

TYBA COACHING CENTRE | टी वाई बी ए कोचिंग सेंटर

Contact / WhatsApp: 6367029449 • NCERT Class 10 Mathematics Complete Comprehensive Notes & Solutions

Chapter 8: Introduction to Trigonometry

अध्याय 8: त्रिकोणमिति का परिचय

Section 1: Detailed Concepts, Standard Ratios & Identities

खंड 1: विस्तृत अवधारणाएँ, मानक अनुपात एवं सर्वसमिकाएँ

8.1 Trigonometric Ratios in a Right-Angled Triangle / 8.1 समकोण त्रिभुज में त्रिकोणमितीय अनुपात

[English Theory]: The word 'Trigonometry' is derived from Greek words 'tri' (meaning three), 'gon' (meaning sides) and 'metron' (meaning measure). It is the study of relationships between the sides and angles of a triangle.

In a right triangle ABC, right-angled at B, with reference to acute angle A:

- **sine of $\angle A$ ($\sin A$):** Side opposite to $\angle A$ / Hypotenuse = BC / AC .
- **cosine of $\angle A$ ($\cos A$):** Side adjacent to $\angle A$ / Hypotenuse = AB / AC .
- **tangent of $\angle A$ ($\tan A$):** Side opposite / Side adjacent = $BC / AB = \sin A / \cos A$.
- **cosecant of $\angle A$ ($\operatorname{cosec} A$):** $1 / \sin A = \text{Hypotenuse} / \text{Opposite} = AC / BC$.
- **secant of $\angle A$ ($\sec A$):** $1 / \cos A = \text{Hypotenuse} / \text{Adjacent} = AC / AB$.
- **cotangent of $\angle A$ ($\cot A$):** $1 / \tan A = \text{Adjacent} / \text{Opposite} = AB / BC$.

Important Property: The trigonometric ratios of an angle do not depend on the size of the triangle, but only on the angle itself.

[हिंदी सिद्धांत]: शब्द 'त्रिकोणमिति' की व्युत्पत्ति ग्रीक शब्दों 'ट्राई' (तीन), 'गोन' (भुजा) और 'मेट्रॉन' (माप) से हुई है। यह एक त्रिभुज की भुजाओं और कोणों के बीच के संबंधों का अध्ययन है।

समकोण त्रिभुज ABC में कोण B = 90° होने पर, न्यूनकोण A के संदर्भ में:

- $\sin A = \text{सम्मुख भुजा (लंब)} / \text{कर्ण} = BC / AC$
- $\cos A = \text{संलग्न भुजा (आधार)} / \text{कर्ण} = AB / AC$
- $\tan A = \text{लंब} / \text{आधार} = BC / AB = \sin A / \cos A$
- $\operatorname{cosec} A = \text{कर्ण} / \text{लंब} = 1 / \sin A$
- $\sec A = \text{कर्ण} / \text{आधार} = 1 / \cos A$
- $\cot A = \text{आधार} / \text{लंब} = 1 / \tan A$

विशेष नियम: किसी कोण के त्रिकोणमितीय अनुपात त्रिभुज के आकार पर नहीं, केवल उस कोण के माप पर निर्भर करते हैं।

8.2 Values of Trigonometric Ratios for $0^\circ, 30^\circ, 45^\circ, 60^\circ$, and 90° / 8.2 विशिष्ट कोणों ($0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$) के त्रिकोणमितीय अनुपातों के मान

[English Theory]: Geometric Derivations:

- **For 45° :** In an isosceles right triangle ABC with $AB = BC = a$, $AC = a\sqrt{2}$. Thus $\sin 45^\circ = a/(a\sqrt{2}) = 1/\sqrt{2}$, $\cos 45^\circ = 1/\sqrt{2}$, $\tan 45^\circ = 1$.
- **For 30° and 60° :** In an equilateral triangle of side $2a$ with altitude $AD = a\sqrt{3}$: in right $\triangle ABD$, $\sin 30^\circ = BD/AB = a/(2a) = 1/2$, $\cos 30^\circ = \sqrt{3}/2$, $\tan 30^\circ = 1/\sqrt{3}$. Similarly $\sin 60^\circ = \sqrt{3}/2$, $\cos 60^\circ = 1/2$, $\tan 60^\circ = \sqrt{3}$.
- **For 0° and 90° :** As $\angle A$ approaches 0° , side $BC \rightarrow 0$ and $AC \rightarrow AB \Rightarrow \sin 0^\circ = 0$, $\cos 0^\circ = 1$, $\tan 0^\circ = 0$. As $\angle A$ approaches 90° , $AB \rightarrow 0$ and $BC \rightarrow AC \Rightarrow \sin 90^\circ = 1$, $\cos 90^\circ = 0$, $\tan 90^\circ$ is Not Defined.

[हिंदी सिद्धांत]: ज्यामितीय निगमन:

- **45° के लिए:** समद्विबाहु समकोण त्रिभुज (भुजा a) में $\sin 45^\circ = \cos 45^\circ = 1/\sqrt{2}$, $\tan 45^\circ = 1$ ।
- **30° और 60° के लिए:** $2a$ भुजा वाले समबाहु त्रिभुज के शीर्षलंब ($a\sqrt{3}$) से: $\sin 30^\circ = 1/2$, $\cos 30^\circ = \sqrt{3}/2$, $\tan 30^\circ = 1/\sqrt{3}$; तथा $\sin 60^\circ = \sqrt{3}/2$, $\cos 60^\circ = 1/2$, $\tan 60^\circ = \sqrt{3}$ ।
- **0° और 90° के लिए:** सीमांत स्थिति में $\sin 0^\circ = 0$, $\cos 0^\circ = 1$; तथा $\sin 90^\circ = 1$, $\cos 90^\circ = 0$, $\tan 90^\circ$ अपरिभाषित है।

8.3 Fundamental Trigonometric Identities / 8.3 मूलभूत त्रिकोणमितीय सर्वसमिकाएँ

[English Theory]: An equation involving trigonometric ratios of an angle is called a **trigonometric identity** if it is true for all values of the angles involved.

Derivations from Pythagoras Theorem in $\triangle ABC$ ($AB^2 + BC^2 = AC^2$):

1. Dividing by AC^2 : $(AB/AC)^2 + (BC/AC)^2 = 1 \Rightarrow \cos^2 A + \sin^2 A = 1$ (True for $0^\circ \leq A \leq 90^\circ$).
2. Dividing by AB^2 : $1 + (BC/AB)^2 = (AC/AB)^2 \Rightarrow 1 + \tan^2 A = \sec^2 A$ (True for $0^\circ \leq A < 90^\circ$).
3. Dividing by BC^2 : $(AB/BC)^2 + 1 = (AC/BC)^2 \Rightarrow \cot^2 A + 1 = \operatorname{cosec}^2 A$ (True for $0^\circ < A \leq 90^\circ$).

[हिंदी सिद्धांत]: त्रिकोणमितीय अनुपातों से संबंधित ऐसा समीकरण जो संबद्ध कोणों के सभी मानों के लिए सत्य हो, **त्रिकोणमितीय सर्वसमिका** कहलाता है।

पाइथागोरस प्रमेय ($AB^2 + BC^2 = AC^2$) से निगमन:

1. AC^2 से भाग देने पर: $\sin^2 A + \cos^2 A = 1$ ($0^\circ \leq A \leq 90^\circ$ के लिए सत्य)।
2. AB^2 से भाग देने पर: $1 + \tan^2 A = \sec^2 A$ ($0^\circ \leq A < 90^\circ$ के लिए सत्य)।
3. BC^2 से भाग देने पर: $1 + \cot^2 A = \operatorname{cosec}^2 A$ ($0^\circ < A \leq 90^\circ$ के लिए सत्य)।

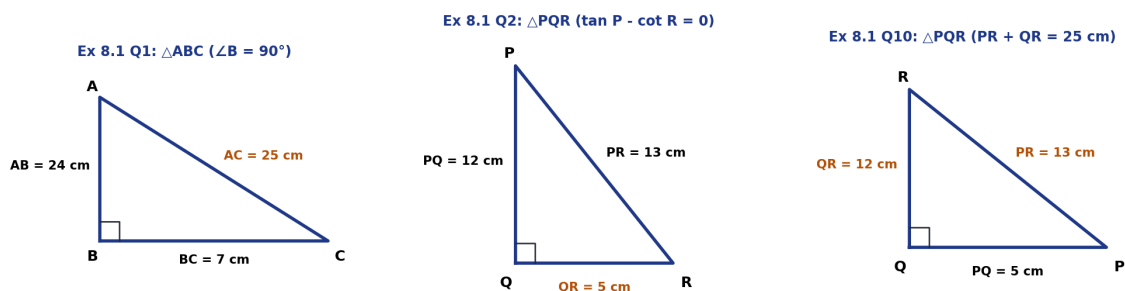


Fig 8.1: Right-Angled Triangles for Ex 8.1 Q1 (24, 7, 25), Q2 (12, 5, 13), and Q10 (5, 12, 13) — TYBA Coaching Centre

Section 2: Textbook Solved Examples with Step-by-Step Bilingual Solutions

खंड 2: पाठ्यपुस्तक के सभी हल किए गए उदाहरण (विस्तृत चरणबद्ध हल)

Textbook Example 1: Given $\tan A = 4/3$, Find Other Ratios / उदाहरण 1: $\tan A = 4/3$ दिया होने पर अन्य अनुपात

[Problem - EN]: Given $\tan A = 4/3$, find the other trigonometric ratios of the angle A.

[प्रश्न - HI]: यदि $\tan A = 4/3$, तो कोण A के अन्य त्रिकोणमितीय अनुपात ज्ञात कीजिए।

Step-by-Step Solution (English):

Let $\triangle ABC$ be a right triangle with $\angle B = 90^\circ$.

$\tan A = BC / AB = 4/3$. Let $BC = 4k$ and $AB = 3k$, where $k > 0$.

By Pythagoras theorem: $AC = \sqrt{AB^2 + BC^2} = \sqrt{(3k)^2 + (4k)^2} = \sqrt{9k^2 + 16k^2} = \sqrt{25k^2} = 5k$.

Therefore:

- $\sin A = BC / AC = 4k / 5k = 4/5$
- $\cos A = AB / AC = 3k / 5k = 3/5$
- $\cot A = 1 / \tan A = 3/4$
- $\operatorname{cosec} A = 1 / \sin A = 5/4$
- $\sec A = 1 / \cos A = 5/3$.

चरणबद्ध हल (हिंदी):

माना समकोण त्रिभुज ABC में $\angle B = 90^\circ$ है।

$\tan A = BC/AB = 4/3 \Rightarrow BC = 4k, AB = 3k$ ।

पाइथागोरस प्रमेय से: $AC = \sqrt{(3k)^2 + (4k)^2} = 5k$ ।

अतः $\sin A = 4/5, \cos A = 3/5, \cot A = 3/4, \operatorname{cosec} A = 5/4, \sec A = 5/3$ ।

Textbook Example 8: Evaluating $\sin(A - B)$ and $\cos(A + B)$ / उदाहरण 8: $\sin(A - B)$ और $\cos(A + B)$ का मान

[Problem - EN]: If $\sin(A - B) = 1/2$, $\cos(A + B) = 1/2$; $0^\circ < A + B \leq 90^\circ$, $A > B$, find A and B.

[प्रश्न - HI]: यदि $\sin(A - B) = 1/2$, $\cos(A + B) = 1/2$; $0^\circ < A + B \leq 90^\circ$, $A > B$, तो A और B ज्ञात कीजिए।

Step-by-Step Solution (English):

Since $\sin(A - B) = 1/2 = \sin 30^\circ \Rightarrow A - B = 30^\circ$ —(1).

Since $\cos(A + B) = 1/2 = \cos 60^\circ \Rightarrow A + B = 60^\circ$ —(2).

Adding (1) and (2): $2A = 90^\circ \Rightarrow A = 45^\circ$.

Subtracting (1) from (2): $2B = 30^\circ \Rightarrow B = 15^\circ$.

Hence, **$A = 45^\circ$ and $B = 15^\circ$** .

चरणबद्ध हल (हिंदी):

चूँकि $\sin(A - B) = 1/2 \Rightarrow A - B = 30^\circ$ ।

चूँकि $\cos(A + B) = 1/2 \Rightarrow A + B = 60^\circ$ ।

जोड़ने पर: $2A = 90^\circ \Rightarrow A = 45^\circ$ । घटाने पर: $2B = 30^\circ \Rightarrow B = 15^\circ$ ।

Textbook Example 10: Proving $(\sec A + \tan A)(1 - \sin A) = \cos A$ / उदाहरण 10: सर्वसमिका $(\sec A + \tan A)(1 - \sin A) = \cos A$ का प्रमाण

[Problem - EN]: Prove that $(\sec A + \tan A)(1 - \sin A) = \cos A$.

[प्रश्न - HI]: सिद्ध कीजिए कि $(\sec A + \tan A)(1 - \sin A) = \cos A$ ।

Step-by-Step Solution (English):

LHS = $(\sec A + \tan A)(1 - \sin A)$

= $(1/\cos A + \sin A/\cos A)(1 - \sin A)$

= $[(1 + \sin A) / \cos A](1 - \sin A)$

= $(1 - \sin^2 A) / \cos A$

= $\cos^2 A / \cos A$ (using $\sin^2 A + \cos^2 A = 1$)

= **$\cos A = \text{RHS}$** . Hence Proved!

चरणबद्ध हल (हिंदी):

LHS = $(1/\cos A + \sin A/\cos A)(1 - \sin A) = [(1 + \sin A)/\cos A](1 - \sin A)$

= $(1 - \sin^2 A) / \cos A = \cos^2 A / \cos A = \mathbf{\cos A = RHS}$ । (इति सिद्धम्)

Section 3: Exercise 8.1 & 8.2 Complete Step-by-Step Solutions

खंड 3: प्रश्नावली 8.1 एवं 8.2 के संपूर्ण चरणबद्ध हल

Exercise 8.1 — Complete Solutions (Q1 to Q11) / प्रश्नावली 8.1 — संपूर्ण हल

[EN] Detailed solutions for Exercise 8.1 Q1 to Q11.

[HI] प्रश्नावली 8.1 के प्रश्न 1 से 11 के संपूर्ण चरणबद्ध हल:

- **Q1 (In $\triangle ABC$, $\angle B = 90^\circ$, $AB = 24$ cm, $BC = 7$ cm — Fig 8.1 Left):**
 $AC = \sqrt{(24^2 + 7^2)} = \sqrt{625} = 25$ cm.
 (i) $\sin A = 7/25$, $\cos A = 24/25$ • (ii) $\sin C = 24/25$, $\cos C = 7/25$.
- **Q2 (Fig 8.13 / Fig 8.1 Middle: $PQ = 12$ cm, $PR = 13$ cm):** $QR = \sqrt{(169 - 144)} = 5$ cm $\Rightarrow \tan P - \cot R = 5/12 - 5/12 = 0$.
- **Q3 ($\sin A = 3/4$):** $AB = \sqrt{(16 - 9)} = \sqrt{7} \Rightarrow \cos A = \sqrt{7}/4$, $\tan A = 3/\sqrt{7}$.
- **Q4 ($15 \cot A = 8 \Rightarrow \cot A = 8/15$):** $AC = \sqrt{(64 + 225)} = 17 \Rightarrow \sin A = 15/17$, $\sec A = 17/8$.

- **Q5 (sec $\theta = 13/12$):** Opp = $\sqrt{169-144} = 5 \Rightarrow \sin \theta = 5/13, \cos \theta = 12/13, \tan \theta = 5/12, \cot \theta = 12/5, \operatorname{cosec} \theta = 13/5$.
- **Q6 (cos A = cos B in right Δ):** AC/AB = BC/AB $\Rightarrow AC = BC \Rightarrow \angle A = \angle B$.
- **Q7 (cot $\theta = 7/8$):** (i) $(1-\sin^2 \theta)/(1-\cos^2 \theta) = \cot^2 \theta = 49/64$ • (ii) $\cot^2 \theta = 49/64$.
- **Q8 (3 cot A = 4):** LHS = $(1-9/16)/(1+9/16) = 7/25$; RHS = $\cos^2 A - \sin^2 A = 16/25 - 9/25 = 7/25 \Rightarrow$ **Yes (बराबर है)**.
- **Q9 (tan A = $1/\sqrt{3}$):** $\angle A = 30^\circ, \angle C = 60^\circ \Rightarrow$ (i) $\sin A \cos C + \cos A \sin C = \sin(A+C) = \sin 90^\circ = 1$ • (ii) $\cos A \cos C - \sin A \sin C = \cos 90^\circ = 0$.
- **Q10 (PR + QR = 25 cm, PQ = 5 cm — Fig 8.1 Right):** QR = 12 cm, PR = 13 cm $\Rightarrow \sin P = 12/13, \cos P = 5/13, \tan P = 12/5$.
- **Q11 (True/False):** (i) **False** ($\tan 60^\circ = \sqrt{3} > 1$) • (ii) **True** ($\sec A \geq 1, 12/5 = 2.4$) • (iii) **False** • (iv) **False** • (v) **False** ($\sin \theta \leq 1$).

Final Answer / उत्तर: All 11 questions of Ex 8.1 solved and verified.

Exercise 8.2 — Complete Solutions (Q1 to Q4) / प्रश्नावली 8.2 — संपूर्ण हल

[EN] Detailed solutions for Exercise 8.2 Q1 to Q4.

[HI] विशिष्ट कोणों के मानों पर आधारित प्रश्नावली 8.2 के प्रश्न 1 से 4 के संपूर्ण हल:

- **Q1:**
 - (i) $\sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ = (\sqrt{3}/2)^2 + (1/2)^2 = 3/4 + 1/4 = 1$.
 - (ii) $2 \tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ = 2(1) + 3/4 - 3/4 = 2$.
 - (iii) $\cos 45^\circ / (\sec 30^\circ + \operatorname{cosec} 30^\circ) = (1/\sqrt{2}) / (2/\sqrt{3} + 2) = (3\sqrt{2} - \sqrt{6}) / 8$.
 - (iv) $(\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ) / (\sec 30^\circ + \cos 60^\circ + \cot 45^\circ) = (43 - 24\sqrt{3}) / 11$.
 - (v) $[5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ] / (\sin^2 30^\circ + \cos^2 30^\circ) = [5(1/4) + 4(4/3) - 1] / 1 = 67/12$.
- **Q2 (MCQs):** (i) $2 \tan 30^\circ / (1 + \tan^2 30^\circ) = \sqrt{3}/2 = \sin 60^\circ$ (Option A) • (ii) $(1 - \tan^2 45^\circ) / (1 + \tan^2 45^\circ) = 0 =$ Option D • (iii) $\sin 2A = 2 \sin A$ holds for **A = 0°** (Option A) • (iv) $2 \tan 30^\circ / (1 - \tan^2 30^\circ) = \sqrt{3} = \tan 60^\circ$ (Option C).
- **Q3:** $\tan(A+B) = \sqrt{3}$ ($A+B=60^\circ$), $\tan(A-B) = 1/\sqrt{3}$ ($A-B=30^\circ$) $\Rightarrow$ **A = 45°, B = 15°**.
- **Q4:** (i) **False** • (ii) **True** • (iii) **False** • (iv) **False** • (v) **True**.

Final Answer / उत्तर: All parts of Ex 8.2 solved and verified.

Section 4: Exercise 8.3 Complete Step-by-Step Solutions (All 10 Identity Proofs)

खंड 4: प्रश्नावली 8.3 के संपूर्ण चरणबद्ध हल (सभी 10 सर्वसमिका उपपत्तियाँ)

Exercise 8.3 — Complete Solutions & Proofs (Q1 to Q4) / प्रश्नावली 8.3 — संपूर्ण हल एवं उपपत्तियाँ

[EN] Detailed solutions and complete algebraic proofs for Exercise 8.3 Q1 to Q4 (including all 10 identities of Q4).

[HI] प्रश्नावली 8.3 के सभी 10 सर्वसमिकाओं सहित प्रश्न 1 से 4 के संपूर्ण हल:

- **Q1:** $\sin A = 1 / \sqrt{1 + \cot^2 A}$, $\sec A = \sqrt{1 + \cot^2 A} / \cot A$, $\tan A = 1 / \cot A$.
- **Q2:** $\cos A = 1/\sec A$; $\tan A = \sqrt{\sec^2 A - 1}$; $\cot A = 1/\sqrt{\sec^2 A - 1}$; $\sin A = \sqrt{\sec^2 A - 1}/\sec A$; $\operatorname{cosec} A = \sec A/\sqrt{\sec^2 A - 1}$.
- **Q3:** (i) $9(\sec^2 A - \tan^2 A) = 9$ (Option B) • (ii) $(1 + \tan \theta + \sec \theta)(1 + \cot \theta - \operatorname{cosec} \theta) = 2$ (Option C) • (iii) $(\sec A + \tan A)(1 - \sin A) = \cos A$ (Option D) • (iv) $(1 + \tan^2 A)/(1 + \cot^2 A) = \tan^2 A$ (Option D).

• **Q4 (All 10 Proofs):**

(i) $\text{LHS} = [(1 - \cos \theta) / \sin \theta]^2 = (1 - \cos \theta)^2 / (1 - \cos^2 \theta) = (1 - \cos \theta) / (1 + \cos \theta) = \text{RHS}.$

(ii) $\text{LHS} = [\cos^2 A + (1 + \sin A)^2] / [\cos A(1 + \sin A)] = 2(1 + \sin A) / [\cos A(1 + \sin A)] = 2 \sec A = \text{RHS}.$

(iii) $\text{LHS} = \tan \theta / (1 - \cot \theta) + \cot \theta / (1 - \tan \theta) = (\sin^3 \theta - \cos^3 \theta) / [\sin \theta \cos \theta (\sin \theta - \cos \theta)] = 1 + \sec \theta \operatorname{cosec} \theta = \text{RHS}.$

(iv) $\text{LHS} = (1 + \sec A) / \sec A = 1 + \cos A = (1 - \cos^2 A) / (1 - \cos A) = \sin^2 A / (1 - \cos A) = \text{RHS}.$

(v) Dividing by $\sin A$: $(\cot A - 1 + \operatorname{cosec} A) / (\cot A + 1 - \operatorname{cosec} A) = \operatorname{cosec} A + \cot A = \text{RHS}.$

(vi) $\sqrt{[(1 + \sin A) / (1 - \sin A)]} = (1 + \sin A) / \cos A = \sec A + \tan A = \text{RHS}.$

(vii) $[\sin \theta (1 - 2 \sin^2 \theta)] / [\cos \theta (2 \cos^2 \theta - 1)] = \tan \theta (\cos 2\theta / \cos 2\theta) = \tan \theta = \text{RHS}.$

(viii) $\sin^2 A + \operatorname{cosec}^2 A + 2 + \cos^2 A + \sec^2 A + 2 = 5 + (1 + \cot^2 A) + (1 + \tan^2 A) = 7 + \tan^2 A + \cot^2 A = \text{RHS}.$

(ix) $\text{LHS} = (\cos^2 A / \sin A)(\sin^2 A / \cos A) = \sin A \cos A$; $\text{RHS} = 1 / (\sin A / \cos A + \cos A / \sin A) = \sin A \cos A \Rightarrow \text{LHS} = \text{RHS}.$

(x) $(1 + \tan^2 A) / (1 + \cot^2 A) = \sec^2 A / \operatorname{cosec}^2 A = \tan^2 A$; and $[(1 - \tan A) / (1 - \cot A)]^2 = \tan^2 A = \text{RHS}.$

Final Answer / उत्तर: All questions of Ex 8.3 and all 10 identity proofs completed.

Section 5: Chapter Summary & Master Values Table

खंड 5: अध्याय सारांश एवं विशिष्ट कोणों की तालिका

$\angle A$	0°	30°	45°	60°	90°
$\sin A$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos A$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\tan A$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	Not Defined
$\operatorname{cosec} A$	Not Defined	2	$\sqrt{2}$	$2/\sqrt{3}$	1
$\sec A$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	Not Defined
$\cot A$	Not Defined	$\sqrt{3}$	1	$1/\sqrt{3}$	0

© TYBA Coaching Centre (टी वाई बी ए कोचिंग सेंटर) — All Rights Reserved. For Admissions, Regular Batch Coaching & Doubt Clearing Sessions, Contact: **6367029449**.