

TYBA COACHING CENTRE | टी वाई बी ए कोचिंग सेंटर

Contact / WhatsApp: 6367029449 • NCERT Class 10 Mathematics Complete Comprehensive Notes & Solutions

Chapter 11: Areas Related to Circles

अध्याय 11: वृत्तों से संबंधित क्षेत्रफल

Section 1: Detailed Concepts, Formulas & Circular Geometry

खंड 1: विस्तृत अवधारणाएँ, सूत्र एवं वृत्तीय ज्यामिति

11.1 Review of Perimeter & Area of a Circle / 11.1 वृत्त का परिमाप एवं क्षेत्रफल — पुनरावलोकन

[English Theory]: • **Perimeter (Circumference):** The distance covered by travelling once around a circle:

$$\text{Circumference} = 2\pi r = \pi d.$$

• **The Number π :** The ratio of circumference to diameter is a constant denoted by π (an irrational number with approximate rational value $22/7$ or decimal $3.14159...$).

• **Area of a Circle:** $\text{Area} = \pi r^2$.

• **Area of a Semicircle:** $\frac{1}{2}\pi r^2$; **Area of a Quadrant:** $\frac{1}{4}\pi r^2$.

• **Area of Circular Ring (Concentric Circles of radii R and r):** $\pi(R^2 - r^2)$.

[हिंदी सिद्धांत]: • **परिमाप (परिधि):** वृत्त के चारों ओर एक बार चलने में तय की गई दूरी: **परिधि** $= 2\pi r = \pi d$ ।

• **संख्या π :** परिधि और व्यास का अनुपात सदैव एक अचर होता है जिसे π से व्यक्त करते हैं (अपरिमित संख्या, लगभग मान $22/7$ अथवा 3.14)।

• **वृत्त का क्षेत्रफल:** $\text{क्षेत्रफल} = \pi r^2$ ।

• **अर्धवृत्त का क्षेत्रफल:** $\frac{1}{2}\pi r^2$; **चतुर्थांश (Quadrant):** $\frac{1}{4}\pi r^2$ ।

• **वृत्ताकार वलय (Ring) का क्षेत्रफल:** $\pi(R^2 - r^2)$ ।

11.2 Areas of Sector and Segment of a Circle / 11.2 त्रिज्यखंड और वृत्तखंड के क्षेत्रफल

[English Theory]: • **Sector of a Circle (त्रिज्यखंड):** The portion (or part) of the circular region enclosed by two radii and the corresponding arc.

1. **Length of an Arc of a sector of angle θ :**

$$l = (\theta / 360^\circ) \times 2\pi r.$$

2. **Area of Minor Sector of angle θ :**

$$\text{Area} = (\theta / 360^\circ) \times \pi r^2 \quad (\text{or } \frac{1}{2} \times l \times r).$$

3. **Area of Major Sector:**

$$\text{Area} = \pi r^2 - \text{Area of Minor Sector} = [(360^\circ - \theta) / 360^\circ] \times \pi r^2.$$

• **Segment of a Circle (वृत्तखंड):** The portion of the circular region enclosed between a chord and the corresponding arc.

1. **Area of Minor Segment:**

$$\text{Area of Minor Segment} = \text{Area of Minor Sector} - \text{Area of } \triangle OAB.$$

• For $\angle AOB = 60^\circ$: $\text{Area}(\triangle OAB) = (\sqrt{3} / 4)r^2$ (Equilateral triangle).

• For $\angle AOB = 90^\circ$: $\text{Area}(\triangle OAB) = \frac{1}{2}r^2$ (Right-angled triangle).

• For any angle θ : $\text{Area}(\triangle OAB) = \frac{1}{2}r^2 \sin \theta$.

2. **Area of Major Segment:**

$$\text{Area of Major Segment} = \pi r^2 - \text{Area of Minor Segment}.$$

[हिंदी सिद्धांत]: • **त्रिज्यखंड (Sector):** दो त्रिज्याओं और संगत चाप से घिरा वृत्ताकार क्षेत्र का भाग।

1. **चाप की लंबाई (Arc Length):** $l = (\theta / 360^\circ) \times 2\pi r$ ।

2. **लघु त्रिज्यखंड का क्षेत्रफल:** $(\theta / 360^\circ) \times \pi r^2$ (अथवा $\frac{1}{2} \times l \times r$)।

3. दीर्घ त्रिज्यखंड का क्षेत्रफल: πr^2 – लघु त्रिज्यखंड का क्षेत्रफल।

• **वृत्तखंड (Segment):** एक जीवा और संगत चाप के बीच घिरा वृत्ताकार क्षेत्र।

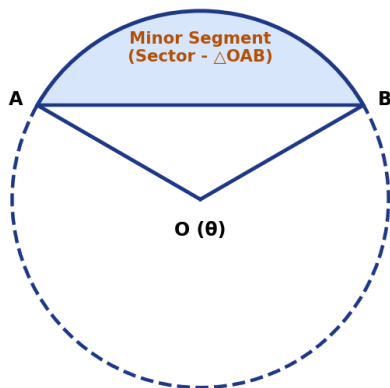
1. लघु वृत्तखंड का क्षेत्रफल = त्रिज्यखंड का क्षेत्रफल – $\triangle OAB$ का क्षेत्रफल।

• $\angle AOB = 60^\circ$ होने पर: $\triangle OAB$ का क्षेत्रफल = $(\sqrt{3} / 4)r^2$ ।

• $\angle AOB = 90^\circ$ होने पर: $\triangle OAB$ का क्षेत्रफल = $\frac{1}{2}r^2$ ।

2. दीर्घ वृत्तखंड का क्षेत्रफल = πr^2 – लघु वृत्तखंड का क्षेत्रफल।

Ex 11.1 Q4-Q7: Minor Sector & Minor Segment



Ex 11.1 Q8: Grazing Area in 15 m Square Field

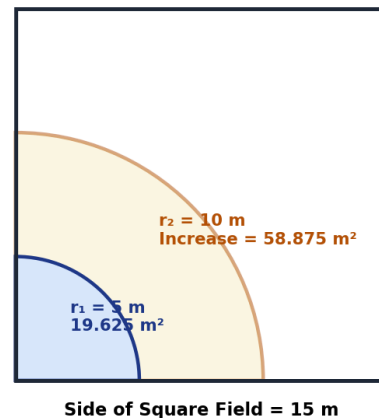


Fig 11.1: Minor Sector & Segment Geometry (Sector – Triangle OAB) and Grazing Area in 15m Square Field (Ex 11.1 Q8) — TYBA Coaching Centre

Section 2: Textbook Solved Examples with Step-by-Step Bilingual Solutions

खंड 2: पाठ्यपुस्तक के सभी हल किए गए उदाहरण (विस्तृत चरणबद्ध हल)

Textbook Example 1: Sector of a Circle of Radius 4 cm and Angle 30° / उदाहरण 1: 4 cm त्रिज्या और 30° कोण वाले त्रिज्यखंड का क्षेत्रफल

[Problem - EN]: Find the area of the sector of a circle with radius 4 cm and of angle 30° . Also, find the area of the corresponding major sector (Use $\pi = 3.14$).

[प्रश्न - HI]: त्रिज्या 4 cm वाले एक वृत्त के त्रिज्यखंड का क्षेत्रफल ज्ञात कीजिए, जिसका कोण 30° है। साथ ही, संगत दीर्घ त्रिज्यखंड का क्षेत्रफल भी ज्ञात कीजिए ($\pi = 3.14$ का प्रयोग कीजिए)।

Step-by-Step Solution (English):

Given $r = 4$ cm, $\theta = 30^\circ$.

Area of minor sector = $(\theta / 360^\circ) \times \pi r^2 = (30 / 360) \times 3.14 \times 4 \times 4 = (1/12) \times 3.14 \times 16 = 50.24 / 12 = 4.19 \text{ cm}^2$ (approx).

Area of corresponding major sector = πr^2 – Area of minor sector
 $= 3.14 \times 16 - 4.19 = 50.24 - 4.19 = 46.05 \text{ cm}^2$ (or $(330/360) \times 3.14 \times 16 = 46.1 \text{ cm}^2$).

चरणबद्ध हल (हिंदी):

यहाँ $r = 4$ cm, $\theta = 30^\circ$ ।

लघु त्रिज्यखंड का क्षेत्रफल = $(30/360) \times 3.14 \times 16 = 4.19 \text{ cm}^2$ ।

दीर्घ त्रिज्यखंड का क्षेत्रफल = $3.14 \times 16 - 4.19 = 46.05 \text{ cm}^2$ ।

Textbook Example 2: Area of Segment AYB with $r = 21$ cm & Angle 120° / उदाहरण 2: 21 cm त्रिज्या और 120° कोण वाले वृत्तखंड AYB का क्षेत्रफल

[Problem - EN]: Find the area of the segment AYB shown in Fig. 11.6, if radius of the circle is 21 cm and $\angle AOB = 120^\circ$. (Use $\pi = 22/7$)

[प्रश्न - HI]: आकृति में दर्शाए गए वृत्तखंड AYB का क्षेत्रफल ज्ञात कीजिए, यदि वृत्त की त्रिज्या 21 cm है और $\angle AOB = 120^\circ$ है। ($\pi = 22/7$ लीजिए)

Step-by-Step Solution (English):

Area of segment AYB = Area of sector OAYB – Area of $\triangle OAB$.

Area of sector OAYB = $(120 / 360) \times (22 / 7) \times 21 \times 21 = (1/3) \times 22 \times 3 \times 21 = 462 \text{ cm}^2$.

To find Area of $\triangle OAB$:

Draw $OM \perp AB$. Then OM bisects AB and $\angle AOB$ ($\angle AOM = 60^\circ$).

In right $\triangle OMA$: $OM = OA \cos 60^\circ = 21 \times (1/2) = 21/2 \text{ cm}$.

$AM = OA \sin 60^\circ = 21 \times (\sqrt{3} / 2) = 21\sqrt{3} / 2 \text{ cm} \Rightarrow AB = 2 \times AM = 21\sqrt{3} \text{ cm}$.

Area($\triangle OAB$) = $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 21\sqrt{3} \times (21/2) = 441\sqrt{3} / 4 \text{ cm}^2$.

Therefore, Area of segment AYB = $(462 - 441\sqrt{3} / 4) \text{ cm}^2 = (21/4)(88 - 21\sqrt{3}) \text{ cm}^2$.

चरणबद्ध हल (हिंदी):

वृत्तखंड AYB का क्षेत्रफल = त्रिज्यखंड OAYB का क्षेत्रफल – $\triangle OAB$ का क्षेत्रफल।

त्रिज्यखंड का क्षेत्रफल = $(120/360) \times (22/7) \times 21 \times 21 = 462 \text{ cm}^2$ ।

$\triangle OAB$ में $OM \perp AB$ खींचने पर: $OM = 21/2 \text{ cm}$ तथा $AB = 21\sqrt{3} \text{ cm}$ ।

$\text{ar}(\triangle OAB) = \frac{1}{2} \times 21\sqrt{3} \times 21/2 = 441\sqrt{3} / 4 \text{ cm}^2$ ।

अतः वृत्तखंड का क्षेत्रफल = $(462 - 441\sqrt{3} / 4) \text{ cm}^2$ ।

Section 3: Exercise 11.1 Complete Step-by-Step Solutions (All 14 Problems)

खंड 3: प्रश्नावली 11.1 के सभी 14 प्रश्नों के संपूर्ण चरणबद्ध हल

Exercise 11.1 — Questions 1 to 5 / प्रश्नावली 11.1 — प्रश्न 1 से 5

[EN] Detailed solutions for Exercise 11.1 Q1 to Q5 (Unless stated otherwise, use $\pi = 22/7$).

[HI] प्रश्नावली 11.1 के प्रश्न 1 से 5 तक के संपूर्ण हल:

- **Q1 (Sector $r = 6 \text{ cm}$, $\theta = 60^\circ$):** Area = $(60/360) \times (22/7) \times 6 \times 6 = (1/6) \times (22/7) \times 36 = 132 / 7 \text{ cm}^2$.
- **Q2 (Quadrant with Circumference = 22 cm):** $2\pi r = 22 \Rightarrow 2(22/7)r = 22 \Rightarrow r = 7/2 \text{ cm}$.
Area of quadrant = $\frac{1}{4}\pi r^2 = (1/4) \times (22/7) \times (7/2) \times (7/2) = 77 / 8 \text{ cm}^2$.
- **Q3 (Minute hand of length 14 cm in 5 minutes):** Angle in 60 min = $360^\circ \Rightarrow$ in 5 min, $\theta = (360/60) \times 5 = 30^\circ$.
Area swept = $(30/360) \times (22/7) \times 14 \times 14 = (1/12) \times (22/7) \times 196 = 154 / 3 \text{ cm}^2$.
- **Q4 (Chord of $r = 10 \text{ cm}$ subtends 90° , $\pi = 3.14$):**
(i) Minor segment = Area of sector – Area of $\triangle OAB = (90/360) \times 3.14 \times 100 - \frac{1}{2} \times 10 \times 10 = 78.5 - 50 = 28.5 \text{ cm}^2$.
(ii) Major sector = $(270/360) \times 3.14 \times 100 = (3/4) \times 314 = 235.5 \text{ cm}^2$.
- **Q5 (Circle of $r = 21 \text{ cm}$, $\theta = 60^\circ$):**
(i) Length of arc = $(60/360) \times 2 \times (22/7) \times 21 = (1/6) \times 132 = 22 \text{ cm}$.
(ii) Area of sector = $(60/360) \times (22/7) \times 21 \times 21 = 231 \text{ cm}^2$.
(iii) Area of equilateral $\triangle OAB = (\sqrt{3}/4) \times 21^2 = 441\sqrt{3} / 4 \text{ cm}^2$.
Area of segment = $(231 - 441\sqrt{3} / 4) \text{ cm}^2$.

Final Answer / उत्तर: Q1: $132/7 \text{ cm}^2$ • Q2: $77/8 \text{ cm}^2$ • Q3: $154/3 \text{ cm}^2$ • Q4: (i) 28.5 cm^2 (ii) 235.5 cm^2 • Q5: 22 cm , 231 cm^2 , $(231 - 441\sqrt{3}/4) \text{ cm}^2$

Exercise 11.1 — Questions 6 to 10 / प्रश्नावली 11.1 — प्रश्न 6 से 10

[EN] Detailed solutions for Exercise 11.1 Q6 to Q10.

[HI] प्रश्नावली 11.1 के प्रश्न 6 से 10 तक के संपूर्ण हल:

- **Q6 ($r = 15$ cm, $\theta = 60^\circ$, $\pi = 3.14$, $\sqrt{3} = 1.73$):**

Sector area = $(60/360) \times 3.14 \times 225 = 117.75$ cm².

Area of $\triangle OAB = (\sqrt{3}/4) \times 225 = (1.73 \times 225) / 4 = 97.3125$ cm².

- **Minor segment** = $117.75 - 97.3125 = 20.4375$ cm².

- **Major segment** = $\pi r^2 - 20.4375 = 3.14 \times 225 - 20.4375 = 706.5 - 20.4375 = 686.0625$ cm².

- **Q7 ($r = 12$ cm, $\theta = 120^\circ$, $\pi = 3.14$, $\sqrt{3} = 1.73$):**

Sector area = $(120/360) \times 3.14 \times 144 = 150.72$ cm².

Area of $\triangle OAB = \frac{1}{2} \times 12 \times 12 \times \sin 120^\circ = 72 \times (\sqrt{3}/2) = 36\sqrt{3} = 36 \times 1.73 = 62.28$ cm².

Area of segment = $150.72 - 62.28 = 88.44$ cm².

- **Q8 (Horse in 15 m square field — Fig 11.1 Right, $\pi = 3.14$):**

Corner of square has angle 90° .

(i) Grazing area for rope $r_1 = 5$ m: $(90/360) \times 3.14 \times 5^2 = (1/4) \times 78.5 = 19.625$ m².

(ii) For rope $r_2 = 10$ m: $(1/4) \times 3.14 \times 100 = 78.5$ m².

Increase in grazing area = $78.5 - 19.625 = 58.875$ m².

- **Q9 (Silver brooch $d = 35$ mm, 5 diameters / 10 equal sectors):**

(i) Wire required = Circumference + $5 \times$ diameter = $(22/7) \times 35 + 5(35) = 110 + 175 = 285$ mm.

(ii) Area of each of the 10 sectors = $(1/10) \times (22/7) \times (35/2)^2 = 385 / 4$ mm² = 96.25 mm².

- **Q10 (Umbrella with 8 equally spaced ribs, $r = 45$ cm):**

Total area of flat circle = $(22/7) \times 45 \times 45 = 44550 / 7$ cm².

Area between two consecutive ribs (1 of 8 sectors) = $(1/8) \times (44550 / 7) = 22275 / 28$ cm² (approx. 795.54 cm²).

Final Answer / उत्तर: Q6: 20.4375 cm², 686.0625 cm² • Q7: 88.44 cm² • Q8: 19.625 m², 58.875 m² • Q9: 285 mm, 385/4 mm² • Q10: 22275/28 cm²

Exercise 11.1 — Questions 11 to 14 / प्रश्नावली 11.1 — प्रश्न 11 से 14

[EN] Detailed solutions for Exercise 11.1 Q11 to Q14.

[HI] प्रश्नावली 11.1 के प्रश्न 11 से 14 तक के संपूर्ण हल:

- **Q11 (Two non-overlapping car wipers, blade 25 cm, angle 115°):**

Area cleaned by 1 wiper = $(115/360) \times (22/7) \times 25 \times 25$.

Total area cleaned by both wipers = $2 \times (115/360) \times (22/7) \times 625 = 158125 / 126$ cm² (approx. 1254.96 cm²).

- **Q12 (Lighthouse sector $\theta = 80^\circ$, distance $r = 16.5$ km, $\pi = 3.14$):**

Area of sea warned = $(80/360) \times 3.14 \times (16.5)^2 = (2/9) \times 3.14 \times 272.25 = 189.97$ km².

- **Q13 (Round table cover with 6 equal designs, $r = 28$ cm, $\sqrt{3} = 1.7$):**

Each design is a minor segment with central angle $\theta = 360^\circ / 6 = 60^\circ$.

Area of 1 sector = $(60/360) \times (22/7) \times 28 \times 28 = (1/6) \times 2464 = 1232 / 3$ cm².

Area of 1 equilateral $\triangle = (\sqrt{3}/4) \times 28^2 = (1.7/4) \times 784 = 333.2$ cm².

Area of 6 designs = $6 \times [(1232/3) - 333.2] = 2464 - 1999.2 = 464.8$ cm².

Cost of making designs at ₹ 0.35 per cm² = $464.8 \times 0.35 = ₹ 162.68$.

- **Q14 (Area of sector of angle p and radius R):**

$$\text{Area} = (p / 360) \times \pi R^2 = (p / 720) \times 2\pi R^2 \Rightarrow \text{Option (D)}.$$

Final Answer / उत्तर: Q11: 158125/126 cm² • Q12: 189.97 km² • Q13: ₹ 162.68 • Q14: Option (D)

Section 4: Chapter Summary & Key Circular Formulas

खंड 4: अध्याय सारांश एवं प्रमुख वृत्तीय सूत्र

Quantity	Formula	हिंदी विवरण
Arc Length	$l = (\theta/360^\circ) \times 2\pi r$	त्रिज्यखंड के चाप की लंबाई
Sector Area	$A = (\theta/360^\circ) \times \pi r^2$	त्रिज्यखंड का क्षेत्रफल
Segment Area	$A_{\text{sector}} - A_{\text{triangle}}$	लघु वृत्तखंड = त्रिज्यखंड – त्रिभुज

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