

TYBA COACHING CENTRE | टी वाई बी ए कोचिंग सेंटर

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Chapter 13: Statistics

अध्याय 13: सांख्यिकी

Section 1: Detailed Concepts, Statistical Measures & Formulas

खंड 1: विस्तृत अवधारणाएँ, सांख्यिकीय माप एवं सूत्र

13.1 Mean of Grouped Data (Direct, Assumed Mean & Step-Deviation) / 13.1 वर्गीकृत आँकड़ों का माध्य (प्रत्यक्ष, कल्पित माध्य एवं पग-विचलन विधियाँ)

[English Theory]: The **mean (माध्य)** of observations is the average value. For grouped frequency distributions with class mark $x_i = (\text{Upper Limit} + \text{Lower Limit}) / 2$ and frequency f_i :

1. Direct Method (प्रत्यक्ष विधि):

$\bar{x} = \Sigma f_i x_i / \Sigma f_i$ — Recommended when numerical values of x_i and f_i are small.

2. Assumed Mean Method (कल्पित माध्य विधि):

Choose a central class mark as assumed mean **a**. Deviation $d_i = x_i - a$.

$$\bar{x} = a + (\Sigma f_i d_i / \Sigma f_i).$$

3. Step-Deviation Method (पग-विचलन विधि):

If all d_i have a common factor (class size h), let $u_i = (x_i - a) / h$.

$$\bar{x} = a + h \times (\Sigma f_i u_i / \Sigma f_i).$$

Examination Tip: All three methods yield the exact same mathematical mean.

[हिंदी सिद्धांत]: वर्गीकृत बारंबारता बंटन के लिए, वर्ग चिह्न $x_i = (\text{उच्च सीमा} + \text{निम्न सीमा}) / 2$:

1. प्रत्यक्ष विधि: $\bar{x} = \Sigma f_i x_i / \Sigma f_i$ (जब मान छोटे हों)।

2. कल्पित माध्य विधि: कल्पित माध्य a मानकर विचलन $d_i = x_i - a$: $\bar{x} = a + (\Sigma f_i d_i / \Sigma f_i)$ ।

3. पग-विचलन विधि: वर्ग माप h से भाग देकर $u_i = (x_i - a) / h$: $\bar{x} = a + h \times (\Sigma f_i u_i / \Sigma f_i)$ ।

तीनों विधियों से सदैव एक ही समान उत्तर प्राप्त होता है।

13.2 Mode & 13.3 Median of Grouped Data with Empirical Relation / 13.2 बहुलक एवं 13.3 माध्यक तथा आनुभविक संबंध

[English Theory]: • **Mode of Grouped Data (बहुलक):** The value among the observations which has maximum frequency.

Modal Class: The class interval having the highest frequency.

$$\text{Mode} = l + [(f_1 - f_0) / (2f_1 - f_0 - f_2)] \times h$$

where l = lower limit of modal class, h = class size, f_1 = frequency of modal class, f_0 = frequency of preceding class, f_2 = frequency of succeeding class.

• **Median of Grouped Data (माध्यक):** The value of the middle observation dividing the data into two halves.

Median Class: The class interval whose cumulative frequency (cf) is just greater than or equal to $n / 2$ (where $n = \Sigma f_i$).

$$\text{Median} = l + [(n/2 - cf) / f] \times h$$

where l = lower limit of median class, cf = cumulative frequency of class preceding the median class, f = frequency of median class, h = class size.

• **Empirical Relationship between the Three Measures of Central Tendency:**

3 Median = Mode + 2 Mean (3 माध्यक = बहुलक + 2 माध्य).

[हिंदी सिद्धांत]: • **बहुलक (Mode):** अधिकतम बारंबारता वाला प्रेक्षण।

बहुलक वर्ग: सबसे अधिक बारंबारता वाला वर्ग अंतराल।

बहुलक = $l + \left[\frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \right] \times h$

• **माध्यक (Median):** वह मान जो व्यवस्थित आँकड़ों को दो बराबर भागों में बाँटता है।

माध्यक वर्ग: वह वर्ग जिसकी संचयी बारंबारता (cf) $n/2$ से ठीक बड़ी या बराबर हो।

माध्यक = $l + \left[\frac{(n/2 - cf)}{f} \right] \times h$

• **तीनों केंद्रीय प्रवृत्ति के मापकों में संबंध:**

3 माध्यक = बहुलक + 2 माध्य।

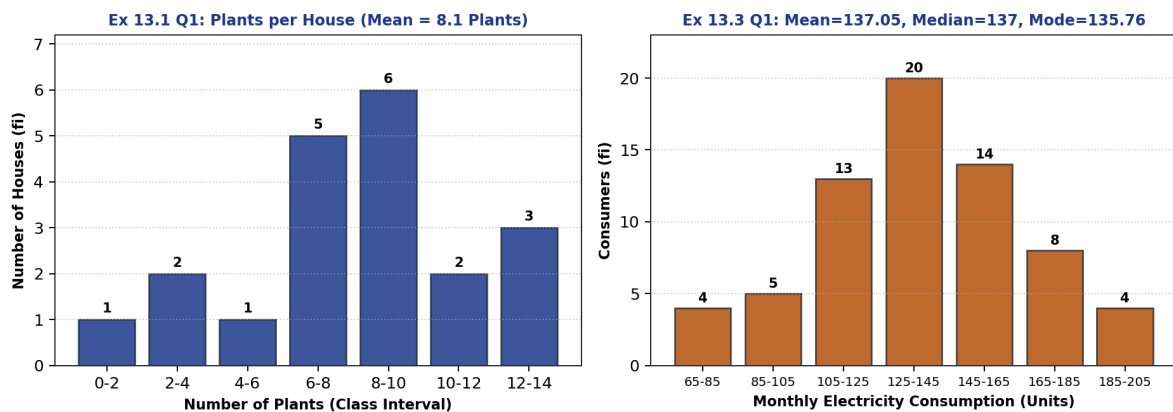


Fig 13.1: Frequency Distributions: Number of Plants per House (Ex 13.1 Q1) and Monthly Electricity Consumption showing Mean=137.05, Median=137, Mode=135.76 (Ex 13.3 Q1) — TYBA Coaching Centre

Section 2: Textbook Solved Examples with Step-by-Step Bilingual Solutions

खंड 2: पाठ्यपुस्तक के सभी हल किए गए उदाहरण (विस्तृत चरणबद्ध हल)

Textbook Example 2: Mean Percentage of Female Teachers in 35 States / उदाहरण 2: 35 राज्यों में महिला शिक्षकों का प्रतिशत (माध्य)

[Problem - EN]: The table gives the percentage distribution of female teachers in the primary schools of rural areas of various states and union territories (U.T.) of India. Find the mean percentage of female teachers using all three methods (Class intervals: 15-25, 25-35, 35-45, 45-55, 55-65, 65-75, 75-85; Frequencies: 6, 11, 7, 4, 4, 2, 1).

[प्रश्न - HI]: भारत के 35 राज्यों/केंद्रशासित प्रदेशों के ग्रामीण क्षेत्रों के प्राथमिक विद्यालयों में महिला शिक्षकों का प्रतिशत बंटन दिया है। तीनों विधियों द्वारा महिला शिक्षकों का माध्य प्रतिशत ज्ञात कीजिए।

Step-by-Step Solution (English):

Class marks x_i : 20, 30, 40, 50, 60, 70, 80.

$\Sigma f_i = 35$. Let assumed mean $a = 50$, class size $h = 10$.

$u_i = (x_i - 50)/10$: -3, -2, -1, 0, 1, 2, 3.

$\Sigma f_i u_i = 6(-3) + 11(-2) + 7(-1) + 4(0) + 4(1) + 2(2) + 1(3) = -18 - 22 - 7 + 0 + 4 + 4 + 3 = -36$.

Mean $\bar{x} = a + h(\Sigma f_i u_i / \Sigma f_i) = 50 + 10(-36 / 35) = 50 - 360/35 = 50 - 10.29 = 39.71\%$.

चरणबद्ध हल (हिंदी):

वर्ग चिह्न x_i : 20, 30, 40, 50, 60, 70, 80। कल्पित माध्य $a = 50$, $h = 10$ ।

$\Sigma f_i = 35$ तथा $\Sigma f_i u_i = -36$ ।

माध्य $\bar{x} = 50 + 10(-36/35) = 50 - 10.29 = 39.71\%$ ।

Textbook Example 5: Mode of Bowling Wickets in 10 Cricket Matches / उदाहरण 5: 10 क्रिकेट मैचों में गेंदबाजों द्वारा लिए गए विकेटों का बहुलक

[Problem - EN]: A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household (Family size: 1-3, 3-5, 5-7, 7-9, 9-11; Frequency: 7, 8, 2, 2, 1). Find the mode of this data.

[प्रश्न - HI]: विद्यार्थियों के एक समूह द्वारा 20 परिवारों पर किए गए सर्वेक्षण से परिवार के सदस्यों की संख्या की बारंबारता सारणी दी गई है। इस आँकड़े का बहुलक ज्ञात कीजिए।

Step-by-Step Solution (English):

Maximum frequency is 8, which corresponds to class interval 3-5.

Therefore, Modal class = 3-5.

Lower limit $l = 3$, class size $h = 2$, $f_1 = 8$, $f_0 = 7$, $f_2 = 2$.

Mode $= l + \left[\frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \right] \times h$

$= 3 + \left[\frac{(8 - 7)}{(2 \times 8 - 7 - 2)} \right] \times 2 = 3 + \left[\frac{1}{(16 - 9)} \right] \times 2 = 3 + \frac{2}{7} = 3 + 0.286 = \mathbf{3.286}$.

चरणबद्ध हल (हिंदी):

अधिकतम बारंबारता 8 है, जो वर्ग 3-5 की है। अतः बहुलक वर्ग = 3-5।

यहाँ $l = 3$, $h = 2$, $f_1 = 8$, $f_0 = 7$, $f_2 = 2$ ।

बहुलक $= 3 + \left[\frac{(8 - 7)}{(16 - 9)} \right] \times 2 = 3 + \frac{2}{7} = \mathbf{3.286}$ ।

Textbook Example 7: Median Marks of 53 Students / उदाहरण 7: 53 विद्यार्थियों के प्राप्तांकों का माध्यक

[Problem - EN]: A survey regarding the heights (in cm) of 51 girls of Class X of a school was conducted and the data was obtained (Height less than 140: 4; less than 145: 11; less than 150: 29; less than 155: 40; less than 160: 46; less than 165: 51). Find the median height.

[प्रश्न - HI]: किसी स्कूल की कक्षा X की 51 लड़कियों की ऊँचाइयों का सर्वेक्षण किया गया (140 से कम: 4; 145 से कम: 11; 150 से कम: 29; 155 से कम: 40; 160 से कम: 46; 165 से कम: 51)। माध्यक ऊँचाई ज्ञात कीजिए।

Step-by-Step Solution (English):

Frequencies for classes: 135-140: 4; 140-145: 7; 145-150: 18; 150-155: 11; 155-160: 6; 160-165: 5.

Total $n = 51 \Rightarrow n/2 = 25.5$.

The cumulative frequency just greater than 25.5 is 29, which belongs to class interval 145-150.

Therefore, Median class = 145-150 ($l = 145$, $cf = 11$, $f = 18$, $h = 5$).

Median $= l + \left[\frac{(n/2 - cf)}{f} \right] \times h = 145 + \left[\frac{(25.5 - 11)}{18} \right] \times 5 = 145 + \left(\frac{14.5}{18} \right) \times 5 = 145 + \frac{72.5}{18} = 145 + 4.03 = \mathbf{149.03 \text{ cm}}$.

चरणबद्ध हल (हिंदी):

बारंबारताएँ: 140-145: 7; 145-150: 18; 150-155: 11... $n/2 = 25.5$ ।

25.5 से बड़ी संचयी बारंबारता 29 है, जो वर्ग 145-150 की है। अतः माध्यक वर्ग = 145-150।

माध्यक $= 145 + \left[\frac{(25.5 - 11)}{18} \right] \times 5 = 145 + 4.03 = \mathbf{149.03 \text{ cm}}$ ।

Section 3: Exercise 13.1, 13.2 & 13.3 Complete Step-by-Step Solutions

खंड 3: प्रश्नावली 13.1, 13.2 एवं 13.3 के संपूर्ण चरणबद्ध हल

Exercise 13.1 (Mean) — Questions 1 to 9 / प्रश्नावली 13.1 (माध्य) — प्रश्न 1 से 9

[EN] Detailed solutions for all 9 problems of Exercise 13.1.

[HI] प्रश्नावली 13.1 के सभी 9 प्रश्नों के संपूर्ण हल:

- **Q1 (Plants in 20 houses — Fig 13.1 Left):** x_i : 1, 3, 5, 7, 9, 11, 13; f_i : 1, 2, 1, 5, 6, 2, 3. $\Sigma f_i = 20$, $\Sigma f_i x_i = 162 \Rightarrow \text{Mean} = 162/20 = \mathbf{8.1 \text{ plants}}$ (Direct method used as values are small).
- **Q2 (Wages of 50 workers):** $a = 550$, $h = 20$, $\Sigma f_i = 50$, $\Sigma f_i u_i = -12 \Rightarrow \text{Mean} = 550 + 20(-12/50) = 550 - 4.80 = \mathbf{₹ 545.20}$.
- **Q3 (Pocket allowance mean = ₹ 18, find missing frequency f):**
 x_i : 12, 14, 16, 18, 20, 22, 24; f_i : 7, 6, 9, 13, f , 5, 4. $\Sigma f_i = 44 + f$.
 Let $a = 18$. Then $\Sigma f_i d_i = 7(-6) + 6(-4) + 9(-2) + 13(0) + f(2) + 5(4) + 4(6) = -42 - 24 - 18 + 0 + 2f + 20 + 24 = 2f - 40$.
 Since mean = 18, $a + (\Sigma f_i d_i)/(\Sigma f_i) = 18 \Rightarrow 2f - 40 = 0 \Rightarrow \mathbf{f = 20}$.
- **Q4 (Heartbeats of 30 women):** $a = 75.5$, $h = 3$, $\Sigma f_i = 30$, $\Sigma f_i u_i = 4 \Rightarrow \text{Mean} = 75.5 + 3(4/30) = 75.5 + 0.4 = \mathbf{75.9}$.
- **Q5 (Mangoes in 400 boxes):** $a = 57$, $h = 3$, $\Sigma f_i = 400$, $\Sigma f_i u_i = 25 \Rightarrow \text{Mean} = 57 + 3(25/400) = \mathbf{57.19 \text{ mangoes}}$.
- **Q6 (Food expenditure of 25 households):** $a = 225$, $h = 50$, $\Sigma f_i = 25$, $\Sigma f_i u_i = -7 \Rightarrow \text{Mean} = 225 + 50(-7/25) = 225 - 14 = \mathbf{₹ 211}$.
- **Q7 (SO₂ concentration in 30 localities):** $a = 0.10$, $h = 0.04$, $\Sigma f_i = 30$, $\Sigma f_i u_i = -1 \Rightarrow \text{Mean} = 0.10 + 0.04(-1/30) = \mathbf{0.099 \text{ ppm}}$.
- **Q8 (Absentee record of 40 students):** Direct method (unequal class sizes): $\Sigma f_i = 40$, $\Sigma f_i x_i = 499 \Rightarrow \text{Mean} = 499/40 = \mathbf{12.48 \text{ days}}$.
- **Q9 (Literacy rate of 35 cities):** $a = 70$, $h = 10$, $\Sigma f_i = 35$, $\Sigma f_i u_i = -2 \Rightarrow \text{Mean} = 70 + 10(-2/35) = 70 - 0.57 = \mathbf{69.43\%}$.

Final Answer / उत्तर: Q1: 8.1 • Q2: ₹ 545.20 • Q3: f = 20 • Q4: 75.9 • Q5: 57.19 • Q6: ₹ 211 • Q7: 0.099 ppm • Q8: 12.48 days • Q9: 69.43%

Exercise 13.2 (Mode & Mean) — Questions 1 to 6 / प्रश्नावली 13.2 (बहुलक एवं माध्य) — प्रश्न 1 से 6

[EN] Detailed solutions for Exercise 13.2 Q1 to Q6.

[HI] प्रश्नावली 13.2 के प्रश्न 1 से 6 तक के संपूर्ण हल:

- **Q1 (Hospital Patients):** Modal class 35–45 ($l=35$, $f_1=23$, $f_0=21$, $f_2=14$, $h=10$).
 $\text{Mode} = 35 + [(23 - 21)/(46 - 21 - 14)] \times 10 = 35 + (2/11) \times 10 = \mathbf{36.8 \text{ years}}$; $\text{Mean} = 2830 / 80 = \mathbf{35.37 \text{ years}}$.
- **Q2 (Lifetimes of 225 components):** Modal class 60–80 ($l=60$, $f_1=61$, $f_0=52$, $f_2=38$, $h=20$).
 $\text{Mode} = 60 + [(61 - 52)/(122 - 90)] \times 20 = 60 + (9/32) \times 20 = \mathbf{65.625 \text{ hours}}$.
- **Q3 (Monthly expenditure of 200 families):** Modal class 1500–2000 ($l=1500$, $f_1=40$, $f_0=24$, $f_2=33$, $h=500$).
 $\text{Mode} = 1500 + (16/23) \times 500 = \mathbf{₹ 1847.83}$; $\text{Mean} = 2750 + 500(-35/200) = \mathbf{₹ 2662.50}$.
- **Q4 (Student-Teacher Ratio across 35 States):** Modal class 30–35 ($l=30$, $f_1=10$, $f_0=9$, $f_2=3$, $h=5$).
 $\text{Mode} = 30 + (1/8) \times 5 = \mathbf{30.6}$; $\text{Mean} = 1022.5 / 35 = \mathbf{29.2}$.
- **Q5 (Runs scored by top batsmen):** Modal class 4000–5000 ($l=4000$, $f_1=18$, $f_0=4$, $f_2=9$, $h=1000$).
 $\text{Mode} = 4000 + (14/23) \times 1000 = \mathbf{4608.7 \text{ runs}}$.
- **Q6 (Cars in 100 periods):** Modal class 40–50 ($l=40$, $f_1=20$, $f_0=12$, $f_2=11$, $h=10$).
 $\text{Mode} = 40 + (8/17) \times 10 = \mathbf{44.7 \text{ cars}}$.

Final Answer / उत्तर: Q1: Mode=36.8 yrs, Mean=35.37 yrs • Q2: 65.625 hrs • Q3: Mode=₹ 1847.83, Mean=₹ 2662.50 • Q4: Mode=30.6, Mean=29.2 • Q5: 4608.7 runs • Q6: 44.7 cars

Exercise 13.3 (Median) — Questions 1 to 7 / प्रश्नावली 13.3 (माध्यक) — प्रश्न 1 से 7

[EN] Detailed solutions for Exercise 13.3 Q1 to Q7.

[HI] प्रश्नावली 13.3 के प्रश्न 1 से 7 तक के संपूर्ण हल:

- **Q1 (Electricity consumption of 68 consumers — Fig 13.1 Right):**
 $n/2 = 34 \Rightarrow$ Median class = 125–145 ($l=125$, $cf=22$, $f=20$, $h=20$).
Median = $125 + [(34 - 22)/20] \times 20 = 137$ units.
 Mean = 137.05 units; Mode = 135.76 units. (All three measures are approximately equal).
- **Q2 (Given Median = 28.5, Total $n = 60$, find x and y):**
 $45 + x + y = 60 \Rightarrow x + y = 15$ —(1). Median class = 20–30 ($l=20$, $cf=5+x$, $f=20$, $h=10$):
 $28.5 = 20 + [(30 - 5 - x)/20] \times 10 \Rightarrow 8.5 = (25 - x)/2 \Rightarrow 25 - x = 17 \Rightarrow x = 8$, and $y = 15 - 8 = 7$.
- **Q3 (100 policy holders):** $n/2 = 50 \Rightarrow$ Median class 35–40 ($l=35$, $cf=45$, $f=33$, $h=5$) $\Rightarrow$ **Median age = $35 + (5/33) \times 5 = 35.76$ years.**
- **Q4 (Lengths of 40 leaves — continuous intervals 117.5–126.5...):**
 $n/2 = 20 \Rightarrow$ Median class 144.5–153.5 ($l=144.5$, $cf=17$, $f=12$, $h=9$) $\Rightarrow$ **Median length = $144.5 + (3/12) \times 9 = 146.75$ mm.**
- **Q5 (400 neon lamps):** $n/2 = 200 \Rightarrow$ Median class 3000–3500 ($l=3000$, $cf=130$, $f=86$, $h=500$) $\Rightarrow$ **Median life = $3000 + (70/86) \times 500 = 3406.98$ hours.**
- **Q6 (Letters in 100 surnames):** Median class 7–10 ($l=7$, $cf=36$, $f=40$, $h=3$) $\Rightarrow$ **Median = 8.05, Mean = 8.32, Modal size = 7.88.**
- **Q7 (Weights of 30 students):** $n/2 = 15 \Rightarrow$ Median class 55–60 ($l=55$, $cf=13$, $f=6$, $h=5$) $\Rightarrow$ **Median weight = $55 + (2/6) \times 5 = 56.67$ kg.**

Final Answer / उत्तर: Q1: Med=137, Mean=137.05, Mode=135.76 • Q2: $x=8$, $y=7$ • Q3: 35.76 yrs • Q4: 146.75 mm • Q5: 3406.98 hrs • Q6: Med=8.05, Mean=8.32, Mode=7.88 • Q7: 56.67 kg

Section 4: Chapter Summary & Statistical Comparison**खंड 4: अध्याय सारांश एवं सांख्यिकीय सूत्रों की तुलना**

Measure	Core Formula	When to Prefer
Mean ($\bar{x}$)	$\Sigma f_i x_i / \Sigma f_i$	When every single data value is equally significant.
Mode	$l + [(f_1 - f_0)/(2f_1 - f_0 - f_2)] \times h$	When finding the most popular, recurring or frequent value.
Median	$l + [(n/2 - cf)/f] \times h$	When extreme outlier values would distort the mean.

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