TYBA COACHING CENTRE, AJMER

RMS / RMC CLASS VI MATHEMATICS • CHAPTER 20: EXPONENTS & POWERS — COMPLETE BILINGUAL NOTES

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Part 1: Introduction to Exponents & Exponential Form (घातांक का परिचय) — Basics

1.1 What is an Exponent? Very large numbers (like the distance from Earth to the Sun: 1,50,00,00,000 km, or mass of Earth) are difficult to read, write, and compare. Exponents provide a compact way to express repeated multiplication of the same number. If a number **a** is multiplied by itself **n** times, we write it as a^n. Here, **a** is the **Base** and **n** is the **Exponent** (or Index/Power).

घातांक क्या है? बहुत बड़ी संख्याओं को आसानी से पढ़ने, लिखने और तुलना करने के लिए घातांकीय रूप का उपयोग होता है। जब किसी संख्या **a** को स्वयं से **n** बार गुणा किया जाए, तो इसे a^n लिखते हैं (**a** = आधार, **n** = घातांक)।

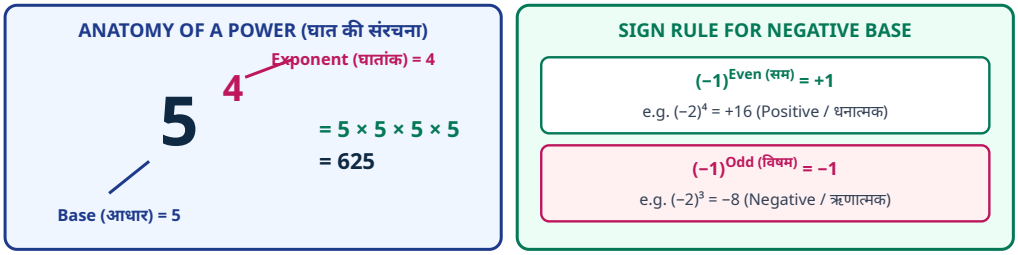

Figure 1: Base & Exponent Anatomy (आधार व घातांक) and Negative Base Sign Rule (ऋणात्मक आधार का नियम)

1.2 Reading Powers & Special Powers: 10^4 is read as "10 raised to the power 4". Power 2 is called **Square (वर्ग)**: $7^2 = 7 \times 7 = 49$. Power 3 is called **Cube (घन)**: $4^3 = 4 \times 4 \times 4 = 64$. Any number with exponent 1 equals itself: $a^1 = a$ (e.g., $15^1 = 15$).

घात 2 को 'वर्ग' और घात 3 को 'घन' कहते हैं। किसी भी संख्या की घात 1 का मान स्वयं वही संख्या होती है ($a^1 = a$)।

Example 1: Write 7 × 7 × 7 × 7 × 7 in exponential form and identify its base and exponent.
Solution: 7 is multiplied 5 times ⇒ $\mathbf{7^5}$. Base = **7**, Exponent = **5**.

7 को 5 बार गुणा किया गया है, अतः घातांकीय रूप = $\mathbf{7^5}$ (आधार = 7, घातांक = 5)।

Example 2: Express a × a × b × b × b × c in exponential form.
Solution: a appears 2 times, b appears 3 times, c appears 1 time ⇒ $\mathbf{a^2b^3c}$.

a दो बार, b तीन बार और c एक बार है ⇒ $\mathbf{a^2b^3c}$।

Example 3: Find the value of $3^4 + 2^5$.
Solution: $3^4 = 3 \times 3 \times 3 \times 3 = 81$; $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$. Sum = 81 + 32 = **113**.

$3^4 = 81$ और $2^5 = 32$, दोनों का योग = 81 + 32 = **113**।

Example 4: Express 343 as a power of 7.
Solution: 343 = 7 × 49 = 7 × 7 × 7 = $\mathbf{7^3}$.

343 = 7 × 7 × 7 = $\mathbf{7^3}$।

Part 2: Powers with Negative Bases (ऋणात्मक आधार की घातें) — Sign Rules

2.1 Negative Base Rule: When a negative number is raised to an **Even (सम)** power, the result is always **Positive (+)** because minus signs pair up: $(-a)^{even} = +a^{even}$. When a negative number is raised to an **Odd (विषम)** power, the result is always **Negative (−)**: $(-a)^{odd} = -a^{odd}$. Also note the bracket trap: $(-3)^2 = (-3)(-3) = +9$, whereas $-3^2 = -(3 \times 3) = -9$!

ऋणात्मक आधार का नियम: ऋणात्मक संख्या पर **सम (Even)** घात होने पर उत्तर **धनात्मक (+)** आता है, जबकि **विषम (Odd)** घात होने पर उत्तर **ऋणात्मक (−)** आता है। कोष्ठक का ध्यान रखें: $(-3)^2 = +9$, परंतु $-3^2 = -9$ होता है।

Example 5: Evaluate $(-2)^5$ and $(-3)^4$.
Solution: $(-2)^5 = -(2^5) =$ **−32** (odd power). $(-3)^4 = +(3^4) =$ **81** (even power).

$(-2)^5 =$ **−32** (विषम घात) और $(-3)^4 =$ **81** (सम घात)।

Example 6: Find the value of $(-1)^{101} + (-1)^{100} - (-1)^{99}$.
Solution: $(-1) + (+1) - (-1) = -1 + 1 + 1 =$ **1**.

विषम घात पर −1 और सम घात पर +1: $(-1) + 1 - (-1) =$ **1**।

Example 7: Evaluate $(-4)^3 \times (-5)^2$.
Solution: $(-4)^3 = -64$; $(-5)^2 = 25$. Product $= -64 \times 25 =$ **−1600**.

$(-64) \times 25 =$ **−1600**।

Example 8: Express −125 in exponential form with base −5.
Solution: $-125 = (-5) \times (-5) \times (-5) =$ **$(-5)^3$**.

$-125 = (-5) \times (-5) \times (-5) =$ **$(-5)^3$**।

Part 3: Laws of Exponents (घातांक के नियम) — Core Formulas

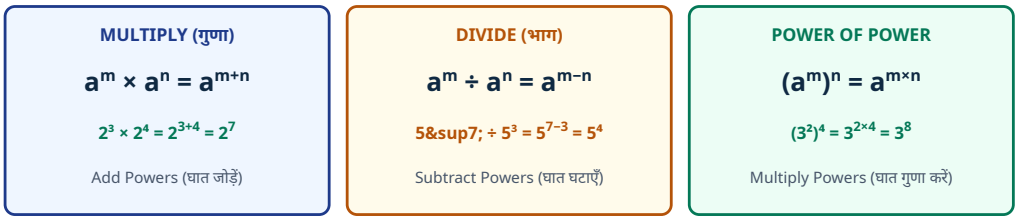

Figure 2: Three Core Laws of Exponents (घातांक के तीन मुख्य नियम)

3.1 Five Fundamental Laws (for non-zero integers a, b):
- **Law 1 (Product, Same Base):** $a^m \times a^n = a^{m+n}$
- **Law 2 (Quotient, Same Base):** $a^m \div a^n = a^{m-n}$ (when $m > n$)
- **Law 3 (Power of Power):** $(a^m)^n = a^{m \times n}$
- **Law 4 (Product, Same Exponent):** $a^m \times b^m = (a \times b)^m$
- **Law 5 (Quotient, Same Exponent):** $a^m \div b^m = (a / b)^m$

समान आधार पर गुणा में घातें जुड़ती हैं (m+n), भाग में घातें घटती हैं (m−n), और घात के ऊपर घात होने पर घातों का गुणा (m×n) होता है। समान घात होने पर आधारों का गुणा या भाग करके वही घात लगा देते हैं।

Example 9: Simplify using laws of exponents: $3^4 \times 3^5 \times 3^2$.
Solution: Base is 3 in all terms $\Rightarrow 3^{4+5+2} =$ **3^{11}**.

समान आधार 3 है, अतः घातें जोड़ने पर $3^{4+5+2} =$ **3^{11}**।

Example 10: Simplify: $8^{12} \div 8^{7}$.
Solution: $8^{12-7} = \mathbf{8^{5}}$.

भाग में घातें घटाने पर $8^{12-7} = \mathbf{8^{5}}$।

Example 11: Simplify: $(5^{3})^{4} \div 5^{7}$.
Solution: $(5^{3})^{4} = 5^{3\times4} = 5^{12}$. Then $5^{12} \div 5^{7} = 5^{12-7} = \mathbf{5^{5}}$.

पहले घात का गुणा 5^{12}, फिर 5^{7} से भाग देने पर $5^{12-7} = \mathbf{5^{5}}$।

Example 12: Simplify: $2^{5} \times 5^{5}$.
Solution: Same exponent 5 ⇒ $(2 \times 5)^{5} = \mathbf{10^{5}} = 1,00,000$.

समान घात 5 होने के कारण $(2 \times 5)^{5} = \mathbf{10^{5}}$।

Example 13: Simplify: $(12^{4}) \div (4^{4})$.
Solution: $(12 / 4)^{4} = 3^{4} = \mathbf{81}$.

$(12 \div 4)^{4} = 3^{4} = \mathbf{81}$।

Example 14: Simplify and express in exponential form: $(2^{3} \times 2^{4})^{2}$.
Solution: Inside bracket: $2^{3+4} = 2^{7}$. Then $(2^{7})^{2} = 2^{7\times2} = \mathbf{2^{14}}$.

कोष्ठक में 2^{7}, फिर $(2^{7})^{2} = \mathbf{2^{14}}$।

Part 4: Zero Exponent & Negative Exponents (शून्य और ऋणात्मक घातांक) — High-Yield RMS

4.1 Zero Exponent ($a^{0} = 1$): For any non-zero number **a**, $\mathbf{a^{0} = 1}$. Proof: $a^{m} \div a^{m} = a^{m-m} = a^{0}$, and $a^{m}/a^{m} = 1 \Rightarrow a^{0} = 1$. Watch out: $5^{0} + 6^{0} = 1 + 1 = 2$ (NOT 11 and NOT 0).
4.2 Negative Exponent ($a^{-n} = 1/a^{n}$): A negative exponent means take the reciprocal of the base and make the exponent positive. For fractions: $\mathbf{(a/b)^{-n} = (b/a)^{n}}$.

शून्य घातांक: किसी भी अशून्य संख्या की घात 0 का मान **1** होता है ($a^{0} = 1$)। **ऋणात्मक घातांक:** घात को धनात्मक बनाने के लिए आधार का व्युत्क्रम (reciprocal) कर देते हैं: $a^{-n} = 1/a^{n}$ तथा $(a/b)^{-n} = (b/a)^{n}$।

Example 15: Find the value of $(7^{0} + 8^{0} + 9^{0}) \times 10^{0}$.
Solution: $(1 + 1 + 1) \times 1 = 3 \times 1 = \mathbf{3}$.

$(1 + 1 + 1) \times 1 = \mathbf{3}$।

Example 16: Evaluate $4^{-2} + 2^{-3}$.
Solution: $1/4^{2} + 1/2^{3} = 1/16 + 1/8 = 1/16 + 2/16 = \mathbf{3/16}$.

$1/16 + 1/8 = (1 + 2)/16 = \mathbf{3/16}$।

Example 17: Evaluate $(3/5)^{-2}$.
Solution: Reciprocal of 3/5 is 5/3 ⇒ $(5/3)^{2} = 25/9 = \mathbf{25/9}$.

आधार पलटने पर $(5/3)^{2} = \mathbf{25/9}$।

Example 18: Simplify: $(2^{-1} \times 4^{-1}) \div 2^{-2}$.
Solution: $(1/2 \times 1/4) \div (1/4) = (1/8) \times 4 = 4/8 = \mathbf{1/2}$.

$(1/2 \times 1/4) \div (1/4) = 1/8 \times 4 = \mathbf{1/2}$।

Part 5: Prime Factorization & Simplification of Expressions (अभाज्य गुणनखंड एवं सरलीकरण) — Exam Favorites

5.1 Converting Composite Numbers to Prime Powers: In RMS CET, numbers like 4, 8, 9, 12, 16, 25, 27, 32, 64, 81, 125 are mixed in fractions. Always convert composite bases into prime bases first (e.g., $4 = 2^2$, $8 = 2^3$, $9 = 3^2$, $12 = 2^2 \times 3$, $27 = 3^3$), then apply laws of exponents!

भाज्य संख्याओं (जैसे 4, 8, 9, 12, 27) को पहले अभाज्य आधारों (2, 3, 5) की घातों में बदलें, फिर घातांक के नियम लगाएँ।

Example 19: Express 432 as a product of prime factors in exponential form.
Solution: $432 = 16 \times 27 = 2^4 \times 3^3$.

432 के अभाज्य गुणनखंड = $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = \mathbf{2^4 \times 3^3}$।

Example 20: Express 1800 in exponential form using prime factorization.
Solution: $1800 = 18 \times 100 = (2 \times 3^2) \times (2^2 \times 5^2) = \mathbf{2^3 \times 3^2 \times 5^2}$.

$1800 = 8 \times 9 \times 25 = \mathbf{2^3 \times 3^2 \times 5^2}$।

Example 21: Simplify: $(2^5 \times 9^2) \div (8 \times 3^3)$.
Solution: Write $9 = 3^2$ and $8 = 2^3 \Rightarrow (2^5 \times (3^2)^2) / (2^3 \times 3^3) = (2^5 \times 3^4) / (2^3 \times 3^3) = 2^{5-3} \times 3^{4-3} = 2^2 \times 3^1 = 4 \times 3 = \mathbf{12}$.

9 को 3^2 और 8 को 2^3 लिखने पर $(2^5 \times 3^4) / (2^3 \times 3^3) = 2^2 \times 3 = \mathbf{12}$।

Example 22: Simplify: $(3^5 \times 10^5 \times 25) \div (5^7 \times 6^5)$.
Solution: Factorize $10 = 2 \times 5$, $25 = 5^2$, $6 = 2 \times 3$:
Numerator $= 3^5 \times 2^5 \times 5^5 \times 5^2 = 2^5 \times 3^5 \times 5^7$.
Denominator $= 5^7 \times 2^5 \times 3^5$. Everything cancels out $\Rightarrow \mathbf{1}$!

अंश व हर दोनों में $2^5 \times 3^5 \times 5^7$ बनता है, अतः कटकर उत्तर **1** आएगा।

Example 23: Simplify: $(2^8 \times a^5) \div (4^3 \times a^3)$.
Solution: $4^3 = (2^2)^3 = 2^6 \Rightarrow 2^{8-6} \times a^{5-3} = 2^2 a^2 = \mathbf{4a^2}$.

$4^3 = 2^6$, अतः $2^{8-6} \times a^{5-3} = \mathbf{4a^2}$।

Part 6: Comparing Powers & Solving Exponential Equations (तुलना व समीकरण) — RMS CET Level

6.1 Comparing Large Powers: To compare two powers like 2^{10} and 10^2, or 2^{30} and 3^{20}: either compute small values directly, or make their exponents equal (e.g., $2^{30} = (2^3)^{10} = 8^{10}$ and $3^{20} = (3^2)^{10} = 9^{10} \Rightarrow 3^{20} > 2^{30}$).

6.2 Solving for Unknown Exponent (x): If $\mathbf{a^x = a^y}$ (with base $a \neq 0, 1, -1$), then $\mathbf{x = y}$. Convert both sides to the exact same base!

तुलना एवं समीकरण: बड़ी घातों की तुलना के लिए घातांक को समान बनाएँ। यदि दोनों पक्षों का आधार समान हो ($a^x = a^y$), तो उनके घातांक भी बराबर ($x = y$) होते हैं।

Example 24: Which is greater: 2^6 or 6^2?
Solution: $2^6 = 64$, $6^2 = 36$. Therefore, $\mathbf{2^6 > 6^2}$.

$2^6 = 64$ और $6^2 = 36$, अतः $\mathbf{2^6}$ बड़ा है।

Example 25: Compare 4×10^{14} and 3×10^{17}.
Solution: Compare powers of 10 first: $10^{17} > 10^{14} \Rightarrow$ **$3 \times 10^{17} > 4 \times 10^{14}$**.

10 की घात 17 बड़ी है, अतः **3×10^{17}** बड़ा है।

Example 26: Find x if $5^{2x+1} \div 25 = 125$.
Solution: $25 = 5^2$, $125 = 5^3 \Rightarrow 5^{2x+1-2} = 5^3 \Rightarrow 5^{2x-1} = 5^3 \Rightarrow 2x - 1 = 3 \Rightarrow 2x = 4 \Rightarrow$ **$x = 2$**.

$5^{2x-1} = 5^3 \Rightarrow 2x - 1 = 3 \Rightarrow$ **$x = 2$**।

Example 27: Find m if $(2/3)^4 \times (2/3)^5 = (2/3)^{2m+1}$.
Solution: LHS $= (2/3)^{4+5} = (2/3)^9$. Equating exponents: $2m + 1 = 9 \Rightarrow 2m = 8 \Rightarrow$ **$m = 4$**.

$4 + 5 = 9 = 2m + 1 \Rightarrow 2m = 8 \Rightarrow$ **$m = 4$**।

Part 7: Standard Form (Scientific Notation) & Expanded Form (मानक व विस्तारित रूप) Notation

Usual / Standard Number	Standard Form ($k \times 10^n$)	Decimal Shift Rule
59,85,000	5.985×10^6	Decimal moves 6 places left
7,00,40,000	7.004×10^7	Decimal moves 7 places left
3,00,000,000 (Speed of light m/s)	3×10^8	Decimal moves 8 places left
1,49,60,00,00,000 m (Sun-Earth)	1.496×10^{11}	Decimal moves 11 places left

Example 28: Write 6,59,50,000 in standard form.
Solution: Place decimal after 6 $\Rightarrow$ 6.595. Count digits after 6 (7 digits) $\Rightarrow$ **6.595×10^7**.

6 के बाद दशमलव लगाने पर 7 अंक शेष हैं, अतः **6.595×10^7**।

Example 29: Write 4.07×10^5 in usual form.
Solution: Move decimal 5 places right: 407000 = **4,07,000**.

दशमलव को 5 स्थान दाईं ओर ले जाने पर **4,07,000** प्राप्त होगा।

Example 30: Write 2,79,404 in expanded form using powers of 10.
Solution: $2 \times 10^5 + 7 \times 10^4 + 9 \times 10^3 + 4 \times 10^2 + 0 \times 10^1 + 4 \times 10^0$.

10 की घातों में विस्तार: $2{\times}10^5 + 7{\times}10^4 + 9{\times}10^3 + 4{\times}10^2 + 0{\times}10^1 + 4{\times}10^0$।

Example 31: Find the number from expanded form: $8 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 4 \times 10^1 + 5 \times 10^0$.
Solution: 80000 + 6000 + 0 + 40 + 5 = **86,045**.

80000 + 6000 + 40 + 5 = **86,045**।

Example 32 (RMS Word Problem): A bacterium doubles every hour. If there is 1 bacterium initially, how many bacteria will there be after 8 hours?
Solution: After 1 hr = 2^1, 2 hrs = 2^2, ..., 8 hrs = 2^8 = **256 bacteria**.

प्रत्येक घंटे दुगुना होने पर 8 घंटे बाद 2^8 = **256 जीवाणु** होंगे।

Part 8: Quick-Reference Powers Table & Advanced RMS Practice (त्वरित तालिका व उच्च स्तरीय अभ्यास) — Speed Booster

Base	n = 1	n = 2	n = 3	n = 4	n = 5	n = –1	n = –2
2	2	4	8	16	32	1/2	1/4
3	3	9	27	81	243	1/3	1/9
5	5	25	125	625	3125	1/5	1/25
10	10	100	1,000	10,000	1,00,000	0.1	0.01

Example 33 (Comparing Large Exponents): Which is greater: 2^{30} or 3^{20}?
Solution: Make exponents equal: $2^{30} = (2^3)^{10} = 8^{10}$, and $3^{20} = (3^2)^{10} = 9^{10}$. Since 9 > 8, $\mathbf{3^{20} > 2^{30}}$.

घातांक को 10 बनाने पर $2^{30} = 8^{10}$ और $3^{20} = 9^{10}$, अत: $\mathbf{3^{20}}$ बड़ा है।

Example 34 (Negative Exponent of a Fraction): Find the value of $(2/5)^{-3} \times (2/5)^5$.
Solution: Same base 2/5 ⇒ $(2/5)^{-3+5} = (2/5)^2 = \mathbf{4/25}$.

समान आधार पर घातें जोड़ने पर $(2/5)^{-3+5} = (2/5)^2 = \mathbf{4/25}$।

Example 35 (Solving with Negative Exponent): Find x if $3^{-x} = 1/81$.
Solution: $1/81 = 1/3^4 = 3^{-4}$. Equating exponents: $-x = -4 \Rightarrow \mathbf{x = 4}$.

$1/81 = 3^{-4}$, अतः तुलना करने पर **x = 4**।

Example 36 (Mixed Zero & Negative Powers): Evaluate $(6^0 - 4^0) \times (5^2 + 3^{-1})$.
Solution: $6^0 = 1$ and $4^0 = 1 \Rightarrow (1 - 1) = 0$. Zero multiplied by anything is **0**!

$6^0 - 4^0 = 1 - 1 = 0$, और 0 से किसी भी संख्या का गुणा **0** होता है।

Example 37 (Algebraic Variables with Exponents): Simplify $(a^3b^4)^2 \div (a^2b^3)$.
Solution: $(a^3b^4)^2 = a^6b^8$. Dividing by a^2b^3 gives $a^{6-2}b^{8-3} = \mathbf{a^4b^5}$.

घात गुणा करने पर a^6b^8, फिर भाग देने पर $\mathbf{a^4b^5}$।

Example 38 (Power of 10 Application): If 1 km = 10^3 m and 1 m = 10^2 cm, express 5 km in cm using powers of 10.
Solution: 5 km = 5×10^3 m = $5 \times 10^3 \times 10^2$ cm = $\mathbf{5 \times 10^5}$ **cm**.

5 किमी = $5 \times 10^3 \times 10^2$ सेमी = $\mathbf{5 \times 10^5}$ **सेमी**।

TYBA Master Checklist for RMS CET: (1) Memorize 2^1 to 2^{10}, 3^1 to 3^6, 5^1 to 5^5. (2) Remember $a^0 = 1$. (3) Watch negative sign with even/odd exponents. (4) Always convert composite bases to prime factors before simplifying!