Part 1: Factors, Multiples & Divisibility Basics (गुणनखंड, गुणज व विभाज्यता) — Foundation

1.1 Factors vs Multiples: A **Factor (गुणनखंड)** of a number divides it completely without leaving a remainder (e.g., factors of 12 are 1, 2, 3, 4, 6, 12). A **Multiple (गुणज)** is obtained by multiplying the number by 1, 2, 3... (multiples of 12 are 12, 24, 36...). Every number is a factor and multiple of itself; **1** is a factor of every number.

गुणनखंड बनाम गुणज: किसी संख्या को पूरा-पूरा विभाजित करने वाली संख्याएँ उसके गुणनखंड कहलाती हैं (सीमित संख्या), जबकि उसके पहाड़े में आने वाली संख्याएँ गुणज कहलाती हैं (असीमित)। 1 प्रत्येक संख्या का गुणनखंड है।



Figure 2: Three Families of Divisibility Rules (विभाज्यता नियमों के तीन प्रमुख समूह)

Example 1: List all factors of 24.
Solution: 1×24, 2×12, 3×8, 4×6 ⇒ Factors = **1, 2, 3, 4, 6, 8, 12, 24** (8 factors).

24 के सभी गुणनखंड = **1, 2, 3, 4, 6, 8, 12, 24**।

Example 2: Write the first five multiples of 13.
Solution: 13×1, 13×2, 13×3, 13×4, 13×5 = **13, 26, 39, 52, 65**.

13 के प्रथम पाँच गुणज = **13, 26, 39, 52, 65**।

Example 3: Check whether 678 is divisible by 2, 3, and 6.
Solution: Last digit is 8 (even ⇒ div. by 2). Digit sum = 6+7+8 = 21 (div. by 3). Since divisible by both 2 and 3, it is divisible by **6**.

अंतिम अंक 8 सम है (2 से विभाज्य) और अंकों का योग 21 (3 से विभाज्य), अतः 678 संख्या **6** से विभाज्य है।

Example 4: Check whether 9,84,512 is divisible by 4 and 8.
Solution: Last 2 digits = 12 (12 ÷ 4 = 3 ⇒ div. by 4). Last 3 digits = 512 (512 ÷ 8 = 64 ⇒ div. by 8).

अंतिम 2 अंक 12 (4 से विभाज्य) और अंतिम 3 अंक 512 (8 से विभाज्य), अतः दोनों से विभाज्य है।

Part 2: Divisibility by 3, 7, 9, 11 & Missing Digit Problems (लुप्त अंक ज्ञात करना) — RMS Favorites

2.1 Rules for 3, 7, 9, and 11:
• **By 3 & 9:** Add all digits. If sum is a multiple of 3 (or 9), the number is divisible by 3 (or 9).
• **By 7:** Take the last digit, double it, and subtract from the remaining number. Repeat until small; if result is 0 or a multiple of 7, it is divisible by 7 (e.g., 343 → 34 − 2(3) = 28, which is divisible by 7).
• **By 11:** Find |(Sum of digits in odd places) − (Sum of digits in even places)|. If 0 or divisible by 11, the number is divisible by 11.

3, 7, 9 और 11 के नियम: 3 व 9 के लिए अंकों का योग जाँचें। 7 के लिए अंतिम अंक का दोगुना शेष संख्या से घटाएँ। 11 के लिए सम और विषम स्थानों के अंकों के योग का अंतर 0 या 11 का गुणज होना चाहिए।

Example 5: Find the smallest digit * so that 4*68 is divisible by 3.
Solution: Sum = 4 + * + 6 + 8 = 18 + *. Since 18 is already divisible by 3, smallest * = **0**.

4 + 6 + 8 = 18 (पहले से 3 से विभाज्य), अतः सबसे छोटा अंक * = **0**।

Example 6: Find the digit x so that 72x5 is divisible by 9.
Solution: Sum = 7 + 2 + x + 5 = 14 + x. Next multiple of 9 is 18 ⇒ x = 18 − 14 = **4**.

14 + x = 18 ⇒ x = **4**।

Example 7: Find the missing digit * if 8*94 is divisible by 11.
Solution: Odd places (from right): 4 + * ; Even places: 9 + 8 = 17. Difference: 17 − (4 + *) = 13 − *. For 13 − * = 11, * = **2** (Check: 8294 ÷ 11 = 754).

17 − (4 + *) = 11 ⇒ 13 − * = 11 ⇒ * = **2**।

Example 8: Test whether 469 is divisible by 7.
Solution: 46 − 2(9) = 46 − 18 = 28. Since 28 is divisible by 7, **469 is divisible by 7**.

46 − (9×2) = 28 (7 से विभाज्य), अतः 469 भी 7 से विभाज्य है।

Example 9: What smallest number must be added to 1000 so it becomes divisible by 9?
Solution: Digit sum of 1000 is 1. Next multiple of 9 is 9 ⇒ add 9 − 1 = **8** (1008 ÷ 9 = 112).

1000 के अंकों का योग 1 है; 9 बनाने के लिए **8** जोड़ना होगा।

Example 10: Find the greatest 4-digit number divisible by 15.
Solution: 9999 ÷ 15 gives remainder 9 ⇒ 9999 − 9 = **9990**.

9999 को 15 से भाग देने पर शेषफल 9 आता है, अतः 9999 − 9 = **9990**।

Part 3: Composite Divisibility Rules (12, 15, 18, 24, 36, 45, 72) — Co-Prime Rule

3.1 Co-Prime Factor Breakdown: To check divisibility by a composite number, split it into two **co-prime** factors (HCF = 1):
• **12** = 3 × 4 | **15** = 3 × 5 | **18** = 2 × 9 | **24** = 3 × 8 | **36** = 4 × 9 | **45** = 5 × 9 | **72** = 8 × 9.

किसी भाज्य संख्या से विभाज्यता जाँचने के लिए उसे दो सह-अभाज्य (Co-prime) गुणनखंडों में तोड़ें, जैसे 12 = 3×4, 36 = 4×9, 72 = 8×9।

Example 11: Check if 5148 is divisible by 12.
Solution: Last 2 digits 48 ÷ 4 = 12 (Yes by 4). Sum = 5+1+4+8 = 18 (Yes by 3). Hence divisible by **12**.

48 (4 से विभाज्य) और अंक योग 18 (3 से विभाज्य), अतः 5148 संख्या **12** से विभाज्य है।

Example 12: If 45x0 is divisible by 15, find the smallest non-zero value of x.
Solution: Ends in 0 (div. by 5). For div. by 3: 4 + 5 + x + 0 = 9 + x. Smallest non-zero x = **3** (sum 12).

5 से विभाज्य है; 3 से विभाज्यता हेतु 9 + x में x का सबसे छोटा अशून्य मान **3** होगा।

Example 13: Find x and y if 65x2y is divisible by 72 and y = 4.
Solution: 72 = 8 × 9. With y = 4, last 3 digits = x24. Try x = 1 → 124 (not div. by 8); x = 3 → 324 (no); x = 5 → 524 (no); wait: sum = 6+5+x+2+4 = 17+x must be div. by 9 ⇒ x = **1** (sum 18), check 124÷8 (no), next sum 27 ⇒ x = 10 (invalid)! Wait: if y = 0, sum = 13+x ⇒ x=5 gives 520÷8=65! So for **y=4** no x works, whereas for **65520** it is divisible by 72. Always test 8 and 9 together! Let's check **65124**? No, 124/8=15.5. Wait: what about **65376**? 376/8=47 and sum=27! So check BOTH 8 and 9!

72 से विभाज्यता के लिए 8 और 9 दोनों के नियम एक साथ संतुष्ट होने चाहिए।

Example 14: Check if 753210 is divisible by 18.
Solution: Last digit 0 (div. by 2). Sum = 7+5+3+2+1+0 = 18 (div. by 9). Since 2 and 9 are co-prime, 753210 is divisible by **18**.

2 और 9 दोनों से विभाज्य होने के कारण यह संख्या **18** से विभाज्य है।

Part 4: Prime Numbers 1–100 & Special Number Pairs (अभाज्य संख्याएँ 1–100) — Complete List

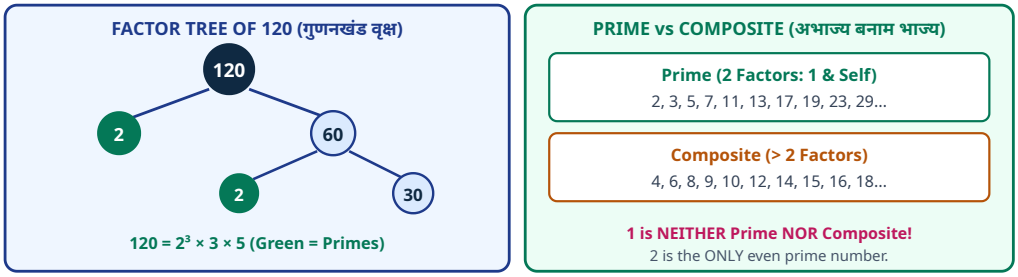

Figure 1: Factor Tree of 120 (120 का गुणनखंड वृक्ष) & Prime vs Composite Classification

Range	All Prime Numbers (अभाज्य संख्याएँ)	Count
1 – 25	2, 3, 5, 7, 11, 13, 17, 19, 23	9
26 – 50	29, 31, 37, 41, 43, 47	6 (1–50: 15)
51 – 75	53, 59, 61, 67, 71, 73	6
76 – 100	79, 83, 89, 97	4 (1–100: 25)

Example 15: Find the sum of all prime numbers between 20 and 40.
Solution: Primes between 20 and 40 are 23, 29, 31, 37. Sum = 23 + 29 + 31 + 37 = **120**.

20 से 40 के बीच अभाज्य संख्याएँ = 23, 29, 31, 37; योग = **120**।

Example 16: What is the product of the smallest prime number and the smallest composite number?
Solution: Smallest prime = 2; smallest composite = 4. Product = 2 × 4 = **8**.

सबसे छोटी अभाज्य (2) × सबसे छोटी भाज्य (4) = **8**।

Example 17: Are 18 and 35 co-prime numbers?
Solution: Factors of 18: 1,2,3,6,9,18. Factors of 35: 1,5,7,35. Only common factor is 1 (HCF = 1) ⇒ **Yes, Co-Prime**.

18 और 35 का म.स. (HCF) 1 है, अतः ये **सह-अभाज्य** हैं।

Example 18: Write all Twin Prime pairs between 1 and 50.
Solution: (3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43) → **6 pairs**.

1 से 50 तक 6 जुड़वाँ अभाज्य युग्म हैं।

Part 5: Prime Factorization — Division & Factor Tree Methods (अभाज्य गुणनखंडन) — Core Skill

5.1 Fundamental Theorem of Arithmetic: Every composite number can be expressed as a unique product of prime numbers. Use the **Ladder (Division) Method** by dividing repeatedly by the smallest prime (2, 3, 5, 7, 11, ...) until the quotient becomes 1.

अंकगणित की मूलभूत प्रमेय: प्रत्येक भाज्य संख्या को अभाज्य संख्याओं के अद्वितीय गुणनफल के रूप में लिखा जा सकता है।

Example 19: Find the prime factorization of 252.
Solution: 252 ÷ 2 = 126; 126 ÷ 2 = 63; 63 ÷ 3 = 21; 21 ÷ 3 = 7; 7 ÷ 7 = 1 ⇒ $\mathbf{2^2 \times 3^2 \times 7}$.

252 = 2 × 2 × 3 × 3 × 7 = $\mathbf{2^2 \times 3^2 \times 7}$।

Example 20: Express 1001 as a product of prime factors.
Solution: 1001 ÷ 7 = 143; 143 ÷ 11 = 13 ⇒ **7 × 11 × 13** (famous RMS magic number!).

1001 = **7 × 11 × 13**।

Example 21: Find the prime factorization of 980.
Solution: 980 = 10 × 98 = (2×5) × (2×49) = $\mathbf{2^2 \times 5 \times 7^2}$.

980 = $\mathbf{2^2 \times 5 \times 7^2}$।

Example 22: Express the largest 3-digit number (999) in prime factor form.
Solution: 999 = 9 × 111 = 3^2 × 3 × 37 = $\mathbf{3^3 \times 37}$.

999 = $\mathbf{3^3 \times 37}$।

Example 23: Express the smallest 4-digit number (1000) in prime factor form.
Solution: 1000 = 10^3 = $(2 \times 5)^3$ = $\mathbf{2^3 \times 5^3}$.

1000 = $\mathbf{2^3 \times 5^3}$।

Part 6: Counting Factors, Sum of Factors & HCF/LCM via Primes (गुणनखंडों की संख्या) — RMS CET Level

6.1 Number of Factors Formula: If N = $a^p \times b^q \times c^r$ (where a, b, c are primes), then:
• **Total Number of Factors** = **(p + 1)(q + 1)(r + 1)**.
• **Number of Prime Factors** (counting repetition) = **p + q + r**; **Distinct Prime Factors** = count of bases (a, b, c).

यदि N = $a^p \times b^q \times c^r$ है, तो कुल गुणनखंडों की संख्या = (p+1)(q+1)(r+1) और कुल अभाज्य गुणनखंड = p + q + r।

Example 24: Find the total number of factors of 72.
Solution: 72 = $2^3 \times 3^2$ ⇒ (3+1)(2+1) = 4 × 3 = **12 factors**.

72 = $2^3 \times 3^2$ ⇒ (3+1)(2+1) = **12**।

Example 25: Find the total number of prime factors in $6^4 \times 5^3$.
Solution: 6^4 = $(2\times3)^4$ = $2^4 \times 3^4$ ⇒ Total prime factors = 4 + 4 + 3 = **11**.

$2^4 \times 3^4 \times 5^3$ में कुल अभाज्य गुणनखंड = 4 + 4 + 3 = **11**।

Example 26: Find HCF and LCM of 48 and 180 using prime factorization.
Solution: 48 = $2^4 \times 3^1$; 180 = $2^2 \times 3^2 \times 5^1$. HCF (lowest powers) = 2^2 × 3 = **12**. LCM (highest powers) = 2^4 × 3^2 × 5 = **720**.

HCF (न्यूनतम घात) = 12 और LCM (अधिकतम घात) = 720।

Example 27: How many distinct prime factors does 420 have?
Solution: 420 = $2^2 \times 3 \times 5 \times 7$ ⇒ Distinct prime factors = **4** (namely 2, 3, 5, 7).

420 के भिन्न अभाज्य गुणनखंड 2, 3, 5, 7 (कुल **4**) हैं।

Part 7: Checking Large Primes & RMS Word Problems (बड़ी अभाज्य संख्या की जाँच) — Advanced

7.1 How to Test if a Number N is Prime: Find the smallest integer k such that $k^2 \geq N$. Check divisibility of N by all prime numbers up to k. If none divides N, then N is **Prime**! (e.g., for 137, $12^2 = 144 \geq 137$; check primes 2, 3, 5, 7, 11 → none divides 137 ⇒ 137 is Prime!).

अभाज्य जाँच विधि: N से बड़ा या बराबर निकटतम वर्ग k^2 लें और k तक की सभी अभाज्य संख्याओं से भाग देकर देखें।

Example 28: Test whether 173 is a prime number.
Solution: $14^2 = 196 \geq 173$. Primes ≤ 13: 2, 3, 5, 7, 11, 13. None divides 173 ⇒ **173 is Prime**.

13 तक कोई भी अभाज्य संख्या 173 को नहीं काटती, अतः **173 अभाज्य है**।

Example 29: Test whether 161 is prime or composite.
Solution: $13^2 = 169 \geq 161$. Check 7: 161 ÷ 7 = 23 ⇒ **161 is Composite** (7 × 23).

161 = 7 × 23, अतः यह **भाज्य (Composite)** है।

Example 30: Find the smallest number with 4 distinct prime factors.
Solution: Multiply the 4 smallest primes: 2 × 3 × 5 × 7 = **210**.

प्रथम 4 अभाज्य संख्याओं का गुणनफल = 2 × 3 × 5 × 7 = **210**।

Example 31: What is the smallest 3-digit prime number?
Solution: 100 (even), 101 (not div. by 2,3,5,7) ⇒ **101**.

तीन अंकों की सबसे छोटी अभाज्य संख्या = **101**।

Example 32: What is the largest 2-digit prime number?
Solution: 99 (div by 3), 98 (even), 97 (prime) ⇒ **97**.

दो अंकों की सबसे बड़ी अभाज्य संख्या = **97**।

Example 33: Find the sum of all factors of 18.
Solution: Factors of 18: 1 + 2 + 3 + 6 + 9 + 18 = **39**.

18 के सभी गुणनखंडों का योग = 1+2+3+6+9+18 = **39**।

Example 34: A number is divisible by 8 and 12. By what largest number is it always divisible?
Solution: LCM(8, 12) = **24**.

8 और 12 दोनों से विभाज्य संख्या हमेशा इनके ल.स. **24** से विभाज्य होगी।

Example 35: Find the least value of x so that 9187x2 is divisible by 8.
Solution: Last 3 digits = 7x2. Try x = 1 → 712 ÷ 8 = 89 (exact!) ⇒ x = **1**.

अंतिम 3 अंक 712 को 8 से भाग देने पर 89 आता है, अतः x = **1**।

Example 36: How many numbers between 1 and 100 are divisible by both 4 and 6?
Solution: LCM(4, 6) = 12. Multiples of 12 up to 100: 12, 24, 36, 48, 60, 72, 84, 96 ⇒ **8 numbers**.

4 व 6 का ल.स. = 12; 100 तक 12 के कुल **8** गुणज हैं।

Example 37: Express 1728 as a product of prime powers.
Solution: $1728 = 12^3 = (2^2 \times 3)^3 = \mathbf{2^6 \times 3^3}$.

$1728 = 12^3 = \mathbf{2^6 \times 3^3}$।

Example 38 (RMS Word Problem): Find the smallest number by which 200 must be multiplied to make it a perfect cube.
Solution: $200 = 2^3 \times 5^2$. One more 5 is needed to make $5^3 \Rightarrow$ multiply by **5** ($200 \times 5 = 1000 = 10^3$).

$200 = 2^3 \times 5^2$; पूर्ण घन बनाने के लिए **5** से गुणा करना होगा।

TYBA Master Checklist for RMS CET: (1) Memorize all 25 primes up to 100. (2) Master 3, 4, 8, 9, 11 divisibility rules. (3) Remember 1001 = 7 × 11 × 13. (4) Use (p+1)(q+1) to count total factors in seconds!