TYBA COACHING CENTRE, AJMER

RMS / RMC CLASS VI MATHEMATICS • CHAPTER 17: GEOMETRY: LINES & ANGLES (ज्यामिति: रेखाएं एवं कोण) — MASTER NOTES

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1. FUNDAMENTALS OF EUCLIDEAN GEOMETRY (ज्यामिति की मूल अवधारणाएँ) — Section 1

1.1 Point, Line, Line Segment & Ray:

• **Point (बिंदु):** A fine dot made by a sharp pencil. It has zero length, zero breadth, and zero thickness—only a position.

• **Line Segment (रेखाखंड):** Part of a line with **2 fixed endpoints** and a definite measurable length. It is the shortest distance between two points.

• **Ray (किरण):** Starts at **1 fixed endpoint** (initial point) and extends infinitely in one direction (like a sun ray).

• **Line (रेखा):** Extends endlessly in both directions; it has **0 endpoints** and no fixed length.

बिंदु में लंबाई-चौड़ाई नहीं होती। रेखाखंड के 2 अंत बिंदु, किरण का 1 अंत बिंदु और रेखा के 0 अंत बिंदु होते हैं।

1.2 Collinear Points & Intersecting/Concurrent Lines:

• **Collinear Points (संरेख बिंदु):** Three or more points lying on the same straight line.

• **Concurrent Lines (संगामी रेखाएँ):** Three or more lines passing through the same single point.

• **Golden Rule:** Infinitely many lines pass through **1 point**, whereas **only ONE unique line** passes through **2 distinct points**.

एक ही रेखा पर स्थित बिंदु संरेख कहलाते हैं। एक बिंदु से अनंत रेखाएँ और दो बिंदुओं से केवल एक अद्वितीय रेखा गुजरती है।

2. ANGLES & THEIR CLASSIFICATION (कोण और उनके प्रकार) — Section 2

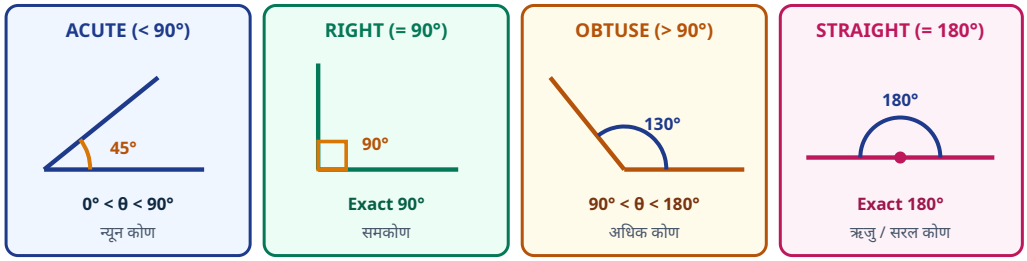


Figure 1: Four Major Types of Angles (कोणों के प्रमुख प्रकार)

2.1 Complete Classification of Angles (0° to 360°):

• **Zero Angle (शून्य कोण):** Exactly **0°** (both arms overlap).

• **Acute Angle (न्यून कोण):** Greater than 0° and less than 90° (e.g., 30°, 45°, 89°).

• **Right Angle (समकोण):** Exactly **90°** (1/4 of a complete turn; formed by perpendicular lines).

• **Obtuse Angle (अधिक कोण):** Greater than 90° and less than 180° (e.g., 91°, 120°, 175°).

• **Straight Angle (ऋजु / सरल कोण):** Exactly **180°** (1/2 of a complete turn; forms a straight line).

• **Reflex Angle (प्रतिवर्ती कोण):** Greater than 180° and less than 360° (e.g., 210°, 270°, 350°).

• **Complete Angle (संपूर्ण कोण):** Exactly **360°** (1 full revolution).

शून्य (0°), न्यून (0°–90°), समकोण (90°), अधिक (90°–180°), ऋजु (180°), प्रतिवर्ती (180°–360°), संपूर्ण (360°)।

Solved Example 1 (Classify Angles): Classify: (a) 89.5° (b) 180° (c) 245° (d) 90°.
Solution: (a) 89.5° < 90° ⇒ **Acute Angle**; (b) 180° ⇒ **Straight Angle**; (c) 180° < 245° < 360° ⇒ **Reflex Angle**; (d) 90° ⇒ **Right Angle**.

हल: (a) न्यून कोण; (b) ऋजु कोण; (c) प्रतिवर्ती कोण; (d) समकोण।

Solved Example 2 (Reflex Angle): Find the reflex angle corresponding to an acute angle of 65°.
Solution: Reflex angle = 360° − 65° = **295°**.

हल: प्रतिवर्ती कोण = 360° − 65° = 295°।

3. COMPLEMENTARY & SUPPLEMENTARY ANGLES (पूरक एवं संपूरक कोण) Section 3

3.1 Definitions & Shortcuts:

• **Complementary Angles (पूरक कोण):** Two angles whose sum is **90°**. Complement of θ = **90° − θ**.
• **Supplementary Angles (संपूरक कोण):** Two angles whose sum is **180°**. Supplement of θ = **180° − θ**.

जिन दो कोणों का योग 90° हो वे पूरक (Complement = 90° − θ), तथा जिनका योग 180° हो वे संपूरक (Supplement = 180° − θ) कहलाते हैं।

Solved Example 3 (Complement & Supplement): Find the complement of 42° and supplement of 128°.
Solution: Complement = 90° − 42° = **48°**. Supplement = 180° − 128° = **52°**.

हल: पूरक = 90° − 42° = 48°; संपूरक = 180° − 128° = 52°।

Solved Example 4 (Self-Equal Angle): Which angle is equal to (a) its complement (b) its supplement?
Solution: (a) x + x = 90° ⇒ 2x = 90° ⇒ **45°**. (b) x + x = 180° ⇒ 2x = 180° ⇒ **90°**.

हल: (a) अपने पूरक के बराबर कोण = 45°; (b) अपने संपूरक के बराबर कोण = 90°।

Solved Example 5 (Difference Given): Two complementary angles differ by 14°. Find both angles.
Solution: Let angles be x and (90 − x). Given: x − (90 − x) = 14 ⇒ 2x = 104 ⇒ x = **52°**, other = 90 − 52 = **38°**.

हल: x − (90 − x) = 14 ⇒ 2x = 104 ⇒ कोण 52° और 38° हैं।

Solved Example 6 (Ratio of Supplementary Angles): Two supplementary angles are in the ratio 4 : 5. Find the larger angle.
Solution: 4x + 5x = 180° ⇒ 9x = 180° ⇒ x = 20°. Larger angle = 5 × 20° = **100°**.

हल: 4x + 5x = 180° ⇒ x = 20°। बड़ा कोण = 5 × 20° = 100°।

4. ADJACENT ANGLES, LINEAR PAIR & VERTICALLY OPPOSITE ANGLES Section 4

4.1 Angle Pair Properties:

• **Adjacent Angles (आसन्न कोण):** Share a common vertex and a common arm, with non-common arms on opposite sides.
• **Linear Pair (रैखिक युग्म):** Two adjacent angles whose non-common arms form a straight line. Their sum is always **180°**.
• **Angles Around a Point:** Sum of all angles meeting at a single point is always **360°**.
• **Vertically Opposite Angles (शीर्षाभिमुख कोण):** Formed when two straight lines cross (like an X). Opposite angles are **always equal**!

रैखिक युग्म का योग 180°, एक बिंदु के चारों ओर के कोणों का योग 360°, और शीर्षाभिमुख कोण हमेशा बराबर होते हैं।

Solved Example 7 (Linear Pair): Two angles form a linear pair. If one angle is (3x + 10)° and the other is (2x + 20)°, find x and both angles.
Solution: (3x + 10) + (2x + 20) = 180 ⇒ 5x + 30 = 180 ⇒ 5x = 150 ⇒ x = **30°**. Angles = **100°** and **80°**.

हल: 5x + 30 = 180 ⇒ x = 30°। कोण = 100° और 80°।

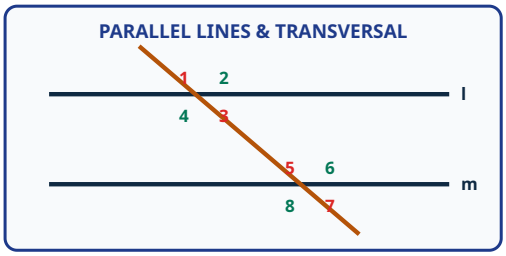
Solved Example 8 (Angles Around a Point): Four angles around a point are 70°, 110°, 95°, and x°. Find x.
Solution: 70 + 110 + 95 + x = 360 ⇒ 275 + x = 360 ⇒ x = 360 − 275 = **85°**.

हल: 275° + x = 360° ⇒ x = 85°।

Solved Example 9 (Intersecting Lines): Two lines intersect such that one angle is 62°. Find the other three angles.
Solution: Vertically opposite angle = **62°**. Adjacent linear pair angle = 180° − 62° = **118°**, and its opposite = **118°**.

हल: शीर्षाभिमुख कोण = 62°; शेष दोनों कोण = 180° − 62° = 118°।

5. PARALLEL LINES & A TRANSVERSAL (समांतर रेखाएँ व तिर्यक छेदी रेखा) Section 5



GOLDEN ANGLE RULES (जब l ∥ m)

- Corresponding (संगत): ∠1=∠5, ∠2=∠6
- Alt. Interior (एकांतर): ∠3=∠5, ∠4=∠6
- Co-Interior (सह-अंतः): ∠3+∠6 = 180°
- Vert. Opposite (शीर्षाभिमुख): ∠1=∠3, ∠2=∠4
- Linear Pair (रैखिक युग्म): ∠1+∠2 = 180°

Figure 2: Parallel Lines Cut by a Transversal (समांतर रेखाएँ व तिर्यक छेदी रेखा)

5.1 Transversal Rules (When Line l ∥ Line m): When a transversal cuts two parallel lines, 8 angles are formed:
• **Corresponding Angles (संगत कोण):** Same position at each intersection (F-shape) → **Equal** (∠1=∠5, ∠2=∠6, ∠3=∠7, ∠4=∠8).
• **Alternate Interior Angles (एकांतर अंतः कोण):** Opposite sides between parallel lines (Z-shape) → **Equal** (∠3=∠5, ∠4=∠6).
• **Co-Interior / Allied Angles (सह-अंतः कोण):** Same side inside parallel lines (C-shape) → **Sum = 180°** (∠3+∠6 = 180°, ∠4+∠5 = 180°).

जब दो समांतर रेखाओं को तिर्यक रेखा काटे: संगत कोण बराबर, एकांतर अंतः कोण बराबर, तथा एक ही ओर के अंतः कोणों का योग 180° होता है।

Solved Example 10 (Transversal Angle): Two parallel lines are cut by a transversal. If one interior angle is 75°, find its co-interior angle and alternate interior angle.
Solution: Alternate interior angle = **75°** (equal). Co-interior angle = 180° − 75° = **105°**.

हल: एकांतर अंतः कोण = 75° (बराबर); सह-अंतः कोण = 180° − 75° = 105°।

Solved Example 11 (Co-Interior Ratio): Two co-interior angles between parallel lines are in the ratio 2 : 7. Find both angles.
Solution: 2x + 7x = 180° ⇒ 9x = 180° ⇒ x = 20°. Angles are **40°** and **140°**.

हल: 2x + 7x = 180° ⇒ x = 20°। कोण = 40° और 140°।

6. CLOCK ANGLES, COMPASS DIRECTIONS & POLYGON ANGLES Section 6

6.1 Clock & Direction Angle Formulas (घड़ी व दिशा के कोण):
• **Hour Hand Speed:** 360° in 12 hours = **30° per hour** (0.5° per minute).
• **Minute Hand Speed:** 360° in 60 minutes = **6° per minute**.
• **Compass Directions:** North, East, South, West are at **90°** intervals. Halfway directions (NE, SE, SW, NW) are at **45°**!
• **Triangle Angle Sum:** Sum of 3 interior angles of ANY triangle = **180°**; Quadrilateral = **360°**.

घड़ी में 1 घंटे की सुई 30° और मिनट सुई 6° प्रति मिनट घूमती है। मुख्य दिशाओं के बीच 90° का कोण होता है। त्रिभुज के तीनों कोणों का योग 180° होता है।

Solved Example 12 (Clock at 4:00): What is the smaller angle between the hour and minute hands of a clock at 4:00 PM?
Solution: At 4:00, the hands are 4 hour-divisions apart. Angle = 4 × 30° = **120°** (Obtuse Angle).

हल: 4 बजे दोनों सुइयों के बीच 4 भाग हैं, अतः कोण = 4 × 30° = 120° (अधिक कोण)।

Solved Example 13 (Compass Turn): You are facing North and turn 135° clockwise. Which direction are you facing now?
Solution: North → East is 90° clockwise; + 45° more reaches **South-East (SE)**.

हल: उत्तर से 90° दक्षिणावर्त पूर्व और +45° आगे दक्षिण-पूर्व (South-East) दिशा होगी।

Solved Example 14 (Triangle Third Angle): Two angles of a triangle are 55° and 68°. Find the third angle.
Solution: Third angle = 180° − (55° + 68°) = 180° − 123° = **57°**.

हल: तीसरा कोण = 180° − (55° + 68°) = 57°।

7. RMS CET ADVANCED WORD PROBLEMS (EXAMPLES 15–22) Section 7

Solved Example 15 (Twice its Complement): Find an angle that is twice its complement.
Solution: Let angle = x, complement = 90 − x. Given x = 2(90 − x) ⇒ x = 180 − 2x ⇒ 3x = 180 ⇒ x = **60°**.

हल: x = 2(90 − x) ⇒ 3x = 180 ⇒ x = 60°।

Solved Example 16 (One-Third of Supplement): Find an angle that is one-third of its supplement.
Solution: Let angle = x, supplement = 180 − x. x = (180 − x)/3 ⇒ 3x = 180 − x ⇒ 4x = 180 ⇒ x = **45°**.

हल: 3x = 180 − x ⇒ 4x = 180 ⇒ x = 45°।

Solved Example 17 (Number of Line Segments): How many line segments can be named using 4 collinear points A, B, C, D?
Solution: Formula: n(n − 1) ÷ 2 = (4 × 3) ÷ 2 = **6 line segments** (AB, AC, AD, BC, BD, CD).

हल: कुल रेखाखंड = n(n−1)/2 = (4 × 3)/2 = 6 रेखाखंड।

Solved Example 18 (Revolution Fraction): How many right angles are there in 1½ complete revolutions?
Solution: 1 revolution = 4 right angles. So 1.5 revolutions = 1.5 × 4 = **6 right angles** (540°).

हल: 1 चक्कर = 4 समकोण; 1.5 चक्कर = 6 समकोण।

Solved Example 19 (Exterior Angle of Triangle): If two interior opposite angles of a triangle are 45° and 65°, find the exterior angle.
Solution: Exterior angle = Sum of two interior opposite angles = 45° + 65° = **110°**.

हल: बाह्य कोण = 45° + 65° = 110°।

Solved Example 20 (Supplementary & Complementary Combined): An angle is 40°. Find the difference between its supplement and its complement.
Solution: Supplement = 180 − 40 = 140°; Complement = 90 − 40 = 50°. Difference = 140 − 50 = **90°** (always 90°!).

हल: 140° − 50° = 90° (किसी भी कोण के संपूरक व पूरक का अंतर सदैव 90° होता है!)।

8. COMMON EXAM TRAPS & 100% SCORE CHECKLIST Section 8

Common RMS Exam Mistake (सामान्य गलती)	Correct Rule (सही नियम)
Confusing Complementary (90°) with Supplementary (180°)	**C**omplementary = **90°**; **S**upplementary = **180°**

Common RMS Exam Mistake (सामान्य गलती)	Correct Rule (सही नियम)
Calling a 180° angle an obtuse angle	180° is a **Straight Angle** (Obtuse is strictly 90° < θ < 180°)
Saying a ray has 2 endpoints	Line = **0** endpoints, Ray = **1** endpoint, Segment = **2** endpoints
Saying co-interior angles are equal	Co-interior angles add up to **180°** (only Alternate/Corresponding are equal)

9. ADVANCED POLYGON, CLOCK & COMPASS SOLVED BANK (EXAMPLES 21–25) Section 9

Solved Example 21 (Polygon Interior Angle Sum): Find the sum of all interior angles of a pentagon (5 sides) and a hexagon (6 sides).
Solution: Formula: **(n − 2) × 180°**. For Pentagon (n=5): (5 − 2) × 180° = **540°**. For Hexagon (n=6): (6 − 2) × 180° = **720°**.

हल: बहुभुज के अंतः कोणों का योग = (n − 2) × 180°। पंचभुज = 540°; षट्भुज = 720°।

Solved Example 22 (Clock Hands at 9:30): Through how many degrees does the minute hand move between 9:00 AM and 9:25 AM?
Solution: Minute hand moves **6° per minute**. In 25 minutes, it moves 25 × 6° = **150°**.

हल: मिनट की सुई 1 मिनट में 6° घूमती है। 25 मिनट में = 25 × 6° = 150°।

Solved Example 23 (Equilateral & Isosceles Triangle Angles): In an isosceles triangle, the two base angles are equal and the vertical angle is 80°. Find each base angle.
Solution: Let each base angle be x. Then x + x + 80° = 180° ⇒ 2x = 100° ⇒ x = **50°**.

हल: 2x + 80° = 180° ⇒ 2x = 100° ⇒ प्रत्येक आधार कोण = 50°।

Solved Example 24 (Parallel Lines & Algebra): Two parallel lines are cut by a transversal such that a pair of corresponding angles are (4x − 10)° and (3x + 15)°. Find x and the angle.
Solution: Corresponding angles are equal: 4x − 10 = 3x + 15 ⇒ x = **25°**. Angle = 4(25) − 10 = **90°**.

हल: संगत कोण बराबर होते हैं: 4x − 10 = 3x + 15 ⇒ x = 25°; कोण = 90°।

Solved Example 25 (Compass Double Turn): Riya faces East. She turns 90° clockwise, then 180° anti-clockwise. Which direction is she facing now?
Solution: East + 90° CW = South. South + 180° ACW = **North**!

हल: पूर्व से 90° दक्षिणावर्त = दक्षिण; दक्षिण से 180° वामावर्त = उत्तर (North)!

10. HIGH-YIELD RMS CET GEOMETRY REVISION BANK & EXTRA EXAMPLES Section 10

Solved Example 26 (Clock Hands at 2:00 PM & 8:00 PM): Find the smaller angle between the clock hands at 2:00 PM and at 8:00 PM.
Solution: At 2:00 PM, 2 divisions → 2 × 30° = **60°**. At 8:00 PM, smaller gap is 4 divisions (8 to 12) → 4 × 30° = **120°**.

हल: 2:00 बजे = 2 × 30° = 60°; 8:00 बजे छोटा कोण = 4 × 30° = 120°।

Solved Example 27 (Three Angles on a Straight Line): Three angles (2x)°, (3x + 10)°, and (4x − 10)° lie on a straight line. Find x.
Solution: 2x + 3x + 10 + 4x − 10 = 180° ⇒ 9x = 180° ⇒ x = **20°**.

हल: 9x = 180° ⇒ x = 20°।

Solved Example 28 (Number of Rays & Triangles): Find the sum of exterior angles of any convex polygon.
Solution: For ANY convex polygon (triangle, quadrilateral, pentagon, hexagon), the sum of all exterior angles is always **360°**!

हल: किसी भी उत्तल बहुभुज के सभी बाह्य कोणों का योग सदैव 360° होता है।

10.1 Quick One-Liner Geometry Facts (परीक्षा के महत्वपूर्ण तथ्य):
• **Euclid** is known as the **Father of Geometry** (authored *The Elements*).
• **Bisector (समद्विभाजक):** A ray or line that divides an angle or segment into two equal halves.
• **Perpendicular Bisector:** Cuts a line segment into two equal parts at exactly **90°**.
• **Acute-angled triangle:** All 3 angles < 90°; **Right-angled:** One angle = 90°; **Obtuse-angled:** One angle > 90°.

यूक्लिड को ज्यामिति का जनक कहा जाता है। लंब समद्विभाजक रेखाखंड को 90° पर दो बराबर भागों में बाँटता है।

Narayan Sir's Final Revision Mantra: (1) Difference between supplement and complement of ANY acute angle is always **90°**. (2) At 3:00 and 9:00 clock hands make **90°**; at 6:00 they make **180°**. (3) Vertically opposite, alternate interior, and corresponding angles (for parallel lines) are ALWAYS equal!

अंतिम मंत्र: (1) किसी भी न्यून कोण के संपूरक और पूरक का अंतर हमेशा 90° होता है। (2) समांतर रेखाओं में संगत व एकांतर कोण हमेशा बराबर होते हैं!