

TYBA COACHING CENTRE | टी वाई बी ए कोचिंग सेंटर

Contact / WhatsApp: 6367029449 • NCERT Class 10 Mathematics Complete Comprehensive Notes & Solutions

Chapter 14: Probability

अध्याय 14: प्रायिकता

Section 1: Detailed Concepts, Definitions & Axioms

खंड 1: विस्तृत अवधारणाएँ, परिभाषाएँ एवं सिद्धांत

14.1 Classical / Theoretical Probability & Elementary Events / 14.1 सैद्धांतिक प्रायिकता एवं प्रारंभिक घटनाएँ

[English Theory]: • **Theoretical (Classical) Probability:** The probability of an event E, written as $P(E)$, is defined as:

$P(E) = (\text{Number of outcomes favourable to E}) / (\text{Number of all possible outcomes of the experiment})$.

This definition assumes that all the outcomes of the experiment are **equally likely**.

- **Elementary Event:** An event having only one outcome of the experiment. The sum of the probabilities of all the elementary events of an experiment is **1**.
- **Impossible Event:** The probability of an event which is impossible to occur is **0**.
- **Sure (Certain) Event:** The probability of an event which is sure (or certain) to occur is **1**.
- **Range of Probability:** For any event E, $0 \leq P(E) \leq 1$.
- **Complementary Event:** The event 'not E' (denoted by $\bar{E}$): $P(E) + P(\text{not } E) = 1 \Rightarrow P(\bar{E}) = 1 - P(E)$.

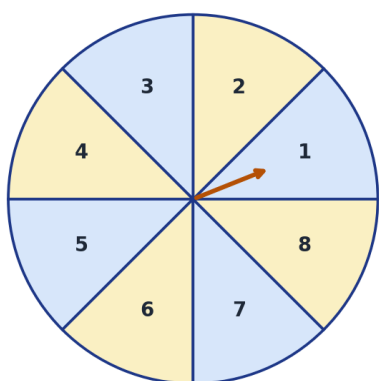
[हिंदी सिद्धांत]: • **सैद्धांतिक (प्रायोगिक) प्रायिकता:** किसी घटना E की प्रायिकता $P(E)$ को इस प्रकार परिभाषित किया जाता है:

$P(E) = (\text{E के अनुकूल परिणामों की संख्या}) / (\text{प्रयोग के सभी संभव परिणामों की संख्या})$ ।

यह परिभाषा इस परिकल्पना पर आधारित है कि प्रयोग के सभी परिणाम **समप्रायिक (Equally likely)** हैं।

- **प्रारंभिक घटना:** वह घटना जिसका केवल एक ही परिणाम हो। किसी प्रयोग की सभी प्रारंभिक घटनाओं की प्रायिकताओं का योग सदैव **1** होता है।
- **असंभव घटना:** जिसकी प्रायिकता **0** होती है।
- **निश्चित घटना:** जिसकी प्रायिकता **1** होती है।
- **प्रायिकता की सीमा:** किसी भी घटना E के लिए, $0 \leq P(E) \leq 1$ ।
- **पूरक घटना:** घटना 'E नहीं' ($\bar{E}$) की प्रायिकता: $P(E) + P(E \text{ नहीं}) = 1$ ।

Ex 14.1 Q12: Game of Chance Wheel (1 to 8)



Ex 14.1 Q20: Circle ($d = 1 \text{ m}$) Inside $3 \text{ m} \times 2 \text{ m}$ Rectangle

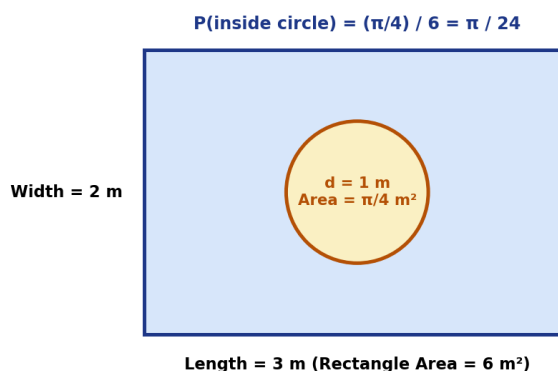


Fig 14.1: Game of Chance Spinning Wheel (Ex 14.1 Q12) and Geometric Probability Circle inside $3 \text{ m} \times 2 \text{ m}$ Rectangle (Ex 14.1 Q20)

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Section 2: Textbook Solved Examples with Step-by-Step Bilingual Solutions

खंड 2: पाठ्यपुस्तक के सभी हल किए गए उदाहरण (विस्तृत चरणबद्ध हल)

Textbook Example 1: Probability of Getting a Head in Coin Toss / उदाहरण 1: सिक्का उछालने पर चित (Head) आने की प्रायिकता

[Problem - EN]: Find the probability of getting a head when a coin is tossed once. Also find the probability of getting a tail.

[प्रश्न - HI]: एक चित प्राप्त करने की प्रायिकता ज्ञात कीजिए, जब एक सिक्के को एक बार उछाला जाता है। साथ ही एक पट प्राप्त करने की भी प्रायिकता ज्ञात कीजिए।

Step-by-Step Solution (English):

Number of possible outcomes = 2 {H, T}. Both outcomes are equally likely.

Let E be the event 'getting a head'. Number of outcomes favourable to E = 1.

$$P(E) = P(\text{head}) = 1 / 2.$$

Similarly, let F be the event 'getting a tail'. Number of outcomes favourable to F = 1.

$$P(F) = P(\text{tail}) = 1 / 2.$$

Check: $P(E) + P(F) = 1/2 + 1/2 = 1$ (Sum of probabilities of all elementary events is 1).

चरणबद्ध हल (हिंदी):

कुल संभव परिणाम = 2 {चित, पट}। दोनों परिणाम समप्रायिक हैं।

चित आने की अनुकूल संख्या = 1 $\Rightarrow P(\text{चित}) = 1 / 2$ ।

पट आने की अनुकूल संख्या = 1 $\Rightarrow P(\text{पट}) = 1 / 2$ ।

जाँच: $1/2 + 1/2 = 1$ ।

Textbook Example 3: Throwing a Die Once (Number Greater than 4 & Less than or Equal to 4) / उदाहरण 3: पासा फेंकने पर 4 से बड़ी संख्या

[Problem - EN]: Suppose we throw a die once. (i) What is the probability of getting a number greater than 4? (ii) What is the probability of getting a number less than or equal to 4?

[प्रश्न - HI]: मान लीजिए हम एक पासे को एक बार फेंकते हैं। (i) 4 से बड़ी संख्या प्राप्त होने की प्रायिकता क्या है? (ii) 4 से छोटी या उसके बराबर संख्या प्राप्त होने की प्रायिकता क्या है?

Step-by-Step Solution (English):

Possible outcomes: $S = \{1, 2, 3, 4, 5, 6\}$. Total number of possible outcomes = 6.

(i) Let E be the event 'getting a number greater than 4'. Favourable outcomes: {5, 6} (2 outcomes).

$$P(E) = 2 / 6 = 1 / 3.$$

(ii) Let F be the event 'getting a number less than or equal to 4'. Favourable outcomes: {1, 2, 3, 4} (4 outcomes).

$$P(F) = 4 / 6 = 2 / 3.$$

Note: F is the event 'not E' $\Rightarrow P(F) = 1 - P(E) = 1 - 1/3 = 2/3$.

चरणबद्ध हल (हिंदी):

कुल परिणाम = 6 {1, 2, 3, 4, 5, 6}।

(i) 4 से बड़ी संख्याएँ: {5, 6} (2 परिणाम) $\Rightarrow P(4 \text{ से बड़ी}) = 2/6 = 1 / 3$ ।

(ii) 4 से छोटी या बराबर: {1, 2, 3, 4} (4 परिणाम) $\Rightarrow P(\leq 4) = 4/6 = 2 / 3$ ।

Textbook Example 4: One Card from a Well-Shuffled Deck of 52 Cards / उदाहरण 4: 52 ताश के पत्तों की गड्डी में से एक पत्ता निकालना

[Problem - EN]: One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will: (i) be an ace, (ii) not be an ace.

[प्रश्न - HI]: अच्छी तरह फेंटी गई 52 पत्तों की एक गड्डी में से एक पत्ता निकाला जाता है। इसकी प्रायिकता परिकलित कीजिए कि यह पत्ता (i) एक इक्का होगा, (ii) एक इक्का नहीं होगा।

Step-by-Step Solution (English):

Total number of possible outcomes = 52.

(i) There are 4 aces in a deck (one of each suit). Number of favourable outcomes = 4.

$$P(\text{Ace}) = 4 / 52 = 1 / 13.$$

(ii) Number of cards that are not aces = $52 - 4 = 48$.

$$P(\text{not an ace}) = 48 / 52 = 12 / 13.$$

Alternatively: $P(\text{not an ace}) = 1 - P(\text{ace}) = 1 - 1/13 = 12/13$.

चरणबद्ध हल (हिंदी):

कुल संभव परिणाम = 52।

(i) गड्डी में कुल 4 इक्के होते हैं। $P(\text{इक्का}) = 4 / 52 = 1 / 13$ ।

(ii) $P(\text{इक्का नहीं}) = 1 - 1/13 = 12 / 13$ ।

Textbook Example 13: Two Dice Thrown Simultaneously (Sum on 2 dice) / उदाहरण 13: दो पासे एक साथ फेंकने पर अंकों का योग

[Problem - EN]: Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is (i) 8? (ii) 13? (iii) less than or equal to 12?

[प्रश्न - HI]: एक नीले पासे और एक सलेटी पासे को एक साथ फेंका जाता है। सभी संभावित परिणाम लिखिए। इसकी क्या प्रायिकता है कि दोनों पासों की संख्याओं का योग (i) 8 है? (ii) 13 है? (iii) 12 से छोटी या उसके बराबर है?

Step-by-Step Solution (English):

When two dice are thrown, each outcome is an ordered pair (x, y) where $x, y \in \{1, 2, 3, 4, 5, 6\}$.

Total number of possible outcomes = $6 \times 6 = 36$.

(i) Outcomes for sum 8: $\{(2,6), (3,5), (4,4), (5,3), (6,2)\}$ (5 outcomes).

$$P(\text{sum } 8) = 5 / 36.$$

(ii) The maximum possible sum is $6 + 6 = 12$. Sum 13 is impossible.

$$P(\text{sum } 13) = 0 / 36 = 0 \text{ (Impossible event)}.$$

(iii) Every possible sum is between 2 and 12. All 36 outcomes are favourable.

$$P(\text{sum} \leq 12) = 36 / 36 = 1 \text{ (Sure event)}.$$

चरणबद्ध हल (हिंदी):

कुल संभव परिणाम = $6 \times 6 = 36$ ।

(i) योग 8 के अनुकूल परिणाम: $\{(2,6), (3,5), (4,4), (5,3), (6,2)\}$ (5 परिणाम) $\Rightarrow P(\text{योग } 8) = 5 / 36$ ।

(ii) अधिकतम योग 12 होता है, अतः योग 13 असंभव है $\Rightarrow P(\text{योग } 13) = 0$ (असंभव घटना)।

(iii) सभी 36 परिणामों का योग 12 या उससे कम है $\Rightarrow P(\leq 12) = 36/36 = 1$ (निश्चित घटना)।

Section 3: Exercise 14.1 Complete Step-by-Step Solutions (All 25 Problems)

खंड 3: प्रश्नावली 14.1 के सभी 25 प्रश्नों के संपूर्ण चरणबद्ध हल

Exercise 14.1 — Questions 1 to 10 / प्रश्नावली 14.1 — प्रश्न 1 से 10

[EN] Detailed solutions for Exercise 14.1 Q1 to Q10.

[HI] प्रश्नावली 14.1 के प्रश्न 1 से 10 के संपूर्ण हल:

- **Q1 (Fill in blanks):** (i) 1 • (ii) 0, impossible event (असंभव घटना) • (iii) 1, sure or certain event (निश्चित घटना) • (iv) 1 • (v) 0, 1.

- **Q2:** Only experiments (iii) **True-false question** and (iv) **Baby born boy or girl** have equally likely outcomes. [(i) and (ii) depend on mechanical/player factors].
- **Q3:** A coin toss has two mutually exclusive and **equally likely** outcomes (Head and Tail), making it completely unbiased and fair.
- **Q4: Option (B) -1.5**, since probability can never be negative ($0 \leq P(E) \leq 1$).
- **Q5:** $P(\text{not } E) = 1 - P(E) = 1 - 0.05 = 0.95$.
- **Q6:** Bag contains lemon candies only: (i) $P(\text{orange candy}) = 0$ (**Impossible**) • (ii) $P(\text{lemon candy}) = 1$ (**Sure event**).
- **Q7:** $P(\text{not same birthday}) = 0.992 \Rightarrow P(\text{same birthday}) = 1 - 0.992 = 0.008$.
- **Q8 (3 red + 5 black = 8 balls):** (i) $P(\text{red}) = 3/8$ • (ii) $P(\text{not red}) = 1 - 3/8 = 5/8$.
- **Q9 (5 red + 8 white + 4 green = 17 marbles):** (i) $P(\text{red}) = 5/17$ • (ii) $P(\text{white}) = 8/17$ • (iii) $P(\text{not green}) = (5 + 8)/17 = 13/17$.
- **Q10 (100×50p + 50×₹1 + 20×₹2 + 10×₹5 = 180 coins):** (i) $P(50p) = 100/180 = 5/9$ • (ii) $P(\text{not ₹5}) = (180 - 10)/180 = 170/180 = 17/18$.

Final Answer / उत्तर: All parts of Ex 14.1 Q1 to Q10 solved and verified.

Exercise 14.1 — Questions 11 to 20 / प्रश्नावली 14.1 — प्रश्न 11 से 20

[EN] Detailed solutions for Exercise 14.1 Q11 to Q20.

[HI] प्रश्नावली 14.1 के प्रश्न 11 से 20 के संपूर्ण हल:

- **Q11 (5 male + 8 female = 13 fish):** $P(\text{male fish}) = 5/13$.
- **Q12 (Spinning wheel 1 to 8 — Fig 14.1 Left):** (i) $P(8) = 1/8$ • (ii) $P(\text{odd: } 1,3,5,7) = 4/8 = 1/2$ • (iii) $P(>2: 3,4,5,6,7,8) = 6/8 = 3/4$ • (iv) $P(<9) = 8/8 = 1$.
- **Q13 (Die thrown once):** (i) $P(\text{prime: } 2,3,5) = 3/6 = 1/2$ • (ii) $P(\text{between 2 & 6: } 3,4,5) = 3/6 = 1/2$ • (iii) $P(\text{odd: } 1,3,5) = 3/6 = 1/2$.
- **Q14 (52 cards deck):** (i) $P(\text{king of red colour: K hearts, K diamonds}) = 2/52 = 1/26$ • (ii) $P(\text{face card: 4 J, 4 Q, 4 K}) = 12/52 = 3/13$ • (iii) $P(\text{red face card}) = 6/52 = 3/26$ • (iv) $P(\text{jack of hearts}) = 1/52$ • (v) $P(\text{spade}) = 13/52 = 1/4$ • (vi) $P(\text{queen of diamonds}) = 1/52$.
- **Q15 (5 cards: 10, J, Q, K, A of diamonds):** (i) $P(\text{queen}) = 1/5$ • (ii) Queen set aside (4 cards left): (a) $P(\text{ace}) = 1/4$, (b) $P(\text{queen}) = 0/4 = 0$.
- **Q16 (12 defective + 132 good = 144 pens):** $P(\text{good pen}) = 132/144 = 11/12$.
- **Q17 (20 bulbs, 4 defective, 16 good):** (i) $P(\text{defective}) = 4/20 = 1/5$ • (ii) Non-defective bulb removed $\Rightarrow$ 19 left, 15 good: $P(\text{not defective}) = 15/19$.
- **Q18 (90 discs numbered 1 to 90):** (i) Two-digit numbers (10 to 90 = 81): $P = 81/90 = 9/10$ • (ii) Perfect squares (1,4,9,16,25,36,49,64,81 = 9): $P = 9/90 = 1/10$ • (iii) Divisible by 5 (18 numbers): $P = 18/90 = 1/5$.
- **Q19 (Die faces: A, B, C, D, E, A):** (i) $P(A) = 2/6 = 1/3$ • (ii) $P(D) = 1/6$.
- **Q20 (Circle d = 1 m inside 3 m × 2 m rectangle — Fig 14.1 Right):**
 Area of rectangle = $3 \times 2 = 6 \text{ m}^2$.
 Area of circle = $\pi r^2 = \pi(1/2)^2 = \pi/4 \text{ m}^2$.
 $P(\text{landing inside circle}) = (\pi/4) / 6 = \pi / 24$.

Final Answer / उत्तर: All parts of Ex 14.1 Q11 to Q20 solved and verified.

Exercise 14.1 — Questions 21 to 25 / प्रश्नावली 14.1 — प्रश्न 21 से 25**[EN] Detailed solutions for Exercise 14.1 Q21 to Q25.****[HI] प्रश्नावली 14.1 के प्रश्न 21 से 25 के संपूर्ण हल:**

- **Q21 (144 pens, 20 defective, 124 good):** (i) $P(\text{buy}) = 124/144 = 31/36$ • (ii) $P(\text{not buy}) = 20/144 = 5/36$.
- **Q22 (Two dice sums):**
 - (i) Probabilities for sums 2 to 12:
 $P(2) = 1/36$, $P(3) = 2/36$, $P(4) = 3/36$, $P(5) = 4/36$, $P(6) = 5/36$, $P(7) = 6/36$, $P(8) = 5/36$, $P(9) = 4/36$, $P(10) = 3/36$, $P(11) = 2/36$, $P(12) = 1/36$.
 - (ii) **No (नहीं):** The student's argument is false because the 11 sums are not equally likely (sum 7 occurs in 6 ways, sum 2 in only 1 way).
- **Q23 (Coin tossed 3 times — 8 outcomes):** HHH, TTT, HHT, HTH, HTT, THH, THT, TTH.
Hanif wins on HHH or TTT (2 outcomes) and loses on the other 6 outcomes.
 $P(\text{losing the game}) = 6 / 8 = 3 / 4$.
- **Q24 (Die thrown twice — 36 outcomes):** Total outcomes where 5 comes up at least once = 11 [(5,1) to (5,6) and (1,5) to (6,5)].
(i) $P(5 \text{ will not come up either time}) = (36 - 11) / 36 = 25 / 36$.
(ii) $P(5 \text{ will come up at least once}) = 11 / 36$.
- **Q25 (Arguments Analysis):**
 - (i) **Incorrect (असत्य):** The outcomes 'two heads', 'two tails', and 'one of each' are not equally likely. 'One of each' occurs in two ways (HT, TH) out of 4 total outcomes, so its probability is $2/4 = 1/2$, not $1/3$.
 - (ii) **Correct (सत्य):** When a fair die is thrown, odd numbers {1, 3, 5} and even numbers {2, 4, 6} are equally likely, each having 3 outcomes out of 6. Therefore, $P(\text{odd}) = 3/6 = 1/2$.

Final Answer / उत्तर: All questions Q21 to Q25 of Ex 14.1 solved and verified.**Section 4: Chapter Summary & Key Probability Axioms****खंड 4: अध्याय सारांश एवं प्रायिकता के प्रमुख सिद्धांत**

Axiom / Property	Mathematical Rule	हिंदी विवरण
Classical Probability	$P(E) = \text{Favourable Outcomes} / \text{Total Outcomes}$	अनुकूल परिणाम / कुल संभव परिणाम
Complementary Rule	$P(\bar{E}) = 1 - P(E)$	$P(E \text{ नहीं}) = 1 - P(E)$
Boundaries	$0 \leq P(E) \leq 1$	प्रायिकता 0 और 1 के बीच होती है

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