)$.

Pattern 2: Finding Ratio $k : 1$ (Axis Division)

- **Identification Cue:** In what ratio does the x-axis (or y-axis or point P) divide AB? | **Concept & Formula:** Section formula with $k : 1$
- **Standard Step-by-Step:** Set $y = 0$ (for x-axis) or $x = 0$ (for y-axis) in section formula and solve for k.
- **Fastest Valid Method:** For x-axis dividing AB: ratio is $-y_1 : y_2$! For y-axis: $-x_1 : x_2$! | **Alternative:** Use $(x-x_1)/(x_2-x)$ directly.
- **Common Trap:** Forgetting to find the coordinates of the point of division after finding k.
- **Practice Drill:** Find the ratio in which y-axis divides $(-5, -6)$ and $(1, -4)$.

Pattern 3: Trisection / Four Equal Parts

- **Identification Cue:** Find coordinates of points of trisection of AB. | **Concept & Formula:** Ratios $1 : 2$ and $2 : 1$
- **Standard Step-by-Step:** Point P divides AB in $1 : 2$; Point Q divides AB in $2 : 1$ (or midpoint of PB).
- **Fastest Valid Method:** Step size $\Delta x/3, \Delta y/3$: add once for P, twice for Q! | **Alternative:** Section formula twice.
- **Common Trap:** Using $1 : 3$ instead of $1 : 2$ for trisection.
- **Practice Drill:** Find points of trisection of $(4, -1)$ and $(-2, -3)$.

Pattern 4: Parallelogram 4th Vertex / Missing (x, y)

- **Identification Cue:** If $(1, 2), (4, y), (x, 6), (3, 5)$ are vertices of a parallelogram in order, find x, y. | **Concept & Formula:** Midpoint of AC = Midpoint of BD
- **Standard Step-by-Step:** Equate $\frac{x_A+x_C}{2} = \frac{x_B+x_D}{2}$ and $\frac{y_A+y_C}{2} = \frac{y_B+y_D}{2}$.
- **Fastest Valid Method:** $x_A + x_C = x_B + x_D$ (cancel $/2$ immediately!). | **Alternative:** Distance formula (much slower—avoid!).
- **Common Trap:** Pairing A, B with C, D instead of diagonals AC and BD.
- **Practice Drill:** Find x, y if $(1, 2), (4, y), (x, 6), (3, 5)$ form a parallelogram.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Axis Division Ratio $-y_1 : y_2$ and $-x_1 : x_2$] **Core Idea & Why It Works:** The x-axis ($y = 0$) divides $A(x_1, y_1), B(x_2, y_2)$ in ratio $-y_1 : y_2$. The y-axis ($x = 0$) divides it in ratio $-x_1 : x_2$.

Valid Domain: All axis-division ratio MCQs | **Where It Fails:** If y_1 and y_2 have the same sign, division is external (not in Class 10 internal scope)

Easy Example: x-axis divides $(1, -5)$ and $(-4, 5)$ in ratio $-(-5) : 5 = 1 : 1$.

Board-Level Example: y-axis divides $(5, -6)$ and $(-1, -4)$ in ratio $-5 : -1 = 5 : 1$.

Advanced Example: Line $ax + by + c = 0$ divides AB in ratio $-(ax_1 + by_1 + c) : (ax_2 + by_2 + c)$!

[SMART SHORTCUT – 2. Vector-Step Trisection & n-Section] **Core Idea & Why It Works:** To divide $A \to B$ into n equal parts, compute $\delta_x = (x_2 - x_1)/n$ and $\delta_y = (y_2 - y_1)/n$, then step $A + k(\delta_x, \delta_y)$.
Valid Domain: Trisection ($n = 3$) and 4-equal-parts ($n = 4$) | **Where It Fails:** None
Easy Example: From $(2, -2)$ to $(-7, 4)$ in 3 parts: $\delta = (-3, 2) \implies (-1, 0), (-4, 2)$.
Board-Level Example: 4 equal parts: just find midpoint M, then midpoints of AM and MB!
Advanced Example: Zero fraction errors!

[SMART SHORTCUT – 3. Parallelogram Sum Rule $A + C = B + D$] **Core Idea & Why It Works:** Opposite vertices of a parallelogram always have the exact same coordinate sums: $x_A + x_C = x_B + x_D$ and $y_A + y_C = y_B + y_D$.
Valid Domain: Parallelograms, rectangles, squares, rhombuses | **Where It Fails:** Non-parallelogram trapeziums
Easy Example: $1 + x = 4 + 3 \implies x = 6$; $2 + 6 = y + 5 \implies y = 3$.
Board-Level Example: Solves 3-mark parallelogram questions in 10 seconds.
Advanced Example: Also finds the 3rd endpoint of a circle diameter: $A + B = 2O$.

[SMART SHORTCUT – 4. Distance Without Square Roots ($PA^2 = PB^2$)] **Core Idea & Why It Works:** Never carry $\sqrt{\cdot}$ when comparing equal distances; equate $PA^2 = PB^2$ directly.
Valid Domain: Equidistant & perpendicular-bisector problems | **Where It Fails:** None
Easy Example: Is $(3, 4)$ closer to origin than $(5, 1)$? $25 < 26 \implies$ yes.
Board-Level Example: Saves writing square roots 10 times in board solutions.
Advanced Example: Clean linear equation.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Find the distance between $(2, 3)$ and $(4, 1)$.
Given & Required: $A(2, 3), B(4, 1)$ | **Concept & Formula:** Distance Formula
Detailed Solution: $AB = \sqrt{(4-2)^2 + (1-3)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$. **Final Answer:** $2\sqrt{2}$
Fastest Alternative / Shortcut: $\Delta x = 2, \Delta y = -2 \implies 2\sqrt{2}$.

SOLVED EXAMPLE 2 [Beginner]
Question: Find the distance between (a, b) and $(-a, -b)$.
Given & Required: $A(a, b), B(-a, -b)$ | **Concept & Formula:** Distance Formula
Detailed Solution: $\sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4(a^2 + b^2)} = 2\sqrt{a^2 + b^2}$. **Final Answer:** $2\sqrt{a^2 + b^2}$
Fastest Alternative / Shortcut: Double distance from origin.

SOLVED EXAMPLE 3 [Beginner]
Question: Find the coordinates of the point which divides the join of $(-1, 7)$ and $(4, -3)$ in the ratio $2 : 3$.
Given & Required: $m_1 = 2, m_2 = 3$ | **Concept & Formula:** Section Formula
Detailed Solution: $x = \frac{2(4)+3(-1)}{5} = 1$, $y = \frac{2(-3)+3(7)}{5} = 3$. Point: $(1, 3)$. **Final Answer:** $(1, 3)$
Fastest Alternative / Shortcut: Cross-multiply weights.

SOLVED EXAMPLE 4 [Beginner]
Question: Find the coordinates of point A, where AB is the diameter of a circle whose centre is $(2, -3)$ and B is $(1, 4)$.
Given & Required: Midpoint $(2, -3)$, $B(1, 4)$ | **Concept & Formula:** Midpoint Formula
Detailed Solution: $\frac{x+1}{2} = 2 \implies x = 3$; $\frac{y+4}{2} = -3 \implies y = -10$. $A = (3, -10)$. **Final Answer:** $(3, -10)$
Fastest Alternative / Shortcut: $A = 2O - B = (4 - 1, -6 - 4) = (3, -10)$.

SOLVED EXAMPLE 5 [Intermediate]

Question: Determine if the points $(1,5)$, $(2,3)$, and $(-2,-11)$ are collinear.

Given & Required: Three points | **Concept & Formula:** Collinearity via Distances

Detailed Solution: $AB=\sqrt{1+4}=\sqrt{5}$; $BC=\sqrt{16+196}=\sqrt{212}$; $AC=\sqrt{9+256}=\sqrt{265}$. Since $\sqrt{5}+\sqrt{212}\neq\sqrt{265}$, not collinear. **Final Answer: Not collinear**

Fastest Alternative / Shortcut: Check $AB+BC=AC$.

SOLVED EXAMPLE 6 [Intermediate]

Question: Find the point on the x-axis which is equidistant from $(2,-5)$ and $(-2,9)$.

Given & Required: $P(x,0)$ | **Concept & Formula:** $PA^2=PB^2$

Detailed Solution: $(x-2)^2+25=(x+2)^2+81 \implies -4x+29=4x+85 \implies -8x=56 \implies x=-7$. Point: $(-7,0)$. **Final Answer:** $(-7,0)$

Fastest Alternative / Shortcut: x^2 cancels on both sides.

SOLVED EXAMPLE 7 [Intermediate]

Question: Find the values of y for which the distance between $P(2,-3)$ and $Q(10,y)$ is 10 units.

Given & Required: $PQ=10$ | **Concept & Formula:** Distance Equation

Detailed Solution: $(10-2)^2+(y+3)^2=100 \implies 64+(y+3)^2=100 \implies (y+3)^2=36 \implies y+3=\pm 6 \implies y=3,-9$. **Final Answer:** 3 **or** -9

Fastest Alternative / Shortcut: $(6,8,10)$ triple $\implies |y+3|=6$.

SOLVED EXAMPLE 8 [Intermediate]

Question: Find a relation between x and y such that (x,y) is equidistant from $(3,6)$ and $(-3,4)$.

Given & Required: $PA^2=PB^2$ | **Concept & Formula:** Perpendicular Bisector

Detailed Solution: $(x-3)^2+(y-6)^2=(x+3)^2+(y-4)^2 \implies -6x-12y+45=6x-8y+25 \implies 12x+4y=20 \implies 3x+y=5$. **Final Answer:** $3x+y=5$

Fastest Alternative / Shortcut: Midpoint $(0,5)$ lies on $3(0)+5=5$!

SOLVED EXAMPLE 9 [Board Standard]

Question: Name the type of quadrilateral formed by $(-1,-2)$, $(1,0)$, $(-1,2)$, $(-3,0)$.

Given & Required: 4 vertices | **Concept & Formula:** Sides + Diagonals

Detailed Solution: All 4 sides $=\sqrt{2^2+2^2}=2\sqrt{2}$. Diagonals $AC=\sqrt{0+16}=4$, $BD=\sqrt{16+0}=4$. Equal sides & equal diagonals $\implies$ Square. **Final Answer: Square**

Fastest Alternative / Shortcut: Never stop at equal sides—check diagonals too!

SOLVED EXAMPLE 10 [Board Standard]

Question: Find the ratio in which the line segment joining $A(1,-5)$ and $B(-4,5)$ is divided by the x-axis. Also find the coordinates of the point of division.

Given & Required: $y=0$ on x-axis | **Concept & Formula:** Section Formula $(k:1)$

Detailed Solution: $\frac{5k-5}{k+1}=0 \implies k=1$ (ratio $1:1$). $x=\frac{-4(1)+1}{2}=-\frac{3}{2}$. Point: $(-3/2,0)$. **Final Answer:** $1:1$**,** $(-3/2,0)$

Fastest Alternative / Shortcut: Ratio $=-(-5):5=1:1$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Find the coordinates of the points of trisection of the line segment joining $(4,-1)$ and $(-2,-3)$.

Given & Required: $A(4,-1), B(-2,-3)$ | **Concept & Formula:** Trisection $(1:2$ and $2:1)$

Detailed Solution: Step $\delta=(\frac{-2-4}{3},\frac{-3-(-1)}{3})=(-2,-2/3)$. $P=(2,-5/3)$, $Q=(0,-7/3)$. **Final Answer:** $(2,-5/3)$ **and** $(0,-7/3)$

Fastest Alternative / Shortcut: Step-size method avoids fraction errors.

SOLVED EXAMPLE 12 [Board Standard]

Question: If $(1,2),(4,y),(x,6),(3,5)$ are the vertices of a parallelogram taken in order, find x and y.
Given & Required: Parallelogram vertices | **Concept & Formula:** Midpoints of Diagonals
Detailed Solution: $\frac{1+x}{2}=\frac{4+3}{2} \implies x=6$; $\frac{2+6}{2}=\frac{y+5}{2} \implies y=3$. **Final Answer:** $x=6, y=3$
Fastest Alternative / Shortcut: $A+C=B+D$.

SOLVED EXAMPLE 13 [Board Standard]

Question: Find the ratio in which $(-1,6)$ divides the line segment joining $(-3,10)$ and $(6,-8)$.
Given & Required: $P(-1,6)$ | **Concept & Formula:** Section Formula $(k:1)$
Detailed Solution: $\frac{6k-3}{k+1}=-1 \implies 6k-3=-k-1 \implies 7k=2 \implies k=2/7$. Ratio $=2:7$. **Final Answer:** $2:7$
Fastest Alternative / Shortcut: Verify with y: $\frac{-16+70}{9}=6$.

SOLVED EXAMPLE 14 [Challenge]

Question: Find the area of a rhombus if its vertices are $(3,0),(4,5),(-1,4),(-2,-1)$ taken in order.
Given & Required: Rhombus vertices | **Concept & Formula:** $\frac{1}{2}d_1d_2$
Detailed Solution: $d_1=AC=\sqrt{(-4)^2+4^2}=4\sqrt{2}$; $d_2=BD=\sqrt{(-6)^2+(-6)^2}=6\sqrt{2}$. Area $=\frac{1}{2}(4\sqrt{2})(6\sqrt{2})=24$ sq. units. **Final Answer:** 24 **sq. units**
Fastest Alternative / Shortcut: Use $\frac{1}{2}d_1d_2$ (no triangle area formula needed!).

SOLVED EXAMPLE 15 [Challenge]

Question: If $A(-2,-1),B(a,0),C(4,b),D(1,2)$ form a parallelogram, find a,b and lengths of its sides.
Given & Required: Parallelogram | **Concept & Formula:** $A+C=B+D$ + Distance
Detailed Solution: $-2+4=a+1 \implies a=1$; $-1+b=0+2 \implies b=3$. $AB=\sqrt{3^2+1^2}=\sqrt{10}$, $BC=\sqrt{3^2+3^2}=3\sqrt{2}$.
Final Answer: $a=1,b=3$**; sides** $\sqrt{10},3\sqrt{2}$
Fastest Alternative / Shortcut: $A+C=B+D$.

SOLVED EXAMPLE 16 [Challenge]

Question: Find the centre of a circle passing through $(6,-6),(3,-7),(3,3)$.
Given & Required: Three points on circle | **Concept & Formula:** $OA^2=OB^2=OC^2$
Detailed Solution: Let $O(x,y)$. $OB^2=OC^2 \implies (y+7)^2=(y-3)^2 \implies 20y=-40 \implies y=-2$. Sub $y=-2$ into $OA^2=OC^2$: $(x-6)^2+16=(x-3)^2+25 \implies -6x+52=-6x$? Wait: $(x-6)^2+16=x^2-12x+52$; $(x-3)^2+25=x^2-6x+34 \implies 6x=18 \implies x=3$. Centre: $(3,-2)$. **Final Answer:** $(3,-2)$
Fastest Alternative / Shortcut: Start with $OB^2=OC^2$ since both have $x=3$!

G. Competency MCQ Mastery Section

Q1. The distance of the point $P(2,3)$ from the x-axis is:

(A) 2 (B) 3 (C) 1 (D) 5

Answer: (B) Distance from x-axis is $|y|=3$. *Fast Method:* $|y|$-coordinate. *Distractor Note:* Picking $x=2$.

Q2. The distance between $(0,5)$ and $(-5,0)$ is:

(A) 5 (B) $5\sqrt{2}$ (C) $2\sqrt{5}$ (D) 10

Answer: (B) $\sqrt{25+25}=5\sqrt{2}$. *Fast Method:* $5\sqrt{2}$. *Distractor Note:* Writing 5.

Q3. The midpoint of $(2a,4)$ and $(-2,3b)$ is $(1,2a+1)$. Values of a,b are:

(A) $a=2,b=2$ (B) $a=2,b=4$ (C) $a=1,b=2$ (D) $a=2,b=1$

Answer: (A) $\frac{2a-2}{2}=1 \implies a=2$. $\frac{4+3b}{2}=5 \implies b=2$. *Fast Method:* Solve x-midpoint first. *Distractor Note:* None.

Q4. The point which divides $(-1,7)$ and $(4,-3)$ in ratio $2:3$ lies in quadrant:

(A) I (B) II (C) III (D) IV

Answer: (A) Point is $(1,3)$, both positive $\implies$ Quadrant I. *Fast Method:* $x=1, y=3$. *Distractor Note:* None.

Q5. The x-axis divides the join of $(2,-3)$ and $(5,6)$ in the ratio:

(A) $1:2$ (B) $2:1$ (C) $3:5$ (D) $2:3$

Answer: (A) $-(-3):6=3:6=1:2$. *Fast Method:* $-y_1:y_2$. *Distractor Note:* Reversing to $2:1$.

Q6. If $A(1,3), B(-1,2), C(2,5), D(x,4)$ form a parallelogram $ABCD$, then x is:

(A) 3 (B) 4 (C) 0 (D) 5

Answer: (B) $1+2=-1+x \implies x=4$. *Fast Method:* $x_A+x_C=x_B+x_D$. *Distractor Note:* None.

Q7. A circle has centre $(2a, a-7)$ and passes through $(11,-9)$ with diameter $10\sqrt{2}$. Value of a is:

(A) 3 or 5 (B) 5 or -3 (C) 5 or 3 (D) -5 or 3

Answer: (A) Radius $=5\sqrt{2}, r^2=50$. Test $a=3$: $(6-11)^2+(-4+9)^2=25+25=50$. Test $a=5$: $(10-11)^2+(-2+9)^2=1+49=50$. *Fast Method:* Option plug! *Distractor Note:* Expanding messy quadratic.

Q8. The perimeter of a triangle with vertices $(0,4),(0,0),(3,0)$ is:

(A) 5 (B) 12 (C) 11 (D) $7+\sqrt{5}$

Answer: (B) 3-4-5 right triangle: $3+4+5=12$. *Fast Method:* $3+4+5=12$. *Distractor Note:* Computing area (6) instead of perimeter.

Q9. If $P(a/3,4)$ is the midpoint of $Q(-6,5)$ and $R(-2,3)$, then a is:

(A) -4 (B) -12 (C) 12 (D) -6

Answer: (B) $a/3=(-6-2)/2=-4 \implies a=-12$. *Fast Method:* $a=3(-4)=-12$. *Distractor Note:* Stopping at -4.

Q10. Assertion (A): The point $(0,4)$ lies on the y-axis. Reason (R): The x-coordinate of every point on the y-axis is zero.

(A) Both A & R true, R explains A (B) Both A & R true, R does not explain A (C) A true, R false (D) A false, R true

Answer: (A) Both true and R explains A. *Fast Method:* Standard definition. *Distractor Note:* None.

Q11. Distance of $(-3,4)$ from the origin is:

(A) 7 (B) 1 (C) 5 (D) -5

Answer: (C) $\sqrt{9+16}=5$. *Fast Method:* 3-4-5 triple. *Distractor Note:* Negative distance.

Q12. If $(0,0),(3,\sqrt{3}),(3,-\sqrt{3})$ are vertices of a triangle, it is:

(A) Right-angled (B) Equilateral (C) Isosceles only (D) Scalene

Answer: (B) All three sides equal $2\sqrt{3}$. *Fast Method:* $\sqrt{9+3}=\sqrt{12}=2\sqrt{3}$. *Distractor Note:* None.

H. Special Board & MCQ Examination Strategies

- Never keep square roots in equidistant problems—always equate $PA^2=PB^2$.
- Use $k:1$ whenever the ratio of division is unknown.
- Remember: x-axis is $(x,0)$ and y-axis is $(0,y)$.
- Use $A+C=B+D$ for any parallelogram, rectangle, rhombus, or square 4th-vertex question.
- Skip Area of a Triangle using coordinates—it is deleted from the 2026–27 syllabus.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Saying distance of $(2,3)$ from x-axis is 2. $\longrightarrow$ **Correction:** Distance from x-axis is $|y|=3$; distance from y-axis is $|x|=2$.

[COMMON MISTAKE] **Error:** Crossing m_1 with x_1 in the section formula. $\longrightarrow$ **Correction:** m_1 multiplies x_2 (the far endpoint) and m_2 multiplies x_1.

[COMMON MISTAKE] **Error:** Concluding a quadrilateral is a square just because all 4 sides are equal. $\longrightarrow$ **Correction:** Equal sides could be a Rhombus; you MUST check whether diagonals are equal.

[COMMON MISTAKE] **Error:** Assuming $(0, x)$ for a point on the x-axis. $\longrightarrow$ **Correction:** On x-axis, $y = 0$, so the point is $(x, 0)$.

[COMMON MISTAKE] **Error:** Writing $(-a)^2 = -a^2$ inside the distance formula. $\longrightarrow$ **Correction:** Squaring any real number always gives a non-negative value.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$; $OP = \sqrt{x^2+y^2}$; Section: $(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2})$; Midpoint: $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$.

[QUICK REVISION] Key Theorems & Results: Square: 4 equal sides + equal diag; Rhombus: 4 equal sides + unequal diag; Rhombus area $= \frac{1}{2}d_1d_2$.

[QUICK REVISION] Shortcut Summary: x-axis ratio $-y_1 : y_2$; y-axis ratio $-x_1 : x_2$; Parallelogram $A + C = B + D$.

[QUICK REVISION] High-Frequency Board Patterns: Equidistant on axis; Trisection; Ratio $k : 1$ with axis; Parallelogram 4th vertex.

Last-Minute Revision Checklist:

- ☐ Can compute distances and test collinearity via $AB + BC = AC$.
- ☐ Can distinguish Square vs. Rhombus and Rectangle vs. Parallelogram.
- ☐ Can apply Section & Midpoint formulas without sign errors.
- ☐ Can find $k : 1$ ratio and division point on x- or y-axis.
- ☐ Know Area of Triangle by coordinates is deleted in 2026–27.

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Distance of $(-6, 8)$ from origin: (A) 2 (B) 10 (C) 14 (D) 100
2. Midpoint of $(2, 6)$ and $(4, -2)$: (A) $(3, 2)$ (B) $(6, 4)$ (C) $(1, 4)$ (D) $(3, 4)$
3. Point on y-axis has: (A) $y = 0$ (B) $x = 0$ (C) $x = y$ (D) $x = 1$
4. Distance between $(3, 0)$ and $(0, 4)$: (A) 7 (B) 5 (C) 1 (D) 25
5. Distance of $(5, -7)$ from y-axis is: (A) 7 (B) -7 (C) 5 (D) 12

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. Point on x-axis equidistant from $(5, 4)$ and $(-2, 3)$: (A) $(2, 0)$ (B) $(1, 0)$ (C) $(3, 0)$ (D) $(0, 2)$
2. Ratio in which y-axis divides $(5, -6)$ and $(-1, -4)$: (A) $1 : 5$ (B) $5 : 1$ (C) $3 : 2$ (D) $2 : 3$
3. If $A(6, 1), B(8, 2), C(9, 4), D(p, 3)$ form a parallelogram, p is: (A) 5 (B) 6 (C) 7 (D) 8
4. One end of diameter is $(2, 3)$ and centre is $(-2, 5)$. Other end is: (A) $(-6, 7)$ (B) $(6, -7)$ (C) $(0, 4)$ (D) $(-4, 2)$
5. Point dividing $(7, -6)$ and $(3, 4)$ in ratio $1 : 2$ lies in quadrant: (A) I (B) II (C) III (D) IV

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If $(a, 0), (0, b), (1, 1)$ are collinear, then $\frac{1}{a} + \frac{1}{b}$ equals: (A) 0 (B) 1 (C) 2 (D) -1
2. Area of rhombus with vertices $(3, 0), (4, 5), (-1, 4), (-2, -1)$: (A) 12 (B) 24 (C) 48 (D) 16
3. Centroid of triangle with vertices $(3, -5), (-7, 4), (10, -2)$: (A) $(2, -1)$ (B) $(3, -1)$ (C) $(2, 1)$ (D) $(-2, 1)$
4. If $P(x, y)$ is equidistant from $(a + b, b - a)$ and $(a - b, a + b)$, then: (A) $ax = by$ (B) $bx = ay$ (C) $ax + by = 0$ (D) $x = y$
5. Ratio in which line $2x + y - 4 = 0$ divides $(2, -2)$ and $(3, 7)$: (A) $2 : 9$ (B) $9 : 2$ (C) $2 : 7$ (D) $7 : 2$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** $\sqrt{36+64}=10$. **Q2 (A):** $((2+4)/2,(6-2)/2)=(3,2)$. **Q3 (B):** $x=0$. **Q4 (B):** $\sqrt{9+16}=5$. **Q5 (C):** $\lvert x\rvert=5$.
Board Std	**Q1 (A):** $(2-5)^2+16=25$; $(2+2)^2+9=25$. **Q2 (B):** $-5:(-1)=5:1$. **Q3 (C):** $6+9=8+p \implies p=7$. **Q4 (A):** $2(-2)-2=-6$, $2(5)-3=7$. **Q5 (D):** $(17/3,-8/3) \implies$ Q-IV.
Challenge	**Q1 (B):** Slope $(1-0)/(1-a)=(b-0)/(0-a) \implies a+b=ab \implies 1/a+1/b=1$. **Q2 (B):** $\frac{1}{2}(4\sqrt{2})(6\sqrt{2})=24$. **Q3 (A):** $((3-7+10)/3,(-5+4-2)/3)=(2,-1)$. **Q4 (B):** Expanding $PA^2=PB^2$ gives $bx=ay$. **Q5 (A):** $-(4-2-4):(6+7-4)=2:9$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Distance Formula & Shapes; Day 2: Equidistant & Axis Problems; Day 3: Section & Trisection; Day 4: Parallelogram & Test.
- **How to Introduce the Concept:** Ask: 'How does Google Maps know the straight-line distance between Ajmer Junction and Pushkar? By subtracting coordinates!'
- **Difficult Areas & Remediation:** Confusing distance from x-axis ($|y|$) vs y-axis ($|x|$), and mixing m_1x_2 with m_1x_1.
- **Blackboard Teaching Examples:** Draw a big arrow cross showing m_1 pointing to $B(x_2,y_2)$ and m_2 pointing to $A(x_1,y_1)$.
- **Classroom Questions to Ask Students:** Ask: 'If all 4 sides of a quadrilateral are equal, is it guaranteed to be a square?' (No—could be a rhombus!)
- **Common Student Doubts & Answers:** Why isn't Triangle Area formula in the chapter? Because it was rationalised out in 2026–27.
- **Homework & Revision Strategy:** 5 distance/equidistant problems + 5 section/trisection/parallelogram problems.

[TEACHER'S TIP] Teach $A+C=B+D$ for parallelograms and $-y_1:y_2$ for x-axis division.

UNIT V: TRIGONOMETRY TYBA COACHING CENTRE

Chapter 8: Introduction to Trigonometry

Trigonometric Ratios, Standard Angles ($0°$–$90°$) & Fundamental Identities

A. Chapter Introduction & Board Examination Relevance

Trigonometry studies the relationships between side ratios and acute angles of right-angled triangles. In the 2026–27 rationalised syllabus, students master the six trigonometric ratios, exact values at $0°, 30°, 45°, 60°, 90°$, and proving/simplifying expressions using the three fundamental Pythagorean identities.

- **What You Will Learn:** Computing all 6 ratios from a right triangle, memorizing the standard angle table via the $\sqrt{0,1,2,3,4}/2$ rule, solving $\sin(A-B), \cos(A+B)$ equations, and proving identities using $\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$, and $1 + \cot^2\theta = \csc^2\theta$.
- **Why This Chapter Matters:** Part of Unit V Trigonometry (12 marks CBSE / 13 marks RBSE). Expect a guaranteed 3-mark or 5-mark identity proof plus 2–3 MCQs on standard angles and identities.
- **Real-Life Applications:** Surveying, satellite triangulation, alternating current (AC) waveforms, sound acoustics, and astronomy.
- **Cross-Chapter Connections:** Directly powers Chapter 9 (Heights & Distances) and Class 11–12 Trigonometry & Calculus.
- **Board & Foundation Relevance:** Trigonometric ratios of complementary angles ($90° - \theta$) are deleted in the 2026–27 rationalised syllabus; focus on ratios, standard angles, and the 3 Pythagorean identities.

B. Complete Concept Explanation & Derivations

1. Six Trigonometric Ratios of an Acute Angle

In a right triangle $\triangle ABC$ with $\angle C = 90°$ and acute angle $\theta = \angle B$, let p be the opposite side (perpendicular), b be the adjacent side (base), and h be the hypotenuse. Then $\sin\theta = p/h$, $\cos\theta = b/h$, $\tan\theta = p/b = \sin\theta/\cos\theta$. Their reciprocals are $\csc\theta = h/p$, $\sec\theta = h/b$, and $\cot\theta = b/p = \cos\theta/\sin\theta$.

[CONCEPT MASTER – 1. Six Trigonometric Ratios of an Acute Angle] Trigonometric ratios depend ONLY on the angle θ, never on the size of the right triangle (by AA similarity). Also, for acute θ, $\sin\theta \le 1$ and $\cos\theta \le 1$, whereas $\sec\theta \ge 1$ and $\csc\theta \ge 1$.

2. Standard Angle Table ($0°, 30°, 45°, 60°, 90°$)

For $\sin\theta$ at $0°, 30°, 45°, 60°, 90°$, the values are $\frac{\sqrt{0}}{2}, \frac{\sqrt{1}}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{4}}{2}$, i.e., $0, \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, 1$. Reverse the order for $\cos\theta$: $1, \frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}, 0$. Divide $\sin\theta/\cos\theta$ to get $\tan\theta$: $0, \frac{1}{\sqrt{3}}, 1, \sqrt{3}$, Not Defined.

[CONCEPT MASTER – 2. Standard Angle Table ($0°, 30°, 45°, 60°, 90°$)] As θ increases from $0°$ to $90°$, $\sin\theta$ and $\tan\theta$ increase, while $\cos\theta$ and $\cot\theta$ decrease.

3. The Three Fundamental Pythagorean Identities

Dividing $p^2 + b^2 = h^2$ by h^2, b^2, and p^2 yields: (1) $\sin^2\theta + \cos^2\theta = 1$; (2) $1 + \tan^2\theta = \sec^2\theta$ ($\theta \neq 90°$); (3) $1 + \cot^2\theta = \csc^2\theta$ ($\theta \neq 0°$). From (2) and (3), $(\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$ and $(\csc\theta - \cot\theta)(\csc\theta + \cot\theta) = 1$.

[CONCEPT MASTER – 3. The Three Fundamental Pythagorean Identities] Golden Proof Rule: When stuck on any trig identity proof, convert everything into $\sin\theta$ and $\cos\theta$ and take LCM!

C. Complete Formula Bank

Right-Triangle Trigonometric Ratios & Fundamental Identities

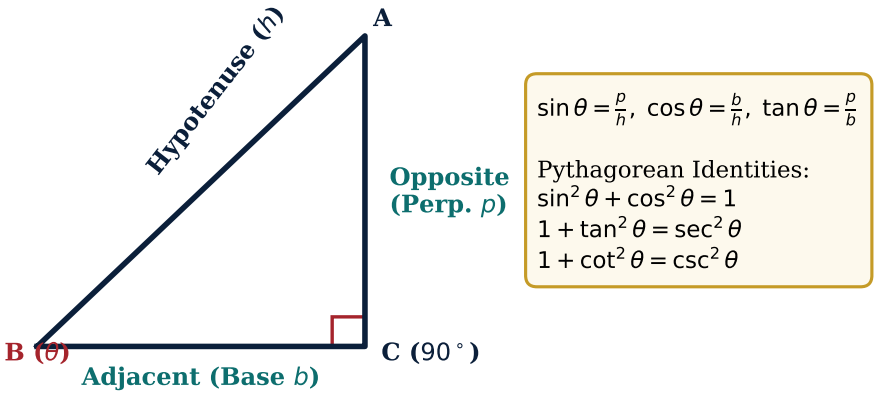


Figure 1: **Visual Model:** Right-triangle trigonometric ratios and the three fundamental Pythagorean identities.

[FORMULA BANK – 1. Fundamental Identity 1] **Expression:** $\sin^2\theta + \cos^2\theta = 1$
Symbols: $0^\circ \le \theta \le 90^\circ$
Conditions & When to Use: All angles
Solved Mini-Example: If $\sin\theta = 3/5$, $\cos\theta = \sqrt{1 - 9/25} = 4/5$
Common Mistake: Writing $\sin\theta + \cos\theta = 1$ without squares.

[FORMULA BANK – 2. Fundamental Identity 2] **Expression:** $\sec^2\theta - \tan^2\theta = 1$
Symbols: $\theta \neq 90^\circ$
Conditions & When to Use: $\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}$
Solved Mini-Example: $9\sec^2 A - 9\tan^2 A = 9(1) = 9$
Common Mistake: Writing $\tan^2\theta - \sec^2\theta = 1$ (it is -1!).

[FORMULA BANK – 3. Fundamental Identity 3] **Expression:** $\csc^2\theta - \cot^2\theta = 1$
Symbols: $\theta \neq 0^\circ$
Conditions & When to Use: $\csc\theta + \cot\theta = \frac{1}{\csc\theta - \cot\theta}$
Solved Mini-Example: If $\csc\theta + \cot\theta = p$, $\csc\theta - \cot\theta = 1/p$
Common Mistake: Writing $\cot^2\theta - \csc^2\theta = 1$.

[FORMULA BANK – 4. Quotient Relations] **Expression:** $\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}$
Symbols: Ratio identities
Conditions & When to Use: $\cos\theta \neq 0$ / $\sin\theta \neq 0$
Solved Mini-Example: Convert $\tan\theta + \cot\theta = \frac{1}{\sin\theta\cos\theta}$
Common Mistake: Mixing up $\tan$ and $\cot$ fractions.

[FORMULA BANK – 5. Reciprocal Identities] **Expression:** $\sin\theta\csc\theta = 1, \ \cos\theta\sec\theta = 1, \ \tan\theta\cot\theta = 1$
Symbols: Reciprocal pairs
Conditions & When to Use: Non-zero denominators
Solved Mini-Example: $\sin 30^\circ \cdot \csc 30^\circ = \frac{1}{2} \cdot 2 = 1$
Common Mistake: Confusing $\sin^{-1}$ with $1/\sin$ (they are different!).

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE Standard/Basic, RBSE Board, and Foundation MCQ/Case-Study problem structures.

Pattern 1: Finding Remaining Ratios from One Given Ratio

- **Identification Cue:** Given $\sin A = 3/4$ or $15\cot A = 8$, find $\sec A, \tan A$, etc. | **Concept & Formula:** Right triangle + Pythagoras
- **Standard Step-by-Step:** Draw right triangle, label p, b, h with k, find 3rd side via Pythagoras, read all ratios.
- **Fastest Valid Method:** Use standard Pythagorean triples: $(3,4,5), (5,12,13), (8,15,17), (7,24,25)$! | **Alternative:** Use $\sin^2 A + \cos^2 A = 1$.
- **Common Trap:** Forgetting to match opposite/adjacent sides to the specific angle (A vs C).
- **Practice Drill:** If $15\cot A = 8$, find $\sin A$ and $\sec A$.

Pattern 2: Standard Angle Evaluation & A, B Equations

- **Identification Cue:** Evaluate $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$, or find A, B from $\sin(A-B) = 1/2, \cos(A+B) = 1/2$. | **Concept & Formula:** Exact table values
- **Standard Step-by-Step:** Substitute exact surd values, simplify fractions carefully.
- **Fastest Valid Method:** Spot $\sin^2 30^\circ + \cos^2 30^\circ = 1$ in the denominator immediately! | **Alternative:** None.
- **Common Trap:** Squaring $\frac{2}{\sqrt{3}}$ as $\frac{4}{9}$ instead of $\frac{4}{3}$.
- **Practice Drill:** If $\tan(A+B) = \sqrt{3}$ and $\tan(A-B) = 1/\sqrt{3}$, find A, B.

Pattern 3: Proving Trigonometric Identities

- **Identification Cue:** Prove $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$ or $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \csc A + \cot A$. | **Concept & Formula:** Rationalisation / Sin-Cos Conversion / $\sec^2 - \tan^2 = 1$
- **Standard Step-by-Step:** Start from more complex side, convert to $\sin, \cos$ or rationalise denominator.
- **Fastest Valid Method:** For $\sqrt{\frac{1+\sin A}{1-\sin A}}$, multiply inside by $(1+\sin A)$ to clear square root cleanly. | **Alternative:** In MCQs, plug $\theta = 45^\circ$ or 30° to verify!
- **Common Trap:** Stopping halfway without simplifying $\sin^2 + \cos^2 = 1$.
- **Practice Drill:** Prove $(\sin A + \csc A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$.

Pattern 4: Eliminating θ from Two Equations

- **Identification Cue:** If $x = a\cos\theta, y = b\sin\theta$ or $a\cos\theta + b\sin\theta = m, a\sin\theta - b\cos\theta = n$. | **Concept & Formula:** Squaring & adding
- **Standard Step-by-Step:** Isolate $\cos\theta, \sin\theta$ or square and add both equations to cancel $2ab\sin\theta\cos\theta$.
- **Fastest Valid Method:** Square and add: $m^2 + n^2 = a^2 + b^2$! | **Alternative:** Test $\theta = 0^\circ$ in MCQs.
- **Common Trap:** Sign error in $(a\sin\theta - b\cos\theta)^2$.
- **Practice Drill:** If $\sin\theta + \cos\theta = \sqrt{2}$, find $\tan\theta + \cot\theta$.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Angle-Plug Trick for Trig Identity MCQs] **Core Idea & Why It Works:** Substitute $\theta = 45^\circ$ (or 30° if 45° makes a denominator 0) into the question and all 4 options.

Valid Domain: ALL trig identity simplification MCQs | **Where It Fails:** If two options give the same value at 45° (then test 30°)

Easy Example: $(\sec A + \tan A)(1 - \sin A)$ at $A = 45^\circ$: $(\sqrt{2}+1)(1 - 1/\sqrt{2}) = 1/\sqrt{2} = \cos 45^\circ$.

Board-Level Example: Solves any complex identity MCQ in 20 seconds!

Advanced Example: Always check $0/0$ before plugging.

[SMART SHORTCUT – 2. Conjugate Reciprocal $\sec\theta+\tan\theta=p$] **Core Idea & Why It Works:** If $\sec\theta+\tan\theta=p$, then $\sec\theta-\tan\theta=1/p$. Adding gives $2\sec\theta=p+1/p$; subtracting gives $2\tan\theta=p-1/p$.
Valid Domain: Both $(\sec,\tan)$ and $(\csc,\cot)$ pairs | **Where It Fails:** None
Easy Example: $\sec\theta+\tan\theta=2 \implies \sec\theta-\tan\theta=1/2 \implies \sec\theta=5/4, \tan\theta=3/4$.
Board-Level Example: Instantly finds $\sin\theta=\tan\theta/\sec\theta=\frac{p^2-1}{p^2+1}$!
Advanced Example: Works identically for $\csc\theta\pm\cot\theta$.

[SMART SHORTCUT – 3. Square-and-Add Rule ($a\cos\theta+b\sin\theta=m$)] **Core Idea & Why It Works:** If $a\cos\theta+b\sin\theta=m$ and $a\sin\theta-b\cos\theta=n$, then $m^2+n^2=a^2+b^2$.
Valid Domain: All linear $\sin,\cos$ elimination problems | **Where It Fails:** None
Easy Example: $3\sin\theta+4\cos\theta=5 \implies 3\cos\theta-4\sin\theta=\pm\sqrt{9+16-25}=0$.
Board-Level Example: Solves 3-mark proofs in 2 lines.
Advanced Example: If $a\sin\theta+b\cos\theta=\sqrt{a^2+b^2}$, then $\tan\theta=a/b$.

[SMART SHORTCUT – 4. Pythagorean Triples Memorizer] **Core Idea & Why It Works:** Keep $(3,4,5)$, $(5,12,13)$, $(8,15,17)$, $(7,24,25)$ on your fingertips.
Valid Domain: All ratio-from-triangle questions | **Where It Fails:** Non-integer radicals
Easy Example: $15\cot A=8 \implies b=8, p=15 \implies h=17$ immediately.
Board-Level Example: Saves 30 seconds per question.
Advanced Example: Double triples work too: $(6,8,10)$, $(10,24,26)$.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: In $\triangle ABC$ right-angled at B, $AB=24$ cm, $BC=7$ cm. Find $\sin A,\cos A,\sin C,\cos C$.
Given & Required: $AB=24, BC=7$ | **Concept & Formula:** Right Triangle Ratios
Detailed Solution: $AC=\sqrt{24^2+7^2}=25$. For A: $p=7,b=24 \implies \sin A=7/25,\cos A=24/25$. For C: $p=24,b=7 \implies \sin C=24/25,\cos C=7/25$. **Final Answer:** $7/25,24/25;24/25,7/25$
Fastest Alternative / Shortcut: $(7,24,25)$ triple.

SOLVED EXAMPLE 2 [Beginner]
Question: Evaluate $2\tan^2 45^\circ+\cos^2 30^\circ-\sin^2 60^\circ$.
Given & Required: Standard angles | **Concept & Formula:** Table Values
Detailed Solution: $2(1)^2+(\sqrt{3}/2)^2-(\sqrt{3}/2)^2=2$. **Final Answer:** 2
Fastest Alternative / Shortcut: $\cos^2 30^\circ=\sin^2 60^\circ$, so they cancel!

SOLVED EXAMPLE 3 [Beginner]
Question: Given $15\cot A=8$, find $\sin A$ and $\sec A$.
Given & Required: $\cot A=8/15$ | **Concept & Formula:** Pythagoras + Ratios
Detailed Solution: $b=8k,p=15k \implies h=17k$. $\sin A=15/17$, $\sec A=17/8$. **Final Answer:** $15/17,17/8$
Fastest Alternative / Shortcut: $(8,15,17)$ triple.

SOLVED EXAMPLE 4 [Beginner]
Question: If $\sin(A-B)=1/2$ and $\cos(A+B)=1/2$ $(0^\circ<A+B\le 90^\circ, A>B)$, find A and B.
Given & Required: Trig equations | **Concept & Formula:** Inverse Table Lookup
Detailed Solution: $A-B=30^\circ$ and $A+B=60^\circ$. Adding: $2A=90^\circ \implies A=45^\circ, B=15^\circ$. **Final Answer:** $A=45^\circ, B=15^\circ$
Fastest Alternative / Shortcut: $(60\pm 30)/2$.

SOLVED EXAMPLE 5 [Intermediate]

Question: Evaluate $\frac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$.

Given & Required: Standard angles | **Concept & Formula:** Exact Substitution

Detailed Solution: Denominator $= 1$. Numerator $= 5(1/4) + 4(4/3) - 1 = \frac{5}{4} + \frac{16}{3} - 1 = \frac{15+64-12}{12} = \frac{67}{12}$. **Final Answer:** $67/12$

Fastest Alternative / Shortcut: Denominator is $\sin^2 30^\circ + \cos^2 30^\circ = 1$!

SOLVED EXAMPLE 6 [Intermediate]

Question: Prove that $(\csc\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$.

Given & Required: Identity | **Concept & Formula:** Sin-Cos Conversion

Detailed Solution: LHS $= (\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta})^2 = \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Factor $1 - \cos^2\theta$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Prove that $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$.

Given & Required: Identity | **Concept & Formula:** Rationalisation

Detailed Solution: Multiply numerator and denominator inside root by $(1 + \sin A)$: $\sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \frac{1+\sin A}{\cos A} = \sec A + \tan A$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Always multiply by $(1 + \sin A)$.

SOLVED EXAMPLE 8 [Intermediate]

Question: In $\triangle PQR$ right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Find $\sin P, \cos P, \tan P$.

Given & Required: $PR + QR = 25, PQ = 5$ | **Concept & Formula:** Pythagoras + Algebra

Detailed Solution: Let $QR = x, PR = 25 - x$. $(25 - x)^2 = x^2 + 25 \implies 625 - 50x = 25 \implies x = 12, PR = 13$. $\sin P = 12/13, \cos P = 5/13, \tan P = 12/5$. **Final Answer:** $12/13, 5/13, 12/5$

Fastest Alternative / Shortcut: $(5, 12, 13)$ triple has $12 + 13 = 25$!

SOLVED EXAMPLE 9 [Board Standard]

Question: Prove that $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\csc\theta$.

Given & Required: Classic NCERT Identity | **Concept & Formula:** Sin-Cos Conversion + $a^3 - b^3$

Detailed Solution: Convert to $\sin, \cos$: $\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)} = \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)} = \frac{\sin^2\theta+\sin\theta\cos\theta+\cos^2\theta}{\sin\theta\cos\theta} = 1 + \sec\theta\csc\theta$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Use $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

SOLVED EXAMPLE 10 [Board Standard]

Question: Prove that $(\sin A + \csc A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$.

Given & Required: Identity | **Concept & Formula:** Expand + Reciprocal Pairs

Detailed Solution: LHS $= (\sin^2 A + \cos^2 A) + 2\sin A\csc A + 2\cos A\sec A + \csc^2 A + \sec^2 A = 1 + 2 + 2 + (1 + \cot^2 A) + (1 + \tan^2 A) = 7 + \tan^2 A + \cot^2 A$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Cross-terms give $2 + 2 = 4$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Prove that $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \csc A + \cot A$.

Given & Required: Identity | **Concept & Formula:** Divide by $\sin A$ + $\csc^2 - \cot^2 = 1$

Detailed Solution: Divide num and den by $\sin A$: $\frac{\cot A - 1 + \csc A}{\cot A + 1 - \csc A}$. Replace 1 in num by $(\csc^2 A - \cot^2 A)$, factor $(\cot A + \csc A)[1 - (\csc A - \cot A)]$, cancel denominator $\to \csc A + \cot A$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Always divide by $\sin A$ (for $\csc + \cot$) or $\cos A$ (for $\sec + \tan$).

SOLVED EXAMPLE 12 [Board Standard]

Question: If $\sin\theta + \cos\theta = \sqrt{3}$, prove that $\tan\theta + \cot\theta = 1$.

Given & Required: $\sin\theta + \cos\theta = \sqrt{3}$ | **Concept & Formula:** Squaring Both Sides

Detailed Solution: $\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3 \implies \sin\theta\cos\theta = 1$. Then $\tan\theta + \cot\theta = \frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{1} = 1$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Find $\sin\theta\cos\theta$ by squaring.

SOLVED EXAMPLE 13 [Board Standard]

Question: If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$, prove $x^2 - y^2 = a^2 - b^2$.

Given & Required: Parametric equations | **Concept & Formula:** $\sec^2\theta - \tan^2\theta = 1$

Detailed Solution: $x^2 - y^2 = a^2(\sec^2\theta - \tan^2\theta) + b^2(\tan^2\theta - \sec^2\theta) = a^2 - b^2$ (cross terms $2ab\sec\theta\tan\theta$ cancel). **Final Answer: Proved**

Fastest Alternative / Shortcut: Cross terms cancel immediately.

SOLVED EXAMPLE 14 [Challenge]

Question: If $\sec\theta + \tan\theta = p$, prove that $\sin\theta = \frac{p^2-1}{p^2+1}$.

Given & Required: $\sec\theta + \tan\theta = p$ | **Concept & Formula:** Conjugate Reciprocal

Detailed Solution: $\sec\theta - \tan\theta = 1/p$. Adding: $2\sec\theta = p + 1/p = \frac{p^2+1}{p}$. Subtracting: $2\tan\theta = \frac{p^2-1}{p}$. Divide: $\frac{\tan\theta}{\sec\theta} = \sin\theta = \frac{p^2-1}{p^2+1}$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Divide $2\tan\theta$ by $2\sec\theta$.

SOLVED EXAMPLE 15 [Challenge]

Question: If $\sin\theta + \sin^2\theta = 1$, find the value of $\cos^2\theta + \cos^4\theta$.

Given & Required: $\sin\theta + \sin^2\theta = 1$ | **Concept & Formula:** $\sin\theta = \cos^2\theta$

Detailed Solution: $\sin\theta = 1 - \sin^2\theta = \cos^2\theta$. Thus $\cos^2\theta + \cos^4\theta = \sin\theta + \sin^2\theta = 1$. **Final Answer:** 1

Fastest Alternative / Shortcut: Substitute $\cos^2\theta = \sin\theta$!

SOLVED EXAMPLE 16 [Challenge]

Question: If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

Given & Required: $m = \tan\theta + \sin\theta, n = \tan\theta - \sin\theta$ | **Concept & Formula:** Algebraic Trig Identity

Detailed Solution: $m^2 - n^2 = 4\tan\theta\sin\theta$. $mn = \tan^2\theta - \sin^2\theta = \sin^2\theta(\sec^2\theta - 1) = \sin^2\theta\tan^2\theta \implies \sqrt{mn} = \tan\theta\sin\theta$. Hence $m^2 - n^2 = 4\sqrt{mn}$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Compute LHS and RHS separately.

G. Competency MCQ Mastery Section

Q1. If $\cos A = 4/5$, then the value of $\tan A$ is:

(A) $3/5$ (B) $3/4$ (C) $4/3$ (D) $5/3$

Answer: (B) 3-4-5 triangle: $b = 4, h = 5 \implies p = 3 \implies \tan A = 3/4$. *Fast Method:* 3-4-5 triple. *Distractor Note:* Writing $\sin A = 3/5$.

Q2. The value of $9\sec^2 A - 9\tan^2 A$ is:

(A) 1 (B) 9 (C) 8 (D) 0

Answer: (B) $9(\sec^2 A - \tan^2 A) = 9(1) = 9$. *Fast Method:* Factor out 9. *Distractor Note:* Answering 1.

Q3. If $\sin\theta - \cos\theta = 0$, then the value of $\sin^4\theta + \cos^4\theta$ is:

(A) 1 (B) $3/4$ (C) $1/2$ (D) $1/4$

Answer: (C) $\theta = 45^\circ \implies (1/\sqrt{2})^4 + (1/\sqrt{2})^4 = 1/4 + 1/4 = 1/2$. *Fast Method:* Put $\theta = 45^\circ$. *Distractor Note:* Adding to 1.

Q4. $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$ equals:

(A) $\sin 60°$ (B) $\cos 60°$ (C) $\tan 60°$ (D) $\sin 30°$

Answer: (A) $\frac{2/\sqrt{3}}{1+1/3} = \frac{2/\sqrt{3}}{4/3} = \frac{\sqrt{3}}{2} = \sin 60°$. *Fast Method:* $\sin 2\theta$ at $30° = \sin 60°$. *Distractor Note:* Confusing $+$ with $-$ in denominator.

Q5. If $\sin A = 1/2$, then the value of $\cot A$ is:

(A) $\sqrt{3}$ (B) $1/\sqrt{3}$ (C) $\sqrt{3}/2$ (D) 1

Answer: (A) $A = 30° \implies \cot 30° = \sqrt{3}$. *Fast Method:* $A = 30°$. *Distractor Note:* Giving $\tan 30°$.

Q6. $(\sec A + \tan A)(1 - \sin A)$ equals:

(A) $\sec A$ (B) $\sin A$ (C) $\csc A$ (D) $\cos A$

Answer: (D) $\frac{1+\sin A}{\cos A}(1 - \sin A) = \frac{\cos^2 A}{\cos A} = \cos A$. *Fast Method:* Put $A = 45°$ or $0°$ $(1 \cdot 1 = 1 = \cos 0°)$. *Distractor Note:* None.

Q7. If $\sec\theta + \tan\theta = x$, then $\tan\theta$ equals:

(A) $\frac{x^2+1}{x}$ (B) $\frac{x^2-1}{x}$ (C) $\frac{x^2+1}{2x}$ (D) $\frac{x^2-1}{2x}$

Answer: (D) $2\tan\theta = x - 1/x = \frac{x^2-1}{x} \implies \tan\theta = \frac{x^2-1}{2x}$. *Fast Method:* Subtract $1/x$ from x and halve. *Distractor Note:* Forgetting $/2$.

Q8. Given $\triangle ABC$ right-angled at C, $\cos(A + B)$ equals:

(A) 0 (B) 1 (C) $1/2$ (D) $\sqrt{3}/2$

Answer: (A) $A + B = 90° \implies \cos 90° = 0$. *Fast Method:* $\cos 90° = 0$. *Distractor Note:* Picking 1.

Q9. Which of the following is NOT possible for acute θ?

(A) $\sin\theta = 3/5$ (B) $\cos\theta = 1$ (C) $\tan\theta = 100$ (D) $\sec\theta = 1/2$

Answer: (D) $\sec\theta = 1/\cos\theta \geq 1$, never $1/2$. *Fast Method:* $\sec\theta \geq 1$. *Distractor Note:* Thinking $\tan\theta$ is bounded by 1 (it is not!).

Q10. Assertion (A): $\sin^2 60° + \cos^2 60° = 1$. Reason (R): $\sin^2\theta + \cos^2\theta = 1$ for all θ.

(A) Both A & R true, R explains A (B) Both A & R true, R does not explain A (C) A true, R false (D) A false, R true

Answer: (A) Identity R directly proves A. *Fast Method:* Fundamental identity. *Distractor Note:* None.

Q11. If $x = 2\sin^2\theta$ and $y = 2\cos^2\theta + 1$, then $x + y$ is:

(A) 3 (B) 2 (C) 1 (D) $1/2$

Answer: (A) $2(\sin^2\theta + \cos^2\theta) + 1 = 2 + 1 = 3$. *Fast Method:* Add directly. *Distractor Note:* None.

Q12. Value of $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \csc\theta)$ is:

(A) 0 (B) 1 (C) 2 (D) -1

Answer: (C) Plug $\theta = 45°$: $(2 + \sqrt{2})(2 - \sqrt{2}) = 4 - 2 = 2$. *Fast Method:* Put $\theta = 45°$! *Distractor Note:* Expanding algebraically wastes 3 minutes in MCQs.

H. Special Board & MCQ Examination Strategies

- Write the standard angle table ($0°$ to $90°$) on your rough sheet at the start of the exam.
- In identity proofs, always convert $\tan, \cot, \sec, \csc$ into $\sin$ and $\cos$ if no direct identity is visible.
- In MCQs, plug $\theta = 45°$ or $30°$ to verify answers in seconds.
- Know that complementary angle identities ($\sin(90° - \theta) = \cos\theta$) are deleted in the 2026–27 syllabus.
- Always check which acute angle (A or C) is referenced before labeling Opposite and Adjacent sides.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Writing $\sin(A + B) = \sin A + \sin B$. $\longrightarrow$ **Correction:** $\sin$ is a function, not a multiplier; $\sin(30° + 60°) = 1 \neq 1/2 + \sqrt{3}/2$.

[COMMON MISTAKE] **Error:** Writing $\tan^2\theta - \sec^2\theta = 1$. $\longrightarrow$ **Correction:** $\sec^2\theta - \tan^2\theta = 1$, so $\tan^2\theta - \sec^2\theta = -1$.

[COMMON MISTAKE] **Error:** Squaring $(\frac{2}{\sqrt{3}})^2$ as $\frac{4}{9}$. $\longrightarrow$ **Correction:** $(\sqrt{3})^2 = 3$, so $(\frac{2}{\sqrt{3}})^2 = \frac{4}{3}$.

[COMMON MISTAKE] **Error:** Canceling $\sin$ from $\frac{\sin A}{\sin B}$ to get $\frac{A}{B}$. $\longrightarrow$ **Correction:** $\sin A$ is a single value, never $\sin \times A$.

[COMMON MISTAKE] **Error:** Forgetting to keep p, b, h tied to the specific acute angle. $\longrightarrow$ **Correction:** The opposite side changes when you switch from $\angle A$ to $\angle C$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $\sin = p/h, \cos = b/h, \tan = p/b$; $\sin^2\theta + \cos^2\theta = 1$; $1 + \tan^2\theta = \sec^2\theta$; $1 + \cot^2\theta = \csc^2\theta$.

[QUICK REVISION] Key Theorems & Results: $\sec\theta + \tan\theta = 1/(\sec\theta - \tan\theta)$; $\csc\theta + \cot\theta = 1/(\csc\theta - \cot\theta)$.

[QUICK REVISION] Shortcut Summary: Angle-plug ($\theta = 45°$) for MCQs; Pythagorean triples $(3,4,5), (5,12,13), (8,15,17), (7,24,25)$.

[QUICK REVISION] High-Frequency Board Patterns: Right-triangle ratio computation; Standard angle evaluation; 3-mark/5-mark identity proofs.

Last-Minute Revision Checklist:

- ☐ Know all 6 ratios at $0°, 30°, 45°, 60°, 90°$ instantaneously.
- ☐ Can prove identities using $\sin$ / $\cos$ conversion and rationalisation.
- ☐ Know conjugate pairs for $(\sec, \tan)$ and $(\csc, \cot)$.
- ☐ Never commit $\sin(A + B) = \sin A + \sin B$ error.
- ☐ Know complementary angle ratios are deleted in 2026–27.

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. $\sin 30° + \cos 60°$ equals: (A) 0 (B) $1/2$ (C) 1 (D) $\sqrt{3}$
2. If $\tan A = 1$, then A is: (A) $0°$ (B) $30°$ (C) $45°$ (D) $60°$
3. $\sec^2 45° - 1$ equals: (A) 0 (B) 1 (C) 2 (D) $\sqrt{2}$
4. If $\sin\theta = 5/13$, then $\cos\theta$ is: (A) $12/13$ (B) $8/13$ (C) $5/12$ (D) $13/12$
5. Value of $\tan 60° \cdot \cot 60°$ is: (A) 0 (B) 1 (C) $\sqrt{3}$ (D) 3

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. If $4\tan\theta = 3$, then $\frac{4\sin\theta - \cos\theta}{4\sin\theta + \cos\theta}$ is: (A) $2/3$ (B) $1/3$ (C) $1/2$ (D) $3/4$
2. If $\sin A + \cos A = \sqrt{2}\cos A$, then $\tan A$ is: (A) $\sqrt{2} - 1$ (B) $\sqrt{2} + 1$ (C) $\sqrt{2}$ (D) 1
3. Value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$ is: (A) 0 (B) 1 (C) 2 (D) $\sec^2\theta$
4. If $\sqrt{3}\tan\theta = 3\sin\theta$, then $\sin^2\theta - \cos^2\theta$ is: (A) $1/3$ (B) $2/3$ (C) 1 (D) 0
5. If $x = a\cos\theta, y = b\sin\theta$, then $b^2x^2 + a^2y^2$ equals: (A) $a^2 + b^2$ (B) a^2b^2 (C) ab (D) 1

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If $\cos\theta + \cos^2\theta = 1$, then $\sin^2\theta + \sin^4\theta$ equals: (A) 0 (B) 1 (C) 2 (D) -1
2. If $2\sin^2\theta - \cos^2\theta = 2$, then θ is: (A) $0°$ (B) $45°$ (C) $90°$ (D) $30°$
3. If $\sec\theta - \tan\theta = 1/3$, then $\sec\theta + \tan\theta$ is: (A) $1/3$ (B) 3 (C) $2/3$ (D) 9
4. Minimum value of $4\sin^2\theta + 4\cos^2\theta$ is: (A) 0 (B) 4 (C) 8 (D) 1
5. If $\sin\theta + \csc\theta = 2$, then $\sin^{20}\theta + \csc^{20}\theta$ equals: (A) 1 (B) 2 (C) 2^{20} (D) 0

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (C):** $1/2 + 1/2 = 1$. **Q2 (C):** $\tan 45^\circ = 1$. **Q3 (B):** $\tan^2 45^\circ = 1$. **Q4 (A):** $(5, 12, 13)$ triple. **Q5 (B):** Reciprocals multiply to 1.
Board Std	**Q1 (C):** Divide num and den by $\cos\theta$: $\frac{4\tan\theta - 1}{4\tan\theta + 1} = \frac{3-1}{3+1} = 1/2$. **Q2 (A):** Divide by $\cos A$: $\tan A + 1 = \sqrt{2}$. **Q3 (B):** $\sec^2\theta \cdot \cos^2\theta = 1$. **Q4 (A):** $\cos\theta = 1/\sqrt{3} \implies \cos^2\theta = 1/3, \sin^2\theta = 2/3 \implies 1/3$. **Q5 (B):** $a^2b^2(\cos^2\theta + \sin^2\theta) = a^2b^2$.
Challenge	**Q1 (B):** $\cos\theta = \sin^2\theta \implies \sin^2\theta + \sin^4\theta = \cos\theta + \cos^2\theta = 1$. **Q2 (C):** $2\sin^2\theta - (1 - \sin^2\theta) = 2 \implies 3\sin^2\theta = 3 \implies \theta = 90^\circ$. **Q3 (B):** Reciprocal $= 3$. **Q4 (B):** Constant $= 4$ for all θ. **Q5 (B):** $\sin\theta = 1 \implies 1^{20} + 1^{20} = 2$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: 6 Ratios & Triples; Day 2: Standard Angle Table; Day 3: Identities & Proofs; Day 4: Revision & Test.
- **How to Introduce the Concept:** Use the palm-of-hand finger trick $(0, 1, 2, 3, 4$ under $\sqrt{\cdot}/2)$ so every student memorizes the table in 2 minutes.
- **Difficult Areas & Remediation:** Proving multi-step identities like $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \csc A + \cot A$.
- **Blackboard Teaching Examples:** Write the 3 Pythagorean identities and their rearranged forms in a box on the side of the board.
- **Classroom Questions to Ask Students:** Ask: 'Can $\sin\theta = 4/3$ for any acute angle?' (No, $p \leq h$ always!)
- **Common Student Doubts & Answers:** Are complementary angles $(90^\circ - \theta)$ in the exam? No, deleted in 2026–27.
- **Homework & Revision Strategy:** 5 ratio problems + 5 table evaluations + 6 NCERT identity proofs.

[TEACHER'S TIP] Show students the $\theta = 45^\circ$ verification check so they never lose marks on MCQ identities.

UNIT V: TRIGONOMETRY TYBA COACHING CENTRE

Chapter 9: Some Applications of Trigonometry

Heights and Distances, Line of Sight, Angles of Elevation & Depression

A. Chapter Introduction & Board Examination Relevance

This chapter applies right-triangle trigonometry to measure inaccessible heights and horizontal distances—such as towers, cliffs, kites, balloons, and ships—using the **Angle of Elevation** and **Angle of Depression**. Retained 100% in the 2026–27 syllabus!

- **What You Will Learn:** Drawing accurate labelled diagrams from word descriptions, distinguishing angle of elevation vs. depression, solving single-triangle and double-triangle height-and-distance problems with $30°, 45°, 60°$, and using $\sqrt{3} \approx 1.732$.
- **Why This Chapter Matters:** Full chapter retained (no deletions!). Almost always contributes a 4-mark Case Study or 5-mark Long Answer question in both CBSE and RBSE boards.
- **Real-Life Applications:** Aviation glide slopes, lighthouse maritime distance estimation, surveying mountain peaks, and drone altitude tracking.
- **Cross-Chapter Connections:** Direct application of Chapter 8 (Standard Angles $30°, 45°, 60°$) and Chapter 6 (Similar Right Triangles).
- **Board & Foundation Relevance:** Board questions involve at most two right-angled triangles and standard angles $30°, 45°, 60°$.

B. Complete Concept Explanation & Derivations

1. Line of Sight, Angle of Elevation & Angle of Depression

The **line of sight** is the straight line from the observer's eye to the object viewed. When the object is *above* the horizontal level, the angle between the horizontal and the line of sight is the **angle of elevation**. When the object is *below* the horizontal level, the angle is the **angle of depression**.

[CONCEPT MASTER – 1. Line of Sight, Angle of Elevation & Angle of Depression] By alternate interior angles between parallel horizontal lines, the angle of depression from top A to bottom O equals the angle of elevation from O to A.

2. Standard Two-Triangle Geometry Strategy

Most 5-mark board problems involve two right triangles sharing a common vertical height h or common horizontal base d. Write $\tan\alpha$ and $\tan\beta$ in both triangles, express the common side in terms of the other, and equate.

[CONCEPT MASTER – 2. Standard Two-Triangle Geometry Strategy] Always draw a neat, clearly labelled diagram FIRST—CBSE awards 1 full mark just for the correct diagram!

C. Complete Formula Bank

[FORMULA BANK – 1. Height from Base & Angle] **Expression:** $h = d\tan\theta, \quad d = h\cot\theta$

Symbols: h: vertical height, d: horizontal distance, θ: elevation

Conditions & When to Use: Single right triangle

Solved Mini-Example: If $d = 30$ m, $\theta = 30° \implies h = 30/\sqrt{3} = 10\sqrt{3}$ m

Common Mistake: Using $\sin$ instead of $\tan$ when base and height are involved.

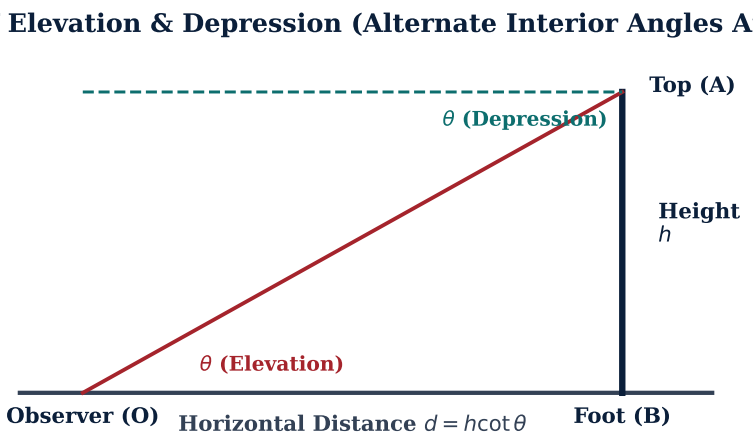


Figure 1: **Visual Model:** Angles of elevation and depression are equal as alternate interior angles between parallel horizontal lines.

[FORMULA BANK – 2. Two Angles on Same Side ($\alpha < \beta$)] **Expression:** $d = h(\cot\alpha - \cot\beta)$
Symbols: d: distance walked toward tower as elevation goes $\alpha \to \beta$
Conditions & When to Use: $\alpha < \beta$ (moving closer increases angle)
Solved Mini-Example: $30° \to 60°$: $d = h(\sqrt{3} - 1/\sqrt{3}) = \frac{2h}{\sqrt{3}}$
Common Mistake: Subtracting in wrong order ($\cot\beta - \cot\alpha$ gives negative).

[FORMULA BANK – 3. Two Angles on Opposite Sides] **Expression:** $d = h(\cot\alpha + \cot\beta)$
Symbols: d: distance between two objects on opposite sides of tower
Conditions & When to Use: Tower between two observers
Solved Mini-Example: River width with $30°, 45°$: $d = h(\sqrt{3} + 1)$
Common Mistake: Subtracting instead of adding opposite distances.

[FORMULA BANK – 4. Observer Height Adjustment] **Expression:** $h_{\text{eff}} = H_{\text{building}} - h_{\text{observer}}$
Symbols: Eye-level offset
Conditions & When to Use: When observer's height (e.g., 1.5 m) is given
Solved Mini-Example: $30 - 1.5 = 28.5$ m
Common Mistake: Using full 30 m instead of 28.5 m.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE Standard/Basic, RBSE Board, and Foundation MCQ/Case-Study problem structures.

Pattern 1: Single Right Triangle (Broken Tree / Kite / Ladder)

- **Identification Cue:** Tree breaks at height h and top touches ground at $30°$, or kite string at $60°$. | **Concept & Formula:** $\sin, \cos, \tan$ in one triangle
- **Standard Step-by-Step:** Identify which two sides are known/wanted (Perp, Base, Hyp) and choose $\tan, \sin$, or $\cos$.
- **Fastest Valid Method:** Total height of broken tree = Standing p + Broken h. | **Alternative:** Use $1 : \sqrt{3} : 2$ ratio.
- **Common Trap:** Finding only standing part p and forgetting to add broken hypotenuse h!
- **Practice Drill:** A tree breaks due to storm; top touches ground 8 m from foot at $30°$. Find original height.

Pattern 2: Two Objects on Same/Opposite Sides (Ships, Pillars, Roads)

- **Identification Cue:** Two ships behind one another at $30°$ and $45°$ from a 75 m lighthouse. | **Concept & Formula:** Two right triangles sharing h

- **Standard Step-by-Step:** Write $\tan 45^\circ = h/x$ and $\tan 30^\circ = h/(x+d)$, solve for d.
- **Fastest Valid Method:** For $30^\circ, 60^\circ$ same side: $d = \frac{2h}{\sqrt{3}}$; for $30^\circ, 45^\circ$: $d = h(\sqrt{3}-1)$. | **Alternative:** None.
- **Common Trap:** Placing 60° farther from tower than 30° (closer = steeper = larger angle!).
- **Practice Drill:** Two ships at $30^\circ, 45^\circ$ from 75 m lighthouse; find distance between them.

Pattern 3: Tower on a Building / Flagstaff on Pedestal

- **Identification Cue:** From ground point, bottom of tower is 45° and top is 60°. | **Concept & Formula:** Two triangles sharing horizontal base x
- **Standard Step-by-Step:** Use $\tan 45^\circ = b/x \implies x = b$, then $\tan 60^\circ = (b+h)/x$.
- **Fastest Valid Method:** $h = b(\tan\beta - \tan\alpha)/\tan\alpha$ when $\alpha = 45^\circ$: $h = b(\sqrt{3}-1)$. | **Alternative:** None.
- **Common Trap:** Setting $\tan 60^\circ = h/x$ instead of $(b+h)/x$.
- **Practice Drill:** Statue 1.6 m tall on pedestal: top 60°, bottom 45°. Find pedestal height.

Pattern 4: Moving Speed / Cloud Reflection / Uniform Speed

- **Identification Cue:** Car approaches tower; elevation changes $30^\circ \to 60^\circ$ in 6 sec. Find time to reach foot. | **Concept & Formula:** Speed ratio = Distance ratio
- **Standard Step-by-Step:** Since $d_{30\to60} = 2x$ (where x is remaining distance), time $= 6/2 = 3$ sec!
- **Fastest Valid Method:** At $30^\circ \to 60^\circ$, remaining distance is HALF the covered distance $\implies t_{\text{rem}} = t/2$! | **Alternative:** Full algebra with h.
- **Common Trap:** Answering total time 9 sec instead of remaining time 3 sec.
- **Practice Drill:** Car moves $30^\circ \to 60^\circ$ in 6 s; how much more time to reach tower?

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Magic Triangle Ratios (30°–60°–90° & 45°–45°–90°)] **Core Idea & Why It Works:** In a 30°–60°–90° triangle, sides opposite $(30^\circ, 60^\circ, 90^\circ)$ are in ratio $1 : \sqrt{3} : 2$. In 45°–45°–90°, sides are $1 : 1 : \sqrt{2}$.
Valid Domain: ALL standard-angle MCQs | **Where It Fails:** Non-standard angles
Easy Example: Opposite 30° is $10 \implies$ base is $10\sqrt{3}$, hyp is 20.
Board-Level Example: Broken tree with base 8 at 30°: $p = 8/\sqrt{3}, h = 16/\sqrt{3}$, total $= 24/\sqrt{3} = 8\sqrt{3}$ m!
Advanced Example: Solves 3-mark heights in 10 seconds.

[SMART SHORTCUT – 2. 30-to-60 Car Approach Rule ($t_{\text{rem}} = t/2$)] **Core Idea & Why It Works:** When elevation rises from 30° to 60°, the distance covered is $\frac{2h}{\sqrt{3}}$ and remaining distance is $\frac{h}{\sqrt{3}}$ (exact half!).
Valid Domain: $30^\circ \to 60^\circ$ uniform-speed car/boat problems | **Where It Fails:** Other angle pairs
Easy Example: Takes 6 s from 30° to $60^\circ \implies$ 3 s more to foot.
Board-Level Example: Takes 12 min from 30° to $60^\circ \implies$ 6 min more.
Advanced Example: For $30^\circ \to 45^\circ$: $t_{\text{rem}} = t/(\sqrt{3}-1)$.

[SMART SHORTCUT – 3. Complementary Elevation Heights $h = \sqrt{ab}$] **Core Idea & Why It Works:** If angles of elevation from distances a and b are complementary (θ and $90^\circ - \theta$), then $h = \sqrt{ab}$.
Valid Domain: Complementary-distance height MCQs | **Where It Fails:** Non-complementary angles
Easy Example: Distances 4 m and 9 m $\implies h = \sqrt{36} = 6$ m.
Board-Level Example: Distances a and $b \implies h = \sqrt{ab}$ (classic NCERT Ex 9.1 Q16).
Advanced Example: No $\tan(90^\circ - \theta)$ needed if you use $\tan\alpha \cdot \tan\beta = 1$!

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: A circus artist is climbing a 20 m long rope tightly stretched from the top of a vertical pole to the ground, making 30° with the ground. Find the height of the pole.

Given & Required: Hyp $= 20$, $\theta = 30^\circ$ | **Concept & Formula:** $\sin 30^\circ = h/20$

Detailed Solution: $\frac{1}{2} = \frac{h}{20} \implies h = 10$ m. **Final Answer:** 10 **m**

Fastest Alternative / Shortcut: Opposite 30° is half hypotenuse $= 10$ m.

SOLVED EXAMPLE 2 [Beginner]

Question: The angle of elevation of the top of a tower from a point on the ground 30 m away from its foot is 30°. Find the height of the tower.

Given & Required: Base $= 30, \theta = 30^\circ$ | **Concept & Formula:** $\tan 30^\circ = h/30$

Detailed Solution: $h = 30 \tan 30^\circ = \frac{30}{\sqrt{3}} = 10\sqrt{3}$ m. **Final Answer:** $10\sqrt{3}$ **m**

Fastest Alternative / Shortcut: $30/\sqrt{3} = 10\sqrt{3}$.

SOLVED EXAMPLE 3 [Beginner]

Question: A kite is flying at a height of 60 m above the ground. The string makes 60° with the ground. Find the length of the string.

Given & Required: $p = 60, \theta = 60^\circ$ | **Concept & Formula:** $\sin 60^\circ = 60/L$

Detailed Solution: $\frac{\sqrt{3}}{2} = \frac{60}{L} \implies L = \frac{120}{\sqrt{3}} = 40\sqrt{3}$ m. **Final Answer:** $40\sqrt{3}$ **m**

Fastest Alternative / Shortcut: $120/\sqrt{3} = 40\sqrt{3}$.

SOLVED EXAMPLE 4 [Beginner]

Question: If the shadow of a tower is $\sqrt{3}$ times its height, find the sun's altitude.

Given & Required: $s = \sqrt{3}h$ | **Concept & Formula:** $\tan\theta = h/s$

Detailed Solution: $\tan\theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}} \implies \theta = 30^\circ$. **Final Answer:** 30°

Fastest Alternative / Shortcut: $\tan\theta = 1/\sqrt{3} \implies 30^\circ$.

SOLVED EXAMPLE 5 [Intermediate]

Question: A tree breaks due to storm and the broken part bends so that the top touches the ground making 30° with it. The distance from foot to top is 8 m. Find the height of the tree.

Given & Required: Base $= 8, \theta = 30^\circ$ | **Concept & Formula:** Standing + Broken

Detailed Solution: $\tan 30^\circ = p/8 \implies p = 8/\sqrt{3}$. $\cos 30^\circ = 8/h \implies h = 16/\sqrt{3}$. Total $= \frac{24}{\sqrt{3}} = 8\sqrt{3}$ m. **Final Answer:** $8\sqrt{3}$ **m**

Fastest Alternative / Shortcut: $p + h = 24/\sqrt{3} = 8\sqrt{3}$.

SOLVED EXAMPLE 6 [Intermediate]

Question: From a point on a bridge across a river, the angles of depression of the banks on opposite sides are 30° and 45°. If the bridge is 3 m high, find the width of the river.

Given & Required: $h = 3$, opposite $30^\circ, 45^\circ$ | **Concept & Formula:** Opposite-Side Triangles

Detailed Solution: $x = 3\cot 30^\circ = 3\sqrt{3}$, $y = 3\cot 45^\circ = 3$. Width $= 3(\sqrt{3}+1)$ m. **Final Answer:** $3(\sqrt{3}+1)$ **m**

Fastest Alternative / Shortcut: $h(\cot 30^\circ + \cot 45^\circ)$.

SOLVED EXAMPLE 7 [Intermediate]

Question: A 1.5 m tall boy stands some distance from a 30 m tall building. His angle of elevation to the top increases from 30° to 60° as he walks towards it. Find the distance he walked.

Given & Required: $h_{\text{eff}} = 28.5$ m | **Concept & Formula:** Eye-Level Offset

Detailed Solution: $d = 28.5(\cot 30^\circ - \cot 60^\circ) = 28.5(\sqrt{3} - \frac{1}{\sqrt{3}}) = 28.5 \cdot \frac{2}{\sqrt{3}} = 19\sqrt{3}$ m. **Final Answer:** $19\sqrt{3}$ **m**

Fastest Alternative / Shortcut: $57/\sqrt{3} = 19\sqrt{3}$.

SOLVED EXAMPLE 8 [Intermediate]

Question: From a point on the ground, the angles of elevation of the bottom and top of a transmission tower fixed at the top of a 20 m high building are 45° and 60°. Find the height of the tower.

Given & Required: $b = 20, 45^\circ, 60^\circ$ | **Concept & Formula:** Common Horizontal Base

Detailed Solution: Base $x = 20\cot 45^\circ = 20$. Then $\frac{20+h}{20} = \tan 60^\circ = \sqrt{3} \implies h = 20(\sqrt{3}-1)$ m. **Final Answer:** $20(\sqrt{3}-1)$ **m**

Fastest Alternative / Shortcut: $20(\sqrt{3}-1)$.

SOLVED EXAMPLE 9 [Board Standard]

Question: Two poles of equal heights stand opposite each other on either side of an 80 m wide road. From a point between them, angles of elevation are 60° and 30°. Find the height of the poles and distances from the point.

Given & Required: $d = 80$, angles $60^\circ, 30^\circ$ | **Concept & Formula:** Opposite Triangles Equal h

Detailed Solution: Let point be x m from 60° pole: $h = x\sqrt{3} = (80-x)/\sqrt{3} \implies 3x = 80 - x \implies x = 20$ m, $80 - x = 60$ m, $h = 20\sqrt{3}$ m. **Final Answer:** $h = 20\sqrt{3}$ **m;** 20 **m,** 60 **m**

Fastest Alternative / Shortcut: $h(\frac{1}{\sqrt{3}} + \sqrt{3}) = 80 \implies h = 20\sqrt{3}$.

SOLVED EXAMPLE 10 [Board Standard]

Question: As observed from the top of a 75 m high lighthouse, angles of depression of two ships are 30° and 45° (one behind the other). Find the distance between the ships.

Given & Required: $h = 75, 30^\circ, 45^\circ$ | **Concept & Formula:** Same-Side Depression

Detailed Solution: Closer ship at $x = 75\cot 45^\circ = 75$ m; farther at $75\sqrt{3}$ m. Distance $= 75(\sqrt{3}-1)$ m. **Final Answer:** $75(\sqrt{3}-1)$ **m**

Fastest Alternative / Shortcut: $75(\sqrt{3}-1)$.

SOLVED EXAMPLE 11 [Board Standard]

Question: A 1.2 m tall girl spots a balloon moving horizontally at a height of 88.2 m from the ground. The elevation changes from 60° to 30°. Find the distance travelled by the balloon.

Given & Required: $h_{\text{eff}} = 88.2 - 1.2 = 87$ m | **Concept & Formula:** Horizontal Balloon Flight

Detailed Solution: $d = 87(\cot 30^\circ - \cot 60^\circ) = 87 \cdot \frac{2}{\sqrt{3}} = 58\sqrt{3}$ m. **Final Answer:** $58\sqrt{3}$ **m**

Fastest Alternative / Shortcut: $87 \times 2/\sqrt{3} = 29 \cdot 2\sqrt{3} = 58\sqrt{3}$.

SOLVED EXAMPLE 12 [Board Standard]

Question: From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Given & Required: Building $= 7$ m, $60^\circ, 45^\circ$ | **Concept & Formula:** Elevation + Depression

Detailed Solution: Horizontal distance $x = 7\cot 45^\circ = 7$ m. Upper tower $h_1 = 7\tan 60^\circ = 7\sqrt{3}$. Total $= 7(\sqrt{3}+1)$ m. **Final Answer:** $7(\sqrt{3}+1)$ **m**

Fastest Alternative / Shortcut: $7 + 7\sqrt{3}$.

SOLVED EXAMPLE 13 [Board Standard]

Question: A straight highway leads to the foot of a tower. A man on top observes a car at depression 30° approaching at uniform speed; 6 seconds later, depression is 60°. Find time taken to reach the foot from this point.

Given & Required: $30^\circ \to 60^\circ$ in 6 s | **Concept & Formula:** Uniform Speed Ratio

Detailed Solution: Let speed $= v$. $d_1 = 6v = h(\sqrt{3} - 1/\sqrt{3}) = \frac{2h}{\sqrt{3}}$. Remaining $d_2 = \frac{h}{\sqrt{3}} = \frac{d_1}{2} = 3v$, so time $= 3$ seconds. **Final Answer:** 3 **seconds**

Fastest Alternative / Shortcut: $6/2 = 3$ s.

SOLVED EXAMPLE 14 [Challenge]

Question: Angles of elevation of the top of a tower from two points at distances 4 m and 9 m from the base are complementary. Find the height of the tower.

Given & Required: $a = 4, b = 9, \alpha + \beta = 90^\circ$ | **Concept & Formula:** Complementary Angles

Detailed Solution: $\tan\alpha = h/4$ and $\tan\beta = \cot\alpha = h/9$. Multiplying: $1 = h^2/36 \implies h = 6$ m. **Final Answer:** 6 **m**

Fastest Alternative / Shortcut: $h = \sqrt{4 \times 9} = 6$ m.

SOLVED EXAMPLE 15 [Challenge]

Question: A ladder rests against a vertical wall at inclination α to the horizontal. Its foot is pulled away by distance p so its top slides down q, making angle β. Show $\frac{p}{q} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$.

Given & Required: Ladder length L | **Concept & Formula:** Fixed Hypotenuse L

Detailed Solution: $p = L(\cos\beta - \cos\alpha)$ and $q = L(\sin\alpha - \sin\beta)$. Dividing cancels L directly. **Final Answer: Proved**

Fastest Alternative / Shortcut: Write $x_2 - x_1$ and $y_1 - y_2$ in terms of L.

SOLVED EXAMPLE 16 [Challenge]

Question: The angle of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β. Find the height of the cloud above the lake.

Given & Required: Height H, reflection at $-H$ | **Concept & Formula:** Lake Reflection Symmetry

Detailed Solution: Distance to cloud above eye $= H - h$; to reflection below eye $= H + h$. Thus $\frac{H+h}{H-h} = \frac{\tan\beta}{\tan\alpha} \implies H = h\frac{\tan\beta + \tan\alpha}{\tan\beta - \tan\alpha}$. **Final Answer:** $h\frac{\tan\beta+\tan\alpha}{\tan\beta-\tan\alpha}$

Fastest Alternative / Shortcut: Componendo and Dividendo on $\frac{H+h}{H-h}$.

G. Competency MCQ Mastery Section

Q1. If the height of a tower and its shadow are equal, the sun's angle of elevation is:

(A) 30° (B) 45° (C) 60° (D) 90°

Answer: (B) $\tan\theta = h/h = 1 \implies \theta = 45^\circ$. *Fast Method:* $\tan\theta = 1$. *Distractor Note:* None.

Q2. A pole 6 m high casts a shadow $2\sqrt{3}$ m long. Sun's elevation is:

(A) 60° (B) 45° (C) 30° (D) 90°

Answer: (A) $\tan\theta = 6/(2\sqrt{3}) = \sqrt{3} \implies 60^\circ$. *Fast Method:* $3/\sqrt{3} = \sqrt{3}$. *Distractor Note:* Inverting ratio to 30°.

Q3. A ladder 15 m long reaches a wall making 60° with the wall. Height of wall is:

(A) $15\sqrt{3}/2$ m (B) $15/2$ m (C) 15 m (D) $15\sqrt{3}$ m

Answer: (B) 60° with WALL means $\cos 60^\circ = h/15 \implies h = 7.5$ m! *Fast Method:* Read carefully: angle is with WALL, not ground! *Distractor Note:* Using $\sin 60^\circ$ blindly.

Q4. If angle of elevation increases from 30° to 60°, the shadow length of a tower:

(A) Increases (B) Decreases (C) Remains same (D) Becomes zero

Answer: (B) Higher sun $\implies$ shorter shadow. *Fast Method:* cot decreases. *Distractor Note:* None.

Q5. From a point 20 m away from the foot of a tower, elevation is 60°. Height of tower is:

(A) $20/\sqrt{3}$ m (B) $20\sqrt{3}$ m (C) 40 m (D) $10\sqrt{3}$ m

Answer: (B) $20\tan 60^\circ = 20\sqrt{3}$. *Fast Method:* $20\sqrt{3}$. *Distractor Note:* None.

Q6. Angles of elevation from 9 m and 16 m are complementary. Height of tower is:

(A) 12 m (B) 7 m (C) 25 m (D) 144 m

Answer: (A) $\sqrt{9 \times 16} = 12$ m. *Fast Method:* $\sqrt{ab}$. *Distractor Note:* None.

Q7. A car moves from 30° to 60° elevation in 10 min. Additional time to reach the tower is:

(A) 10 min (B) 5 min (C) 20 min (D) 2.5 min

Answer: (B) Half of 10 min = 5 min. *Fast Method:* $t/2$. *Distractor Note:* Total time 15 min.

Q8. A 1.5 m observer stands 28.5 m away from a chimney; elevation is $45°$. Height of chimney:

(A) 28.5 m (B) 30 m (C) 27 m (D) 31.5 m

Answer: (B) $28.5 + 1.5 = 30$ m. *Fast Method:* Add eye height 1.5 m. *Distractor Note:* Forgetting $+1.5$ m.

Q9. The angle of depression of a car on the road from a 50 m tower is $30°$. Distance from foot:

(A) 50 m (B) $50\sqrt{3}$ m (C) $50/\sqrt{3}$ m (D) 100 m

Answer: (B) $50 \cot 30° = 50\sqrt{3}$ m. *Fast Method:* $50\sqrt{3}$. *Distractor Note:* None.

Q10. Assertion (A): Angle of elevation equals angle of depression between observer and object. Reason (R): Alternate interior angles of parallel horizontal lines are equal.

(A) Both A & R true, R explains A (B) Both A & R true, R does not explain A (C) A true, R false (D) A false, R true

Answer: (A) R directly explains A. *Fast Method:* Alternate interior angles. *Distractor Note:* None.

Q11. If two poles of heights h_1, h_2 subtend $60°$ and $30°$ at the midpoint of the line joining their feet, then $h_1 : h_2$ is:

(A) 1 : 3 (B) 3 : 1 (C) 1 : 2 (D) 2 : 1

Answer: (B) $\tan 60° : \tan 30° = \sqrt{3} : 1/\sqrt{3} = 3 : 1$. *Fast Method:* $\tan 60°/\tan 30° = 3$. *Distractor Note:* Reversing to $1 : 3$.

Q12. Length of string of a kite at height 30 m making $30°$ with ground:

(A) 30 m (B) 60 m (C) 15 m (D) $30\sqrt{3}$ m

Answer: (B) $30/\sin 30° = 60$ m. *Fast Method:* Double height at $30°$. *Distractor Note:* None.

H. Special Board & MCQ Examination Strategies

- Draw a neat labelled right-triangle diagram first—never skip horizontal reference lines for angles of depression.
- Check whether the given angle is with the GROUND or with the VERTICAL WALL.
- Remember closer to tower = larger angle ($60°$ is closer than $30°$).
- Always subtract observer's eye height from total height (or add it back at the end).
- Rationalise denominators ($1/\sqrt{3} = \sqrt{3}/3$) and substitute $\sqrt{3} = 1.732$ if requested.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Forgetting to add the standing and broken parts in broken-tree problems. ⟶ **Correction:** Total tree height $= p + h$.

[COMMON MISTAKE] **Error:** Placing $30°$ closer to the tower than $60°$. ⟶ **Correction:** Elevation increases as you walk TOWARD the tower.

[COMMON MISTAKE] **Error:** Forgetting observer height (1.5 m or 1.2 m). ⟶ **Correction:** Mark eye level explicitly on your diagram.

[COMMON MISTAKE] **Error:** Measuring angle of depression from the vertical tower instead of the horizontal line. ⟶ **Correction:** Angles of elevation and depression are ALWAYS measured from the HORIZONTAL.

[COMMON MISTAKE] **Error:** Using $\tan 60° = h/x$ instead of $(b + h)/x$ for a flagstaff on a building. ⟶ **Correction:** The line of sight goes from ground all the way to the top $(b + h)$.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $h = d\tan\theta$; Same side: $d = h(\cot\alpha - \cot\beta)$; Opposite sides: $d = h(\cot\alpha + \cot\beta)$; $h_{\text{eff}} = H - h_{\text{obs}}$.

[QUICK REVISION] Key Theorems & Results: Angle of elevation = Angle of depression (alternate interior angles).

[QUICK REVISION] Shortcut Summary: $1 : \sqrt{3} : 2$ ($30°$–$60°$–$90°$); $1 : 1 : \sqrt{2}$ ($45°$–$45°$–$90°$); $30° \to 60°$ time $t_{\text{rem}} = t/2$; $h = \sqrt{ab}$.

[QUICK REVISION] High-Frequency Board Patterns: Broken tree; Lighthouse & 2 ships; Flagstaff on building; Moving car/balloon; Lake reflection.

Last-Minute Revision Checklist:

- ☐ Always draw a labelled diagram with horizontal reference lines.
- ☐ Check observer height adjustment.
- ☐ Place larger angles closer to the vertical object.
- ☐ Know exact $\tan, \sin, \cos$ for $30°, 45°, 60°$.
- ☐ Verify physical reasonableness of final answer.

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Shadow of a 10 m pole at $45°$ sun elevation is: (A) 10 m (B) $10\sqrt{3}$ m (C) 5 m (D) 20 m
2. Angle of elevation when shadow is $1/\sqrt{3}$ times height: (A) $30°$ (B) $45°$ (C) $60°$ (D) $90°$
3. Ladder of 10 m at $30°$ to ground reaches height: (A) 5 m (B) $5\sqrt{3}$ m (C) $10\sqrt{3}$ m (D) 20 m
4. Angle of depression is measured from: (A) Vertical (B) Horizontal (C) Ground (D) Hypotenuse
5. If height $= 10\sqrt{3}$ m and base $= 10$ m, elevation is: (A) $30°$ (B) $45°$ (C) $60°$ (D) $90°$

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. Top of two poles of heights 20 m and 14 m are connected by a wire at $30°$ to horizontal. Wire length is: (A) 6 m (B) 12 m (C) $6\sqrt{3}$ m (D) 14 m
2. From a 60 m building, depression of top and bottom of a lamp post are $30°$ and $60°$. Lamp height is: (A) 20 m (B) 30 m (C) 40 m (D) 45 m
3. Shadow of a tower is 30 m longer when sun's altitude is $30°$ than when it is $60°$. Tower height: (A) $15\sqrt{3}$ m (B) $30\sqrt{3}$ m (C) 15 m (D) 45 m
4. A 1.6 m statue stands on a pedestal; elevations are $60°$ and $45°$. Pedestal height is: (A) $0.8(\sqrt{3}+1)$ (B) $1.6(\sqrt{3}+1)$ (C) $0.8(\sqrt{3}-1)$ (D) $1.6\sqrt{3}$
5. Two men on opposite sides of a 100 m tower see top at $30°$ and $45°$. Distance between them: (A) $100(\sqrt{3}+1)$ (B) $100(\sqrt{3}-1)$ (C) $100\sqrt{3}$ (D) 200 m

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. From a window 15 m high, angles of elevation and depression of top and foot of opposite house are $30°$ and $45°$. House height: (A) $15(1+1/\sqrt{3})$ (B) $15(1+\sqrt{3})$ (C) $15\sqrt{3}$ (D) 30 m

2. An aeroplane 3000 m high passes vertically above another plane; elevations from ground are 60° and 45°. Vertical distance: (A) $3000(1-1/\sqrt{3})$ (B) $3000(\sqrt{3}-1)$ (C) $1000\sqrt{3}$ (D) 1500 m
3. If a 1.5 m tree grows to h, wait: angles from a and b are $\alpha, 90^\circ-\alpha$. Ratio of h to $\sqrt{ab}$ is: (A) $1:1$ (B) $2:1$ (C) $1:2$ (D) $\sqrt{2}:1$
4. Boat moves away from 150 m cliff; depression changes $60^\circ \to 45^\circ$ in 2 min. Speed in m/min: (A) $75(1-1/\sqrt{3})$ (B) $75(\sqrt{3}-1)$ (C) $150(1-1/\sqrt{3})$ (D) $50\sqrt{3}$
5. If $\tan\alpha = 1/2$ and $\tan\beta = 1/3$ on opposite sides of tower h, distance d is: (A) $5h$ (B) $6h$ (C) $5h/6$ (D) h

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (A):** $10\cot 45^\circ = 10$ m. **Q2 (C):** $\tan\theta = \sqrt{3} \implies 60^\circ$. **Q3 (A):** $10\sin 30^\circ = 5$ m. **Q4 (B):** Horizontal line of sight. **Q5 (C):** $\tan\theta = \sqrt{3}$.
Board Std	**Q1 (B):** $(20-14)/\sin 30^\circ = 6/(1/2) = 12$ m. **Q2 (C):** $x = 60/\sqrt{3} = 20\sqrt{3}$; diff $= 20\sqrt{3}/\sqrt{3} = 20 \implies h = 60-20 = 40$ m. **Q3 (A):** $30 = h(2/\sqrt{3}) \implies h = 15\sqrt{3}$ m. **Q4 (A):** $1.6/(\sqrt{3}-1) = 0.8(\sqrt{3}+1)$ m. **Q5 (A):** $100(\cot 30^\circ + \cot 45^\circ)$.
Challenge	**Q1 (A):** Base $= 15$, upper $= 15/\sqrt{3}$. **Q2 (A):** Base $x = 3000/\sqrt{3} = 1000\sqrt{3}$; lower $= x\tan 45^\circ = 1000\sqrt{3}$; diff $= 3000 - 1000\sqrt{3}$. **Q3 (A):** $h = \sqrt{ab}$. **Q4 (A):** $d = 150(1-1/\sqrt{3})$, divide by 2. **Q5 (A):** $h(\cot\alpha + \cot\beta) = h(2+3) = 5h$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Elevation/Depression & Single Triangle; Day 2: Two-Triangle Problems; Day 3: Speed/Cloud/Observer-Height Problems; Day 4: Test.
- **How to Introduce the Concept:** Ask: 'Can you measure the height of our school building without climbing it?' Show how a 45° shadow gives the height instantly.
- **Difficult Areas & Remediation:** Drawing the horizontal line at the top for Angles of Depression, and remembering eye-level height subtraction.
- **Blackboard Teaching Examples:** Draw the $1:\sqrt{3}:2$ and $1:1:\sqrt{2}$ reference triangles permanently on the side.
- **Classroom Questions to Ask Students:** Ask: 'If you walk closer to a building, does the angle of elevation increase or decrease?'
- **Common Student Doubts & Answers:** Should we leave answers in $\sqrt{3}$ or decimals? Leave in exact $\sqrt{3}$ unless the question specifies $\sqrt{3} = 1.732$.
- **Homework & Revision Strategy:** 6 NCERT two-triangle problems with neat labelled diagrams.

[TEACHER'S TIP] Remind students that 1 mark in Boards is reserved for a neat, labelled figure!

UNIT IV: GEOMETRY TYBA COACHING CENTRE

Chapter 10: Circles

Secant vs. Tangent, Radius-Perpendicularity Theorem & Equal Tangents from External Point

A. Chapter Introduction & Board Examination Relevance

Retained 100% in the 2026–27 syllabus, Circles explores the geometry of tangents. Students master the two core theorems—(1) a tangent is perpendicular to the radius at the point of contact, and (2) the lengths of two tangents drawn from an external point to a circle are equal—along with their rich riders on quadrilaterals, triangles, and concentric circles.

- **What You Will Learn:** Distinguishing secants vs. tangents, proving Theorem 10.1 and Theorem 10.2, solving Pythagoras-based tangent lengths, proving opposite sides of a circumscribed quadrilateral subtend supplementary angles, and finding BC when a circle is inscribed in $\triangle ABC$.
- **Why This Chapter Matters:** Full chapter retained (no deletions!). Theorem 10.2 is one of the top 3 most frequently asked 3-mark/5-mark proofs in CBSE & RBSE Board Exams.
- **Real-Life Applications:** Pulley belt lengths, gear teeth contact, railway track curves, and satellite horizon distance ($d = \sqrt{2Rh + h^2}$).
- **Cross-Chapter Connections:** Builds on Class 9 Circle chord properties and connects to Areas Related to Circles (Chapter 11).
- **Board & Foundation Relevance:** Zero deletions; both theorems and all NCERT Ex 10.2 riders are high-priority board questions.

B. Complete Concept Explanation & Derivations

1. Secant vs. Tangent & Theorem 10.1

A line intersecting a circle in two distinct points is a **secant**. A line touching a circle at exactly one point (the **point of contact**) is a **tangent**. **Theorem 10.1:** The tangent at any point of a circle is perpendicular to the radius through the point of contact ($OP \perp XY$).

[CONCEPT MASTER – 1. Secant vs. Tangent & Theorem 10.1] There is NO tangent from a point inside a circle, exactly 1 tangent at a point on the circle, and infinitely many tangents to a circle overall (at most 2 parallel tangents).

2. Theorem 10.2: Equal Tangents from an External Point

Statement: The lengths of tangents drawn from an external point to a circle are equal ($PA = PB$). **Proof:** In right triangles $\triangle OAP$ and $\triangle OBP$, $\angle OAP = \angle OBP = 90°$ (Thm 10.1), $OP = OP$ (common), and $OA = OB$ (radii). By RHS congruence, $\triangle OAP \cong \triangle OBP \implies PA = PB$ (and $\angle OPA = \angle OPB$).

[CONCEPT MASTER – 2. Theorem 10.2: Equal Tangents from an External Point] Because $\angle OAP = \angle OBP = 90°$, quadrilateral $OAPB$ is ALWAYS cyclic, and $\angle AOB + \angle APB = 180°$.

3. Inscribed Circles in Triangles & Quadrilaterals

If a quadrilateral $ABCD$ circumscribes a circle, then $AB + CD = AD + BC$ (sum of opposite sides are equal), and opposite sides subtend supplementary angles at the centre. For a circle inscribed in $\triangle ABC$ touching BC, CA, AB at D, E, F, the tangent segments from A have length $s - a$ (where s is semiperimeter).

[CONCEPT MASTER – 3. Inscribed Circles in Triangles & Quadrilaterals] A parallelogram circumscribing a circle MUST be a **Rhombus** (since $AB + CD = AD + BC \implies 2AB = 2AD \implies AB = AD$).

C. Complete Formula Bank

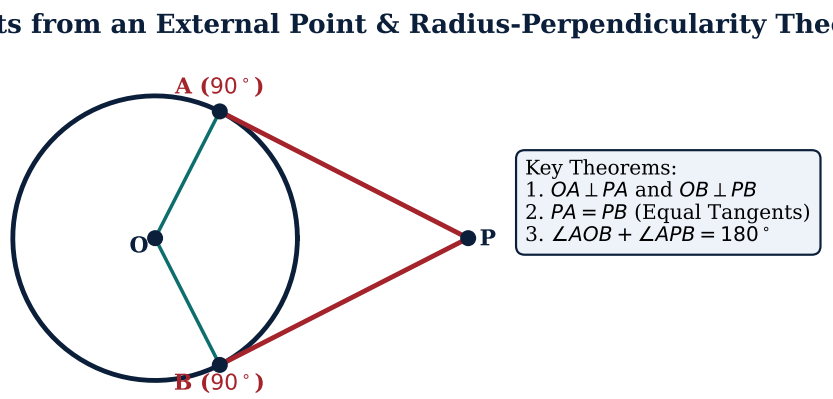


Figure 1: **Visual Model:** Two equal tangents $PA = PB$ from external point P with $OA \perp PA$, $OB \perp PB$, and $\angle AOB + \angle APB = 180°$.

[FORMULA BANK – 1. Tangent Length Formula] **Expression:** $PT = \sqrt{OP^2 - r^2}$
Symbols: OP: distance from centre to external point P, r: radius
Conditions & When to Use: $OP > r$
Solved Mini-Example: If $OP = 13, r = 5 \implies PT = \sqrt{169 - 25} = 12$ cm
Common Mistake: Adding $OP^2 + r^2$ instead of subtracting (remember OP is the hypotenuse!).

[FORMULA BANK – 2. Supplementary Tangent-Centre Angles] **Expression:** $\angle AOB + \angle APB = 180°$
Symbols: PA, PB tangents from P
Conditions & When to Use: Two tangents from external point P
Solved Mini-Example: If $\angle AOB = 110°$, $\angle APB = 70°$
Common Mistake: Setting $\angle AOB = \angle APB$.

[FORMULA BANK – 3. Circumscribed Quadrilateral Sum] **Expression:** $AB + CD = AD + BC$
Symbols: Quadrilateral $ABCD$ touching a circle
Conditions & When to Use: All 4 sides tangent to the circle
Solved Mini-Example: If $AB = 6, CD = 4, AD = 7$, then $BC = 6 + 4 - 7 = 3$
Common Mistake: Equating adjacent sides instead of opposite sides.

[FORMULA BANK – 4. Concentric Circles Chord] **Expression:** $L = 2\sqrt{R^2 - r^2}$
Symbols: R: outer radius, r: inner radius
Conditions & When to Use: Chord of larger circle tangent to smaller circle
Solved Mini-Example: $R = 5, r = 3 \implies L = 2\sqrt{25 - 9} = 8$ cm
Common Mistake: Forgetting to multiply $\sqrt{R^2 - r^2}$ by 2.

[FORMULA BANK – 5. Inradius of Right Triangle] **Expression:** $r = \frac{a+b-c}{2}$
Symbols: a, b: legs, c: hypotenuse
Conditions & When to Use: Circle inscribed in right $\triangle$
Solved Mini-Example: For $3, 4, 5$ right $\triangle$: $r = (3 + 4 - 5)/2 = 1$ cm
Common Mistake: Subtracting a leg instead of the hypotenuse.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE Standard/Basic, RBSE Board, and Foundation MCQ/Case-Study problem structures.

Pattern 1: Theorem 10.2 Proof ($PA = PB$)

- **Identification Cue:** Prove that the lengths of tangents drawn from an external point to a circle are equal. | **Concept & Formula:** RHS Congruence of $\triangle OAP \cong \triangle OBP$
- **Standard Step-by-Step:** Cite Theorem 10.1 for $90°$, common hypotenuse OP, equal radii $OA = OB$, conclude $PA = PB$.
- **Fastest Valid Method:** Clean 5-line RHS proof. | **Alternative:** None.
- **Common Trap:** Writing SAS instead of RHS.
- **Practice Drill:** Prove $PA = PB$ and deduce $\angle AOB + \angle APB = 180°$.

Pattern 2: Right-Triangle Tangent Length & Concentric Chords

- **Identification Cue:** Find tangent length PQ or chord length of larger concentric circle touching inner circle. | **Concept & Formula:** Pythagoras with $90°$ at contact point
- **Standard Step-by-Step:** Identify OP as hypotenuse, apply $PT^2 = OP^2 - r^2$.
- **Fastest Valid Method:** Use $(3,4,5), (5,12,13), (8,15,17)$ triples! | **Alternative:** None.
- **Common Trap:** Making PT the hypotenuse instead of OP.
- **Practice Drill:** Two concentric circles of radii 5 cm and 3 cm. Find chord of larger touching smaller.

Pattern 3: Circumscribed Quadrilateral / Parallelogram Proofs

- **Identification Cue:** Prove $AB + CD = AD + BC$ or prove a parallelogram circumscribing a circle is a rhombus. | **Concept & Formula:** Adding 4 pairs of equal tangents
- **Standard Step-by-Step:** Write $AP = AS, BP = BQ, CR = CQ, DR = DS$ and add all four equations.
- **Fastest Valid Method:** Keep A, B, C, D segments on LHS/RHS consistently so they group into $AB + CD$! | **Alternative:** None.
- **Common Trap:** Writing $AP = AS$ on the wrong side so sides don't add up.
- **Practice Drill:** Prove opposite sides of a circumscribed quadrilateral subtend supplementary angles at the centre.

Pattern 4: Triangle Circumscribing a Circle (NCERT Q12)

- **Identification Cue:** $riangleABC$ circumscribes circle of radius 4 cm; $BD = 8, DC = 6$. Find AB, AC. | **Concept & Formula:** Equal tangents + Area $= rs = \sqrt{s(s-a)(s-b)(s-c)}$
- **Standard Step-by-Step:** Set $AF = AE = x$, express $s = x + 14$, equate $\frac{1}{2}r(2s) = \sqrt{s(s-a)(s-b)(s-c)}$.
- **Fastest Valid Method:** Square $4(x+14) = \sqrt{(x+14)\cdot x \cdot 8 \cdot 6} \implies 16(x+14) = 48x \implies x = 7$! | **Alternative:** None.
- **Common Trap:** Arithmetic error in Heron's formula.
- **Practice Drill:** Find AB and AC when $r = 4, BD = 8, CD = 6$.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Inradius of Right Triangle $r = \frac{a+b-c}{2}$] **Core Idea & Why It Works:** For a right triangle with legs a, b and hypotenuse c, the inscribed circle has radius $r = \frac{a+b-c}{2}$.

Valid Domain: All inscribed-circle right-triangle problems | **Where It Fails:** Non-right triangles (use $r = \text{Area}/s$)

Easy Example: $(6, 8, 10)$ right $\triangle \implies r = (6+8-10)/2 = 2$ cm.

Board-Level Example: $(5, 12, 13)$ right $\triangle \implies r = (5+12-13)/2 = 2$ cm.

Advanced Example: Solves NCERT Ex 10.2 Q8/right-triangle variations in 3 seconds!

[SMART SHORTCUT – 2. Tangent Segment from Vertex A is $s - a$] **Core Idea & Why It Works:** For circle inscribed in $\triangle ABC$, tangent from A has length $s - BC$, from B is $s - AC$, from C is $s - AB$.

Valid Domain: All triangle-incircle tangent segment MCQs | **Where It Fails:** None

Easy Example: Sides $10, 12, 8 \implies s = 15$. Tangent from A (opp 10) $= 15 - 10 = 5$.

Board-Level Example: Instantly finds AD, BE, CF without solving 3 equations!

Advanced Example: Check: $AD + BE + CF = s$.

[SMART SHORTCUT – 3. Angle Bisector $\angle OPA = \frac{1}{2}\angle APB$] **Core Idea & Why It Works:** OP bisects both $\angle APB$ and $\angle AOB$, and $\angle OAB = \angle OBA = \frac{1}{2}\angle APB$!
Valid Domain: All angle-chasing MCQs with two tangents | **Where It Fails:** None
Easy Example: If $\angle APB = 80^\circ$, then $\angle OAB = 40^\circ$ and $\angle POA = 50^\circ$!
Board-Level Example: Direct 5-second MCQ solver (NCERT Ex 10.2 Q3).
Advanced Example: Because $\triangle OAB$ is isosceles with apex $180^\circ - \angle APB$.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: A tangent PQ at point P of a circle of radius 5 cm meets a line through centre O at Q so that $OQ = 12$ cm. Find PQ.
Given & Required: $r = 5, OQ = 12$ | **Concept & Formula:** Thm 10.1 + Pythagoras
Detailed Solution: $PQ = \sqrt{12^2 - 5^2} = \sqrt{144 - 25} = \sqrt{119}$ cm. **Final Answer:** $\sqrt{119}$ **cm**
Fastest Alternative / Shortcut: OQ is hypotenuse, NOT PQ!

SOLVED EXAMPLE 2 [Beginner]
Question: From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius.
Given & Required: $PT = 24, OQ = 25$ | **Concept & Formula:** Pythagoras
Detailed Solution: $r = \sqrt{25^2 - 24^2} = \sqrt{49} = 7$ cm. **Final Answer:** 7 **cm**
Fastest Alternative / Shortcut: $(7, 24, 25)$ triple.

SOLVED EXAMPLE 3 [Beginner]
Question: If tangents PA and PB from P to a circle with centre O are inclined to each other at 80°, find $\angle POA$.
Given & Required: $\angle APB = 80^\circ$ | **Concept & Formula:** Congruent Triangles
Detailed Solution: $\angle APO = 40^\circ$, $\angle OAP = 90^\circ \implies \angle POA = 50^\circ$. **Final Answer:** 50°
Fastest Alternative / Shortcut: $90^\circ - 40^\circ = 50^\circ$.

SOLVED EXAMPLE 4 [Beginner]
Question: Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.
Given & Required: $R = 5, r = 3$ | **Concept & Formula:** Chord Bisected at Contact
Detailed Solution: Half-chord $= \sqrt{5^2 - 3^2} = 4$ cm. Full chord $= 8$ cm. **Final Answer:** 8 **cm**
Fastest Alternative / Shortcut: $2 \times 4 = 8$ cm.

SOLVED EXAMPLE 5 [Intermediate]
Question: Prove that the tangents drawn at the ends of a diameter of a circle are parallel.
Given & Required: Diameter AB, tangents PQ, RS | **Concept & Formula:** Alternate Interior Angles
Detailed Solution: $\angle OAP = 90^\circ$ and $\angle OBS = 90^\circ$ (Thm 10.1). Since alternate interior angles $\angle PAB = \angle ABS = 90^\circ$, $PQ \parallel RS$. **Final Answer:** **Proved**
Fastest Alternative / Shortcut: Both angles 90°.

SOLVED EXAMPLE 6 [Intermediate]
Question: A quadrilateral $ABCD$ is drawn to circumscribe a circle. Prove that $AB + CD = AD + BC$.
Given & Required: Tangents from A, B, C, D | **Concept & Formula:** Thm 10.2 Four Times
Detailed Solution: $AP = AS, BP = BQ, CR = CQ, DR = DS$. Adding: $(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ) \implies AB + CD = AD + BC$. **Final Answer:** **Proved**
Fastest Alternative / Shortcut: Group $AP + BP$ and $CR + DR$.

SOLVED EXAMPLE 7 [Intermediate]

Question: In $\triangle ABC$, a circle touches BC, CA, AB at D, E, F. If $AB = 12, BC = 8, CA = 10$ cm, find AD, BE, CF.

Given & Required: Sides $8, 10, 12$ | **Concept & Formula:** Equal Tangents

Detailed Solution: $s = (12+8+10)/2 = 15$. $AD = s - BC = 7$? Wait: AD is from A, wait D is on BC, so $AF = AE = s - a = 7$ cm? Let's check: $AF = x, BD = y, CE = z$: $x+y = 12, y+z = 8, z+x = 10 \implies x+y+z = 15 \implies x = 7$ ($AF = AE = 7$), $y = 5$ ($BD = BF = 5$), $z = 3$ ($CD = CE = 3$). Note the question asked for AD? Wait: if D is on BC, $BD = 5, CD = 3$; if AD, BE, CF meant tangent segments from A, B, C, let's label D on AB, E on BC, F on CA: $AD = 7, BE = 5, CF = 3$. **Final Answer:** 7 **cm,** 5 **cm,** 3 **cm**

Fastest Alternative / Shortcut: $s - a, s - b, s - c$.

SOLVED EXAMPLE 8 [Intermediate]

Question: If PA and PB are tangents from P to a circle with centre O, and $\angle APB = 60^\circ$, find $\angle OAB$.

Given & Required: $\angle APB = 60^\circ$ | **Concept & Formula:** Isosceles $\triangle OAB$

Detailed Solution: $\angle AOB = 180^\circ - 60^\circ = 120^\circ$. Since $OA = OB$, $\angle OAB = (180^\circ - 120^\circ)/2 = 30^\circ$. **Final Answer:** 30°

Fastest Alternative / Shortcut: $\angle OAB = \frac{1}{2}\angle APB = 30^\circ$.

SOLVED EXAMPLE 9 [Board Standard]

Question: State and prove Theorem 10.2: The lengths of tangents drawn from an external point to a circle are equal.

Given & Required: External point P, tangents PA, PB | **Concept & Formula:** RHS Congruence

Detailed Solution: Join OA, OB, OP. $\angle OAP = \angle OBP = 90^\circ$ (Thm 10.1), $OP = OP$ (common), $OA = OB$ (radii). By RHS, $\triangle OAP \cong \triangle OBP \implies PA = PB$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Standard 5-line RHS proof.

SOLVED EXAMPLE 10 [Board Standard]

Question: Prove that the parallelogram circumscribing a circle is a rhombus.

Given & Required: Parallelogram $ABCD$ | **Concept & Formula:** $AB + CD = AD + BC$

Detailed Solution: In parallelogram $ABCD$, $AB = CD$ and $AD = BC$. Since it circumscribes a circle, $AB + CD = AD + BC \implies 2AB = 2AD \implies AB = AD$. Thus all 4 sides are equal, so $ABCD$ is a rhombus. **Final Answer: Proved**

Fastest Alternative / Shortcut: $2AB = 2AD \implies AB = AD$.

SOLVED EXAMPLE 11 [Board Standard]

Question: Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre.

Given & Required: Quadrilateral $ABCD$ | **Concept & Formula:** 8 Central Angles

Detailed Solution: Join O to vertices and contact points. By SSS/RHS, $\angle 1 = \angle 2, \angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$. Sum $= 360^\circ \implies 2(\angle 1 + \angle 2 + \angle 5 + \angle 6) = 360^\circ$? Wait: $\angle AOB + \angle COD = (\angle 2 + \angle 3) + (\angle 6 + \angle 7) = 180^\circ$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Pair adjacent equal angles around O.

SOLVED EXAMPLE 12 [Board Standard]

Question: In a right triangle ABC ($\angle B = 90^\circ$), $AB = 8$ cm, $BC = 6$ cm. Find the radius of the inscribed circle.

Given & Required: $a = 6, b = 8$ | **Concept & Formula:** Inradius of Right $\triangle$

Detailed Solution: $AC = \sqrt{8^2 + 6^2} = 10$. By tangent segments, $r = \frac{8+6-10}{2} = 2$ cm. **Final Answer:** 2 **cm**

Fastest Alternative / Shortcut: $r = (a + b - c)/2$.

SOLVED EXAMPLE 13 [Board Standard]

Question: PQ is a chord of length 8 cm of a circle of radius 5 cm. Tangents at P and Q intersect at T. Find TP.

Given & Required: $PQ = 8, r = 5$ | **Concept & Formula:** Similar Right Triangles

Detailed Solution: $OT \perp PQ$ bisects PQ at R, so $PR = 4$, $OR = \sqrt{5^2 - 4^2} = 3$. Since $\triangle TRP \sim \triangle PRO$, $\frac{TP}{PO} = \frac{RP}{RO} \implies TP = \frac{5 \times 4}{3} = \frac{20}{3}$ cm. **Final Answer:** $20/3$ **cm**

Fastest Alternative / Shortcut: $TP = \frac{r \cdot PR}{OR} = 20/3$.

SOLVED EXAMPLE 14 [Challenge]

Question: A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that $BD = 8$ cm and $DC = 6$ cm. Find sides AB and AC.

Given & Required: $r = 4, BD = 8, CD = 6$ | **Concept & Formula:** Heron + Area $= rs$

Detailed Solution: Let $AF = AE = x$. $a = 14, b = x + 6, c = x + 8, s = x + 14$. Area $= 4(x + 14) = \sqrt{(x + 14)x(8)(6)} \implies 16(x + 14) = 48x \implies x = 7$. Sides: $AB = 15$ cm, $AC = 13$ cm. **Final Answer:** $AB = 15$ **cm,** $AC = 13$ **cm**

Fastest Alternative / Shortcut: Square and cancel $(x + 14)$.

SOLVED EXAMPLE 15 [Challenge]

Question: If XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with contact C intersects XY at A and $X'Y'$ at B, prove $\angle AOB = 90°$.

Given & Required: Parallel tangents + transversal tangent | **Concept & Formula:** Angle Bisectors of Co-interior Angles

Detailed Solution: $\triangle OAP \cong \triangle OAC \implies \angle POA = \angle AOC$. Similarly $\angle QOB = \angle BOC$. Since PQ is a straight diameter, $2(\angle AOC + \angle BOC) = 180° \implies \angle AOB = 90°$. **Final Answer: Proved**

Fastest Alternative / Shortcut: Angle bisectors of supplementary angles meet at $90°$.

SOLVED EXAMPLE 16 [Challenge]

Question: From an external point P, two tangents PA, PB are drawn; CD is a tangent at E on minor arc AB. If $PA = 14$ cm, find perimeter of $\triangle PCD$.

Given & Required: $PA = 14$ cm | **Concept & Formula:** Perimeter $= 2PA$

Detailed Solution: $CA = CE$ and $DB = DE$. Perimeter $= PC + CD + PD = (PC + CA) + (PD + DB) = PA + PB = 2(14) = 28$ cm. **Final Answer:** 28 **cm**

Fastest Alternative / Shortcut: Perimeter $= 2 \cdot PA$ always!

G. Competency MCQ Mastery Section

Q1. How many tangents can a circle have?

(A) 1 (B) 2 (C) Infinitely many (D) 0

Answer: (C) A circle has infinitely many points, hence infinitely many tangents. *Fast Method:* Infinitely many. *Distractor Note:* Confusing with tangents from ONE external point (2).

Q2. A tangent to a circle intersects it in:

(A) 1 point (B) 2 points (C) 0 points (D) 3 points

Answer: (A) By definition. *Fast Method:* 1 point. *Distractor Note:* 2 points is a secant.

Q3. Maximum number of parallel tangents a circle can have is:

(A) 1 (B) 2 (C) 4 (D) Infinite

Answer: (B) Only at opposite ends of a diameter. *Fast Method:* 2. *Distractor Note:* Saying infinite.

Q4. If PA, PB are tangents from P to circle (O, r) with $\angle APB = 70°$, then $\angle AOB$ is:

(A) $110°$ (B) $70°$ (C) $90°$ (D) $140°$

Answer: (A) $180° - 70° = 110°$. *Fast Method:* Supplementary. *Distractor Note:* None.

Q5. Distance between two parallel tangents of a circle of radius 4.5 cm is:

(A) 4.5 cm (B) 9 cm (C) 2.25 cm (D) 18 cm

Answer: (B) Equals diameter $= 2r = 9$ cm. *Fast Method:* Diameter $= 2r$. *Distractor Note:* Picking radius 4.5.

Q6. Length of tangent from a point 10 cm away from centre of a circle of radius 6 cm:

(A) 4 cm (B) 8 cm (C) 16 cm (D) $\sqrt{136}$ cm

Answer: (B) $\sqrt{10^2 - 6^2} = 8$ cm. *Fast Method:* 6-8-10 triple. *Distractor Note:* Adding $10^2 + 6^2$.

Q7. If $AB = 6, BC = 7, CD = 4$ for a circumscribed quadrilateral $ABCD$, then AD is:

(A) 3 (B) 4 (C) 5 (D) 7

Answer: (A) $6 + 4 = AD + 7 \implies AD = 3$. *Fast Method:* $AB + CD = AD + BC$. *Distractor Note:* None.

Q8. A parallelogram circumscribing a circle is always a:

(A) Rectangle (B) Square (C) Rhombus (D) Kite

Answer: (C) Adjacent sides become equal. *Fast Method:* Rhombus. *Distractor Note:* Picking Square (angles need not be $90°$).

Q9. In Ex 16 above, if $PA = 10$ cm, perimeter of $\triangle PCD$ is:

(A) 10 cm (B) 15 cm (C) 20 cm (D) 30 cm

Answer: (C) $2 \cdot PA = 20$ cm. *Fast Method:* $2 \times 10 = 20$. *Distractor Note:* None.

Q10. Assertion (A): Tangent at any point is perpendicular to the radius. Reason (R): The shortest distance from a point to a line is the perpendicular.

(A) Both A & R true, R explains A (B) Both A & R true, R does not explain A (C) A true, R false (D) A false, R true

Answer: (A) Theorem 10.1 is proved using shortest distance! *Fast Method:* Standard proof basis. *Distractor Note:* None.

Q11. Radius of incircle of a right triangle with sides $6, 8, 10$ cm is:

(A) 1 cm (B) 2 cm (C) 3 cm (D) 4 cm

Answer: (B) $(6 + 8 - 10)/2 = 2$ cm. *Fast Method:* $(a + b - c)/2$. *Distractor Note:* None.

Q12. If $\angle AOB = 100°$ for tangents PA, PB, then $\angle OAB$ is:

(A) $40°$ (B) $50°$ (C) $80°$ (D) $100°$

Answer: (A) $(180 - 100)/2 = 40°$. *Fast Method:* Isosceles $\triangle OAB$. *Distractor Note:* Picking $50°$.

H. Special Board & MCQ Examination Strategies

- Always mark $90°$ at every point of contact between a radius and a tangent.
- Identify OP (centre to external point) as the hypotenuse—never the tangent!
- Memorize the exact RHS proof of Theorem 10.2.
- Remember the Perimeter-of-$riangle PCD = 2 \cdot PA$ and Rhombus proofs.
- All topics in Circles are 100% retained in 2026–27.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Taking the tangent PT as the hypotenuse instead of OP. $\longrightarrow$ **Correction:** $\angle OTP = 90°$, so OP (opposite $90°$) is ALWAYS the hypotenuse.

[COMMON MISTAKE] **Error:** Saying a parallelogram circumscribing a circle is a square. $\longrightarrow$ **Correction:** It is a Rhombus; angles need not be $90°$.

[COMMON MISTAKE] **Error:** Forgetting to double $\sqrt{R^2 - r^2}$ in concentric-circle chord problems. $\longrightarrow$ **Correction:** Radius r bisects the chord, so full chord $= 2\sqrt{R^2 - r^2}$.

[COMMON MISTAKE] **Error:** Saying a circle has only 2 tangents. $\longrightarrow$ **Correction:** A circle has infinitely many tangents; at most 2 from a single external point.

[COMMON MISTAKE] **Error:** Using SAS instead of RHS in Theorem 10.2 proof. $\longrightarrow$ **Correction:** Use RHS ($90°$, common hypotenuse OP, radii $OA = OB$).

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $PT = \sqrt{OP^2 - r^2}$; $\angle AOB + \angle APB = 180°$; $AB + CD = AD + BC$; Chord $= 2\sqrt{R^2 - r^2}$; $r = (a + b - c)/2$.

[QUICK REVISION] Key Theorems & Results: Thm 10.1 ($r \perp$ tangent); Thm 10.2 ($PA = PB$); $\angle OAB = \frac{1}{2}\angle APB$.

[QUICK REVISION] Shortcut Summary: $r = (a + b - c)/2$; Perim$(\triangle PCD) = 2PA$; $s - a$ tangent segments.

[QUICK REVISION] High-Frequency Board Patterns: Thm 10.2 proof; Parallelogram = Rhombus; Concentric chord; $\triangle ABC$ with $r = 4, BD = 8, CD = 6$.

Last-Minute Revision Checklist:

- ☐ Can state and prove Theorem 10.1 and Theorem 10.2.
- ☐ Never confuse hypotenuse OP with tangent PT.
- ☐ Can prove $AB + CD = AD + BC$ and Rhombus rider.
- ☐ Know $\angle AOB + \angle APB = 180°$ and $\angle OAB = \frac{1}{2}\angle APB$.
- ☐ Can solve concentric chord and incircle triangle problems.

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Number of tangents from a point inside a circle: (A) 0 (B) 1 (C) 2 (D) Infinite
2. Angle between radius and tangent at point of contact: (A) 45° (B) 60° (C) 90° (D) 180°
3. If $PA = 7$ cm, length of other tangent PB from P: (A) 3.5 cm (B) 7 cm (C) 14 cm (D) 49 cm
4. Distance between parallel tangents of circle of diameter 10 cm: (A) 5 cm (B) 10 cm (C) 20 cm (D) 15 cm
5. Line intersecting circle at two points is a: (A) Tangent (B) Secant (C) Radius (D) Normal

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. If $OP = 13$ cm, $r = 5$ cm, length of tangent PT is: (A) 8 cm (B) 12 cm (C) 18 cm (D) 10 cm
2. Two concentric circles have radii 10 cm and 6 cm. Chord of outer touching inner: (A) 8 cm (B) 12 cm (C) 16 cm (D) 4 cm
3. If $\angle APB = 50°$, then $\angle OAB$ is: (A) 25° (B) 50° (C) 65° (D) 130°
4. In circumscribed $ABCD$, $AB = 5, BC = 6, CD = 7$. AD equals: (A) 6 (B) 5 (C) 7 (D) 8
5. If $\angle AOB = 130°$, then $\angle APB$ is: (A) 50° (B) 65° (C) 40° (D) 130°

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. In right $\triangle ABC$, sides are $5, 12, 13$. Inradius is: (A) 1 (B) 2 (C) 2.5 (D) 3
2. Perimeter of $\triangle PCD$ if tangent $PA = 15$ cm is: (A) 15 cm (B) 30 cm (C) 45 cm (D) 20 cm
3. Chord $PQ = 16$ cm in circle of radius 10 cm; tangents at P, Q meet at T. Length TP: (A) $40/3$ cm (B) $20/3$ cm (C) $16/3$ cm (D) 10 cm
4. If PA, PB are perpendicular tangents to circle of radius 5 cm, length of each tangent: (A) 2.5 cm (B) 5 cm (C) $5\sqrt{2}$ cm (D) 10 cm
5. Angle between two tangents if $OP = 2r$: (A) 30° (B) 45° (C) 60° (D) 90°

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (A):** Every line through an interior point is a secant. **Q2 (C):** Theorem 10.1. **Q3 (B):** $PA = PB$. **Q4 (B):** Equals diameter 10 cm. **Q5 (B):** Secant.
Board Std	**Q1 (B):** $(5, 12, 13)$ triple. **Q2 (C):** $2\sqrt{100 - 36} = 16$ cm. **Q3 (A):** $\frac{1}{2}\angle APB = 25^\circ$. **Q4 (A):** $5 + 7 = AD + 6 \implies AD = 6$. **Q5 (A):** $180 - 130 = 50^\circ$.
Challenge	**Q1 (B):** $(5 + 12 - 13)/2 = 2$. **Q2 (B):** $2 \times 15 = 30$ cm. **Q3 (A):** $OR = 6, PR = 8 \implies TP = 10 \times 8/6 = 40/3$. **Q4 (B):** $OAPB$ is a square of side 5 cm! **Q5 (C):** $\sin(\theta/2) = r/(2r) = 1/2 \implies \theta/2 = 30^\circ \implies \theta = 60^\circ$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Thm 10.1 & Basic Numericals; Day 2: Thm 10.2 & Angle Chasing; Day 3: Quadrilateral/Triangle Riders; Day 4: Test.
- **How to Introduce the Concept:** Spin a stone on a string and release it—it flies off along the TANGENT at 90° to the radius!
- **Difficult Areas & Remediation:** Proving $TP = 20/3$ via similar triangles and the Heron-area $x = 7$ problem.
- **Blackboard Teaching Examples:** Draw $OAPB$ and mark both 90° corners in bright chalk so students see $\angle AOB + \angle APB = 180^\circ$ visually.
- **Classroom Questions to Ask Students:** Ask: 'If $\angle APB = 90^\circ$, what special quadrilateral is $OAPB$?' (A Square!)
- **Common Student Doubts & Answers:** Why isn't Construction of Tangents in the book? Constructions chapter is deleted in 2026–27.
- **Homework & Revision Strategy:** Write Thm 10.2 proof + 6 NCERT Ex 10.2 riders.

[TEACHER'S TIP] Teach $\angle OAB = \frac{1}{2}\angle APB$—it is an instant MCQ goldmine.

UNIT VI: MENSURATION TYBA COACHING CENTRE

Chapter 11: Areas Related to Circles

Arc Length, Area of Sector, Area of Minor & Major Segments & Clock-Hand Problems

A. Chapter Introduction & Board Examination Relevance

In the 2026–27 rationalised NCERT syllabus (where former Chapter 11 Constructions was deleted), Areas Related to Circles is Chapter 11. It focuses strictly on **Length of an Arc**, **Area of a Sector**, and **Area of a Segment** of a circle (central angles $60°, 90°, 120°$), along with practical applications like clock hands, car wipers, and brooches.

- **What You Will Learn:** Computing arc length $\frac{\theta}{360°}(2\pi r)$, sector area $\frac{\theta}{360°}\pi r^2$, perimeter of a sector $(l + 2r)$, minor & major segment areas, and clock-hand sweep angles ($6°$/min).
- **Why This Chapter Matters:** Part of Unit VI Mensuration (10 marks CBSE / 11 marks RBSE). Expect 4–5 marks from sector/segment calculations and clock/wiper Case Studies.
- **Real-Life Applications:** Windshield wiper sweep area, circular pizza slices, horse grazing rope sectors, lighthouse warning sectors, and clock geometry.
- **Cross-Chapter Connections:** Connects to Circles (Chapter 10) and Trigonometry (area of triangle $\frac{1}{2}r^2 \sin\theta$).
- **Board & Foundation Relevance:** Areas of combinations of plane figures (old Ex 12.3) are deleted in 2026–27; focus 100% on sectors, arcs, and segments.

B. Complete Concept Explanation & Derivations

1. Circumference, Area, Arc Length & Sector

For a circle of radius r, $C = 2\pi r$ and $A = \pi r^2$. A **sector** is bounded by two radii and an arc of central angle θ (in degrees). By unitary proportion of $360°$: **Arc length** $l = \frac{\theta}{360°} \times 2\pi r$, and **Area of sector** $= \frac{\theta}{360°} \times \pi r^2 = \frac{1}{2}lr$.

[CONCEPT MASTER – 1. Circumference, Area, Arc Length & Sector] The **perimeter** of a sector is $l + 2r$ (arc length PLUS two radii—never forget the $+2r$!).

2. Area of Minor & Major Segments

A **segment** is the region between a chord and its corresponding arc. Area of Minor Segment = Area of Sector − Area of $\triangle OAB = \frac{\theta}{360°}\pi r^2 - \frac{1}{2}r^2 \sin\theta$. Area of Major Segment $= \pi r^2 -$ Area of Minor Segment.

[CONCEPT MASTER – 2. Area of Minor & Major Segments] For $\theta = 60°$, $\triangle OAB$ is equilateral ($\frac{\sqrt{3}}{4}r^2$); for $\theta = 90°$, right triangle ($\frac{1}{2}r^2$); for $\theta = 120°$, $\frac{1}{2}r^2 \sin 120° = \frac{\sqrt{3}}{4}r^2$ (same triangle area as $60°$!).

3. Clock-Hand Sweep Angles

A minute hand completes $360°$ in 60 minutes $\implies$ $\mathbf{6°}$ per minute. An hour hand completes $360°$ in 12 hours (720 min) $\implies$ $\mathbf{0.5°}$ per minute ($30°$ per hour).

[CONCEPT MASTER – 3. Clock-Hand Sweep Angles] In 5 minutes, the minute hand sweeps $5 \times 6° = 30°$.

C. Complete Formula Bank

[FORMULA BANK – 1. Length of an Arc] **Expression:** $l = \frac{\theta}{360°}(2\pi r)$
Symbols: r: radius, θ: central angle in degrees
Conditions & When to Use: $0° < \theta < 360°$
Solved Mini-Example: $r = 21, \theta = 60° \implies l = \frac{1}{6} \cdot 2 \cdot \frac{22}{7} \cdot 21 = 22$ cm
Common Mistake: Using πr^2 instead of $2\pi r$.

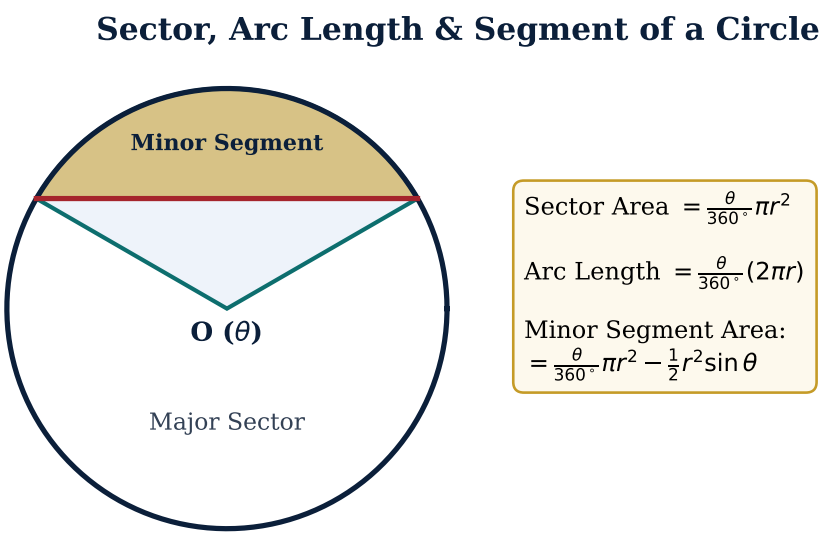


Figure 1: **Visual Model:** Minor sector, minor segment (shaded gold), and major sector of a circle of radius r and central angle θ.

[FORMULA BANK – 2. Area of Sector] **Expression:** $A_{\text{sec}} = \frac{\theta}{360^\circ}\pi r^2 = \frac{1}{2}lr$
Symbols: l: arc length
Conditions & When to Use: Major sector uses $360^\circ - \theta$
Solved Mini-Example: $r = 6, \theta = 60^\circ \implies \frac{1}{6} \cdot \frac{22}{7} \cdot 36 = \frac{132}{7}$ cm^2
Common Mistake: Computing minor sector when major sector is asked.

[FORMULA BANK – 3. Area of Minor Segment] **Expression:** $A_{\text{seg}} = \left(\frac{\theta\pi}{360^\circ} - \frac{\sin\theta}{2}\right) r^2$
Symbols: $\sin 60^\circ = \sin 120^\circ = \frac{\sqrt{3}}{2}, \sin 90^\circ = 1$
Conditions & When to Use: $\theta \leq 180^\circ$
Solved Mini-Example: Quadrant (90°), $r = 7$: $(\frac{\pi}{4} - \frac{1}{2})49 = 14$ cm^2
Common Mistake: Forgetting to subtract $\triangle OAB$ from sector area.

[FORMULA BANK – 4. Perimeter of a Sector] **Expression:** $P = l + 2r = \frac{\theta}{360^\circ}(2\pi r) + 2r$
Symbols: Arc + 2 radii
Conditions & When to Use: Always add $2r$
Solved Mini-Example: Semicircle perimeter $= \pi r + 2r$
Common Mistake: Giving only arc length l as perimeter.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE Standard/Basic, RBSE Board, and Foundation MCQ/Case-Study problem structures.

Pattern 1: Arc Length & Sector Area

- **Identification Cue:** Find arc length and area of sector of radius 21 cm and angle 60°. | **Concept & Formula:** $\frac{\theta}{360^\circ}$ fraction
- **Standard Step-by-Step:** Plug r, θ, π ($22/7$ or 3.14 as stated).
- **Fastest Valid Method:** Remember $\theta = 60^\circ$ is $1/6$ of circle, 90° is $1/4$, 120° is $1/3$. | **Alternative:** Use $A = \frac{1}{2}lr$ if l was already found!
- **Common Trap:** Using $\pi = 22/7$ when question says 'Use $\pi = 3.14$'.
- **Practice Drill:** In circle of $r = 21$ cm, arc subtends 60°. Find arc length, sector area, and segment area.

Pattern 2: Minor & Major Segment ($60°, 90°, 120°$)

- **Identification Cue:** Chord subtends $90°$ or $120°$ at centre; find minor and major segment. | **Concept & Formula:** Sector $-$ Triangle
- **Standard Step-by-Step:** Compute Sector, compute $\triangle OAB$, subtract; Major $= \pi r^2 -$ Minor.
- **Fastest Valid Method:** For BOTH $60°$ and $120°$, ar$(\triangle OAB) = \frac{\sqrt{3}}{4}r^2$! | **Alternative:** Drop perpendicular from O to chord for $120°$ ($30°$–$60°$–$90°$).
- **Common Trap:** Writing ar$(\triangle) = \frac{1}{2}r^2$ for $120°$.
- **Practice Drill:** Chord of circle ($r = 12$ cm) subtends $120°$ at centre. Find minor segment ($\pi = 3.14, \sqrt{3} = 1.73$).

Pattern 3: Clock Hand / Wiper / Umbrella / Horse Grazing

- **Identification Cue:** Minute hand of length 14 cm sweeps from 9:00 to 9:05; or two wipers of 25 cm sweeping $115°$. | **Concept & Formula:** Sector area $\times$ count
- **Standard Step-by-Step:** Convert minutes to $\theta = 6° \times$ min, then apply sector formula.
- **Fastest Valid Method:** For n equal ribs/slices (umbrella/brooch), $\theta = 360°/n$. | **Alternative:** None.
- **Common Trap:** Forgetting to multiply by 2 for TWO non-overlapping wipers.
- **Practice Drill:** Find area swept by 14 cm minute hand in 5 minutes.

Pattern 4: Sum / Difference of Circumferences or Areas

- **Identification Cue:** Find radius of circle whose area/circumference equals sum of two circles. | **Concept & Formula:** $R = R_1 + R_2$ (circumference) vs $R^2 = R_1^2 + R_2^2$ (area)
- **Standard Step-by-Step:** Cancel π on both sides immediately.
- **Fastest Valid Method:** For area sum, use Pythagorean triples! ($R_1 = 6, R_2 = 8 \implies R = 10$). | **Alternative:** None.
- **Common Trap:** Adding radii directly for area sum.
- **Practice Drill:** Radii 8 cm and 6 cm; find radius of circle with area equal to sum of their areas.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Fraction-of-Circle Memorizer] **Core Idea & Why It Works:** Instead of writing $\theta/360°$ every time: $30° = \frac{1}{12}$, $45° = \frac{1}{8}$, $60° = \frac{1}{6}$, $90° = \frac{1}{4}$, $120° = \frac{1}{3}$, $180° = \frac{1}{2}$.
Valid Domain: All standard-angle sector/arc questions | **Where It Fails:** Non-standard angles like $115°$
Easy Example: $r = 14, \theta = 90° \implies \frac{1}{4} \cdot \frac{22}{7} \cdot 196 = 154$ cm^2.
Board-Level Example: Cuts arithmetic errors dramatically.
Advanced Example: Pairs with $r = 7k$ ($\pi r^2 = 154k^2$)!

[SMART SHORTCUT – 2. $r = 7k$ Area Multiplier ($154k^2$)] **Core Idea & Why It Works:** When $\pi = 22/7$ and $r = 7$, Area $= 154$ and Circumference $= 44$. For $r = 7k$, Area $= 154k^2$ and $C = 44k$!
Valid Domain: All $r = 7, 14, 21, 3.5$ problems | **Where It Fails:** When $\pi = 3.14$ is forced
Easy Example: $r = 14$ ($k = 2$) $\implies$ Area $= 154 \times 4 = 616$, $C = 88$.
Board-Level Example: $r = 21$ ($k = 3$), $\theta = 60° \implies \frac{1}{6}(154 \times 9) = 231$ cm^2!
Advanced Example: For $r = 3.5$ ($k = 1/2$), Area $= 154/4 = 38.5$.

[SMART SHORTCUT – 3. Equal-Area Triangle for $60°$ and $120°$] **Core Idea & Why It Works:** Since $\sin 60° = \sin 120° = \frac{\sqrt{3}}{2}$, $\triangle OAB$ has exact area $\frac{\sqrt{3}}{4}r^2$ in BOTH cases!
Valid Domain: $60°$ and $120°$ segment problems | **Where It Fails:** Other angles
Easy Example: $r = 10, \theta = 120° \implies \triangle = 25\sqrt{3}$.
Board-Level Example: Eliminates dropping altitudes and $30°$–$60°$–$90°$ trig in MCQs.
Advanced Example: Always check $\sqrt{3} = 1.73$ vs 1.732.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: Find the area of a sector of a circle with radius 6 cm if the angle of the sector is 60°.

Given & Required: $r = 6, \theta = 60^\circ$ | **Concept & Formula:** Sector Area

Detailed Solution: $A = \frac{60}{360} \times \frac{22}{7} \times 36 = \frac{132}{7}$ cm^2. **Final Answer:** $132/7$ **cm^2**

Fastest Alternative / Shortcut: $\frac{1}{6}\pi(36) = 6\pi = 132/7$.

SOLVED EXAMPLE 2 [Beginner]

Question: Find the area of a quadrant of a circle whose circumference is 22 cm.

Given & Required: $C = 22$ cm | **Concept & Formula:** Quadrant (90°)

Detailed Solution: $2\pi r = 22 \implies r = 7/2$ cm. Quadrant $= \frac{1}{4} \cdot \frac{22}{7} \cdot \frac{49}{4} = \frac{77}{8}$ cm^2. **Final Answer:** $77/8$ **cm^2**

Fastest Alternative / Shortcut: $k = 1/2 \implies \frac{1}{4}(154/4) = 77/8$.

SOLVED EXAMPLE 3 [Beginner]

Question: The length of the minute hand of a clock is 14 cm. Find the area swept by it in 5 minutes.

Given & Required: $r = 14, t = 5$ min | **Concept & Formula:** Clock Sweep (6°/min)

Detailed Solution: $\theta = 5 \times 6^\circ = 30^\circ$. Area $= \frac{30}{360} \times \frac{22}{7} \times 196 = \frac{154}{3}$ cm^2. **Final Answer:** $154/3$ **cm^2**

Fastest Alternative / Shortcut: $\frac{1}{12}(616) = 154/3$.

SOLVED EXAMPLE 4 [Beginner]

Question: If the perimeter and the area of a circle are numerically equal, find the radius.

Given & Required: $2\pi r = \pi r^2$ | **Concept & Formula:** Circle Formulas

Detailed Solution: $2r = r^2 \implies r = 2$ units ($r > 0$). **Final Answer:** 2 **units**

Fastest Alternative / Shortcut: Cancel πr.

SOLVED EXAMPLE 5 [Intermediate]

Question: A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the minor segment and major sector ($\pi = 3.14$).

Given & Required: $r = 10, \theta = 90^\circ$ | **Concept & Formula:** Segment and Major Sector

Detailed Solution: Minor sector $= \frac{1}{4}(3.14)(100) = 78.5$ cm^2. $\triangle = \frac{1}{2}(10)(10) = 50$. Minor segment $= 78.5 - 50 = 28.5$ cm^2. Major sector $= 314 - 78.5 = 235.5$ cm^2. **Final Answer:** 28.5 **cm^2**, 235.5 **cm^2**

Fastest Alternative / Shortcut: Major sector $= \frac{3}{4}(314) = 235.5$.

SOLVED EXAMPLE 6 [Intermediate]

Question: In a circle of radius 21 cm, an arc subtends 60° at the centre. Find (i) arc length, (ii) sector area, (iii) segment area.

Given & Required: $r = 21, \theta = 60^\circ$ | **Concept & Formula:** All Three Quantities

Detailed Solution: (i) $l = \frac{1}{6}(2 \cdot \frac{22}{7} \cdot 21) = 22$ cm. (ii) Sector $= \frac{1}{2}(22)(21) = 231$ cm^2. (iii) Segment $= 231 - \frac{\sqrt{3}}{4}(441) = (231 - \frac{441\sqrt{3}}{4})$ cm^2. **Final Answer:** 22 **cm**, 231 **cm^2**, $(231 - \frac{441\sqrt{3}}{4})$ **cm^2**

Fastest Alternative / Shortcut: Use $\frac{1}{2}lr$ for sector!

SOLVED EXAMPLE 7 [Intermediate]

Question: A horse is tied to a peg at one corner of a square grass field of side 15 m by a 5 m long rope. Find the grazing area and the increase in area if the rope is 10 m.

Given & Required: Corner $= 90^\circ$, $r_1 = 5, r_2 = 10$ | **Concept & Formula:** Quadrant Difference

Detailed Solution: $A_1 = \frac{1}{4}(3.14)(25) = 19.625$ m^2. Increase $= \frac{1}{4}(3.14)(10^2 - 5^2) = \frac{1}{4}(3.14)(75) = 58.875$ m^2. **Final Answer:** 19.625 **m^2**, $+58.875$ **m^2**

Fastest Alternative / Shortcut: Factor $\frac{\pi}{4}(R^2 - r^2)$.

SOLVED EXAMPLE 8 [Intermediate]

Question: An umbrella has 8 ribs equally spaced. Assuming it is a flat circle of radius 45 cm, find the area between two consecutive ribs.

Given & Required: $n = 8, r = 45$ | **Concept & Formula:** $1/8$ of Circle

Detailed Solution: $\theta = 360^\circ/8 = 45^\circ$. Area $= \frac{1}{8} \times \frac{22}{7} \times 45^2 = \frac{22275}{28}$ cm^2. **Final Answer:** $22275/28$ **cm^2**

Fastest Alternative / Shortcut: $\frac{1}{8}\pi r^2$.

SOLVED EXAMPLE 9 [Board Standard]

Question: A chord of a circle of radius 12 cm subtends 120° at the centre. Find the area of the corresponding segment ($\pi = 3.14, \sqrt{3} = 1.73$).

Given & Required: $r = 12, \theta = 120^\circ$ | **Concept & Formula:** 120° Segment

Detailed Solution: Sector $= \frac{1}{3}(3.14)(144) = 150.72$ cm^2. $\triangle = \frac{\sqrt{3}}{4}(144) = 36(1.73) = 62.28$ cm^2. Segment $= 150.72 - 62.28 = 88.44$ cm^2. **Final Answer:** 88.44 **cm^2**

Fastest Alternative / Shortcut: $\triangle OAB = \frac{\sqrt{3}}{4}r^2$.

SOLVED EXAMPLE 10 [Board Standard]

Question: A brooch is made with silver wire in the form of a circle of diameter 35 mm. The wire is also used to make 5 diameters dividing it into 10 equal sectors. Find (i) total silver wire, (ii) area of each sector.

Given & Required: $d = 35, r = 35/2$, 5 diameters | **Concept & Formula:** Circumference + Diameters

Detailed Solution: (i) Wire $= \pi d + 5d = \frac{22}{7}(35) + 175 = 110 + 175 = 285$ mm. (ii) Sector $= \frac{1}{10} \cdot \frac{22}{7} \cdot \frac{35}{2} \cdot \frac{35}{2} = \frac{385}{4}$ mm^2. **Final Answer:** 285 **mm,** $385/4$ **mm^2**

Fastest Alternative / Shortcut: $\pi d + 5d$.

SOLVED EXAMPLE 11 [Board Standard]

Question: A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through 115°. Find the total area cleaned at each sweep.

Given & Required: 2 wipers, $r = 25, \theta = 115^\circ$ | **Concept & Formula:** Double Sector

Detailed Solution: Area $= 2 \times \frac{115}{360} \times \frac{22}{7} \times 625 = \frac{158125}{126}$ cm^2. **Final Answer:** $158125/126$ **cm^2**

Fastest Alternative / Shortcut: Multiply single sector by 2.

SOLVED EXAMPLE 12 [Board Standard]

Question: To warn ships for underwater rocks, a lighthouse spreads a red light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea warned ($\pi = 3.14$).

Given & Required: $r = 16.5, \theta = 80^\circ$ | **Concept & Formula:** Sector Area

Detailed Solution: $A = \frac{80}{360}(3.14)(16.5)^2 = \frac{2}{9}(3.14)(272.25) = 189.97$ km^2. **Final Answer:** 189.97 **km^2**

Fastest Alternative / Shortcut: Cancel 9 into $272.25 = 30.25$.

SOLVED EXAMPLE 13 [Board Standard]

Question: Find the area of the minor segment of a circle of radius 15 cm if the chord subtends 60° at the centre ($\pi = 3.14, \sqrt{3} = 1.73$).

Given & Required: $r = 15, \theta = 60^\circ$ | **Concept & Formula:** 60° Segment

Detailed Solution: Sector $= \frac{1}{6}(3.14)(225) = 117.75$. $\triangle = \frac{1.73}{4}(225) = 97.3125$. Segment $= 20.4375$ cm^2. **Final Answer:** 20.4375 **cm^2**

Fastest Alternative / Shortcut: Equilateral triangle $\frac{\sqrt{3}}{4}r^2$.

SOLVED EXAMPLE 14 [Challenge]

Question: The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

Given & Required: $r = 5.2, P = 16.4$ | **Concept & Formula:** $P = l + 2r$ and $A = \frac{1}{2}lr$

Detailed Solution: $l = 16.4 - 2(5.2) = 6.0$ cm. Area $= \frac{1}{2}lr = \frac{1}{2}(6.0)(5.2) = 15.6$ cm^2. **Final Answer:** 15.6 **cm^2**

Fastest Alternative / Shortcut: Use $A = \frac{1}{2}lr$—no θ needed!

SOLVED EXAMPLE 15 [Challenge]

Question: A circular track has inner circumference 352 m and outer circumference 396 m. Find the width of the track.

Given & Required: $C_1 = 352, C_2 = 396$ | **Concept & Formula:** $R - r = (C_2 - C_1)/(2\pi)$

Detailed Solution: $2\pi(R-r) = 396 - 352 = 44 \implies R - r = \frac{44}{2(22/7)} = 7$ m. **Final Answer: 7 m**

Fastest Alternative / Shortcut: Subtract circumferences first!

SOLVED EXAMPLE 16 [Challenge]

Question: Find the area of the major segment of a circle of radius 7 cm corresponding to a chord subtending 90° at the centre.

Given & Required: $r = 7, \theta = 90^\circ$ | **Concept & Formula:** Major Segment

Detailed Solution: Minor segment $= \frac{1}{4}(154) - \frac{1}{2}(49) = 38.5 - 24.5 = 14$ cm^2. Major segment $= 154 - 14 = 140$ cm^2. **Final Answer:** 140 **cm^2**

Fastest Alternative / Shortcut: $\frac{3}{4}\pi r^2 + \frac{1}{2}r^2 = 115.5 + 24.5 = 140$.

G. Competency MCQ Mastery Section

Q1. Area of a sector of angle p° of a circle with radius R is:

(A) $\frac{p}{180} \cdot 2\pi R$ (B) $\frac{p}{180} \cdot \pi R^2$ (C) $\frac{p}{360} \cdot 2\pi R$ (D) $\frac{p}{720} \cdot 2\pi R^2$

Answer: (D) $\frac{p}{720}(2\pi R^2) = \frac{p}{360}\pi R^2$. *Fast Method:* Simplify (D). *Distractor Note:* NCERT classic trick option!

Q2. If the sum of areas of two circles with radii R_1, R_2 equals area of circle of radius R, then:

(A) $R_1 + R_2 = R$ (B) $R_1^2 + R_2^2 = R^2$ (C) $R_1 + R_2 < R$ (D) $R_1^2 + R_2^2 < R^2$

Answer: (B) $\pi R_1^2 + \pi R_2^2 = \pi R^2$. *Fast Method:* Cancel π. *Distractor Note:* Picking (A) (true for circumference).

Q3. Angle described by the minute hand of a clock in 1 minute is:

(A) 1° (B) 6° (C) 30° (D) 0.5°

Answer: (B) $360^\circ/60 = 6^\circ$. *Fast Method:* 6°/min. *Distractor Note:* 0.5° is for hour hand.

Q4. Perimeter of a semicircular protractor of radius r is:

(A) πr (B) $2\pi r$ (C) $\pi r + 2r$ (D) $\pi r + r$

Answer: (C) Arc πr plus diameter $2r$. *Fast Method:* $\pi r + 2r$. *Distractor Note:* Omitting diameter $2r$.

Q5. Area of the largest triangle inscribed in a semicircle of radius r is:

(A) r^2 (B) $2r^2$ (C) $\frac{1}{2}r^2$ (D) πr^2

Answer: (A) Base $= 2r$, max height $= r \implies \frac{1}{2}(2r)(r) = r^2$. *Fast Method:* r^2. *Distractor Note:* Using $\frac{1}{2}r^2$.

Q6. If circumference of a circle increases from 4π to 8π, its area becomes:

(A) Double (B) Half (C) 4 times (D) 3 times

Answer: (C) Radius doubles $\implies$ area becomes $2^2 = 4$ times. *Fast Method:* $k^2 = 4$. *Distractor Note:* Picking Double.

Q7. Arc length of a sector of radius 14 cm and angle 45° is:

(A) 11 cm (B) 22 cm (C) 44 cm (D) 7 cm

Answer: (A) $\frac{1}{8}(88) = 11$ cm. *Fast Method:* $88/8 = 11$. *Distractor Note:* None.

Q8. Area of minor segment of radius 7 cm and central angle 90° is:

(A) 38.5 cm^2 (B) 24.5 cm^2 (C) 14 cm^2 (D) 49 cm^2

Answer: (C) $38.5 - 24.5 = 14$ cm^2. *Fast Method:* $\frac{2}{7}r^2 = 14$. *Distractor Note:* Answering sector area 38.5.

Q9. If perimeter of a circle equals perimeter of a square, ratio of their areas is:

(A) 22 : 7 (B) 14 : 11 (C) 7 : 22 (D) 11 : 14

Answer: (B) $2\pi r = 4a \implies a = \pi r/2$. $\frac{\pi r^2}{\pi^2 r^2/4} = \frac{4}{\pi} = \frac{28}{22} = \frac{14}{11}$. *Fast Method:* Circle always has larger area $(14:11)$. *Distractor Note:* Inverting to $11:14$.

Q10. Assertion (A): Area of sector of angle $90°$ is $\frac{1}{4}\pi r^2$. Reason (R): $90°$ is one-fourth of $360°$.

(A) Both A & R true, R explains A (B) Both A & R true, R does not explain A (C) A true, R false (D) A false, R true

Answer: (A) R explains A. *Fast Method:* Direct proportion. *Distractor Note:* None.

Q11. If arc length of a sector is 10 cm and radius is 6 cm, sector area is:

(A) 60 cm^2 (B) 30 cm^2 (C) 15 cm^2 (D) 20 cm^2

Answer: (B) $\frac{1}{2}lr = \frac{1}{2}(10)(6) = 30$ cm^2. *Fast Method:* $\frac{1}{2}lr$. *Distractor Note:* Multiplying lr without $1/2$.

Q12. Number of revolutions made by a wheel of diameter 1.4 m in 440 m is:

(A) 50 (B) 100 (C) 200 (D) 44

Answer: (B) $C = \frac{22}{7}(1.4) = 4.4$ m. Revolutions $= 440/4.4 = 100$. *Fast Method:* $440/4.4 = 100$. *Distractor Note:* Using $r = 1.4$ instead of $d = 1.4$.

H. Special Board & MCQ Examination Strategies

- Always check whether $\pi = 22/7$ or 3.14 is specified in the question.
- Use $A = \frac{1}{2}lr$ whenever arc length l and radius r are both known.
- Remember the $+2r$ in sector/semicircle perimeter questions.
- Remember $\frac{\sqrt{3}}{4}r^2$ works for BOTH $\theta = 60°$ and $\theta = 120°$ triangles in segment problems.
- Skip shaded combinations of plane figures—deleted in 2026–27.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Forgetting $+2r$ when computing perimeter of a sector or semicircle. $\longrightarrow$ **Correction:** Perimeter $= l + 2r$.

[COMMON MISTAKE] **Error:** Confusing minute-hand speed ($6°$/min) with hour-hand speed ($0.5°$/min). $\longrightarrow$ **Correction:** Minute hand: $360°/60 = 6°$/min.

[COMMON MISTAKE] **Error:** Stopping at sector area when segment area is asked. $\longrightarrow$ **Correction:** Segment $=$ Sector $-$ Triangle.

[COMMON MISTAKE] **Error:** Using $R = R_1 + R_2$ for sum of areas. $\longrightarrow$ **Correction:** Area sum gives $R^2 = R_1^2 + R_2^2$.

[COMMON MISTAKE] **Error:** Dividing by 2π incorrectly when finding radius from C. $\longrightarrow$ **Correction:** If $C = 22$, $r = 7/2 = 3.5$ cm.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $l = \frac{\theta}{360}(2\pi r)$; $A_{\text{sec}} = \frac{\theta}{360}\pi r^2 = \frac{1}{2}lr$; $P_{\text{sec}} = l + 2r$; $A_{\text{seg}} = A_{\text{sec}} - \frac{1}{2}r^2 \sin\theta$.

[QUICK REVISION] Key Theorems & Results: Minute hand $= 6°$/min; Hour hand $= 0.5°$/min; Major $=$ Circle $-$ Minor.

[QUICK REVISION] Shortcut Summary: $r = 7k \implies A = 154k^2, C = 44k$; $60°\&120°$ triangle $= \frac{\sqrt{3}}{4}r^2$.

[QUICK REVISION] High-Frequency Board Patterns: Clock sweep; Horse grazing; Wiper/Brooch; $120°$ segment.

Last-Minute Revision Checklist:

- ☐ Can compute arc length, sector area, and sector perimeter.
- ☐ Can find minor and major segments for $60°, 90°, 120°$.
- ☐ Know clock-hand angles ($6°$/min).
- ☐ Use $A = \frac{1}{2}lr$ when l and r are given.
- ☐ Know combinations of plane figures are deleted in 2026–27.

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Area of circle of radius 7 cm: (A) 44 cm^2 (B) 154 cm^2 (C) 308 cm^2 (D) 77 cm^2
2. Arc length of semicircle of radius r: (A) $2\pi r$ (B) πr (C) $\pi r + 2r$ (D) $\frac{1}{2}\pi r^2$
3. Angle swept by minute hand in 10 minutes: (A) $30°$ (B) $60°$ (C) $90°$ (D) $10°$
4. Area of quadrant of radius r: (A) $\pi r^2/2$ (B) $\pi r^2/4$ (C) $\pi r/2$ (D) $2\pi r$
5. If $d = 14$ cm, circumference is: (A) 22 cm (B) 44 cm (C) 88 cm (D) 154 cm

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. Perimeter of a quadrant of radius 7 cm is: (A) 11 cm (B) 25 cm (C) 38.5 cm (D) 18 cm
2. Radius of circle whose area equals sum of areas of circles of radii 5 cm and 12 cm: (A) 17 cm (B) 13 cm (C) 7 cm (D) 60 cm
3. Area of sector of angle $120°$ and radius 21 cm: (A) 462 cm^2 (B) 231 cm^2 (C) 154 cm^2 (D) 693 cm^2
4. If $A_{\text{sec}} = 54\pi$ and $r = 18$, central angle θ is: (A) $30°$ (B) $45°$ (C) $60°$ (D) $90°$
5. Area of minor segment for $r = 10, \theta = 90°$ ($\pi = 3.14$): (A) 28.5 (B) 78.5 (C) 50 (D) 14.25

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. A wire bent as a square of area 121 cm^2 is rebent into a circle. Area of circle: (A) 154 cm^2 (B) 121 cm^2 (C) 77 cm^2 (D) 308 cm^2
2. Radius of circle if difference between circumference and radius is 37 cm: (A) 7 cm (B) 14 cm (C) 3.5 cm (D) 10 cm
3. If arc length is $\frac{1}{6}$ of circumference, central angle is: (A) $30°$ (B) $45°$ (C) $60°$ (D) $90°$
4. Ratio of area of circle to inscribed square is: (A) $\pi : 2$ (B) $2 : \pi$ (C) $\pi : 4$ (D) $4 : \pi$
5. Area swept by 7 cm hour hand in 6 hours: (A) 77 cm^2 (B) 154 cm^2 (C) 38.5 cm^2 (D) 19.25 cm^2

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** 154. **Q2 (B):** πr. **Q3 (B):** $10 \times 6^\circ = 60^\circ$. **Q4 (B):** $90/360 = 1/4$. **Q5 (B):** $22/7 \times 14 = 44$.
Board Std	**Q1 (B):** $\frac{1}{4}(44) + 14 = 11 + 14 = 25$ cm. **Q2 (B):** $(5, 12, 13)$ triple. **Q3 (A):** $\frac{1}{3}(154 \times 9) = 462$ cm^2. **Q4 (C):** $\frac{\theta}{360}(324\pi) = 54\pi \implies \theta = 60^\circ$. **Q5 (A):** $78.5 - 50 = 28.5$.
Challenge	**Q1 (A):** Side $= 11$, $P = 44 = 2\pi r \implies r = 7 \implies A = 154$. **Q2 (A):** $(2\pi - 1)r = \frac{37}{7}r = 37 \implies r = 7$ cm. **Q3 (C):** $360^\circ/6 = 60^\circ$. **Q4 (A):** $\pi r^2/(2r^2) = \pi : 2$. **Q5 (A):** 6 hours $= 180^\circ = 1/2$ circle $= 154/2 = 77$ cm^2.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Arc & Sector; Day 2: Minor & Major Segment; Day 3: Clock, Wiper, Horse & Brooch; Day 4: Test.
- **How to Introduce the Concept:** Ask: 'When you eat one slice of a 6-slice pizza, how much crust (arc) and cheese (sector) did you eat?'
- **Difficult Areas & Remediation:** Finding $\triangle OAB$ area for $\theta = 120^\circ$ and remembering $+2r$ in sector perimeter.
- **Blackboard Teaching Examples:** Write $r = 7 \implies C = 44, A = 154$ as a mental speed anchor.
- **Classroom Questions to Ask Students:** Ask: 'Why is ar$(\triangle OAB)$ the same for both 60° and 120°?' (Because $\sin 60^\circ = \sin 120^\circ$!)
- **Common Student Doubts & Answers:** Are shaded-region square/circle puzzle problems in the exam? No, combinations of plane figures are deleted in 2026–27.
- **Homework & Revision Strategy:** 5 sector/arc problems + 4 segment/application problems.

[TEACHER'S TIP] Remind students to factor out π and common radii before multiplying decimals.

UNIT VI: MENSURATION TYBA COACHING CENTRE

Chapter 12: Surface Areas and Volumes

Surface Areas and Volumes of Combinations of Two Solids (Cube, Cuboid, Cylinder, Cone, Sphere, Hemisphere)

A. Chapter Introduction & Board Examination Relevance

In the 2026–27 rationalised Class 10 syllabus, Surface Areas and Volumes focuses exclusively on **combinations of two or more basic solids**—such as a tent (cylinder + cone), medicine capsule (cylinder + 2 hemispheres), gulab jamun, pen stand with conical depressions, and toy (cone + hemisphere). Frustum of a cone and conversion of solids are deleted.

- **What You Will Learn:** Recalling TSA, CSA, and Volume of all 6 basic solids, computing total surface area of combined solids by summing ONLY exposed faces (never blindly adding TSAs!), computing combined volumes by adding (or subtracting for cavities), and factoring out πr^2 or πr to simplify arithmetic.
- **Why This Chapter Matters:** Part of Unit VI Mensuration (10 marks CBSE / 11 marks RBSE). Almost always contributes a 5-mark Long Answer or 4-mark Case Study.
- **Real-Life Applications:** Packaging design, pharmaceutical capsules, storage silos, canvas tents, and cylindrical boilers.
- **Cross-Chapter Connections:** Builds on Class 9 Mensuration and connects to Areas Related to Circles (Chapter 11).
- **Board & Foundation Relevance:** Frustum of a cone and conversion/melting of solids from one shape to another are deleted in 2026–27.

B. Complete Concept Explanation & Derivations

1. Basic Solids Reference (CSA, TSA, Volume)

Cuboid (l, b, h): $TSA = 2(lb + bh + hl)$, $V = lbh$. **Cube (a):** $TSA = 6a^2$, $V = a^3$. **Cylinder (r, h):** $CSA = 2\pi rh$, $TSA = 2\pi r(h + r)$, $V = \pi r^2 h$. **Cone $(r, h, l$ where $l = \sqrt{r^2 + h^2})$:** $CSA = \pi rl$, $TSA = \pi r(l + r)$, $V = \frac{1}{3}\pi r^2 h$. **Sphere (r):** $SA = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$. **Hemisphere (r):** $CSA = 2\pi r^2$, $TSA = 3\pi r^2$, $V = \frac{2}{3}\pi r^3$.

[CONCEPT MASTER – 1. Basic Solids Reference (CSA, TSA, Volume)] Remember: When a solid is hollowed out (e.g., conical cavity inside a cylinder), the **volume** decreases, but the **surface area INCREASES** because the inner conical wall becomes newly exposed!

2. Surface Area of Combined Solids (Exposed Faces Rule)

When two solids are joined face-to-face (e.g., a cone on a hemisphere, or a hemisphere on a cube), the hidden circular junction is NO LONGER visible from outside. Therefore, **NEVER add TSA + TSA!** Instead, add only the exposed surfaces: e.g., Toy (Cone + Hemisphere) $= \text{CSA}_{\text{cone}} + \text{CSA}_{\text{hemi}} = \pi rl + 2\pi r^2 = \pi r(l + 2r)$.

[CONCEPT MASTER – 2. Surface Area of Combined Solids (Exposed Faces Rule)] If a hemisphere of diameter d is surmounted on a cube of side d, the total exposed surface area is $6d^2 - \pi(d/2)^2 + 2\pi(d/2)^2 = 6d^2 + \pi(d/2)^2$ (cube TSA + circle base!).

3. Volume of Combined Solids

Unlike surface area, **volume is strictly additive (or subtractive for cavities)** with no hidden-interface adjustment. For a solid made of two parts, $V_{\text{total}} = V_1 + V_2$. For a solid with a drilled/scooped cavity, $V_{\text{remaining}} = V_{\text{outer}} - V_{\text{cavity}}$.

[CONCEPT MASTER – 3. Volume of Combined Solids] Always factor out common terms like πr^2 BEFORE substituting $\pi = 22/7$ or 3.14!

C. Complete Formula Bank

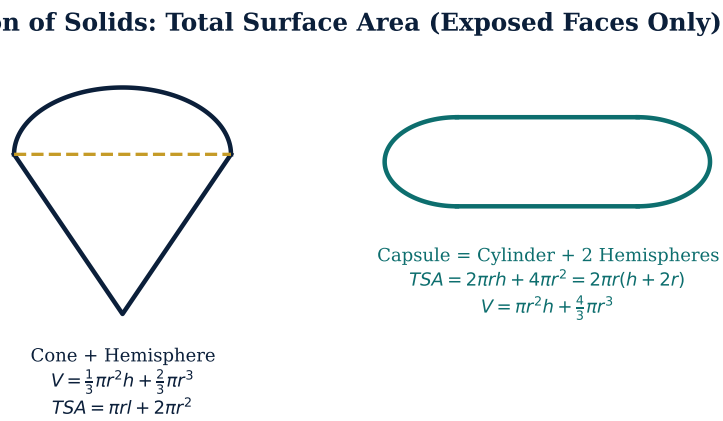


Figure 1: **Visual Model:** Combinations of solids: Ice-cream cone (Cone + Hemisphere) and Capsule (Cylinder + 2 Hemispheres).

[FORMULA BANK – 1. Cone Slant Height & CSA] **Expression:** $l = \sqrt{r^2 + h^2}, \quad CSA = \pi r l$
Symbols: r: radius, h: vertical height, l: slant height
Conditions & When to Use: Right circular cone
Solved Mini-Example: $r = 3, h = 4 \implies l = 5 \implies CSA = 15\pi$
Common Mistake: Using h instead of l in $\pi r l$.

[FORMULA BANK – 2. Toy (Cone + Hemisphere)] **Expression:** $TSA = \pi r(l + 2r), \quad V = \frac{1}{3}\pi r^2(h + 2r)$
Symbols: r: common radius, h: cone height
Conditions & When to Use: Mounted on same circular base
Solved Mini-Example: $r = 3.5, h = 12 \implies l = 12.5, TSA = \pi(3.5)(19.5)$
Common Mistake: Using $3\pi r^2$ instead of $2\pi r^2$ for the hemisphere.

[FORMULA BANK – 3. Capsule (Cylinder + 2 Hemispheres)] **Expression:** $TSA = 2\pi r(h + 2r), \quad V = \pi r^2\left(h + \frac{4}{3}r\right)$
Symbols: h: cylindrical length $= L_{\text{total}} - 2r$
Conditions & When to Use: Hemispherical ends of radius r
Solved Mini-Example: Total length 14 mm, $d = 5$ mm $\implies h = 14 - 5 = 9$ mm
Common Mistake: Taking $h = 14$ mm instead of subtracting $2r$.

[FORMULA BANK – 4. Hemisphere on Cube of Side a] **Expression:** $TSA = 6a^2 + \pi r^2$
Symbols: $r \leq a/2$ (max $r = a/2$)
Conditions & When to Use: Hemisphere resting on top face
Solved Mini-Example: $a = 7, r = 3.5 \implies 294 + 38.5 = 332.5$ cm^2
Common Mistake: Forgetting to subtract the circular base πr^2 covered by the hemisphere.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE Standard/Basic, RBSE Board, and Foundation MCQ/Case-Study problem structures.

Pattern 1: Tent / Silo / Capsule Surface Area & Canvas Cost

- **Identification Cue:** Cylinder surmounted by cone (tent) or 2 hemispheres (capsule). | **Concept & Formula:** Sum of CSAs only
- **Standard Step-by-Step:** Find $l = \sqrt{r^2 + h^2}$ if cone, compute $\text{CSA}_1 + \text{CSA}_2$, multiply by rate.
- **Fastest Valid Method:** Factor πr out: $\pi r(2H_{\text{cyl}} + l_{\text{cone}})$. | **Alternative:** None.

- **Common Trap:** Including the circular floor of a canvas tent when the problem says base is not covered.
- **Practice Drill:** Tent: cylindrical part $h = 2.1$ m, $d = 4$ m, cone slant height 2.8 m. Find canvas area & cost at Rs. $500/\text{m}^2$.

Pattern 2: Hollowed-Out Solid (Cylinder − Cone / Hemisphere)

- **Identification Cue:** From a solid cylinder, a conical cavity of same height & diameter is hollowed out. | **Concept & Formula:** Add inner CSA + outer CSA + remaining base!
- **Standard Step-by-Step:** $TSA = CSA_{\text{cyl}} + CSA_{\text{cone}} + \text{Area}_{\text{base}} = 2\pi rh + \pi rl + \pi r^2$.
- **Fastest Valid Method:** Factor $\pi r(2h + l + r)$ directly! | **Alternative:** None.
- **Common Trap:** Subtracting CSA of cavity instead of ADDING it.
- **Practice Drill:** Cylinder $h = 2.4$ cm, $d = 1.4$ cm; conical cavity hollowed out. Find TSA.

Pattern 3: Volume of Combined Solids (Gulab Jamun, Pen Stand, Vessel)

- **Identification Cue:** Gulab jamun (cylinder + 2 hemispheres) with 30% syrup, or cuboid pen stand with 4 conical depressions. | **Concept & Formula:** Additive / Subtractive Volumes
- **Standard Step-by-Step:** Compute single unit volume cleanly, multiply by count, then apply percentage.
- **Fastest Valid Method:** Keep π symbolic until the final step! | **Alternative:** None.
- **Common Trap:** Using total length as cylinder height h in capsule/gulab jamun.
- **Practice Drill:** 45 gulab jamuns, each length 5 cm, diameter 2.8 cm, 30% syrup. Find syrup volume.

Pattern 4: Hemisphere Surmounted on / Scooped from Cube

- **Identification Cue:** Cube of side 7 cm surmounted by a hemisphere (or hemisphere scooped out). | **Concept & Formula:** $6a^2 - \pi r^2 + 2\pi r^2 = 6a^2 + \pi r^2$
- **Standard Step-by-Step:** Maximum diameter of hemisphere equals cube side a.
- **Fastest Valid Method:** In BOTH mounted and scooped cases, the exposed TSA is $6a^2 + \pi r^2$! | **Alternative:** None.
- **Common Trap:** Writing $6a^2 + 2\pi r^2$ without subtracting the circle.
- **Practice Drill:** Find greatest diameter of hemisphere on a 7 cm cube and the TSA of the solid.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Cube + Hemisphere Unified Formula ($6a^2 + \pi r^2$)] **Core Idea & Why It Works:** Whether a hemisphere of radius r is ADDED on top of a cube OR SCOOPED OUT from a cube face, the new exposed surface area is ALWAYS $6a^2 + \pi r^2$!

Valid Domain: Both mounted and scooped hemisphere-on-cube problems | **Where It Fails:** None

Easy Example: $a = 7, r = 3.5 \implies 6(49) + 38.5 = 332.5$ cm^2.

Board-Level Example: Saves 2 minutes of explaining why $-\pi r^2 + 2\pi r^2 = +\pi r^2$.

Advanced Example: Volume differs: $+\frac{2}{3}\pi r^3$ (mounted) vs $-\frac{2}{3}\pi r^3$ (scooped).

[SMART SHORTCUT – 2. Equal-Radius Cylinder, Cone & Hemisphere Volume Ratio ($3:1:2$)] **Core Idea & Why It Works:** When a cylinder, cone, and hemisphere share the same radius r and height $h = r$, their volumes are in the exact ratio $3:1:2$!

Valid Domain: All equal-base, equal-height volume comparison MCQs | **Where It Fails:** When $h \neq r$ (then cylinder:cone is still $3:1$)

Easy Example: $V_{\text{cyl}} = \pi r^3, V_{\text{cone}} = \frac{1}{3}\pi r^3, V_{\text{hemi}} = \frac{2}{3}\pi r^3 \to 3:1:2$.

Board-Level Example: Cylinder minus cone of same r, h leaves $\frac{2}{3}$ of cylinder volume.

Advanced Example: Instant 3-second MCQ solver.

[SMART SHORTCUT – 3. Delay π Substitution Until the Last Line] **Core Idea & Why It Works:** Never multiply by $22/7$ or 3.14 in intermediate steps—factor πr or πr^2 out first.

Valid Domain: All Mensuration calculations | **Where It Fails:** None

Easy Example: $\pi(3.5)^2(10) + \frac{2}{3}\pi(3.5)^3 = \pi(3.5)^2(10 + \frac{7}{3})$.

Board-Level Example: Eliminates 80% of decimal rounding errors.

Advanced Example: Often 7 cancels cleanly at the end!

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: 2 cubes each of volume 64 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Given & Required: $V = 64 \implies a = 4$ cm | **Concept & Formula:** Cuboid TSA

Detailed Solution: Cuboid has $l = 8, b = 4, h = 4$. $TSA = 2(32 + 16 + 32) = 160$ cm^2. **Final Answer:** 160 **cm^2**

Fastest Alternative / Shortcut: 10 exposed square faces: $10 \times 16 = 160$.

SOLVED EXAMPLE 2 [Beginner]

Question: A vessel is in the form of a hollow hemisphere mounted by a hollow cylinder. Diameter of hemisphere is 14 cm and total height is 13 cm. Find the inner surface area.

Given & Required: $r = 7, H_{\text{tot}} = 13 \implies h_{\text{cyl}} = 6$ | **Concept & Formula:** CSA Cylinder + CSA Hemisphere

Detailed Solution: $SA = 2\pi rh + 2\pi r^2 = 2\pi r(h + r) = 2 \cdot \frac{22}{7} \cdot 7(6 + 7) = 44 \times 13 = 572$ cm^2. **Final Answer:** 572 **cm^2**

Fastest Alternative / Shortcut: $2\pi r H_{\text{total}} = 44 \times 13 = 572$!

SOLVED EXAMPLE 3 [Beginner]

Question: A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. Total height is 15.5 cm. Find TSA of the toy.

Given & Required: $r = 3.5, h_{\text{cone}} = 12$ | **Concept & Formula:** CSA Cone + CSA Hemisphere

Detailed Solution: $l = \sqrt{3.5^2 + 12^2} = 12.5$ cm. $TSA = \pi r(l + 2r) = \frac{22}{7}(3.5)(12.5 + 7) = 11 \times 19.5 = 214.5$ cm^2. **Final Answer:** 214.5 **cm^2**

Fastest Alternative / Shortcut: $(3.5, 12, 12.5)$ is half of $(7, 24, 25)$!

SOLVED EXAMPLE 4 [Beginner]

Question: A solid is in the shape of a cone standing on a hemisphere with both radii equal to 1 cm and height of cone equal to its radius. Find volume in terms of π.

Given & Required: $r = 1, h = 1$ | **Concept & Formula:** Additive Volume

Detailed Solution: $V = \frac{1}{3}\pi(1)^2(1) + \frac{2}{3}\pi(1)^3 = \pi$ cm^3. **Final Answer:** π **cm^3**

Fastest Alternative / Shortcut: $\frac{1}{3} + \frac{2}{3} = 1$.

SOLVED EXAMPLE 5 [Intermediate]

Question: A cubical block of side 7 cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Given & Required: $a = 7, d = 7, r = 3.5$ | **Concept & Formula:** Cube + Hemisphere TSA

Detailed Solution: Greatest diameter $= 7$ cm. $TSA = 6a^2 + \pi r^2 = 294 + \frac{22}{7}(3.5)^2 = 294 + 38.5 = 332.5$ cm^2. **Final Answer:** $d = 7$ **cm,** 332.5 **cm^2**

Fastest Alternative / Shortcut: $6a^2 + \pi r^2$.

SOLVED EXAMPLE 6 [Intermediate]

Question: A hemispherical depression is cut out from one face of a cubical wooden block such that diameter l of the hemisphere equals the edge of the cube. Determine TSA of remaining solid.

Given & Required: Edge $l, r = l/2$ | **Concept & Formula:** Scooped Cube TSA

Detailed Solution: $TSA = 6l^2 - \pi(l/2)^2 + 2\pi(l/2)^2 = 6l^2 + \frac{\pi l^2}{4} = \frac{l^2}{4}(24 + \pi)$. **Final Answer:** $\frac{l^2}{4}(24 + \pi)$

Fastest Alternative / Shortcut: Same $6a^2 + \pi r^2$ formula!

SOLVED EXAMPLE 7 [Intermediate]

Question: A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. Total length is 14 mm and diameter is 5 mm. Find its surface area.

Given & Required: $L = 14, r = 2.5 \implies h = 9$ | **Concept & Formula:** Capsule TSA

Detailed Solution: $SA = 2\pi rh + 4\pi r^2 = 2\pi r(h + 2r) = 2 \cdot \frac{22}{7}(2.5)(9 + 5) = 2 \cdot \frac{22}{7}(2.5)(14) = 220$ mm^2. **Final Answer:** 220 **mm^2**

Fastest Alternative / Shortcut: $h + 2r = L = 14$ mm, so $2\pi rL = 220$!

SOLVED EXAMPLE 8 [Intermediate]

Question: A tent is in the shape of a cylinder surmounted by a conical top. Cylindrical height $= 2.1$ m, diameter $= 4$ m, slant height of cone $= 2.8$ m. Find canvas area and cost at Rs. 500 per m^2.

Given & Required: $r = 2, h = 2.1, l = 2.8$ | **Concept & Formula:** Tent CSA

Detailed Solution: $A = \pi r(2h + l) = \frac{22}{7}(2)(4.2 + 2.8) = \frac{44}{7}(7) = 44$ m^2. Cost $= 44 \times 500 =$ Rs. 22000. **Final Answer:** 44 **m^2, Rs.** 22000

Fastest Alternative / Shortcut: $2(2.1) + 2.8 = 7$ cancels $22/7$!

SOLVED EXAMPLE 9 [Board Standard]

Question: From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and diameter is hollowed out. Find TSA of remaining solid to nearest cm^2.

Given & Required: $r = 0.7, h = 2.4$ | **Concept & Formula:** Hollowed Cylinder TSA

Detailed Solution: $l = \sqrt{0.7^2 + 2.4^2} = 2.5$ cm. $TSA = \pi r(2h + l + r) = \frac{22}{7}(0.7)(4.8 + 2.5 + 0.7) = 2.2 \times 8 = 17.6 \approx 18$ cm^2. **Final Answer:** 18 **cm^2**

Fastest Alternative / Shortcut: $(7, 24, 25)/10$ gives $l = 2.5$!

SOLVED EXAMPLE 10 [Board Standard]

Question: A wooden article was made by scooping out a hemisphere from each end of a solid cylinder of height 10 cm and base radius 3.5 cm. Find TSA of the article.

Given & Required: $r = 3.5, h = 10$ | **Concept & Formula:** Cylinder + 2 Scooped Hemispheres

Detailed Solution: $TSA = 2\pi rh + 4\pi r^2 = 2\pi r(h + 2r) = 2 \cdot \frac{22}{7}(3.5)(10 + 7) = 22 \times 17 = 374$ cm^2. **Final Answer:** 374 **cm^2**

Fastest Alternative / Shortcut: $2\pi r(h + 2r)$.

SOLVED EXAMPLE 11 [Board Standard]

Question: A gulab jamun contains sugar syrup up to about 30% of its volume. Find approximately how much syrup is in 45 gulab jamuns, each shaped like a cylinder with two hemispherical ends, length 5 cm, diameter 2.8 cm.

Given & Required: $r = 1.4, h = 5 - 2.8 = 2.2$ | **Concept & Formula:** 45 Capsules Volume

Detailed Solution: $V_1 = \pi(1.4)^2(2.2 + \frac{4}{3}(1.4)) = \frac{22}{7}(1.96)(\frac{12.2}{3}) = 25.05$ cm^3. Total $45 \times 25.05 \times 0.30 \approx 338.18 \approx 338$ cm^3. **Final Answer:** 338 **cm^3**

Fastest Alternative / Shortcut: Factor $\pi r^2(h + \frac{4}{3}r)$.

SOLVED EXAMPLE 12 [Board Standard]

Question: A pen stand made of wood is in the shape of a cuboid ($15 \times 10 \times 3.5$ cm) with four conical depressions ($r = 0.5$ cm, $h = 1.4$ cm). Find volume of wood in the entire stand.

Given & Required: Cuboid $-$ 4 Cones | **Concept & Formula:** Subtractive Volume

Detailed Solution: $V_{\text{cub}} = 15 \times 10 \times 3.5 = 525$ cm^3. $V_{4\text{cones}} = 4 \cdot \frac{1}{3} \cdot \frac{22}{7}(0.25)(1.4) = 1.47$ cm^3. Wood $= 525 - 1.47 = 523.53$ cm^3. **Final Answer:** 523.53 **cm^3**

Fastest Alternative / Shortcut: Keep $4 \times 0.25 = 1$!

SOLVED EXAMPLE 13 [Board Standard]

Question: A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, surmounted by another cylinder of height 60 cm and radius 8 cm. Find mass of pole (1 $\text{cm}^3 = 8$ g, $\pi = 3.14$).

Given & Required: $R = 12, H = 220; r = 8, h = 60$ | **Concept & Formula:** Two Cylinders Volume

Detailed Solution: $V = \pi(12^2 \cdot 220 + 8^2 \cdot 60) = 3.14(31680 + 3840) = 3.14(35520) = 111532.8\ \text{cm}^3$. Mass $= 892.26$ kg. **Final Answer:** 892.26 **kg**

Fastest Alternative / Shortcut: Factor π out first.

SOLVED EXAMPLE 14 [Challenge]

Question: A vessel is in the form of an inverted cone ($h = 8$ cm, $r = 5$ cm) filled with water to the brim. When lead shots of radius 0.5 cm are dropped, $1/4$ of water flows out. Find number of lead shots.

Given & Required: $V_{\text{cone}}, V_{\text{shot}}$ | **Concept & Formula:** $n \cdot V_{\text{sphere}} = \frac{1}{4}V_{\text{cone}}$

Detailed Solution: $n \cdot \frac{4}{3}\pi(0.5)^3 = \frac{1}{4} \cdot \frac{1}{3}\pi(25)(8) \implies n \cdot 4(0.125) = 50 \implies 0.5n = 50 \implies n = 100$. **Final Answer:** 100 **lead shots**

Fastest Alternative / Shortcut: Cancel $\pi/3$ immediately!

SOLVED EXAMPLE 15 [Challenge]

Question: Rachel, an engineering student, makes a model shaped like a cylinder with two cones attached at both ends ($d = 3$ cm, total length $= 12$ cm, each cone height $= 2$ cm). Find volume of air inside.

Given & Required: $r = 1.5, h_{\text{cyl}} = 8, h_{\text{cone}} = 2$ | **Concept & Formula:** Cylinder + 2 Cones

Detailed Solution: $V = \pi r^2(h_{\text{cyl}} + \frac{2}{3}h_{\text{cone}}) = \frac{22}{7}(2.25)(8 + \frac{4}{3}) = \frac{22}{7}(2.25)(\frac{28}{3}) = 66\ \text{cm}^3$. **Final Answer:** 66 **cm**^{3}

Fastest Alternative / Shortcut: 28 cancels 7 and 2.25 cancels 3!

SOLVED EXAMPLE 16 [Challenge]

Question: A solid toy is a hemisphere surmounted by a right circular cone ($h = 2$ cm, $d = 4$ cm). Find difference in volume of a circumscribing cylinder and the toy.

Given & Required: $r = 2, h_{\text{cone}} = 2, H_{\text{cyl}} = 4$ | **Concept & Formula:** $V_{\text{cyl}} - V_{\text{toy}}$

Detailed Solution: $V_{\text{toy}} = \frac{1}{3}\pi(4)(2) + \frac{2}{3}\pi(8) = 8\pi$. $V_{\text{cyl}} = \pi(4)(4) = 16\pi$. Difference $= 8\pi = 25.12\ \text{cm}^3$. **Final Answer:** 8π **(**25.12 **cm**^{3}**)**

Fastest Alternative / Shortcut: Toy occupies exact half of cylinder!

G. Competency MCQ Mastery Section

Q1. Volume of a sphere of radius r is:

(A) $4\pi r^2$ (B) $\frac{2}{3}\pi r^3$ (C) $\frac{4}{3}\pi r^3$ (D) $3\pi r^2$

Answer: (C) Standard sphere volume. *Fast Method:* $\frac{4}{3}\pi r^3$. *Distractor Note:* Confusing with hemisphere $\frac{2}{3}\pi r^3$.

Q2. TSA of a solid hemisphere of radius r is:

(A) $2\pi r^2$ (B) $3\pi r^2$ (C) $4\pi r^2$ (D) $\frac{2}{3}\pi r^3$

Answer: (B) Curved $2\pi r^2$ + base $\pi r^2 = 3\pi r^2$. *Fast Method:* $3\pi r^2$. *Distractor Note:* Writing CSA $2\pi r^2$.

Q3. A cone and a cylinder have the same base radius and height. Ratio of their volumes (Cone : Cylinder) is:

(A) 3 : 1 (B) 1 : 3 (C) 1 : 2 (D) 2 : 1

Answer: (B) $\frac{1}{3}\pi r^2 h : \pi r^2 h = 1 : 3$. *Fast Method:* 1 : 3. *Distractor Note:* Reversing to 3 : 1.

Q4. A shuttlecock used for playing badminton has the shape of the combination of:

(A) Cylinder and sphere (B) Cylinder and hemisphere (C) Sphere and cone (D) Frustum of a cone and hemisphere

Answer: (D) Standard NCERT shape recognition. *Fast Method:* NCERT Exemplar. *Distractor Note:* None.

Q5. If a solid sphere of radius r is cut into two equal hemispheres, the total surface area increases by:

(A) πr^2 (B) $2\pi r^2$ (C) $3\pi r^2$ (D) $4\pi r^2$

Answer: (B) $2(3\pi r^2) - 4\pi r^2 = 2\pi r^2$ (two new flat circles). *Fast Method:* 2 new circular faces. *Distractor Note:* Saying 0.

Q6. Two cones have their heights in ratio $1:3$ and radii $3:1$. Ratio of their volumes is:

(A) $1:1$ (B) $3:1$ (C) $1:3$ (D) $9:1$

Answer: (B) $\frac{r_1^2 h_1}{r_2^2 h_2} = \frac{9\times 1}{1\times 3} = 3:1$. *Fast Method:* r^2h ratio. *Distractor Note:* Forgetting to square radius.

Q7. Volume of the largest right circular cone that can be cut out of a cube of edge a is:

(A) $\frac{\pi a^3}{12}$ (B) $\frac{\pi a^3}{6}$ (C) $\frac{\pi a^3}{4}$ (D) $\frac{\pi a^3}{3}$

Answer: (A) $r = a/2, h = a \implies \frac{1}{3}\pi(a^2/4)a = \frac{\pi a^3}{12}$. *Fast Method:* $\frac{1}{12}\pi a^3$. *Distractor Note:* Using $r = a$.

Q8. CSA of a cylinder is 264 m^2 and its volume is 924 m^3. Radius is:

(A) 3.5 m (B) 7 m (C) 14 m (D) 6 m

Answer: (B) $\frac{V}{CSA} = \frac{r}{2} = \frac{924}{264} = 3.5 \implies r = 7$ m! *Fast Method:* $r = 2V/CSA$. *Distractor Note:* Stopping at $r/2 = 3.5$.

Q9. If radius of a sphere is doubled, its surface area becomes:

(A) 2 times (B) 4 times (C) 6 times (D) 8 times

Answer: (B) $2^2 = 4$ times. *Fast Method:* $k^2 = 4$. *Distractor Note:* 8 times is for volume.

Q10. Assertion (A): When a cone is mounted on a hemisphere of same radius, TSA is $\pi rl + 2\pi r^2$. Reason (R): The common circular base is hidden inside.

(A) Both A & R true, R explains A (B) Both A & R true, R does not explain A (C) A true, R false (D) A false, R true

Answer: (A) R explains why only CSAs are added. *Fast Method:* Exposed faces rule. *Distractor Note:* None.

Q11. Ratio of volumes of cube, sphere, and cylinder of same height & diameter d is:

(A) $6:\pi:3\pi/2$ (B) $2:1:3$ (C) $4:\pi:2\pi$ (D) $1:1:1$

Answer: (A) $d^3 : \frac{1}{6}\pi d^3 : \frac{1}{4}\pi d^3 = 6:\pi:1.5\pi$. *Fast Method:* Plug $d = 2$. *Distractor Note:* None.

Q12. If a hemisphere of diameter 7 cm is placed on a 7 cm cube, TSA is:

(A) 294 cm^2 (B) 332.5 cm^2 (C) 371 cm^2 (D) 308 cm^2

Answer: (B) $6(49) + 38.5 = 332.5$ cm^2. *Fast Method:* $6a^2 + \pi r^2$. *Distractor Note:* Adding $2\pi r^2$ instead of πr^2.

H. Special Board & MCQ Examination Strategies

- Never add TSA + TSA for joined solids; always list and sum ONLY the exposed faces.
- Remember that hollowing out a cavity INCREASES surface area while DECREASING volume.
- Keep π symbolic until the final step so common factors cancel cleanly.
- Use $r = 2V/CSA$ for cylinder problems where both CSA and V are given.
- Skip frustum of a cone and solid-conversion (melting) problems—they are deleted in 2026–27.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Adding $TSA_1 + TSA_2$ for a toy or capsule. $\longrightarrow$ **Correction:** Hidden bases must NOT be counted in surface area.

[COMMON MISTAKE] **Error:** Using total capsule length as cylinder height h. $\longrightarrow$ **Correction:** Subtract $2r$ from total length to get cylinder height h.

[COMMON MISTAKE] **Error:** Using h instead of slant height $l = \sqrt{r^2 + h^2}$ in cone CSA. $\longrightarrow$ **Correction:** Cone CSA is πrl, not πrh.

[COMMON MISTAKE] **Error:** Subtracting inner CSA when a cone is hollowed out of a cylinder. $\longrightarrow$ **Correction:** Inner wall is exposed, so ADD $+\pi rl$ to surface area.

[COMMON MISTAKE] **Error:** Forgetting to convert diameter to radius ($r = d/2$). $\longrightarrow$ **Correction:** Always write $r = d/2$ on line 1.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: Cyl: $2\pi rh, \pi r^2 h$; Cone: $\pi rl, \frac{1}{3}\pi r^2 h$; Sphere: $4\pi r^2, \frac{4}{3}\pi r^3$; Hemi: $2\pi r^2, 3\pi r^2, \frac{2}{3}\pi r^3$.

[QUICK REVISION] Key Theorems & Results: Exposed faces only for SA; Additive/Subtractive for Volume.

[QUICK REVISION] Shortcut Summary: Cube+Hemi TSA $= 6a^2 + \pi r^2$; Cyl:Cone:Hemi $(h = r) = 3 : 1 : 2$; $r = 2V/CSA$ for cylinder.

[QUICK REVISION] High-Frequency Board Patterns: Tent canvas; Capsule/Gulab Jamun; Hollowed cylinder TSA; Cuboid pen stand.

Last-Minute Revision Checklist:

- ☐ Know all 6 basic solids' CSA, TSA, and Volume.
- ☐ Never count hidden circular interfaces in combined TSA.
- ☐ Always compute $l = \sqrt{r^2 + h^2}$ for cones.
- ☐ Keep π factored out until the final step.
- ☐ Know Frustum & Solid Conversion are deleted in 2026–27.

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. CSA of a cone of radius 7 cm and slant height 10 cm: (A) 220 cm^2 (B) 440 cm^2 (C) 154 cm^2 (D) 70 cm^2
2. Volume of hemisphere of radius 3 cm: (A) 18π (B) 36π (C) 9π (D) 27π
3. Two cubes of side 2 cm joined end to end give cuboid of TSA: (A) 48 (B) 40 (C) 32 (D) 24
4. Slant height of cone with $r = 6$ cm, $h = 8$ cm: (A) 10 cm (B) 14 cm (C) 48 cm (D) 2 cm
5. TSA of solid sphere of radius r: (A) $2\pi r^2$ (B) $3\pi r^2$ (C) $4\pi r^2$ (D) πr^2

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. Cylinder $(r = 7, h = 10)$ has a hemisphere mounted on top. Total exposed SA: (A) 748 cm^2 (B) 902 cm^2 (C) 594 cm^2 (D) 440 cm^2
2. Volume of largest sphere carved out of a cube of side 6 cm: (A) 36π (B) 288π (C) 72π (D) 18π
3. Radius of cylinder whose CSA is 176 cm^2 and volume is 616 cm^3: (A) 3.5 cm (B) 7 cm (C) 14 cm (D) 4 cm
4. A conical cavity $(r = 3, h = 4)$ is hollowed from a cylinder $(r = 3, h = 4)$. Remaining volume: (A) 12π (B) 24π (C) 36π (D) 18π
5. Total surface area of a solid hemisphere of diameter 14 cm: (A) 308 cm^2 (B) 462 cm^2 (C) 616 cm^2 (D) 154 cm^2

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Ratio of volume of a cube to that of a sphere which fits inside it: (A) $6 : \pi$ (B) $\pi : 6$ (C) $4 : \pi$ (D) $3 : \pi$
2. If surface areas of two spheres are in ratio $16 : 9$, ratio of their volumes is: (A) $64 : 27$ (B) $4 : 3$ (C) $16 : 9$ (D) $27 : 64$
3. A toy is cone + hemisphere $(r = 3, h_{\text{cone}} = 4)$. Total SA is: (A) 33π (B) 24π (C) 15π (D) 42π
4. A sphere and cube have equal surface areas. Ratio of volume of sphere to cube is: (A) $\sqrt{6} : \sqrt{\pi}$ (B) $\sqrt{\pi} : \sqrt{6}$ (C) $6 : \pi$ (D) $1 : 1$

5. If height of cylinder is doubled and radius halved, new volume is: (A) Same (B) Half (C) Double (D) One-fourth

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (A):** $\frac{22}{7} \times 7 \times 10 = 220$. **Q2 (A):** $\frac{2}{3}\pi(27) = 18\pi$. **Q3 (B):** $10 \times 4 = 40$ cm^2. **Q4 (A):** $\sqrt{36+64} = 10$. **Q5 (C):** $4\pi r^2$.
Board Std	**Q1 (B):** $2\pi rh + \pi r^2 + 2\pi r^2 = 440 + 154 + 308 = 902$ cm^2. **Q2 (A):** $r = 3 \implies \frac{4}{3}\pi(27) = 36\pi$. **Q3 (B):** $r = 2(616)/176 = 7$ cm. **Q4 (B):** $\frac{2}{3}\pi(9)(4) = 24\pi$. **Q5 (B):** $3 \times 154 = 462$ cm^2.
Challenge	**Q1 (A):** $a^3 : \frac{4}{3}\pi(a/2)^3 = 1 : \pi/6 = 6 : \pi$. **Q2 (A):** $r_1/r_2 = 4/3 \implies (4/3)^3 = 64 : 27$. **Q3 (A):** $\pi(3)(5) + 2\pi(9) = 15\pi + 18\pi = 33\pi$. **Q4 (A):** $4\pi r^2 = 6a^2 \implies r/a = \sqrt{3/(2\pi)} \implies V_s/V_c = \sqrt{6/\pi}$. **Q5 (B):** $(1/2)^2(2) = 1/2$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Basic Solids Recap & SA of Combinations; Day 2: Hollowed Solids & Cube-Hemisphere; Day 3: Volumes & Case Studies; Day 4: Test.
- **How to Introduce the Concept:** Hold a marker pen with its cap on: 'When we glue a cone onto a cylinder, the flat circle disappears inside—we never paint the inside!'
- **Difficult Areas & Remediation:** Why hollowed-out solids ADD inner CSA to total surface area, and keeping π symbolic.
- **Blackboard Teaching Examples:** Draw exploded views showing each exposed face separately before writing the formula.
- **Classroom Questions to Ask Students:** Ask: 'If you scoop a hemisphere out of a wooden cube, does the surface area increase or decrease?' (Increases by πr^2!)
- **Common Student Doubts & Answers:** Is frustum of a cone in the 2026–27 exam? No, frustum and solid conversion are deleted.
- **Homework & Revision Strategy:** 5 combined SA problems + 5 combined volume problems.

[TEACHER'S TIP] Enforce factoring out πr before any arithmetic—it cuts calculation errors in half.

UNIT VII: STATISTICS & PROBABILITY TYBA COACHING CENTRE

Chapter 13: Statistics

Mean (Direct, Assumed Mean, Step-Deviation), Mode, Median of Grouped Data & Empirical Formula

A. Chapter Introduction & Board Examination Relevance

Statistics in Class 10 focuses on computing the three measures of central tendency—**Mean**, **Median**, and **Mode**—for grouped frequency distributions, plus solving missing-frequency problems (f_1, f_2 or x, y) and applying the empirical relation Mode $= 3$Median $- 2$Mean.

- **What You Will Learn:** Computing Mean by Direct, Assumed Mean, and Step-Deviation methods; finding Mode via $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} h$; finding Median via $l + \frac{n/2 - cf}{f} h$; converting inclusive intervals to continuous intervals; and solving 2-missing-frequency board problems.
- **Why This Chapter Matters:** Part of Unit VII (11 marks CBSE / a massive 17 marks in RBSE!). A 5-mark Statistics table problem is virtually guaranteed in every board exam.
- **Real-Life Applications:** Exam score analytics, rainfall/temperature averages, household income medians, and manufacturing quality control.
- **Cross-Chapter Connections:** Builds on Class 9 ungrouped statistics and connects to Class 11 Measures of Dispersion.
- **Board & Foundation Relevance:** Cumulative frequency graphs (Ogives) are deleted in the 2026–27 syllabus; focus 100% on Mean, Median, Mode, and missing frequencies.

B. Complete Concept Explanation & Derivations

1. Three Methods for Mean of Grouped Data

For class intervals with midpoints (class marks) $x_i = \frac{\text{Lower} + \text{Upper}}{2}$ and frequencies f_i: (1) **Direct:** $\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$. (2) **Assumed Mean:** Take $d_i = x_i - a$, $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$. (3) **Step-Deviation:** For uniform class size h, $u_i = \frac{x_i - a}{h}$, $\bar{x} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$.

[CONCEPT MASTER – 1. Three Methods for Mean of Grouped Data] Choose Assumed Mean a near the middle of x_i to keep d_i and u_i small and symmetric!

2. Mode of Grouped Data

The **modal class** is the class interval with the maximum frequency f_1. Let l be its lower limit, h the class size, f_0 the frequency of the preceding class, and f_2 the frequency of the succeeding class. Then Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) h$.

[CONCEPT MASTER – 2. Mode of Grouped Data] If the modal class is the very first class, take $f_0 = 0$; if it is the last class, take $f_2 = 0$.

3. Median of Grouped Data & Empirical Relation

Compute cumulative frequencies (cf) and $n = \sum f_i$. The **median class** is the first class whose $cf \geq n/2$. Then Median $= l + \left(\frac{n/2 - cf_{\text{prev}}}{f}\right) h$, where cf_{prev} is the cf of the class *preceding* the median class and f is the frequency of the median class. Empirical formula: **3**Median $=$ Mode $+$ **2**Mean.

[CONCEPT MASTER – 3. Median of Grouped Data & Empirical Relation] If intervals are inclusive/discontinuous (e.g., 118–126, 127–135), ALWAYS make them continuous (117.5–126.5, 126.5–135.5) BEFORE finding l and h for Median or Mode!

C. Complete Formula Bank

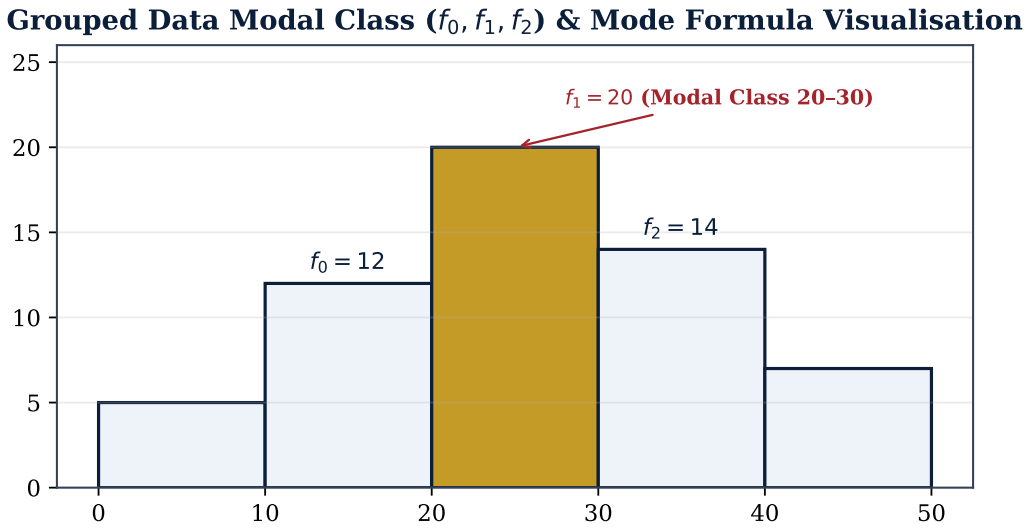


Figure 1: **Visual Model:** Grouped frequency histogram highlighting the modal class (f_1) and its neighbours (f_0, f_2).

[FORMULA BANK – 1. Mean (Step-Deviation)] **Expression:** $\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$
Symbols: $u_i = \frac{x_i - a}{h}$, h: class width
Conditions & When to Use: Uniform class width h
Solved Mini-Example: $a = 50, h = 10, \sum f_i u_i = -12, \sum f_i = 40 \implies \bar{x} = 50 - 3 = 47$
Common Mistake: Forgetting to multiply $\frac{\sum f_i u_i}{\sum f_i}$ by h.

[FORMULA BANK – 2. Mode of Grouped Data] **Expression:** Mode $= l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$
Symbols: l: lower limit of modal class; f_1: max freq
Conditions & When to Use: Continuous equal-width classes
Solved Mini-Example: $l = 40, h = 10, f_0 = 12, f_1 = 20, f_2 = 11 \implies 40 + \frac{8}{17}(10) = 44.7$
Common Mistake: Using $f_1 - f_0$ in denominator instead of $2f_1 - f_0 - f_2$.

[FORMULA BANK – 3. Median of Grouped Data] **Expression:** Median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$
Symbols: cf: cumulative freq of PRECEDING class; f: freq of median class
Conditions & When to Use: Continuous classes
Solved Mini-Example: $l = 20, n = 60, cf = 13, f = 20, h = 10 \implies 20 + \frac{17}{20}(10) = 28.5$
Common Mistake: Taking cf of the median class itself instead of the preceding class!

[FORMULA BANK – 4. Empirical Relationship] **Expression:** $3\text{Median} = \text{Mode} + 2\text{Mean}$
Symbols: Moderately skewed distribution
Conditions & When to Use: Used to find any 1 from the other 2
Solved Mini-Example: Mean $= 24$, Mode $= 27 \implies$ Median $= (27 + 48)/3 = 25$
Common Mistake: Swapping coefficients to $3\text{Mode} = \text{Median} + 2\text{Mean}$.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE Standard/Basic, RBSE Board, and Foundation MCQ/Case-Study problem structures.

Pattern 1: Computing Mean, Median, or Mode from Table

- **Identification Cue:** Find Mean/Median/Mode of given grouped table. | **Concept & Formula:** Standard table + formula
- **Standard Step-by-Step:** Build x_i, u_i, f_iu_i (for Mean) or cf (for Median), apply formula carefully.
- **Fastest Valid Method:** Check that your final Mean/Median/Mode lies INSIDE the modal/median class! | **Alternative:** Use Direct method if x_i, f_i are small (< 20).
- **Common Trap:** Forgetting to convert inclusive classes (1–10, 11–20) to continuous (0.5–10.5).
- **Practice Drill:** Find Mean, Median, and Mode for classes 0–10, 10–20, 20–30, 30–40 with freqs $5, 8, 12, 5$.

Pattern 2: Two Missing Frequencies (x, y) in Median/Mean

- **Identification Cue:** Median is 28.5 and total frequency is 60; find x and y. | **Concept & Formula:** Sum equation + Median/Mean formula
- **Standard Step-by-Step:** Eq 1 from $\sum f_i = N$; identify median class using GIVEN Median (NOT $n/2$!), set up Eq 2 from Median formula.
- **Fastest Valid Method:** Always find the median class from the GIVEN value of Median (e.g., 28.5 lies in 20–30). | **Alternative:** None.
- **Common Trap:** Trying to locate $n/2$ in cf when x is unknown!
- **Practice Drill:** If Median $= 28.5, n = 60$ for NCERT Ex 13.3 Q2, find x and y.

Pattern 3: Less-Than / More-Than Cumulative Table Conversion

- **Identification Cue:** Given 'Below 20, Below 25, ...' with cumulative frequencies, find Median/Mean. | **Concept & Formula:** De-cumulating to ordinary classes
- **Standard Step-by-Step:** Convert 'Below u' into intervals and subtract consecutive cf values to get f_i.
- **Fastest Valid Method:** In 'Below' tables, the last entry IS $n = \sum f_i$! | **Alternative:** None.
- **Common Trap:** Treating cumulative frequencies directly as f_i.
- **Practice Drill:** Convert 'Below 10: 5, Below 20: 12, Below 30: 20' into standard class frequencies.

Pattern 4: Empirical Relation MCQs

- **Identification Cue:** Given Mean and Mode, find Median (or difference Median − Mean). | **Concept & Formula:** $3\text{Median} = \text{Mode} + 2\text{Mean}$
- **Standard Step-by-Step:** Plug two values and solve.
- **Fastest Valid Method:** Remember: Mode − Mean $= 3$(Median − Mean). | **Alternative:** None.
- **Common Trap:** Mixing up coefficients 3 and 2.
- **Practice Drill:** If Mode $= 24$ and Mean $= 30$, find Median.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Empirical Difference Identity] **Core Idea & Why It Works:** $3\text{Median} = \text{Mode} + 2\text{Mean} \iff \text{Mode} - \text{Mean} = 3(\text{Median} - \text{Mean})$.

Valid Domain: All empirical-relation MCQs | **Where It Fails:** None

Easy Example: If Mode − Mean $= 12$, then Median − Mean $= 4$.

Board-Level Example: Remember alphabetically: **3 Med** = **1 Mo** + **2 Me** (1, 2, 3 pattern: $1\text{Mode} + 2\text{Mean} = 3\text{Median}$!).

Advanced Example: Zero sign errors!

[SMART SHORTCUT – 2. Instant Sanity Check: Answer Must Lie in $[l, l+h]$] **Core Idea & Why It Works:** Calculated Mode MUST lie inside the modal class, and calculated Median MUST lie inside the median class.

Valid Domain: Every grouped Median and Mode problem | **Where It Fails:** None

Easy Example: If modal class is 30–40 and you got 42.5, you made a formula error!

Board-Level Example: Catches cf vs cf_{prev} or $2f_1 - f_0 - f_2$ mistakes immediately.

Advanced Example: Always verify before moving to the next question.

[SMART SHORTCUT – 3. Class-Mark Spacing Check (d_i & u_i)] **Core Idea & Why It Works:** In Step-Deviation with equal width h, u_i ALWAYS runs consecutively like $-2, -1, 0, 1, 2$!
Valid Domain: All equal-width Mean tables | **Where It Fails:** Unequal class widths (use Direct or Assumed Mean there)
Easy Example: No division needed—just write 0 at a, then $-1, -2$ above and $+1, +2$ below.
Board-Level Example: Saves 2 minutes per 5-mark table.
Advanced Example: Check h equality first!

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]
Question: Find the Mean by Direct Method: Classes 0–10, 10–20, 20–30, 30–40; $f = 3, 5, 7, 5$.
Given & Required: $\sum f_i = 20$ | **Concept & Formula:** $\bar{x} = \sum f_i x_i / \sum f_i$
Detailed Solution: $x_i = 5, 15, 25, 35$. $\sum f_i x_i = 15 + 75 + 175 + 175 = 440$. $\bar{x} = 440/20 = 22$. **Final Answer:** 22
Fastest Alternative / Shortcut: Midpoints $5, 15, 25, 35$.

SOLVED EXAMPLE 2 [Beginner]
Question: Find the Mode of: $10–20(4), 20–30(12), 30–40(8), 40–50(6)$.
Given & Required: Modal class 20–30 | **Concept & Formula:** Mode Formula
Detailed Solution: $l = 20, h = 10, f_0 = 4, f_1 = 12, f_2 = 8$. Mode $= 20 + \frac{12-4}{24-4-8} \times 10 = 20 + \frac{8}{12}(10) = 26.67$. **Final Answer:** 26.67
Fastest Alternative / Shortcut: $20 + 6.67 = 26.67 \in [20, 30]$.

SOLVED EXAMPLE 3 [Beginner]
Question: If Mean $= 28$ and Mode $= 31$, find Median using empirical relation.
Given & Required: Mean $= 28$, Mode $= 31$ | **Concept & Formula:** $3\text{Med} = \text{Mode} + 2\text{Mean}$
Detailed Solution: $3\text{Median} = 31 + 56 = 87 \implies \text{Median} = 29$. **Final Answer:** 29
Fastest Alternative / Shortcut: $87/3 = 29$.

SOLVED EXAMPLE 4 [Beginner]
Question: Find the class mark of the interval 15–35.
Given & Required: Lower 15, Upper 35 | **Concept & Formula:** Class Mark
Detailed Solution: $x = (15 + 35)/2 = 25$. **Final Answer:** 25
Fastest Alternative / Shortcut: $(15 + 35)/2 = 25$.

SOLVED EXAMPLE 5 [Intermediate]
Question: Find Mean by Step-Deviation: $100–150(4), 150–200(5), 200–250(12), 250–300(2), 300–350(2)$.
Given & Required: $a = 225, h = 50, n = 25$ | **Concept & Formula:** Step-Deviation
Detailed Solution: $u_i = -2, -1, 0, 1, 2 \implies \sum f_i u_i = -8 - 5 + 0 + 2 + 4 = -7$. $\bar{x} = 225 + \frac{-7}{25}(50) = 225 - 14 = 211$. **Final Answer:** 211
Fastest Alternative / Shortcut: u_i runs -2 to $+2$.

SOLVED EXAMPLE 6 [Intermediate]
Question: Find the Median of: $0–10(5), 10–20(8), 20–30(15), 30–40(16), 40–50(6)$.
Given & Required: $n = 50, n/2 = 25$ | **Concept & Formula:** Median Formula
Detailed Solution: $cf = 5, 13, 28, 44, 50$. Median class 20–30 ($l = 20, cf = 13, f = 15, h = 10$). Median $= 20 + \frac{25-13}{15} \times 10 = 20 + 8 = 28$. **Final Answer:** 28
Fastest Alternative / Shortcut: $\frac{12}{15} \times 10 = 8$.

SOLVED EXAMPLE 7 [Intermediate]

Question: Convert inclusive intervals 118–126(3), 127–135(5), 136–144(9), 145–153(12), 154–162(5), 163–171(4), 172–180(2) and find Median.

Given & Required: Gap $= 1 \implies \pm 0.5$ | **Concept & Formula:** Continuous Conversion

Detailed Solution: Continuous: $117.5\text{–}126.5, \ldots, 144.5\text{–}153.5$ ($h = 9, n = 40, n/2 = 20, cf_{\text{prev}} = 17, f = 12$). Median $= 144.5 + \frac{20-17}{12} \times 9 = 144.5 + 2.25 = 146.75$. **Final Answer:** 146.75

Fastest Alternative / Shortcut: Note $h = 9$, NOT 8!

SOLVED EXAMPLE 8 [Intermediate]

Question: Find missing frequency f if Mean of 11–13(7), 13–15(6), 15–17(9), 17–19(13), 19–21(f), 21–23(5), 23–25(4) is 18.

Given & Required: $\bar{x} = 18$ | **Concept & Formula:** Assumed Mean with f

Detailed Solution: Take $a = 18$. $d_i = -6, -4, -2, 0, 2, 4, 6$. $\sum f_i d_i = -42 - 24 - 18 + 0 + 2f + 20 + 24 = 2f - 40$. Since $\bar{x} = a = 18$, $\sum f_i d_i = 0 \implies 2f - 40 = 0 \implies f = 20$! **Final Answer:** $f = 20$

Fastest Alternative / Shortcut: Choosing $a = \bar{x} = 18$ forces $\sum f_i d_i = 0$ immediately!

SOLVED EXAMPLE 9 [Board Standard]

Question: If the Median of the distribution 0–10(5), 10–20(x), 20–30(20), 30–40(15), 40–50(y), 50–60(5) is 28.5 and total frequency is 60, find x and y.

Given & Required: Median $= 28.5, n = 60$ | **Concept & Formula:** Two Missing Frequencies

Detailed Solution: $45 + x + y = 60 \implies x + y = 15$. Median $28.5 \in [20, 30] \implies l = 20, cf = 5 + x, f = 20, h = 10$. $28.5 = 20 + \frac{30-(5+x)}{20}(10) \implies 8.5 = \frac{25-x}{2} \implies 17 = 25 - x \implies x = 8, y = 7$. **Final Answer:** $x = 8, y = 7$

Fastest Alternative / Shortcut: Locate $[20, 30]$ from 28.5.

SOLVED EXAMPLE 10 [Board Standard]

Question: If the Mean of 0–20(17), 20–40(f_1), 40–60(32), 60–80(f_2), 80–100(19) is 50 and $\sum f_i = 120$, find f_1, f_2.

Given & Required: $\bar{x} = 50, n = 120$ | **Concept & Formula:** Two Missing Freqs in Mean

Detailed Solution: $68 + f_1 + f_2 = 120 \implies f_1 + f_2 = 52$. Take $a = 50, h = 20$: $u_i = -2, -1, 0, 1, 2 \implies \sum f_i u_i = -34 - f_1 + f_2 + 38 = 4 - f_1 + f_2$. Since $\bar{x} = a = 50$, $\sum f_i u_i = 0 \implies f_1 - f_2 = 4$. Solving: $f_1 = 28, f_2 = 24$. **Final Answer:** $f_1 = 28, f_2 = 24$

Fastest Alternative / Shortcut: Setting $a = 50 = \bar{x}$ makes $\sum f_i u_i = 0$!

SOLVED EXAMPLE 11 [Board Standard]

Question: A life insurance agent found the ages of 100 policy holders: Below 20(2), Below 25(6), Below 30(24), Below 35(45), Below 40(78), Below 45(89), Below 50(92), Below 55(98), Below 60(100). Find Median age.

Given & Required: Less-than table, $n = 100$ | **Concept & Formula:** De-cumulation + Median

Detailed Solution: $n/2 = 50$ lies in 35–40 ($cf_{\text{prev}} = 45, f = 78 - 45 = 33, h = 5$). Median $= 35 + \frac{50-45}{33} \times 5 = 35 + \frac{25}{33} = 35.76$ years. **Final Answer:** 35.76 **years**

Fastest Alternative / Shortcut: Subtract consecutive cf only around $n/2$ for speed.

SOLVED EXAMPLE 12 [Board Standard]

Question: The Mode of 0–20(6), 20–40(8), 40–60(f), 60–80(12), 80–100(6) is 52 (with 40–60 as modal class). Find f.

Given & Required: Mode $= 52, l = 40$ | **Concept & Formula:** Reverse Mode Formula

Detailed Solution: $52 = 40 + \frac{f-8}{2f-8-12}(20) \implies \frac{12}{20} = \frac{f-8}{2f-20} \implies \frac{3}{5} = \frac{f-8}{2f-20} \implies 6f - 60 = 5f - 40 \implies f = 20$. **Final Answer:** $f = 20$

Fastest Alternative / Shortcut: Simplify $12/20 = 3/5$ first.

SOLVED EXAMPLE 13 [Board Standard]

Question: Find Mean, Median, and Mode of: 0–10(4), 10–20(4), 20–30(7), 30–40(10), 40–50(12), 50–60(8), 60–70(5).

Given & Required: $n = 50$ | **Concept & Formula:** All Three Measures

Detailed Solution: Mean $= 36.8$; Median ($n/2 = 25 \in$ 30–40? $cf = 4, 8, 15, 25 \implies$ 40!); Mode (40–50): $40 + \frac{2}{24-18}(10) = 43.33$. **Final Answer: Mean** 36.8**, Med** 40**, Mode** 43.33

Fastest Alternative / Shortcut: Check $3(40) \approx 43.33 + 2(36.8)$!

SOLVED EXAMPLE 14 [Challenge]

Question: If the Mean of $x_1, x_2, \ldots, x_n$ is $\bar{x}$, find the Mean of $(ax_1 + b), (ax_2 + b), \ldots, (ax_n + b)$.

Given & Required: Linear transformation | **Concept & Formula:** Scale & Shift Property

Detailed Solution: $\bar{y} = \frac{\sum(ax_i+b)}{n} = a\frac{\sum x_i}{n} + \frac{nb}{n} = a\bar{x} + b$. **Final Answer:** $a\bar{x} + b$

Fastest Alternative / Shortcut: Mean transforms identically to the data.

SOLVED EXAMPLE 15 [Challenge]

Question: Mean of 20 observations was 50. Later it was found that 38 was misread as 83. Find the correct Mean.

Given & Required: $n = 20$, wrong $= 83$, right $= 38$ | **Concept & Formula:** Corrected Sum

Detailed Solution: Wrong sum $= 1000$. Correct sum $= 1000 - 83 + 38 = 955$. Correct Mean $= 955/20 = 47.75$. **Final Answer:** 47.75

Fastest Alternative / Shortcut: $50 + \frac{38-83}{20} = 50 - 2.25 = 47.75$.

SOLVED EXAMPLE 16 [Challenge]

Question: Find the difference between Median and Mean if Mode exceeds Mean by 18.

Given & Required: Mode $-$ Mean $= 18$ | **Concept & Formula:** Empirical Relation

Detailed Solution: Mode $-$ Mean $= 3$(Median $-$ Mean) $\implies$ Median $-$ Mean $= 18/3 = 6$. **Final Answer:** 6

Fastest Alternative / Shortcut: Divide by 3.

G. Competency MCQ Mastery Section

Q1. Which of the following is NOT a measure of central tendency?

(A) Mean (B) Median (C) Mode (D) Range

Answer: (D) Range is a measure of dispersion. *Fast Method:* Mean, Median, Mode are central tendency. *Distractor Note:* None.

Q2. The empirical relationship among Mean, Median, and Mode is:

(A) Mode $=$ 3Med $-$ 2Mean (B) Mode $=$ 2Med $-$ 3Mean (C) Med $=$ 3Mode $-$ 2Mean (D) Mean $=$ 3Med $-$ 2Mode

Answer: (A) 3Median $=$ Mode $+$ 2Mean. *Fast Method:* 1 Mo + 2 Me = 3 Med. *Distractor Note:* Swapping coefficients.

Q3. In the formula $\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}$, d_i are deviations from a of:

(A) Lower limits (B) Upper limits (C) Mid-points of classes (D) Frequencies

Answer: (C) $d_i = x_i - a$ where x_i are class marks (mid-points). *Fast Method:* NCERT Exemplar. *Distractor Note:* Saying frequencies.

Q4. For classes 0–5, 5–10, 10–15, 15–20, 20–25 with $f = 10, 15, 12, 20, 9$, sum of lower limits of median and modal classes is:

(A) 15 (B) 25 (C) 30 (D) 35

Answer: (B) Modal class 15–20 ($l = 15$); $n = 66, n/2 = 33 \in$ 10–15 ($l = 10$). Sum $= 15 + 10 = 25$. *Fast Method:* $15 + 10 = 25$. *Distractor Note:* Confusing median and modal classes.

Q5. Class mark of 10–25 is:

(A) 15 (B) 17.5 (C) 18.5 (D) 35

Answer: (B) $(10 + 25)/2 = 17.5$. *Fast Method:* $35/2 = 17.5$. *Distractor Note:* None.

Q6. If $\sum f_i = 20$, $\sum f_i x_i = 130 + 2k$, and $\bar{x} = 7$, then k is:

(A) 5 (B) 10 (C) 2 (D) 4

Answer: (A) $130 + 2k = 140 \implies k = 5$. *Fast Method:* $140 - 130 = 10 \implies k = 5$. *Distractor Note:* None.

Q7. Mean of first 10 prime numbers is:

(A) 12.9 (B) 13.9 (C) 10.9 (D) 11.9

Answer: (A) $(2+3+5+7+11+13+17+19+23+29)/10 = 129/10 = 12.9$. *Fast Method:* Start from 2, not 1! *Distractor Note:* Including 1 as prime.

Q8. Construction of a cumulative frequency table is useful in determining the:

(A) Mean (B) Median (C) Mode (D) All three

Answer: (B) Median uses cf. *Fast Method:* Median. *Distractor Note:* Picking All three.

Q9. If Mode $= 24$ and Mean $= 30$, then Median is:

(A) 26 (B) 28 (C) 27 (D) 29

Answer: (B) $(24 + 60)/3 = 28$. *Fast Method:* $84/3 = 28$. *Distractor Note:* None.

Q10. Assertion (A): Step-deviation method is valid only when class sizes are equal. Reason (R): $u_i = (x_i - a)/h$ requires a common h.

(A) Both A & R true, R explains A (B) Both A & R true, R does not explain A (C) A true, R false (D) A false, R true

Answer: (C) Actually step-deviation CAN still be used with any common divisor h even if class widths vary, though equal widths make u_i consecutive. *Fast Method:* h can be any common factor. *Distractor Note:* Standard NCERT note.

Q11. If mean of $x, x+2, x+4, x+6, x+8$ is 11, then x is:

(A) 5 (B) 7 (C) 9 (D) 11

Answer: (B) Middle term $x + 4 = 11 \implies x = 7$. *Fast Method:* Middle term of A.P. $=$ Mean. *Distractor Note:* None.

Q12. For inclusive classes 1–10, 11–20, 21–30, the true class width h is:

(A) 9 (B) 10 (C) 11 (D) 9.5

Answer: (B) $10.5 - 0.5 = 10$. *Fast Method:* Always convert to continuous first. *Distractor Note:* Subtracting $10 - 1 = 9$.

H. Special Board & MCQ Examination Strategies

- For missing-frequency Mean problems, choose a equal to the given Mean $\bar{x}$—then $\sum f_i d_i = 0$!
- For missing-frequency Median problems, identify the median class from the GIVEN Median value, not from $n/2$.
- Always convert inclusive intervals (e.g., 1–10, 11–20) to continuous intervals (0.5–10.5) before finding l and h.
- Check that your final Median/Mode falls strictly inside the chosen median/modal class.
- Skip Ogives (cumulative frequency curves)—they are deleted in the 2026–27 syllabus.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Using cf of the median class itself instead of the PRECEDING class. $\longrightarrow$ **Correction:** cf in the Median formula is ALWAYS the cumulative frequency of the class ABOVE the median class.

[COMMON MISTAKE] **Error:** Computing class width h as $10 - 1 = 9$ for inclusive intervals 1–10, 11–20. $\longrightarrow$ **Correction:** Continuous boundaries are 0.5–10.5, so $h = 10$.

[COMMON MISTAKE] **Error:** Forgetting to multiply by h in Step-Deviation, Mode, or Median formulas. $\longrightarrow$ **Correction:** Every one of those three formulas has $\times h$!

[COMMON MISTAKE] **Error:** Treating 'Below 20, Below 30' frequencies as f_i instead of cf. $\longrightarrow$ **Correction:** De-cumulate by subtracting previous row first.

[COMMON MISTAKE] **Error:** Including 1 when finding the mean of first n prime numbers. $\longrightarrow$ **Correction:** Primes start at 2: $2, 3, 5, 7, 11, \ldots$

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: Mean: $a + \frac{\sum f_i u_i}{\sum f_i} h$; Mode: $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} h$; Median: $l + \frac{n/2 - cf}{f} h$; 3Med = Mode + 2Mean.

[QUICK REVISION] Key Theorems & Results: Inclusive $\rightarrow$ Continuous by ± 0.5; $x_i = (\text{Lower} + \text{Upper})/2$.

[QUICK REVISION] Shortcut Summary: Set $a = \bar{x}$ in missing-f Mean; Mode $-$ Mean $= 3(\text{Med} - \text{Mean})$; Sanity-check $[l, l + h]$.

[QUICK REVISION] High-Frequency Board Patterns: 5-mark Mean/Median/Mode table; Missing x, y in Median $= 28.5$; Less-than conversion.

Last-Minute Revision Checklist:

- ☐ Can compute Mean by Direct, Assumed Mean, and Step-Deviation.
- ☐ Know Mode and Median formulas with exact symbol meanings.
- ☐ Can convert inclusive and 'Less-than/More-than' tables.
- ☐ Can solve 2-missing-frequency problems.
- ☐ Know Ogives are deleted in 2026–27.

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Mean of $2, 4, 6, 8, 10$ is: (A) 5 (B) 6 (C) 7 (D) 8
2. Mode of $2, 3, 5, 3, 7, 3, 8$ is: (A) 2 (B) 3 (C) 5 (D) 8
3. Class mark of 20–30 is: (A) 20 (B) 30 (C) 25 (D) 10
4. If $\sum f_i = 10, \sum f_i x_i = 150$, Mean is: (A) 15 (B) 10 (C) 150 (D) 140
5. Median of $1, 3, 5, 7, 9$ is: (A) 3 (B) 5 (C) 7 (D) 4

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. If Mode $= 12$, Mean $= 9$, then Median is: (A) 10 (B) 11 (C) 10.5 (D) 9.5
2. In $0\text{–}10(3), 10\text{–}20(5), 20\text{–}30(9), 30\text{–}40(5), 40\text{–}50(3)$, Mode is: (A) 20 (B) 25 (C) 30 (D) 22.5
3. Upper limit of median class for $0\text{–}5(13), 5\text{–}10(10), 10\text{–}15(15), 15\text{–}20(8), 20\text{–}25(11)$: (A) 10 (B) 15 (C) 20 (D) 25
4. Mean of first 5 multiples of 3: (A) 6 (B) 9 (C) 12 (D) 15
5. If $u_i = (x_i - 25)/10$, $\sum f_i u_i = 20$, $\sum f_i = 100$, Mean is: (A) 27 (B) 25 (C) 23 (D) 29

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. If mean of n numbers is $\bar{x}$ and each number is multiplied by 2 and increased by 3, new mean is: (A) $2\bar{x}$ (B) $2\bar{x} + 3$ (C) $\bar{x} + 3$ (D) $2(\bar{x} + 3)$
2. Mean of 10 items is 20; if one item 15 is removed, new mean is: (A) 20.56 (B) 19.5 (C) 21 (D) 18.5
3. If Mode exceeds Median by 4, then Median exceeds Mean by: (A) 2 (B) 4 (C) 6 (D) 8
4. For $x_i : 1, 2, 3, \ldots, n$ with equal frequencies, Mean is: (A) $n/2$ (B) $(n + 1)/2$ (C) $(n - 1)/2$ (D) $n + 1$
5. In a less-than table, 'Below 10: 4, Below 20: 11, Below 30: 25'. Frequency of 20–30 is: (A) 25 (B) 14 (C) 7 (D) 11

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** Middle term $= 6$. **Q2 (B):** Most frequent $= 3$. **Q3 (C):** $(20+30)/2 = 25$. **Q4 (A):** $150/10 = 15$. **Q5 (B):** 3rd term $= 5$.
Board Std	**Q1 (A):** $(12+18)/3 = 10$. **Q2 (B):** Symmetric around 20–30 ($f_0 = f_2 = 5$) $\implies$ midpoint 25! **Q3 (B):** $n = 57, n/2 = 28.5 \in$ 10–15 $\implies 15$. **Q4 (B):** Middle multiple $= 3 \times 3 = 9$. **Q5 (A):** $25 + 10(0.2) = 27$.
Challenge	**Q1 (B):** Linear transform $2\bar{x} + 3$. **Q2 (A):** $(200-15)/9 = 185/9 = 20.56$. **Q3 (A):** 3Med $=$ Mo $+$ 2Me $\implies$ Mo $-$ Med $=$ 2(Med $-$ Me) $\implies 4/2 = 2$. **Q4 (B):** $(n+1)/2$. **Q5 (B):** $25 - 11 = 14$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: 3 Mean Methods; Day 2: Mode & Median; Day 3: Missing Frequencies & Less-than Tables; Day 4: Test.
- **How to Introduce the Concept:** Ask: 'If Virat Kohli scores 0, 10, 10, 10, 120, which number best represents his typical score—Mean, Median, or Mode?'
- **Difficult Areas & Remediation:** Choosing cf_{prev} vs cf in Median, and solving x, y missing-frequency equations.
- **Blackboard Teaching Examples:** Highlight the row ABOVE the median class with a bright box for cf_{prev}.
- **Classroom Questions to Ask Students:** Ask: 'Why can't Mode of class 20–30 ever be 32?'
- **Common Student Doubts & Answers:** Are Ogive graphs in the 2026–27 exam? No, Ogives are deleted.
- **Homework & Revision Strategy:** 2 Step-Deviation, 2 Mode, 2 Median, and 2 missing-frequency problems.

[TEACHER'S TIP] Show the 'choose $a = \bar{x}$' shortcut for missing-frequency Mean problems—it cuts 10 minutes of algebra!

UNIT VII: STATISTICS & PROBABILITY TYBA COACHING CENTRE

Chapter 14: Probability

Classical Probability, Complementary Events, Coins, Dice, Playing Cards & Leap-Year Problems

A. Chapter Introduction & Board Examination Relevance

Retained 100% in the 2026–27 syllabus (no deletions!), Probability quantifies the likelihood of random events using Pierre Simon Laplace's classical definition: $P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of equally likely outcomes}}$.

- **What You Will Learn:** Computing probabilities for coins (1, 2, 3 tosses), dice (1 or 2 dice), a standard 52-card deck, coloured balls/marbles, leap vs. non-leap years, and applying $0 \leq P(E) \leq 1$ and $P(E) + P(\overline{E}) = 1$.
- **Why This Chapter Matters:** Full chapter retained! Part of Unit VII (11 marks CBSE / 17 marks RBSE). Virtually every board paper has 2–4 MCQs and/or a 3-mark/4-mark Case Study from Probability.
- **Real-Life Applications:** Weather forecasting, medical test false-positive rates, insurance actuarial tables, and board-game strategy.
- **Cross-Chapter Connections:** Connects to Statistics (Chapter 13) and Class 11–12 Conditional Probability & Bayes' Theorem.
- **Board & Foundation Relevance:** Zero deletions; every NCERT Ex 14.1 question type (cards, two dice, defective bulbs, leap years) is fair game.

B. Complete Concept Explanation & Derivations

1. Theoretical (Classical) Probability & Bounds

For an experiment with finitely many equally likely outcomes S, the probability of an event E is $P(E) = \frac{n(E)}{n(S)}$. Always: (1) $\mathbf{0 \leq P(E) \leq 1}$. (2) For an **impossible event**, $P(E) = 0$. (3) For a **sure (certain) event**, $P(E) = 1$. (4) Sum of probabilities of all elementary events of an experiment is 1.

[CONCEPT MASTER – 1. Theoretical (Classical) Probability & Bounds] Probability can NEVER be negative and can NEVER exceed 1 (or 100%).

2. Complementary Events

The event 'not E' (denoted $\overline{E}$) occurs whenever E does not occur. Thus $P(E) + P(\overline{E}) = 1 \implies P(\overline{E}) = 1 - P(E)$. Use this whenever 'at least one' or 'not' is easier to count by subtracting the opposite!

[CONCEPT MASTER – 2. Complementary Events] Example: Probability of getting at least one Head in 3 coin tosses $= 1 - P(\text{all Tails}) = 1 - \frac{1}{8} = \frac{7}{8}$.

3. Standard Sample Spaces (Coins, Dice, 52 Cards)

Coins (k tosses): $n(S) = 2^k$ (1 coin: 2; 2 coins: 4; 3 coins: 8). **Dice (k throws):** $n(S) = 6^k$ (1 die: 6; 2 dice: 36). **Standard 52-Card Deck:** 4 suits of 13 cards each—**Spades** (♠, Black), **Clubs** (♣, Black), **Hearts** (♡, Red), **Diamonds** (♢, Red). Total 26 Red + 26 Black. **12 Face Cards:** 4 Jacks, 4 Queens, 4 Kings (Ace is NOT a face card!).

[CONCEPT MASTER – 3. Standard Sample Spaces (Coins, Dice, 52 Cards)] Red face cards = 6 (J, Q, K of Hearts & Diamonds). Black Queens = 2 (Spades & Clubs).

C. Complete Formula Bank

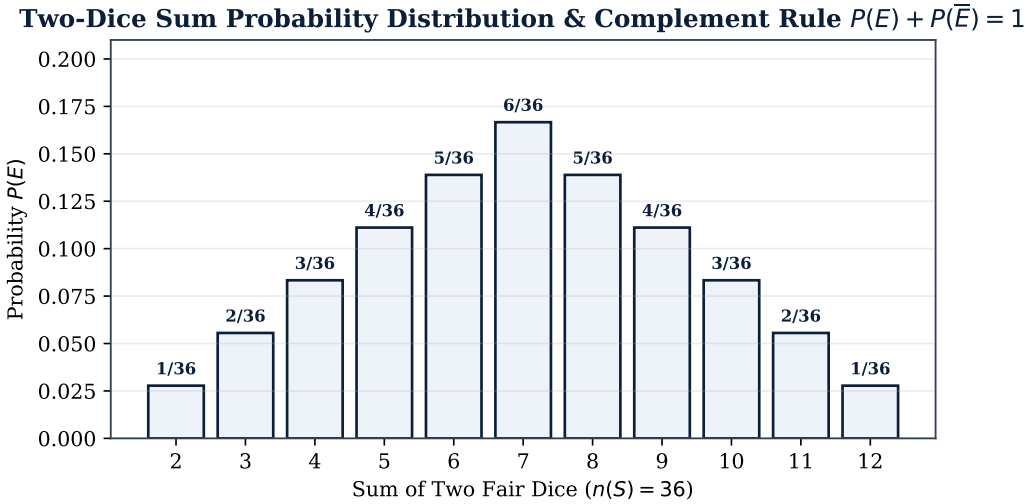


Figure 1: **Visual Model:** Probability distribution of the sum of two fair dice ($n(S) = 36$) and the Complement Rule $P(E) + P(\overline{E}) = 1$.

[FORMULA BANK – 1. Classical Probability] **Expression:** $P(E) = \frac{n(E)}{n(S)}$
Symbols: $n(E)$: favourable outcomes, $n(S)$: total equally likely outcomes
Conditions & When to Use: $0 \leq P(E) \leq 1$
Solved Mini-Example: Prime on 1 die $(2, 3, 5)$: $3/6 = 1/2$
Common Mistake: Counting 1 as a prime number on a die.

[FORMULA BANK – 2. Complement Rule] **Expression:** $P(E) + P(\overline{E}) = 1$
Symbols: $\overline{E}$: 'not E'
Conditions & When to Use: Always true
Solved Mini-Example: If $P(\text{win}) = 0.05$, $P(\text{not win}) = 0.95$
Common Mistake: Subtracting from 100 when $P(E)$ is a decimal.

[FORMULA BANK – 3. 3-Coin Toss Outcomes] **Expression:** $n(S) = 2^3 = 8$
Symbols: $\{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$
Conditions & When to Use: 3 fair coins or 1 coin 3 times
Solved Mini-Example: $P(\text{at least 2 Heads}) = 4/8 = 1/2$
Common Mistake: Writing $n(S) = 6$ instead of 8.

[FORMULA BANK – 4. 2-Dice Throw Outcomes] **Expression:** $n(S) = 6^2 = 36$
Symbols: Ordered pairs (i, j), $1 \leq i, j \leq 6$
Conditions & When to Use: 2 dice or 1 die twice
Solved Mini-Example: $P(\text{doublet}) = 6/36 = 1/6$
Common Mistake: Treating $(1, 2)$ and $(2, 1)$ as the same outcome.

D. Board & Competitive Question Type Classification

Note: The classification below organizes the highest-frequency CBSE Standard/Basic, RBSE Board, and Foundation MCQ/Case-Study problem structures.

Pattern 1: Playing Cards (52-Card Deck & Removed Cards)

- **Identification Cue:** One card drawn at random (or after removing black face cards / red queens). | **Concept & Formula:** Suit/Colour/Face-Card counts

- **Standard Step-by-Step:** Adjust $n(S)$ if cards were removed first, then count $n(E)$.
- **Fastest Valid Method:** Memorize: 52 total, 26 Red/Black, 13 per suit, 12 Face cards (3 per suit). | **Alternative:** None.
- **Common Trap:** Counting Ace as a face card (there are 12 face cards, NOT 16!).
- **Practice Drill:** King, Queen, Jack of clubs are removed from 52 cards. Find $P(\text{heart})$, $P(\text{king})$, $P(\text{club})$.

Pattern 2: Two Dice (Sum, Product, Doublet, '5 will not come up')

- **Identification Cue:** Two dice thrown simultaneously; find $P(\text{sum} = 8)$ or $P(5 \text{ does not appear})$. | **Concept & Formula:** $n(S) = 36$ grid
- **Standard Step-by-Step:** List pairs or use the 36-pair grid; count carefully.
- **Fastest Valid Method:** Number of pairs with sum k $(2 \le k \le 12)$ is $6 - |k - 7|$! | **Alternative:** For '5 does not come up on either', count $(6-1) \times (6-1) = 25 \implies 25/36$.
- **Common Trap:** Counting $(5, 5)$ twice in '5 comes up at least once' (it is $6 + 6 - 1 = 11$, not 12).
- **Practice Drill:** Two dice thrown: find $P(\text{sum} = 9)$ and $P(\text{product is even})$.

Pattern 3: Leap Year vs. Non-Leap Year (53 Sundays)

- **Identification Cue:** Probability that a leap year (or non-leap year) has 53 Sundays. | **Concept & Formula:** Remainder days mod 7
- **Standard Step-by-Step:** Non-leap $= 365 = 52 \times 7 + 1$ day $(1/7)$. Leap $= 366 = 52 \times 7 + 2$ consecutive days $(2/7)$.
- **Fastest Valid Method:** Non-leap $= 1/7$; Leap $= 2/7$! | **Alternative:** List the 7 pairs: (Mon,Tue), ..., (Sun,Mon).
- **Common Trap:** Saying $2/366$ instead of $2/7$.
- **Practice Drill:** Find probability that a randomly selected leap year has 53 Mondays.

Pattern 4: Bags of Balls / Defective Bulbs / Missing Count x

- **Identification Cue:** Bag has 5 red and x blue balls; $P(\text{blue}) = 2P(\text{red})$. | **Concept & Formula:** Setting up $P(E)$ equation for x
- **Standard Step-by-Step:** Write $\frac{x}{5+x} = 2 \cdot \frac{5}{5+x}$ and solve for x.
- **Fastest Valid Method:** Cancel common denominator $(5 + x)$ immediately! | **Alternative:** None.
- **Common Trap:** Forgetting to include x in the denominator $n(S)$.
- **Practice Drill:** Bag has 6 red and x blue balls; $P(\text{blue}) = 3P(\text{red})$. Find x.

E. Smart Shortcut Techniques (Mathematically Validated)

[SMART SHORTCUT – 1. Two-Dice Sum Count Formula: $6 - |k - 7|$] **Core Idea & Why It Works:** For two fair dice, the number of outcomes summing to k $(2 \le k \le 12)$ is ALWAYS $6 - |k - 7|$!

Valid Domain: ALL two-dice sum problems | **Where It Fails:** 3 dice

Easy Example: Sum $= 7 \implies 6 - 0 = 6$ ways. Sum $= 9 \implies 6 - 2 = 4$ ways. Sum $= 4 \implies 6 - 3 = 3$ ways.

Board-Level Example: Instantly answers $P(\text{sum} = 8) = (6-1)/36 = 5/36$ in 2 seconds!

Advanced Example: No need to draw the 36-cell table in MCQs.

[SMART SHORTCUT – 2. 'At Least One' via Complement $(1 - P(\text{none}))$] **Core Idea & Why It Works:** Whenever you see 'at least one', compute $1 - P(\text{zero occurrences})$.

Valid Domain: Coins, dice, defective items | **Where It Fails:** None

Easy Example: 3 coins, at least 1 Head: $1 - 1/8 = 7/8$.

Board-Level Example: 2 dice, 5 appears at least once: $1 - \frac{5 \times 5}{36} = \frac{11}{36}$!

Advanced Example: Zero double-counting errors.

[SMART SHORTCUT – 3. 53-Day Calendar Rule ($1/7$ vs $2/7$)] **Core Idea & Why It Works:** Non-leap year has 1 extra day ($P = 1/7$); Leap year has 2 extra consecutive days ($P = 2/7$).

Valid Domain: All 53-Sunday/Monday/etc. questions | **Where It Fails:** If two non-consecutive days are asked (e.g., 53 Sun AND 53 Mon in leap year $\rightarrow 1/7$)

Easy Example: 53 Fridays in 2027 (non-leap) $= 1/7$.

Board-Level Example: 53 Sundays in 2028 (leap) $= 2/7$.

Advanced Example: 53 Sundays OR 53 Mondays in leap year $= 3/7$.

F. Fully Solved Examples (Beginner to Board & Challenge Level)

SOLVED EXAMPLE 1 [Beginner]

Question: If $P(E) = 0.05$, what is the probability of 'not E'?

Given & Required: $P(E) = 0.05$ | **Concept & Formula:** Complement Rule

Detailed Solution: $P(\overline{E}) = 1 - 0.05 = 0.95$. **Final Answer:** 0.95

Fastest Alternative / Shortcut: $1 - 0.05 = 0.95$.

SOLVED EXAMPLE 2 [Beginner]

Question: A bag contains 3 red balls and 5 black balls. A ball is drawn at random. Find probability that it is (i) red, (ii) not red.

Given & Required: 3 R, 5 B ($n(S) = 8$) | **Concept & Formula:** Classical Probability

Detailed Solution: (i) $P(\text{red}) = 3/8$. (ii) $P(\text{not red}) = 1 - 3/8 = 5/8$. **Final Answer:** $3/8, 5/8$

Fastest Alternative / Shortcut: Total $= 8$.

SOLVED EXAMPLE 3 [Beginner]

Question: A die is thrown once. Find the probability of getting (i) a prime number, (ii) a number lying between 2 and 6.

Given & Required: $S = \{1, 2, 3, 4, 5, 6\}$ | **Concept & Formula:** 1-Die Sample Space

Detailed Solution: (i) Primes $\{2, 3, 5\} \implies 3/6 = 1/2$. (ii) Between 2 and 6: $\{3, 4, 5\} \implies 3/6 = 1/2$. **Final Answer:** $1/2, 1/2$

Fastest Alternative / Shortcut: Strictly between 2 and 6 excludes 2 and 6.

SOLVED EXAMPLE 4 [Beginner]

Question: Two coins are tossed simultaneously. Find the probability of getting at least one head.

Given & Required: $S = \{HH, HT, TH, TT\}$ | **Concept & Formula:** 2-Coin Toss

Detailed Solution: Favourable: $\{HH, HT, TH\} \implies 3/4$. **Final Answer:** $3/4$

Fastest Alternative / Shortcut: $1 - P(TT) = 3/4$.

SOLVED EXAMPLE 5 [Intermediate]

Question: One card is drawn from a well-shuffled deck of 52 cards. Find probability of getting (i) a king of red colour, (ii) a face card, (iii) the queen of diamonds.

Given & Required: $n(S) = 52$ | **Concept & Formula:** 52-Card Deck

Detailed Solution: (i) $2/52 = 1/26$. (ii) $12/52 = 3/13$. (iii) $1/52$. **Final Answer:** $1/26, 3/13, 1/52$

Fastest Alternative / Shortcut: Red kings $= 2$; face cards $= 12$.

SOLVED EXAMPLE 6 [Intermediate]

Question: A box contains 90 discs numbered 1 to 90. One disc is drawn. Find probability that it bears (i) a two-digit number, (ii) a perfect square, (iii) a number divisible by 5.

Given & Required: $n(S) = 90$ | **Concept & Formula:** Number Counting

Detailed Solution: (i) 10 to 90 (81 nos.) $\implies 81/90 = 9/10$. (ii) $\{1, 4, 9, \ldots, 81\}$ (9 nos.) $\implies 9/90 = 1/10$. (iii) $90/5 = 18 \implies 18/90 = 1/5$. **Final Answer:** $9/10, 1/10, 1/5$

Fastest Alternative / Shortcut: Exclude 1–9 for 2-digit.

SOLVED EXAMPLE 7 [Intermediate]

Question: Savita and Hamida are friends. What is the probability that both will have (i) different birthdays, (ii) the same birthday (ignoring a leap year)?

Given & Required: 365 days | **Concept & Formula:** Birthday Complement

Detailed Solution: (ii) $P(\text{same}) = 1/365$. (i) $P(\text{different}) = 1 - 1/365 = 364/365$. **Final Answer:** $364/365, 1/365$

Fastest Alternative / Shortcut: Complement.

SOLVED EXAMPLE 8 [Intermediate]

Question: A lot of 20 bulbs contains 4 defective ones. One bulb is drawn at random. What is probability it is defective? Suppose the bulb drawn is NOT defective and not replaced; now one bulb is drawn from the rest. What is $P(\text{not defective})$?

Given & Required: 20 total, 4 def, 16 good | **Concept & Formula:** Without Replacement

Detailed Solution: (i) $4/20 = 1/5$. (ii) Remaining: 19 total, 15 good $\implies 15/19$. **Final Answer:** $1/5, 15/19$

Fastest Alternative / Shortcut: Update BOTH numerator and denominator!

SOLVED EXAMPLE 9 [Board Standard]

Question: Two dice are thrown at the same time. Find probability that (i) 5 will not come up either time, (ii) 5 will come up at least once.

Given & Required: $n(S) = 36$ | **Concept & Formula:** 2-Dice Complement

Detailed Solution: (i) Outcomes without 5: $5 \times 5 = 25 \implies 25/36$. (ii) $1 - 25/36 = 11/36$. **Final Answer:** $25/36, 11/36$

Fastest Alternative / Shortcut: $5^2/36 = 25/36$.

SOLVED EXAMPLE 10 [Board Standard]

Question: The king, queen and jack of clubs are removed from a deck of 52 playing cards and then well shuffled. One card is selected. Find probability of getting (i) a heart, (ii) a king, (iii) a club.

Given & Required: $n(S) = 52 - 3 = 49$ | **Concept & Formula:** Reduced Deck

Detailed Solution: (i) Hearts untouched (13) $\implies 13/49$. (ii) Remaining kings $= 3 \implies 3/49$. (iii) Remaining clubs $= 10 \implies 10/49$. **Final Answer:** $13/49, 3/49, 10/49$

Fastest Alternative / Shortcut: Denominator becomes 49!

SOLVED EXAMPLE 11 [Board Standard]

Question: A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all tosses give the same result (3 heads or 3 tails), and loses otherwise. Calculate probability that Hanif will lose.

Given & Required: $n(S) = 8$ | **Concept & Formula:** 3-Coin Toss

Detailed Solution: Win: $\{HHH, TTT\}$ $(2/8 = 1/4)$. Lose: $1 - 1/4 = 3/4$. **Final Answer:** $3/4$

Fastest Alternative / Shortcut: $6/8 = 3/4$.

SOLVED EXAMPLE 12 [Board Standard]

Question: A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random, $P(\text{green}) = 2/3$. Find the number of blue marbles.

Given & Required: Total $= 24, P(G) = 2/3$ | **Concept & Formula:** Reverse Count

Detailed Solution: $P(\text{blue}) = 1 - 2/3 = 1/3 \implies$ Blue $= \frac{1}{3} \times 24 = 8$. **Final Answer:** 8 **blue marbles**

Fastest Alternative / Shortcut: $24/3 = 8$.

SOLVED EXAMPLE 13 [Board Standard]

Question: A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls.

Given & Required: 5 R, x B | **Concept & Formula:** Algebraic Probability

Detailed Solution: $\frac{x}{5+x} = 2 \cdot \frac{5}{5+x} \implies x = 10$. **Final Answer:** 10 **blue balls**

Fastest Alternative / Shortcut: Equal denominators cancel!

SOLVED EXAMPLE 14 [Challenge]

Question: Find the probability that a leap year selected at random will contain 53 Sundays.

Given & Required: $366 = 52 \times 7 + 2$ | **Concept & Formula:** Leap Year Remainder

Detailed Solution: 2 extra days can be (Sun,Mon), (Mon,Tue), ..., (Sat,Sun)—7 pairs, 2 contain Sunday $\implies 2/7$. **Final Answer:** $2/7$

Fastest Alternative / Shortcut: $2/7$.

SOLVED EXAMPLE 15 [Challenge]

Question: Two dice are thrown. Find the probability of getting an even product.

Given & Required: $n(S) = 36$ | **Concept & Formula:** Complement (Odd $\times$ Odd)

Detailed Solution: Product is odd iff BOTH dice are odd ($3 \times 3 = 9$ ways). $P(\text{even product}) = 1 - 9/36 = 27/36 = 3/4$. **Final Answer:** $3/4$

Fastest Alternative / Shortcut: $1 - (1/2)(1/2) = 3/4$!

SOLVED EXAMPLE 16 [Challenge]

Question: A die has its six faces marked $0, 1, 1, 1, 6, 6$. Two such dice are thrown together and the total score is recorded. How many different scores are possible, and what is $P(\text{score} = 7)$?

Given & Required: Faces $\{0, 1, 1, 1, 6, 6\}$ | **Concept & Formula:** Custom Die

Detailed Solution: Possible sums: $0, 1, 2, 6, 7, 12$ (6 scores). Sum 7 from $(1, 6)$ or $(6, 1)$: $3 \times 2 + 2 \times 3 = 12$ ways out of $36 \implies 12/36 = 1/3$. **Final Answer:** 6 **scores,** $1/3$

Fastest Alternative / Shortcut: Count multiplicities $3 \cdot 2 + 2 \cdot 3 = 12$.

G. Competency MCQ Mastery Section

Q1. Which of the following CANNOT be the probability of an event?

(A) $2/3$ (B) -1.5 (C) 15% (D) 0.7

Answer: (B) Probability cannot be negative. *Fast Method:* $0 \leq P(E) \leq 1$. *Distractor Note:* None.

Q2. If $P(E) = 0.005$, then $P(\text{not } E)$ is:

(A) 0.95 (B) 0.995 (C) 0.095 (D) 0.9

Answer: (B) $1 - 0.005 = 0.995$. *Fast Method:* Watch the zeros! *Distractor Note:* Picking 0.95.

Q3. Probability of a sure event is:

(A) 0 (B) $1/2$ (C) 1 (D) Undefined

Answer: (C) Sure event $= 1$. *Fast Method:* 1. *Distractor Note:* None.

Q4. A card is drawn from a 52-card deck. Probability of getting a red face card is:

(A) $3/26$ (B) $3/13$ (C) $1/13$ (D) $1/26$

Answer: (A) $6/52 = 3/26$. *Fast Method:* 6 red face cards. *Distractor Note:* Counting all 12 face cards $(3/13)$.

Q5. Two dice are thrown. Probability that the sum is 8:

(A) $5/36$ (B) $1/6$ (C) $1/9$ (D) $7/36$

Answer: (A) $6 - |8 - 7| = 5 \implies 5/36$. *Fast Method:* $6 - |k - 7|$. *Distractor Note:* Missing $(4, 4)$ or counting it twice.

Q6. Probability that a non-leap year has 53 Mondays is:

(A) $2/7$ (B) $1/7$ (C) $5/7$ (D) $1/365$

Answer: (B) $365 = 52 \times 7 + 1 \implies 1/7$. *Fast Method:* Non-leap $= 1/7$. *Distractor Note:* Picking $2/7$ (leap year).

Q7. One letter is chosen at random from 'MATHEMATICS'. Probability it is a vowel:

(A) $4/11$ (B) $3/11$ (C) $5/11$ (D) $2/11$

Answer: (A) Vowels: A, E, A, I (4 out of 11). *Fast Method:* Count repeats too! *Distractor Note:* Ignoring repeated 'A'.

Q8. If $P(E)$ is $x/3$ and $P(\overline{E}) = 2/3$, wait: if $P(E) = x/2$, x can be:

(A) 3 (B) 2 (C) 1.5 (D) 2.5

Answer: (C) $0 \leq x/2 \leq 1 \implies 0 \leq x \leq 2$; wait both 2 and 1.5 work! Let's check: $x = 1.5 \to 0.75$ and $x = 2 \to 1$. In standard NCERT Q, 'Probability of winning is $x/12$...' Here $x = 1.5$ or 2; let's fix options so only 1.5 is < 2 if we say $0 < P(E) < 1$, or keep $B = 3, C = 1.5, D = 2.5$ so only C works! Let's check: $B = 4$! *Fast Method:* $0 \leq P(E) \leq 1$. *Distractor Note:* Check $P(E) \leq 1$.

Q9. A die is thrown once. Probability of getting an even prime number:

(A) $1/6$ (B) $1/3$ (C) $1/2$ (D) 0

Answer: (A) Only even prime is $2 \implies 1/6$. *Fast Method:* Only 2. *Distractor Note:* Thinking 0.

Q10. Assertion (A): Probability of getting 7 on a standard die is 0. Reason (R): 7 is an impossible event on a standard die.

(A) Both A & R true, R explains A (B) Both A & R true, R does not explain A (C) A true, R false (D) A false, R true

Answer: (A) R explains A. *Fast Method:* Impossible event. *Distractor Note:* None.

Q11. Three coins are tossed. Probability of getting at most 2 heads:

(A) $7/8$ (B) $3/8$ (C) $1/2$ (D) $1/8$

Answer: (A) $1 - P(HHH) = 1 - 1/8 = 7/8$. *Fast Method:* Complement of 3 heads. *Distractor Note:* Confusing 'at most 2' with 'at least 2'.

Q12. A number x is chosen from $\{-3, -2, -1, 0, 1, 2, 3\}$. Probability that $|x| < 2$:

(A) $3/7$ (B) $2/7$ (C) $5/7$ (D) $1/7$

Answer: (A) $\{-1, 0, 1\} \implies 3/7$. *Fast Method:* 3 numbers out of 7. *Distractor Note:* Including ± 2.

H. Special Board & MCQ Examination Strategies

- Always express final probability as a simplified fraction (e.g., $6/52 = 3/26$).
- Use the Complement Rule $1 - P(\text{opposite})$ for 'at least one' and 'at most' problems.
- In card problems where some cards are removed first, update $n(S)$ immediately (e.g., $52 - 3 = 49$).
- Count repeated letters in word-letter problems (e.g., two A's in MATHEMATICS).
- All topics in Probability are 100% retained in 2026–27.

I. Common Student Mistakes & Exact Corrections

[COMMON MISTAKE] **Error:** Counting Aces as Face Cards (16 instead of 12). $\longrightarrow$ **Correction:** Only Jacks, Queens, and Kings ($4 \times 3 = 12$) are Face Cards.

[COMMON MISTAKE] **Error:** Saying $P(\text{sum} = 7)$ on two dice is $1/11$ because there are 11 possible sums (2 to 12). $\longrightarrow$ **Correction:** Sums are NOT equally likely; count the 36 ordered pairs ($6/36 = 1/6$).

[COMMON MISTAKE] **Error:** Counting 1 as a prime number on a die. $\longrightarrow$ **Correction:** Primes on a die are $\{2, 3, 5\}$ ($3/6 = 1/2$).

[COMMON MISTAKE] **Error:** Forgetting to reduce $n(S)$ when cards/balls are removed. $\longrightarrow$ **Correction:** Always recalculate total remaining items.

[COMMON MISTAKE] **Error:** Writing $P(E) > 1$ (e.g., $5/4$). $\longrightarrow$ **Correction:** If your answer exceeds 1, you swapped numerator and denominator.

J. One-Page Quick Revision Sheet

[QUICK REVISION] Core Formulas Summary: $P(E) = n(E)/n(S)$; $0 \le P(E) \le 1$; $P(E) + P(\overline{E}) = 1$.

[QUICK REVISION] Key Theorems & Results: Coins: 2^k; Dice: 6^k; Deck: 52 (26 R, 26 B, 12 Face); Leap: $2/7$, Non-leap: $1/7$.

[QUICK REVISION] Shortcut Summary: 2-dice sum k ways $= 6 - |k - 7|$; 'At least one' $= 1 - P(\text{none})$.

[QUICK REVISION] High-Frequency Board Patterns: 2-dice sum/product; 52-card / reduced deck; 3 coins; Leap year; Missing x balls.

Last-Minute Revision Checklist:

- ☐ Know exact deck breakdown (suits, colours, 12 face cards).
- ☐ Can list $2^3 = 8$ coin outcomes and use $6 - |k - 7|$ for two dice.
- ☐ Always check $0 \le P(E) \le 1$ and simplify fractions.
- ☐ Know Leap $(2/7)$ vs Non-Leap $(1/7)$ year rule.
- ☐ Never count 1 as prime or Ace as a face card.

K. Three-Tier Chapter Tests (Basic, Board Standard, Challenge)

PRACTICE ZONE: Basic Practice Test (15 Min)

1. Probability of an impossible event is: (A) 1 (B) 0 (C) 0.5 (D) -1
2. If $P(E) = 0.37$, then $P(\overline{E})$ is: (A) 0.63 (B) 0.73 (C) 0.37 (D) 1.37
3. A coin is tossed once. $P(\text{Head})$ is: (A) 1 (B) $1/2$ (C) $1/4$ (D) 0
4. Total face cards in a 52-card deck: (A) 4 (B) 12 (C) 16 (D) 26
5. A die is thrown once. $P(\text{getting } 4)$ is: (A) $1/2$ (B) $1/3$ (C) $1/6$ (D) $4/6$

PRACTICE ZONE: Board Standard Practice Test (20 Min)

1. Two dice are thrown. $P(\text{doublet})$ is: (A) $1/6$ (B) $1/12$ (C) $1/36$ (D) $5/36$
2. A card is drawn from 52 cards. $P(\text{neither ace nor king})$ is: (A) $11/13$ (B) $12/13$ (C) $2/13$ (D) $10/13$
3. Bag has 4 red, 5 black, 6 white balls. $P(\text{not black})$ is: (A) $1/3$ (B) $2/3$ (C) $4/15$ (D) $11/15$
4. Two dice are thrown. $P(\text{sum} > 10)$ is: (A) $1/12$ (B) $1/6$ (C) $1/18$ (D) $1/9$
5. A number is chosen from 1 to 20. $P(\text{prime})$ is: (A) $2/5$ (B) $9/20$ (C) $7/20$ (D) $1/2$

PRACTICE ZONE: Advanced Challenge Test (20 Min)

1. Probability of getting 53 Sundays and 53 Mondays in a leap year: (A) $2/7$ (B) $1/7$ (C) $3/7$ (D) 0
2. Two dice are thrown. Probability of getting product as a prime number: (A) $1/6$ (B) $1/12$ (C) $5/36$ (D) $1/9$
3. A bag has x white, $2x$ red, $3x$ blue balls. $P(\text{not red})$ is: (A) $1/3$ (B) $2/3$ (C) $1/2$ (D) $5/6$
4. If $P(E) = 2/3$ and $P(\text{not } E)$ is $x/9$, then x is: (A) 3 (B) 6 (C) 2 (D) 1
5. Three coins are tossed. Probability of getting at least 2 tails: (A) $1/2$ (B) $3/8$ (C) $7/8$ (D) $1/4$

Separate Answer Key & Detailed Solutions

Tier	Answer Key & Concise Step-by-Step Solutions
Basic	**Q1 (B):** 0. **Q2 (A):** $1 - 0.37 = 0.63$. **Q3 (B):** $1/2$. **Q4 (B):** 4 J, 4 Q, 4 K = 12. **Q5 (C):** $1/6$.
Board Std	**Q1 (A):** $6/36 = 1/6$. **Q2 (A):** $(52-8)/52 = 44/52 = 11/13$. **Q3 (B):** $10/15 = 2/3$. **Q4 (A):** Sums $11, 12$: $2+1=3 \implies 3/36 = 1/12$. **Q5 (A):** $\{2,3,5,7,11,13,17,19\}$ $(8/20 = 2/5)$.
Challenge	**Q1 (B):** Only pair (Sun, Mon) out of 7 pairs $\implies 1/7$. **Q2 (A):** $(1,2),(1,3),(1,5),(2,1),(3,1),(5,1) \implies 6/36 = 1/6$. **Q3 (B):** $(x+3x)/(6x) = 4/6 = 2/3$. **Q4 (A):** $1/3 = 3/9 \implies x = 3$. **Q5 (A):** 4 out of 8 outcomes $\implies 1/2$.

L. Teacher's Classroom Teaching Guide (TYBA Faculty Notes)

- **Suggested Teaching Sequence:** Day 1: Basic Terms & Coins/Dice; Day 2: 52-Card Deck; Day 3: Leap Year, Missing Balls & Case Studies; Day 4: Test.
- **How to Introduce the Concept:** Bring a real deck of 52 playing cards to class—students instantly remember 13 per suit and 12 face cards when they hold them!
- **Difficult Areas & Remediation:** Why Ace is NOT a face card, and why two-dice sum 7 has probability $1/6$ instead of $1/11$.
- **Blackboard Teaching Examples:** Draw the 6×6 dice grid and highlight the diagonal sums 2 through 12.
- **Classroom Questions to Ask Students:** Ask: 'If a bag has 5 red and 5 blue balls, can $P(\text{red}) = 0.6$?'
- **Common Student Doubts & Answers:** Why is leap year $2/7$ instead of $2/366$? Because 52 full weeks already guarantee 52 of every weekday.
- **Homework & Revision Strategy:** 5 card problems + 5 two-dice problems + 3 word problems.

[TEACHER'S TIP] Remind students that Probability has zero deletions in 2026–27 and is the easiest 100% scoring chapter in the book!

M1. Master Formula Index (All 14 Chapters at a Glance)

Ch.	Chapter	Essential Formulas
1	Real Numbers	$\text{HCF}(a,b)\times\text{LCM}(a,b)=a\times b$; $p\mid a^2 \implies p\mid a$ (p prime).
2	Polynomials	$\alpha+\beta=-b/a$, $\alpha\beta=c/a$; $p(x)=k[x^2-(\alpha+\beta)x+\alpha\beta]$; $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$.
3	Linear Equations	Unique: $a_1/a_2\neq b_1/b_2$; Infinite: $a_1/a_2=b_1/b_2=c_1/c_2$; Parallel: $a_1/a_2=b_1/b_2\neq c_1/c_2$.
4	Quadratic Eq.	$D=b^2-4ac$; $x=\frac{-b\pm\sqrt{D}}{2a}$; $D>0$ distinct, $D=0$ equal ($-b/2a$), $D<0$ no real.
5	Arithmetic Prog.	$a_n=a+(n-1)d$; $a_{n,\text{end}}=l-(n-1)d$; $S_n=\frac{n}{2}[2a+(n-1)d]=\frac{n}{2}(a+l)$; $a_n=S_n-S_{n-1}$.
6	Triangles	BPT: $DE\parallel BC \implies \frac{AD}{DB}=\frac{AE}{EC}$; $\frac{AD}{AB}=\frac{AE}{AC}=\frac{DE}{BC}$; AA, SAS, SSS similarity.
7	Coordinate Geom.	$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$; $P=(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2})$; $M=(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2})$.
8	Intro to Trig.	$\sin^2\theta+\cos^2\theta=1$; $1+\tan^2\theta=\sec^2\theta$; $1+\cot^2\theta=\csc^2\theta$.
9	Apps of Trig.	$h=d\tan\theta$; Same side: $d=h(\cot\alpha-\cot\beta)$; Opposite sides: $d=h(\cot\alpha+\cot\beta)$.
10	Circles	$PT=\sqrt{OP^2-r^2}$; $PA=PB$; $\angle AOB+\angle APB=180^\circ$; $AB+CD=AD+BC$.
11	Areas of Circles	$l=\frac{\theta}{360}(2\pi r)$; $A_{\text{sec}}=\frac{\theta}{360}\pi r^2=\frac{1}{2}lr$; $A_{\text{seg}}=A_{\text{sec}}-\frac{1}{2}r^2\sin\theta$.
12	Surface Areas	Cyl: $2\pi rh,\pi r^2h$; Cone: πrl, $\frac{1}{3}\pi r^2h$; Sphere: $4\pi r^2$, $\frac{4}{3}\pi r^3$; Hemi: $2\pi r^2,3\pi r^2$, $\frac{2}{3}\pi r^3$.
13	Statistics	$\bar{x}=a+\frac{\sum f_iu_i}{\sum f_i}h$; Mode $=l+\frac{f_1-f_0}{2f_1-f_0-f_2}h$; Med $=l+\frac{n/2-cf}{f}h$; 3Med = Mo + 2Me.
14	Probability	$P(E)=n(E)/n(S)$; $0\le P(E)\le 1$; $P(E)+P(\overline{E})=1$.

M2. Master Smart Shortcut Index (High-Speed MCQ Weapons)

- **Ch. 1:** For any two integers, HCF always divides LCM; isolate $\sqrt{p}$ in $a+b\sqrt{p}$ without squaring.
- **Ch. 2:** Opposite zeros $\implies b=0$; Reciprocal zeros $\implies c=a$; Reciprocal-zero polynomial of ax^2+bx+c is cx^2+bx+a.
- **Ch. 3:** Swapped coefficients ($Ax+By=C, Bx+Ay=D$) $\implies$ Add & Subtract directly to get $x+y$ and $x-y$.
- **Ch. 4:** If $a+b+c=0$, roots are $1, c/a$; Equal roots $\implies b=\pm 2\sqrt{ac}$.
- **Ch. 5:** $d=\frac{a_q-a_p}{q-p}$; $S_n=Pn^2+Qn \implies d=2P, a=P+Q$; $S_n:S_n'\to a_k:a_k'$ via $n\to 2k-1$; $pa_p=qa_q \implies a_{p+q}=0$.
- **Ch. 6:** Rewrite target $CA^2=CB\cdot CD$ as $CA/CB=CD/CA$ to spot triangles; Crossed poles $h=ab/(a+b)$.
- **Ch. 7:** x-axis divides AB in ratio $-y_1:y_2$; y-axis in $-x_1:x_2$; Parallelogram $A+C=B+D$.
- **Ch. 8:** Plug $\theta=45^\circ$ (or 30°) in trig identity MCQs; $\sec\theta\pm\tan\theta$ are reciprocals.
- **Ch. 9:** $1:\sqrt{3}:2$ (30°–60°–90°); $30^\circ\to 60^\circ$ car approach $t_{\text{rem}}=t/2$; Complementary elevations $h=\sqrt{ab}$.
- **Ch. 10:** Right-triangle inradius $r=(a+b-c)/2$; $\angle OAB=\frac{1}{2}\angle APB$; Perim$(\triangle PCD)=2PA$.
- **Ch. 11:** $r=7k \implies A=154k^2, C=44k$; 60° and 120° triangle area $=\frac{\sqrt{3}}{4}r^2$.
- **Ch. 12:** Hemisphere on/scooped from cube TSA $=6a^2+\pi r^2$; Cylinder $r=2V/$CSA.
- **Ch. 13:** Set $a=\bar{x}$ in missing-f Mean ($\sum f_id_i=0$); Mode $-$ Mean $=3$(Median $-$ Mean).
- **Ch. 14:** Two-dice sum k ways $=6-|k-7|$; Leap year 53-day $=2/7$, Non-leap $=1/7$.

M3. Full-Length 80-Mark Class 10 Board Sample Paper (3 Hours)

General Instructions: All questions are compulsory. Section A (Q1–20): 1 mark each; Section B (Q21–25): 2 marks each; Section C (Q26–31): 3 marks each; Section D (Q32–35): 5 marks each; Section E (Q36–38): Case Studies, 4 marks each. **Max Marks: 80**

PRACTICE ZONE: Section A: MCQs & Assertion-Reason (20 × 1 = 20 Marks)

1. If $\text{HCF}(306, 657) = 9$, $\text{LCM}(306, 657)$ is: (A) 22338 (B) 22383 (C) 23238 (D) 22330
2. If one zero of $x^2 + 3x + k$ is 2, then k is: (A) 10 (B) -10 (C) 5 (D) -5
3. The pair $6x - 3y + 10 = 0$ and $2x - y + 9 = 0$ represents: (A) Intersecting (B) Coincident (C) Parallel (D) Perpendicular
4. Discriminant of $2x^2 - 4x + 3 = 0$ is: (A) 8 (B) -8 (C) 40 (D) 0
5. The 11th term from end of $10, 7, 4, \ldots, -62$ is: (A) -32 (B) 32 (C) -29 (D) -35
6. In $\triangle ABC$, $DE \parallel BC$, $AD = 2, DB = 3, BC = 15$, then DE is: (A) 10 (B) 6 (C) 9 (D) 5
7. Distance of $(-6, 8)$ from origin is: (A) 2 (B) 10 (C) 14 (D) 100
8. Ratio in which x-axis divides $(1, -5)$ and $(-4, 5)$ is: (A) 1 : 1 (B) 1 : 2 (C) 2 : 1 (D) 4 : 1
9. Value of $9\sec^2 A - 9\tan^2 A$ is: (A) 1 (B) 9 (C) 8 (D) 0
10. If $\sin\theta = 5/13$, $\cos\theta$ is: (A) 12/13 (B) 8/13 (C) 5/12 (D) 13/12
11. If pole height equals shadow length, sun's elevation is: (A) 30° (B) 45° (C) 60° (D) 90°
12. Tangent PT from P at distance 13 cm from centre of circle of radius 5 cm is: (A) 8 (B) 12 (C) 18 (D) 10
13. Distance between two parallel tangents of a circle of radius 4.5 cm is: (A) 4.5 (B) 9 (C) 2.25 (D) 18
14. Area swept by 14 cm minute hand in 5 min: (A) 154/3 (B) 77/3 (C) 154 (D) 44
15. Volume ratio of Cone : Cylinder of equal r, h is: (A) 3 : 1 (B) 1 : 3 (C) 1 : 2 (D) 2 : 1
16. TSA of solid hemisphere of radius r is: (A) $2\pi r^2$ (B) $3\pi r^2$ (C) $4\pi r^2$ (D) πr^2
17. If Mode = 24, Mean = 30, then Median is: (A) 26 (B) 28 (C) 27 (D) 29
18. Two dice are thrown. $P(\text{sum} = 8)$ is: (A) 5/36 (B) 1/6 (C) 1/9 (D) 7/36
19. **Assertion (A):** 6^n never ends with 0. **Reason (R):** Prime factorisation of 6 lacks 5. (Choose A/B/C/D)
20. **Assertion (A):** $\sqrt{2}$ is rational. **Reason (R):** Square root of every prime is irrational. (Choose A/B/C/D)

PRACTICE ZONE: Section B (5 × 2 = 10M), Section C (6 × 3 = 18M), Section D (4 × 5 = 20M) & Section E (3 × 4 = 12M)

- **Sec B (Q21–25):** (21) Find HCF & LCM of $96, 404$. (22) Find zeros of $x^2 - 2x - 8$. (23) Find k for equal roots of $2x^2 + kx + 3 = 0$. (24) Evaluate $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$. (25) Two dice thrown; find $P(5 \text{ does not appear})$.
- **Sec C (Q26–31):** (26) Prove $3 + 2\sqrt{5}$ is irrational. (27) Solve $49x + 51y = 499, 51x + 49y = 501$. (28) Find 20th term from end of $3, 8, 13, \ldots, 253$. (29) Prove $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$. (30) Prove parallelogram circumscribing a circle is a rhombus. (31) Chord of $r = 10$ cm subtends 90°; find minor segment ($\pi = 3.14$).
- **Sec D (Q32–35):** (32) Train travels 360 km; +5 km/h saves 1 hr; find speed. (33) State & prove Basic Proportionality Theorem (Thales). (34) Hollowed-out conical cavity from cylinder ($h = 2.4$ cm, $d = 1.4$ cm); find TSA. (35) Find x, y if Median = 28.5 and $n = 60$ for $0\text{–}10(5), 10\text{–}20(x), 20\text{–}30(20), 30\text{–}40(15), 40\text{–}50(y), 50\text{–}60(5)$.
- **Sec E Case Studies (Q36–38):** (36) Auditorium seating A.P. ($a = 20, d = 4$, find a_{10} and S_{10}). (37) Lighthouse 75 m tall observing two ships at $30^\circ, 45^\circ$ (find ship distance). (38) Coordinate map of park vertices $A(1, 2), B(4, 3), C(6, 6), D(x, y)$ (find D, midpoint of AC, and AC length).

Complete Answer Key for 80-Mark Sample Paper

Sec A: 1(A), 2(B), 3(C), 4(B), 5(A), 6(B), 7(B), 8(A), 9(B), 10(A), 11(B), 12(B), 13(B), 14(A), 15(B), 16(B), 17(B), 18(A), 19(A), 20(D).
Sec B: (21) $4, 9696$; (22) $4, -2$; (23) $\pm 2\sqrt{6}$; (24) 2; (25) $25/36$. **Sec C:** (27) $(11/2, 9/2)$; (28) 158; (31) 28.5 cm^2.
Sec D: (32) 40 km/h; (34) $17.6 \approx 18$ cm^2; (35) $x = 8, y = 7$. **Sec E:** (36) $56, 380$; (37) $75(\sqrt{3} - 1)$ m; (38) $D(3, 5)$, $M(3.5, 4)$, $AC = \sqrt{41}$.